APPENDIX A: Calculations of Gear System with 1:10 Ratio

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MAC 2015, Autocollimator Callibration

9 July 2015

SAND2015-5621C

10 Teeth: 20 Teeth
Ratio: 1/2

Gear Set	Reduction
1	0.5
2	0.25
3	0.125
4	0.0625
5	0.03125
6	0.015625
7	0.0078125
8	0.0039063
9	0.0019531
10	0.0009766
11	0.0004883
12	0.0002441
13	0.0001221
14	6.104E-05
15	3.052E-05
16	1.526E-05
17	7.629E-06
18	3.815E-06
19	1.907E-06
20	9.537E-07
21	4.768E-07
22	2.384E-07
23	1.192E-07
24	5.96E-08
25	2.98E-08
26	1.49E-08
27	7.451E-09
28	3.725E-09
29	1.863E-09
30	9.313E-10

6 Teeth: 60 Teeth
Ratio: 1/10

Gear Set	Reduction
1	0.10
2	0.01
3	0.001
4	0.0001
5	0.00001
6	0.000001
7	1E-07
8	1E-08
9	1E-09
10	1E-10

6 Tooth: 120 Teeth
Ratio: 1/20

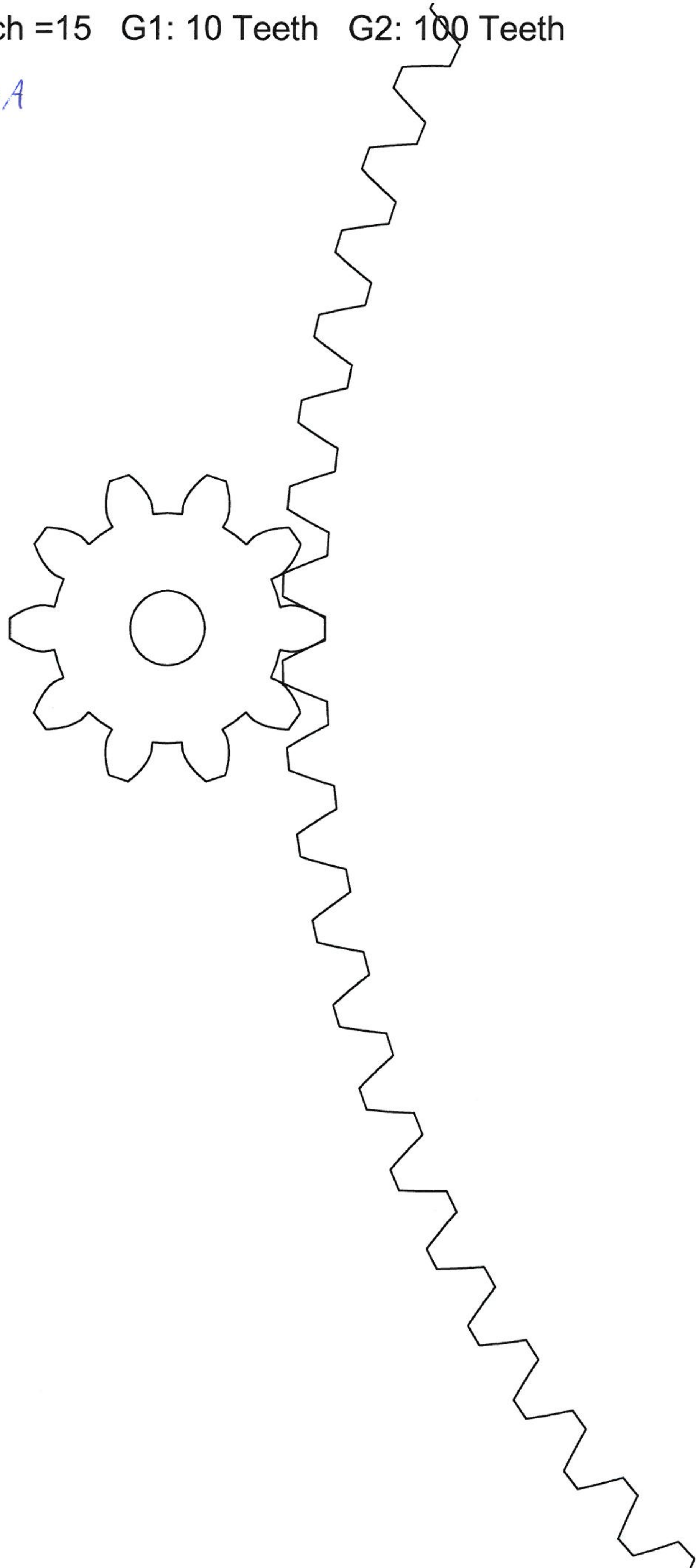
Gear Set	Reduction
1	0.05
2	0.0025
3	0.000125
4	6.25E-06
5	3.13E-07
6	1.56E-08
7	7.81E-10

1 Tooth: 100 Teeth
Ratio: 1/100

Gear Set	Reduction
1	0.01
2	0.0001
3	0.000001
4	1E-08
5	1E-10

Angle = 25   Pitch =15   G1: 10 Teeth   G2: 100 Teeth   210 mm

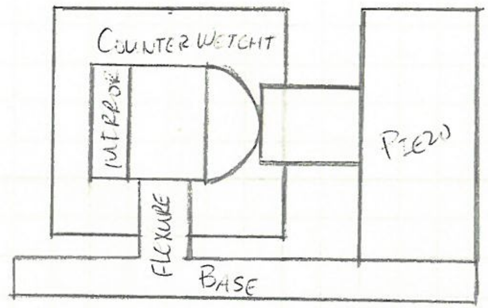
Appendix A

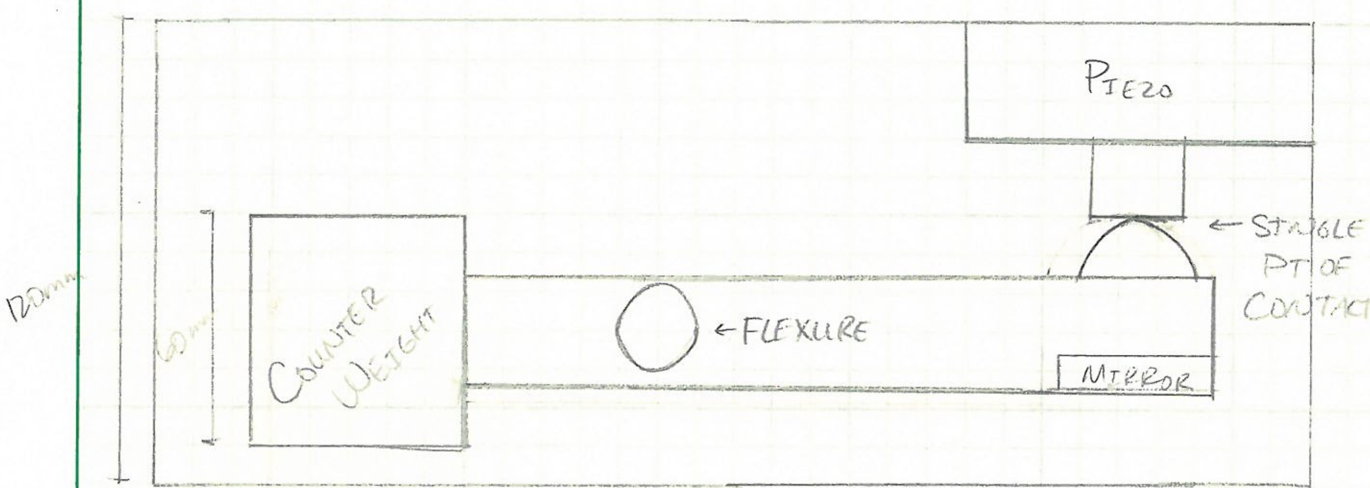


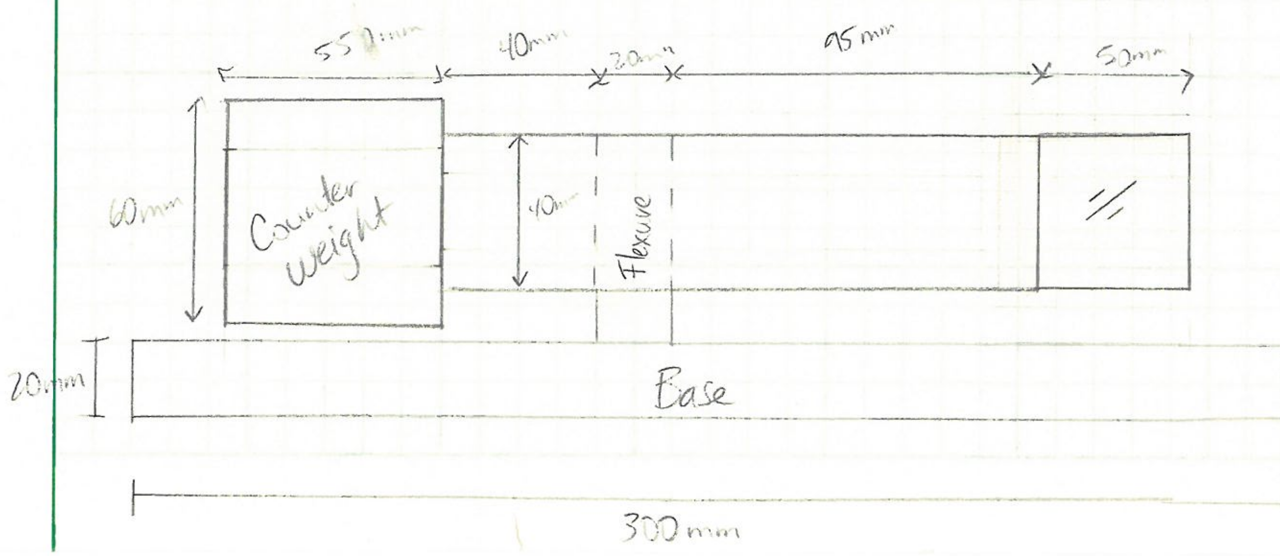
Scale → 1 block = 10mm

Notes

- Sine arm is 75mm shorter than HPSAG. May need to adjust.
- Entire plan is smaller than HPSAG, smaller by ≈20 mm on each dimension



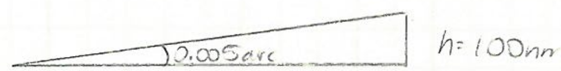




Sine Bar length calculation

GIVEN: $\alpha = 0.005$ arc sec
$h = 100\,nm$

$\alpha = \frac{h}{L}$



FIND: L

SOLUTION: $0.005 = \frac{100\,nm}{L}$

$1\,mm = 1{,}000{,}000\,nm$

$L = 20000\,nm \Rightarrow 0.02\,mm$ ← can't build that and make it operable.

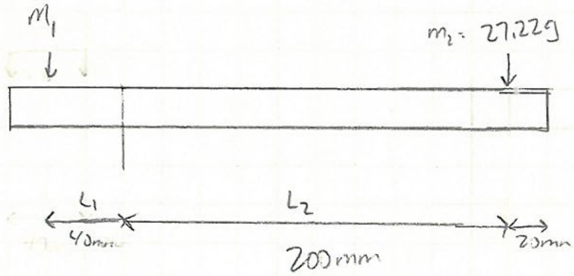
If we make sine arm 200mm ($\times 10^4$) w/ same angle

$0.005 = \frac{h}{200\,mm}$

$h = 1\,mm \rightarrow 1{,}000{,}000\,nm$

Assuming a 2" diameter mirror w/ a weight of 0.06 lbs is attached to the end of the sign bar.

diameter = 2" → 50.8mm
weight = 0.06 lb → 27.2155 g



1 block = 20mm

$m_1 L_1 = m_2 L_2$
$m_1 (40\,mm) = (27.22g)(200\,mm)$

$m_1 = 136.1g$ ← need counterweight applied w/ this mass

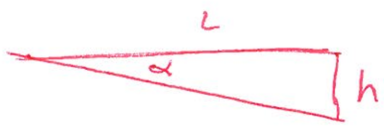
Appendix B

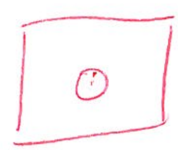
put sine bar on
edge of the weight applied
↳ or have designated barrier plate

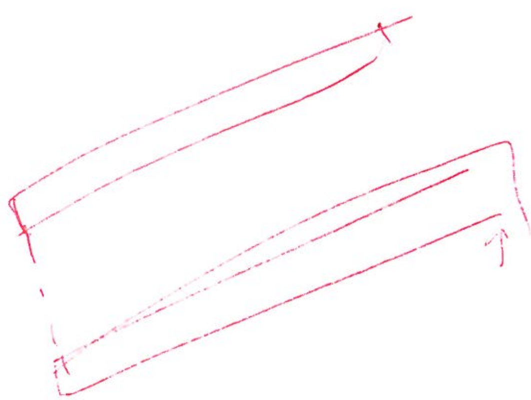
Lever Arm:	250 mm
Initial Deflection	5 mm
Beyond Flexure:	50 mm

	Beyond Flexure Displacement	Angle (deg)	Radian	Nanoradian
Arm 1	1	0.02	0.000349	349065.8504
Arm 2	0.02	0.0004	6.98E-06	6981.317008
Arm 3	0.0004	0.000008	1.4E-07	139.6263402
Arm 4	0.000008	0.00000016	2.79E-09	2.792526803
Arm 5	0.00000016	3.2E-09	5.59E-11	0.055850536
Arm 6	3.2E-09	6.4E-11	1.12E-12	0.001117011
Arm 7	6.4E-11	1.28E-12	2.23E-14	2.23402E-05
Arm 8	1.28E-12	2.56E-14	4.47E-16	4.46804E-07
Arm 9	2.56E-14	5.12E-16	8.94E-18	8.93609E-09
Arm 10	5.12E-16	1.024E-17	1.79E-19	1.78722E-10

ratio $\frac{\text{Lever dis}}{250m} = \frac{x}{50mm}$ $\frac{\text{BF displace}}{50mm}$ $deg \cdot \frac{\pi}{180}$ $rad \cdot 10^9$







Appendix B

Lever Arm:	450	mm
Initial Deflection	5	mm
Beyond Flexure:	50	mm

	Beyond Flexure Displacement	Angle (deg)	Radian	Nanoradian
Arm 1	0.555555556	0.011111111	0.000194	193925.4724
Arm 2	0.00617284	0.000123457	2.15E-06	2154.727472
Arm 3	6.85871E-05	1.37174E-06	2.39E-08	23.94141635
Arm 4	7.62079E-07	1.52416E-08	2.66E-10	0.266015737
Arm 5	8.46754E-09	1.69351E-10	2.96E-12	0.00295573
Arm 6	9.40838E-11	1.88168E-12	3.28E-14	3.28414E-05
Arm 7	1.04538E-12	2.09075E-14	3.65E-16	3.64905E-07
Arm 8	1.16153E-14	2.32306E-16	4.05E-18	4.0545E-09
Arm 9	1.29059E-16	2.58117E-18	4.5E-20	4.505E-11
Arm 10	1.43399E-18	2.86797E-20	5.01E-22	5.00556E-13

Appendix B

Lever Arm:	550	mm
Initial Deflection	5	mm
Beyond Flexure:	50	mm

	Beyond Flexure Displacement	Angle (deg)	Radian	Nanoradian
Arm 1	0.454545455	0.009090909	0.000159	158666.2956
Arm 2	0.004132231	8.26446E-05	1.44E-06	1442.420869
Arm 3	3.75657E-05	7.51315E-07	1.31E-08	13.11291699
Arm 4	3.41507E-07	6.83013E-09	1.19E-10	0.119208336
Arm 5	3.10461E-09	6.20921E-11	1.08E-12	0.001083712
Arm 6	2.82237E-11	5.64474E-13	9.85E-15	9.85193E-06
Arm 7	2.56579E-13	5.13158E-15	8.96E-17	8.9563E-08
Arm 8	2.33254E-15	4.66507E-17	8.14E-19	8.14209E-10
Arm 9	2.12049E-17	4.24098E-19	7.4E-21	7.4019E-12
Arm 10	1.92772E-19	3.85543E-21	6.73E-23	6.729E-14

Appendix B

Mirror Weights

Diameter	Thickness	Weight (lb)	Difference between values	
2	3/16"	0.06		ESTIMATE
3	3/16"	0.12	0.06	
4	3/16"	0.21	0.09	
5	3/16"	0.33	0.12	
6	3/16"	0.48	0.15	

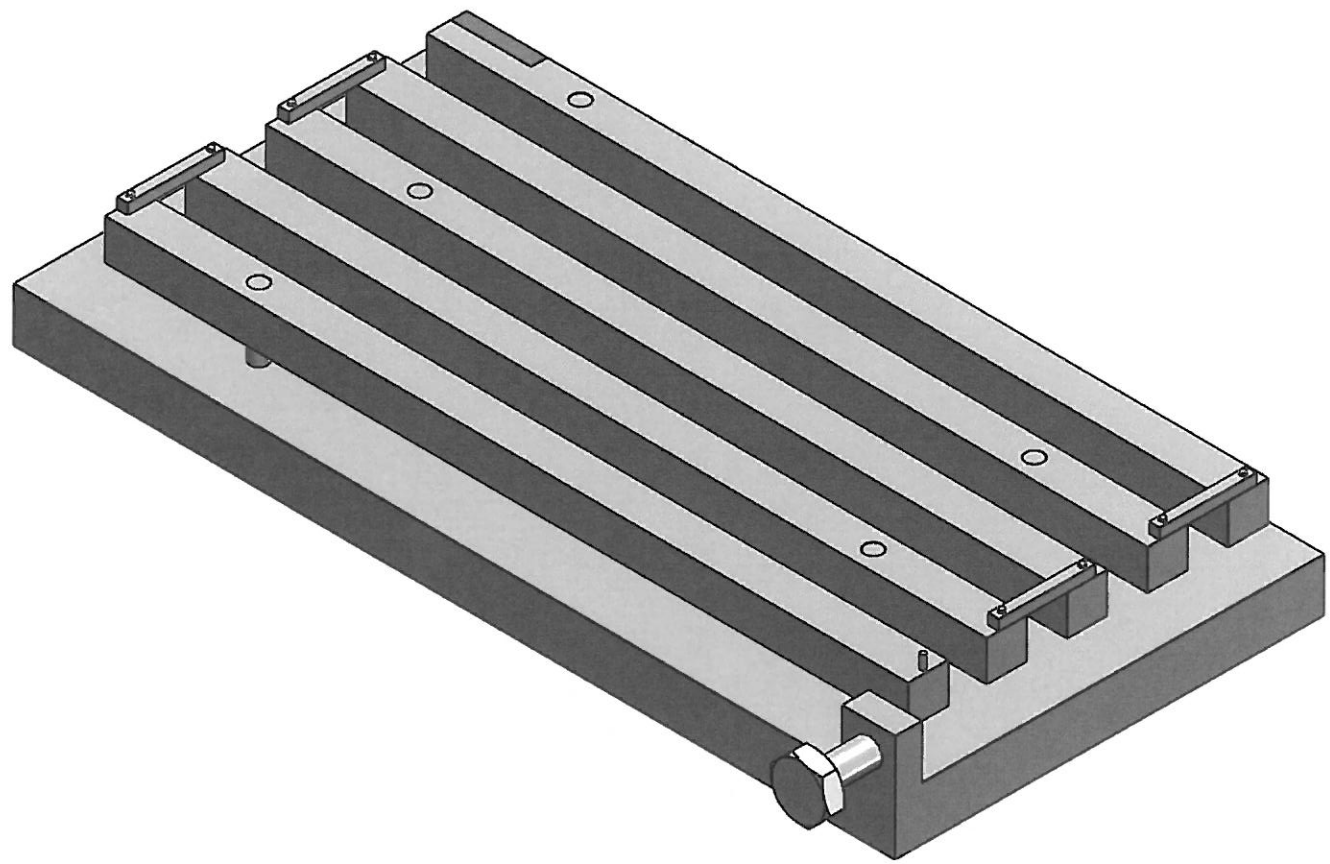
**Values are all in Customary Units

Source: http://www.dullesglassandmirror.com/glass-weight-calculator.aspx

Lever Arm:	250 mm
Initial Deflection from Gage Block	5 mm
Beyond Flexure:	50 mm

	Beyond Flexure Displacement	Angle (rad)	Nanoradians	
Arm 1	1	0.02	20000000	
Arm 2	0.02	0.0004	400000	
Arm 3	0.0004	0.000008	8000	
Arm 4	0.000008	0.00000016	160	
Arm 5	0.00000016	3.2E-09	3.2	
Arm 6	3.2E-09	6.4E-11	0.064	
Arm 7	6.4E-11	1.28E-12	2.23402E-14	2.23E-05
Arm 8	1.28E-12	2.56E-14	4.46804E-16	4.47E-07
Arm 9	2.56E-14	5.12E-16	8.93609E-18	8.94E-09
Arm 10	5.12E-16	1.024E-17	1.78722E-19	1.79E-10

Calculation of displacment on the opposite side of the flexure (solve for x)	*Calculate angle on side beyond flexure*	*Convert radian to nanoradians*
$\frac{5\ mm}{250mm} = \frac{x\ mm}{50\ mm}$	$\alpha = \frac{h}{L}$ $Angle = \frac{Displacement}{Lever\ Length}$	$nano = rad * 10^9$



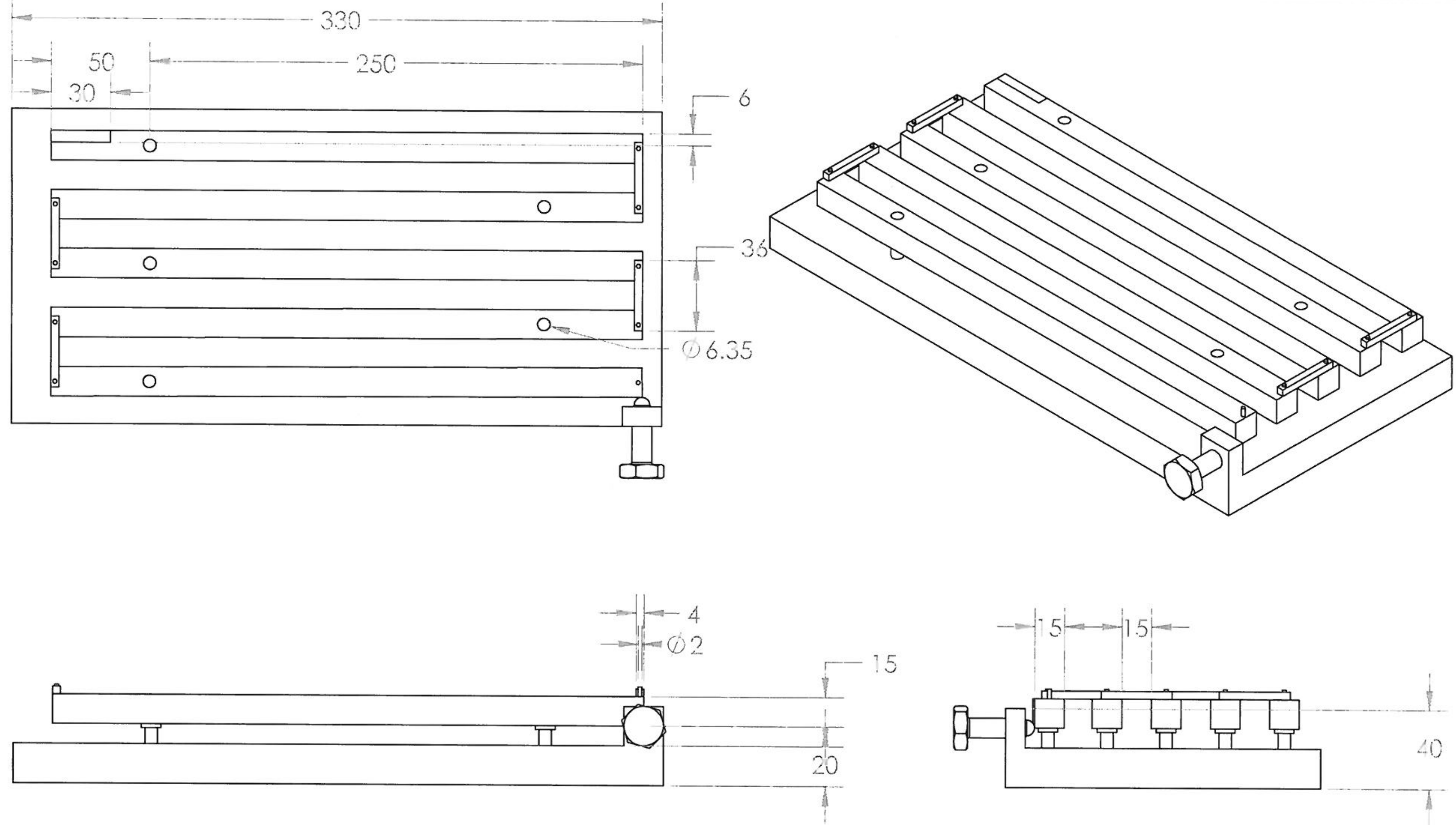
SCALE 1 : 2

UNLESS OTHERWISE SPECIFIED:		NAME	DATE	APPENDIX D: SolidWorks of Final Design
DIMENSIONS ARE IN MILLIMETERS TOLERANCES: FRACTIONAL± ANGULAR: MACH± BEND ± TWO PLACE DECIMAL ± THREE PLACE DECIMAL ±	DRAWN	KK	22 June 15	TITLE:
	CHECKED			Autocollimator Calibrator Assembly
	ENG APPR.			
	MFG APPR.			
INTERPRET GEOMETRIC TOLERANCING PER:	Q.A.			
MATERIAL 1060 Aluminum Alloy	COMMENTS: Fluxure used is 0.26 in diamter cross-flex hinge from Riverhawk Flex Pivots			SIZE A DWG. NO. 1 REV
FINISH				
DO NOT SCALE DRAWING				SCALE: 1:2 WEIGHT: SHEET 1 OF 2

NEXT ASSY	USED ON
APPLICATION	

PROPRIETARY AND CONFIDENTIAL

5 4 3 2 1



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INTERPRET GEOMETRIC TOLERANCING PER:	Q.A.				
MATERIAL 1060 Aluminum Alloy	COMMENTS: Fluxure used is 0.26 in diamter cross-flex hinge from Riverhawk Flex Pivots			SIZE A DWG. NO. 1	REV
FINISH					
DO NOT SCALE DRAWING				SCALE: 1:3 WEIGHT:	SHEET 2 OF 2

NEXT ASSY | USED ON

APPLICATION

PROPRIETARY AND CONFIDENTIAL

5 4 3 2 1

Torsional Spring Rate

Calculation: $\frac{0.0145\ lb\text{-}in}{1\ degree} * \frac{1\ degree}{0.0174\ radians} * \frac{1\ N\text{-}m}{8.85076\ lb\text{-}in} = \frac{N\text{-}m}{radians}$

0.0145	57.47126437	0.112984648	**0.094153873**

Calculation of Torque Applied by Screw

Ratio, angles from APPDX B: $\frac{X\ N\text{-}m}{radians} = \frac{0.094154\ N\text{-}m}{radian}$

X= **0.0018831**

Conversion to N-mm

Ratio: $\frac{N\text{-}m}{radian} * \frac{1000\ N\text{-}mm}{N\text{-}m}$

τ (N-mm): **1.8830775**

Calculation of Force Applied by Displacement

Equation: $\tau = F * r * sin\theta$

r= 250 mm, θ=90*

F (N)= **0.0084254**

Deflection of Long Portion of Lever

Assuming that the lever arm works like a cantilever with point load applied at end

Moment of Intertia

**Assuming rotation around x axis, rectangular shape

Mass of Entire Lever

Density (d) (g/mm^3)	0.0027	$Mass = Density * Volume$
Volume (mm^3)	67500	Mass (g)= **182.25**

Moment of Inertia

$$I_x = \frac{1}{12} * m * (h^2 + w^2)$$

Height (mm)	15	
Width (mm)	15	MoI (g-mm^2)= **3417.1875**

Deflection of a Cantilever

$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end

P (g)	0.8593943	
L (mm)	250	Deflection (mm)= **0.018712179**
E (N/mm^2)	70000	

Deflection of Connection Piece, Axial Loading and in Tension

Assuming that the point load applied by the actuator is the same felt by the connection piece, causing axial loading and deflection

Deflection of an Axial Member

$$\delta = \frac{PL}{AE}$$

P (g)	0.8593943	
L (mm)	36	Deflection (mm)= **2.76234E-05**
E (N/mm^2)	70000	
A (mm^2)	16	

Deflection of Short Portion of Lever	Assuming that the load applied by the connection piece is the same as applied by the actuator		
	Deflection of a Cantilever		
	P (g)	0.8593943	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end
	L (mm)	50	Deflection (mm)= **0.000149697**
	E (N/mm^2)	70000	

Torsional Spring Rate	**Calculation:** $\frac{0.0145\ lb\text{-}in}{1\ degree} * \frac{1\ degree}{0.0174\ radians} * \frac{1\ N\text{-}m}{8.85076\ lb\text{-}in} = \frac{N\text{-}m}{radians}$
	0.0145 · 57.47126437 · 0.112984648 · **0.094153873**
Calculation of Torque Applied by Screw	**Ratio, angles from APPDX B:** $\frac{X\ N\text{-}m}{radians} = \frac{0.094154\ N\text{-}m}{radian}$
	X= **3.766E-05**
Conversion to N-mm	**Ratio:** $\frac{N\text{-}m}{radian} * \frac{1000\ N\text{-}mm}{N\text{-}m}$
	τ (N-mm): **0.0376615**
Calculation of Force Applied by Displacement	**Equation:** $\tau = F * r * sin\theta$ **r= 250 mm, θ=90***
	F (N)= **0.0001685**

Deflection of Long Portion of Lever

Assuming that the lever arm works like a cantilever with point load applied at end

Moment of Intertia — **Assuming rotation around x axis, rectangular shape

Mass of Entire Lever		
Density (d) (g/mm^3)	0.0027	$Mass = Density * Volume$
Volume (mm^3)	67500	Mass (g)= **182.25**

Moment of Inertia		
Height (mm)	15	$I_x = \frac{1}{12} * m * (h^2 + w^2)$
Width (mm)	15	MoI (g-mm^2)= **3417.1875**

Deflection of a Cantilever		
P (g)	0.0171879	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end
L (mm)	250	Deflection (mm)= **0.000374244**
E (N/mm^2)	70000	

Deflection of Connection Piece, Axial Loading and in Tension

Assuming that the point load applied by the actuator is the same felt by the connection piece, causing axial loading and deflection

Deflection of an Axial Member		
P (g)	0.0171879	$\delta = \frac{PL}{AE}$
L (mm)	36	Deflection (mm)= **5.52468E-07**
E (N/mm^2)	70000	
A (mm^2)	16	

Deflection of Short Portion of Lever	Assuming that the load applied by the connection piece is the same as applied by the actuator		
	Deflection of a Cantilever		
	P (g)	0.0171879	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end
	L (mm)	50	Deflection (mm)= **2.99395E-06**
	E (N/mm^2)	70000	

Torsional Spring Rate

Calculation: $\frac{0.0145\ lb-in}{1\ degree} * \frac{1\ degree}{0.0174\ radians} * \frac{1\ N-m}{8.85076\ lb-in} = \frac{N-m}{radians}$

0.0145	57.47126437	0.112984648	**0.094153873**

Calculation of Torque Applied by Screw

Ratio, angles from APPDX B: $\frac{X\ N-m}{radians} = \frac{0.094154\ N-m}{radian}$

X= **7.532E-07**

Conversion to N-mm

Ratio: $\frac{N-m}{radian} * \frac{1000\ N-mm}{N-m}$

τ (N-mm): **0.0007532**

Calculation of Force Applied by Displacement

Equation: $\tau = F * r * sin\theta$

r= 250 mm, θ=90*

F (N)= **3.37E-06**

Deflection of Long Portion of Lever

Assuming that the lever arm works like a cantilever with point load applied at end

Moment of Intertia

**Assuming rotation around x axis, rectangular shape

Mass of Entire Lever

Density (d) (g/mm^3)	0.0027	
Volume (mm^3)	67500	

$$Mass = Density * Volume$$

Mass (g)= **182.25**

Moment of Inertia

Height (mm)	15
Width (mm)	15

$$I_x = \frac{1}{12} * m * (h^2 + w^2)$$

MoI (g-mm^2)= **3417.1875**

Deflection of a Cantilever

P (g)	0.0003438
L (mm)	250
E (N/mm^2)	70000

$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end

Deflection (mm)= **7.48487E-06**

Deflection of Connection Piece, Axial Loading and in Tension

Assuming that the point load applied by the actuator is the same felt by the connection piece, causing axial loading and deflection

Deflection of an Axial Member

P (g)	0.0003438
L (mm)	36
E (N/mm^2)	70000
A (mm^2)	16

$$\delta = \frac{PL}{AE}$$

Deflection (mm)= **1.10494E-08**

Deflection of Short Portion of Lever	Assuming that the load applied by the connection piece is the same as applied by the actuator			
	Deflection of a Cantilever			
	P (g)	0.0003438	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end	
	L (mm)	50	Deflection (mm)=	**5.9879E-08**
	E (N/mm^2)	70000		

Torsional Spring Rate

Calculation: $\frac{0.0145\ lb\text{-}in}{1\ degree} * \frac{1\ degree}{0.0174\ radians} * \frac{1\ N\text{-}m}{8.85076\ lb\text{-}in} = \frac{N\text{-}m}{radians}$

0.0145 | 57.47126437 | 0.112984648 | **0.094153873**

Calculation of Torque Applied by Screw

Ratio, angles from APPDX B: $\frac{X\ N\text{-}m}{radians} = \frac{0.094154\ N\text{-}m}{radian}$

X= **1.506E-08**

Conversion to N-mm

Ratio: $\frac{N\text{-}m}{radian} * \frac{1000\ N\text{-}mm}{N\text{-}m}$

τ (N-mm): **1.506E-05**

Calculation of Force Applied by Displacement

Equation: $\tau = F * r * sin\theta$

r= 250 mm, θ=90*

F (N)= **6.74E-08**

Deflection of Long Portion of Lever

Assuming that the lever arm works like a cantilever with point load applied at end

Moment of Intertia

**Assuming rotation around x axis, rectangular shape

Mass of Entire Lever

Density (d) (g/mm^3)	0.0027	$Mass = Density * Volume$	
Volume (mm^3)	67500	Mass (g)=	**182.25**

Moment of Inertia

$$I_x = \frac{1}{12} * m * (h^2 + w^2)$$

Height (mm)	15		
Width (mm)	15	MoI (g-mm^2)=	**3417.1875**

Deflection of a Cantilever

$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end

P (g)	6.875E-06		
L (mm)	250	Deflection (mm)=	**1.49697E-07**
E (N/mm^2)	70000		

Deflection of Connection Piece, Axial Loading and in Tension

Assuming that the point load applied by the actuator is the same felt by the connection piece, causing axial loading and deflection

Deflection of an Axial Member

$$\delta = \frac{PL}{AE}$$

P (g)	6.875E-06		
L (mm)	36	Deflection (mm)=	**2.20987E-10**
E (N/mm^2)	70000		
A (mm^2)	16		

Deflection of Short Portion of Lever	Assuming that the load applied by the connection piece is the same as applied by the actuator		
	Deflection of a Cantilever		
	P (g)	6.875E-06	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end
	L (mm)	50	Deflection (mm)= **1.19758E-09**
	E (N/mm^2)	70000	

Torsional Spring Rate

Calculation: $\frac{0.0145\ lb-in}{1\ degree} * \frac{1\ degree}{0.0174\ radians} * \frac{1\ N-m}{8.85076\ lb-in} = \frac{N-m}{radians}$

0.0145 57.47126437 0.112984648 **0.094153873**

Calculation of Torque Applied by Screw

Ratio, angles from APPDX B: $\frac{X\ N-m}{radians} = \frac{0.094154\ N-m}{radian}$

X= **3.013E-10**

Conversion to N-mm

Ratio: $\frac{N-m}{radian} * \frac{1000\ N-mm}{N-m}$

τ (N-mm): **3.013E-07**

Calculation of Force Applied by Displacement

Equation: $\tau = F * r * sin\theta$

r= 250 mm, θ=90*

F (N)= **1.348E-09**

Deflection of Long Portion of Lever

Assuming that the lever arm works like a cantilever with point load applied at end

Moment of Intertia **Assuming rotation around x axis, rectangular shape

Mass of Entire Lever

$$Mass = Density * Volume$$

Density (d) (g/mm^3)	0.0027	Mass (g)=	**182.25**
Volume (mm^3)	67500		

Moment of Inertia

$$I_x = \frac{1}{12} * m * (h^2 + w^2)$$

Height (mm)	15		
Width (mm)	15	MoI (g-mm^2)=	**3417.1875**

Deflection of a Cantilever

$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end

P (g)	1.375E-07		
L (mm)	250	Deflection (mm)=	**2.99395E-09**
E (N/mm^2)	70000		

Deflection of Connection Piece, Axial Loading and in Tension

Assuming that the point load applied by the actuator is the same felt by the connection piece, causing axial loading and deflection

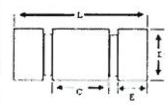
Deflection of an Axial Member

$$\delta = \frac{PL}{AE}$$

P (g)	1.375E-07		
L (mm)	36	Deflection (mm)=	**4.41974E-12**
E (N/mm^2)	70000		
A (mm^2)	16		

Deflection of Short Portion of Lever	Assuming that the load applied by the connection piece is the same as applied by the actuator		
	Deflection of a Cantilever		
	P (g)	1.375E-07	$\Delta_{max} = \frac{PL^3}{3EI}$ at the free end
	L (mm)	50	Deflection (mm)= **2.39516E-11**
	E (N/mm^2)	70000	

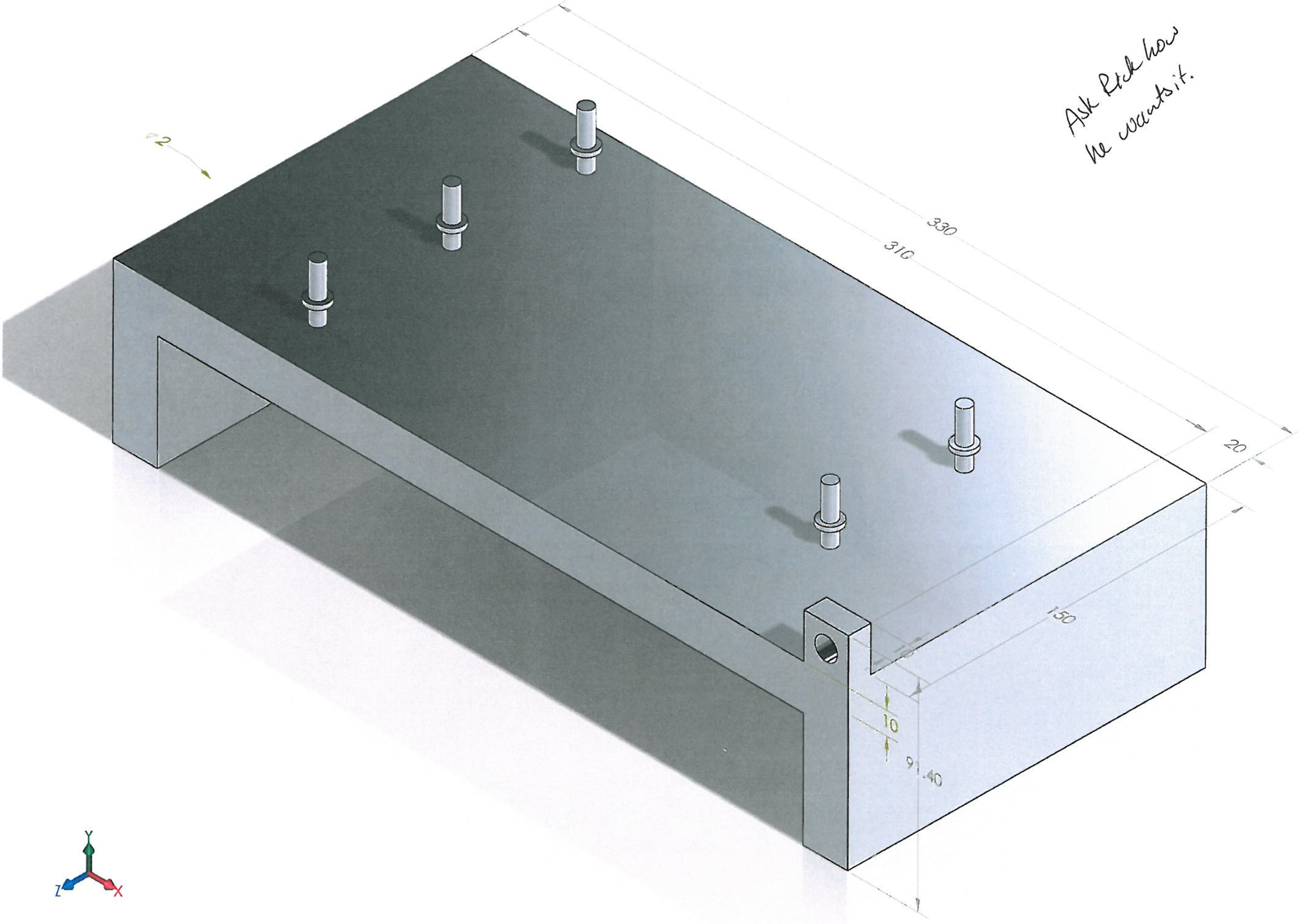
APPENDIX F: FLEXURE VALUES

Nominal Outside Diameter (Inch) D 0 -0.0005	Torsional Spring Rate (in-lb) (Degree) See Note 2	Radial Load Capacity (Pounds) Load at Center of "C" See Note 1		Axial Load Capacity (Pounds)				Catalog Number
		Vc	Vt	Pa	L ±0.003	E ±0.005	C 0.005 -0.015	Series 6000 Double-Ended
	0.0140	28	28	24.5				6004-400
	0.0017	17.7	25	12.3				6004-600
0.125	0.0002	2.2	4.7	5.9	0.2	0.045	0.085	6004-800
	0.0279	44	44	38.7				6005-400
	0.0035	27.6	39	19.4				6005-600
0.1562	0.0004	3.5	7.4	9.7	0.25	0.057	0.11	6005-800
	0.0473	63	63	56.4				6006-400
	0.0057	39.6	56	27.9				6006-600
0.1875	0.0007	4.9	9	13.9	0.3	0.067	0.13	6006-800
	0.1141	113	113	99				6008-400
	0.0143	70.7	100	49.4				6008-600
0.25	0.0018	8.5	19	24.7	0.4	0.09	0.175	6008-800
	0.2234	176	176	154				6010-400
	0.0286	110	156	77.5				6010-600
0.3125	0.0036	14	29	38.8	0.5	0.112	0.22	6010-800
	0.3840	253	253	223				6012-400
	0.0480	159	225	112				6012-600
0.375	0.0058	19.8	42	55.4	0.6	0.135	0.265	6012-800
	0.9080	450	450	392				6016-400
	0.1134	283	400	196				6016-600
0.5	0.0142	35.4	75	98	0.8	0.18	0.355	6016-800
	1.8500	703	703	615				6020-400
	0.2321	442	625	308				6020-600
0.625	0.0295	55	117	155	1	0.225	0.445	6020-800
	3.1800	1013	1013	884				6024-400
	0.3980	363	900	442				6024-600
0.75	0.0500	78	169	221	1.2	0.27	0.535	6024-800
	7.5200	1800	1800	1570				6032-400
	0.9390	1131	1600	785				6032-600
1	0.1175	141	300	392	1.6	0.37	0.735	6032-800

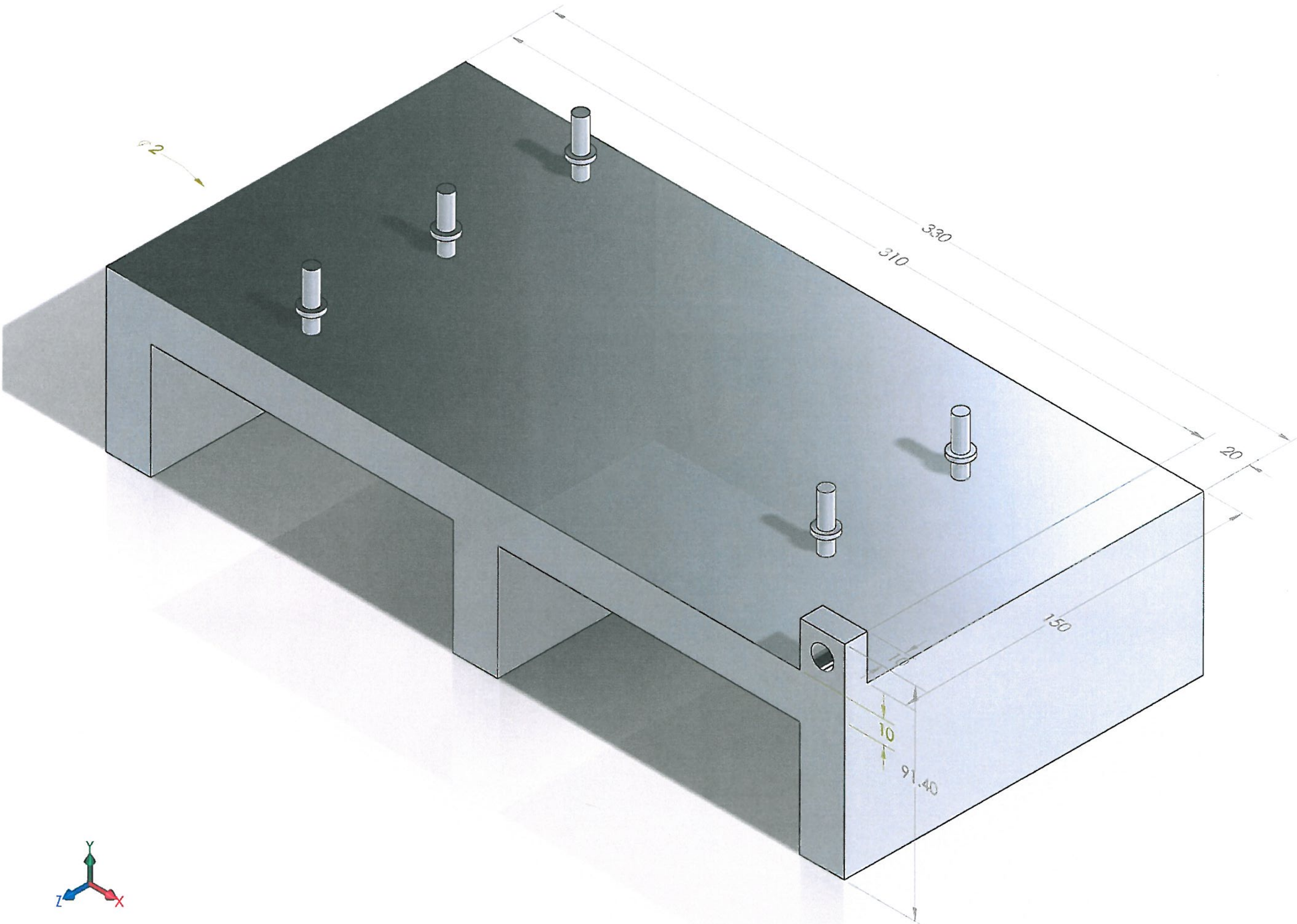
Notes:

1. Pounds at zero deflection based on pure radial load. Performance of pivot is a function of number of cycles, angular travel, and loading. Must use life cycle curves for selection of the proper pivot. When the load is applied directly through a single
2. At zero load, Torsional Spring Rate may change with radial load.

Appendix 6: Calibrator Base Suggestion



Appendix 6: Calibration Base Suggestions



Appendix G: Calibration Base Suggestion