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Wavefront Correction Using Micromirror Arrays: Comparing the Efficacy of Tip-Tilt-Piston and Piston-Only Micromirror Arrays

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Abstract: Micromirrors arrays can be used to correct residual wavefront aberrations in certain optical systems. The aberration correction capability of arrays of piston-only and piston-tip-tilt micromirrors are compared. Sandia's micromirror fabrication program is discussed and two example systems are presented.

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1. Introduction

A single-actuator micromirror can only introduce a piston wavefront correction. Thus an array of them can correct a wavefront error in a step-wise fashion. A micromirror mounted on three actuators with stroke normal to the substrate can introduce piston, tip, and tilt. An array of these micromirrors can produce a piece-wise continuous approximation to both the displacement and slope of the wavefront. Increasing the actuator count thusly is likely to be worth the added complexity.

An array of N piston-tip-tilt micromirrors can reduce the root-mean-square wavefront error (RMS WFE) proportional to $1/N$ (for $N > 25$). An array of piston-only micromirrors only reduces the RMS WFE proportional to $1/\sqrt{N}$. So for example, an array of $N = 1000$ piston-tip-tilt micromirrors reduces the RMS WFE by a factor of *31X* more than 1000 piston-only micromirrors. There are 3 actuators driving each piston-tip-tilt micromirror, so based on actuator count, these micromirrors are still 10X better at reducing RMS WFE.

Arrays of 91-element and 61-element piston-tip-tilt micromirrors have been fabricated by Sandia National Lab. and are now under test. Our preliminary results confirm the efficacy of the piston-tip-tilt approach.

Two example systems are presented: The first is an ophthalmic instrument that uses the micromirror array to correct up to ten diopters of defocus and aberration. The other is an astronomical telescope where the micromirror array corrects for atmospheric turbulence.

2. Wavefront Correction—System Calculations

The wavefront error (WFE) of any symmetric optical system can be described in terms of Zernike aberrations. Presented below are the reductions in RMS WFE of the Zernike terms possible using four different micromirror array configurations[1]. The coefficients have been calculated for piston-only and piston-tip-tilt actuation and for square arrays of micromirrors versus hexagonal arrays.

Table 1 shows how well the lower order Zernike aberrations can be corrected, including tilt, defocus, and the third-order aberrations. These expressions indicate how much the RMS WFE of individual Zernike aberrations can be reduced with an array of "N" micromirrors. Note that for large N the residual WFE for piston-only micromirrors is inversely proportional to $N^{0.5}$, while the aberration reduction of a piston-tip-tilt array is inversely proportional to N. Also note that hexagonal arrays correct Zernike aberrations about 10% better than square arrays.

One can roughly calculate how well an array of micromirrors can correct the aberration in an optical system using the information in Table I. First select the type of array (i.e. sq. p-t-t), and then knowing the number of micromirrors in the array (N), calculate how much the RMS WFE of each of the Zernike terms can be reduced (Table I). Next, the RMS Zernike wavefront contributions for the system can be calculated using any of the commercial ray trace program. These values can be multiplied by the terms in Table I and then squared and summed. The square root of this sum is a good estimate of the residual RMS wavefront error after the system has been optimally corrected by the micromirror array.

Table 1. Normalized residual RMS wavefront errors for Zernike Aberrations

Zernike term	sq. p-t-t	hex p-t-t	sq. piston-only	hex piston-only
Z2 & Z3 tilt	0	0	$[1.05/N]^{0.5}$	$N^{-0.5}$
Z4 defocus	1.12/N	1/N	$[1.18/N+1.25/N^2]^{0.5}$	$[1.12/N+1/N^2]^{0.5}$
Z5 astigmatism	0.92/N	1/N	$[0.26/N+0.84/N^2]^{0.5}$	$[0.25/N+1/N^2]^{0.5}$
Z6 astig. @ 45°	1.45/N	1/N	$[0.26/N+2.1/N^2]^{0.5}$	$[0.25/N+1/N^2]^{0.5}$
Z7 & Z8 coma	3.90/N	3.5/N	$[18/N+15/N^2]^{0.5}$	$[14/N+12/N^2]^{0.5}$
Z9 3rd spherical	10.8/N	9.3/N	$[31/N+117/N^2]^{0.5}$	$[30/N+87/N^2]^{0.5}$
Z16 5th spherical	64.0/N	54/N	$[276/N+4050/N^2]^{0.5}$	$[264/N+2930/N^2]^{0.5}$

Figure 1 shows how much the image quality of an aberrated optical system can be improved when a micromirror array is added. More specifically, it shows how much the RMS WFE of Zernike aberrations can be reduced as a function of the number of micromirrors (*N*) in the array. The plot shows the improvement possible for defocus (*Z4*) and 3rd and 5th order spherical aberration (*Z9* & *Z16*) for hexagonal arrays of piston-only and piston-tip-tilt micromirrors. As above in Table 1, these values are normalized to the uncorrected RMS WFEs for the Zernike aberrations.

It is clear from Figure 1 that piston-tip-tilt micromirrors are far more effective at reducing Zernike aberrations than are piston-only micromirrors. Thirty-two p-t-t mirrors can reduce defocus as well as 1024 piston-only mirrors. Furthermore, (as seen in Fig. 1), an array of 1024 piston only micromirrors cannot reduce 3rd and 5th spherical aberration by enough to be useful (5X & 2X), though 1024 piston-tip-tilt micromirrors can (90X & 25X).

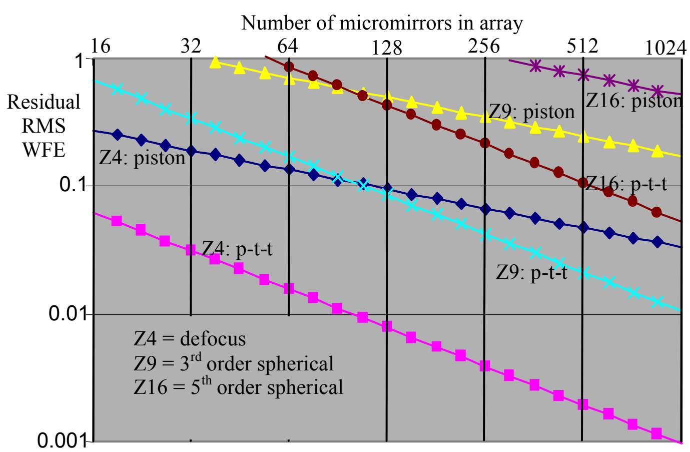

Fig. 1. Residual RMS wave front error after correction has been applied using a square micromirror array. Subscripts "$_{piston}$" and "$_{p-t-t}$" refer to piston-only and piston-tip-tilt micromirror arrays.

3. Recent Developments in Sandia's Micromirror Fabrication Program

A 3.9-mm array of 91 piston-tip-tilt micromirrors and the electronics control board are shown in Figure 2. The chip was packaged in a 256 pin PGA using automated die attach and wire bonding equipment. The array of 0.5-mm micromirrors was fabricated at Sandia using the SUMMiT™ V process. The micromirrors are driven by three linear actuators, each with a stroke of ~8-µm, which allow each micromirror to move axially and to be tipped and tilted.

Sandia has several research efforts that are under way: 1.) the mirror curvature introduced when the metal reflective coating is applied is being compensated by pre-bending the mirrors before coating, 2.) advanced micromirrors with 27-µm stroke are being developed and have been demonstrated, and 3.) on-chip, under-micromirror electronics are being prototyped.

Figure 3 shows 3D images of 1st and 3rd order aberrations created with a 61-micromirror array (Different than Fig. 2.). The actuators under these micromirrors each have a 27-µm stroke. The images presented in Fig. 3 shows wavefront data taken on each mirror segment and subsequently stitched together. This data was taken with a WYKO white-light interferometer.

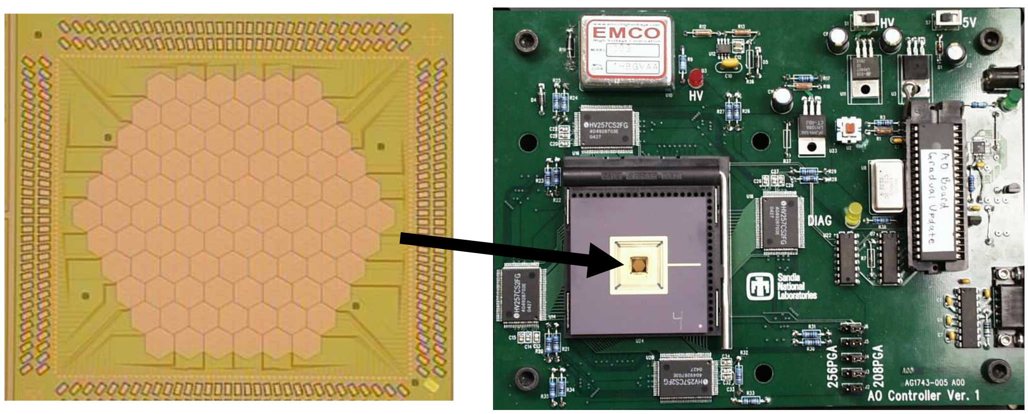

Fig. 2. Photo of 91 element hexagonal piston-tip-tilt mirror array with a ~4 mm active aperture packaged on controller board.

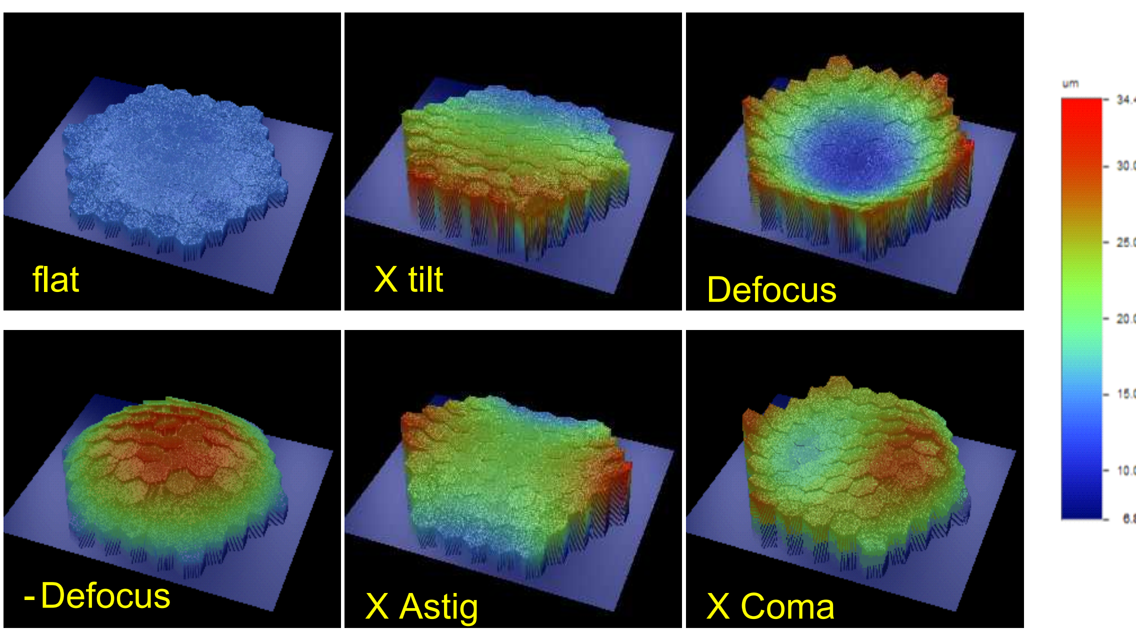

Fig. 3: Zernike aberrations created with an array of 61 piston-tip-tilt micromirrors with 27-µm stroke

4. Examples of Design

If one designs an optical system containing a micromirror array for wavefront correction, how many micromirrors are needed and how much stroke is needed? Perhaps the best way to illuminate this problem is by presenting a pair of examples:

- An ophthalmic instrument to simulate corrective lenses
- Air turbulence compensation in an astronomical telescope

The ophthalmic instrument is assumed to duplicate the correction of spectacles with 10 diopters of power and cylinder. If the correction were all defocus, it would be equivalent to a ±100-mm focal length lens. If the pupil is dilated to a 7-mm diameter, the maximum peak-to-valley wavefront error would be 70 µm. Thus the range of micromirror travel is half, or 35 µm (the range of motion of the micromirrors). The RMS WFE (wavefront error) is $2\sqrt{3}$ times[5] smaller or 10.1 µm. To calculate the number of micromirrors that are required, we must determine an acceptable image quality. This will lead us to an acceptable amount of WFE that can remain after the micromirror array has been optimally tuned.

How good does the wavefront error need to be? A Strehl ratio of 80% is considered to be "diffraction limited" for optical instruments, including the human eye. This is equal to a required RMS WFE of $\lambda/14$, or 35 nm. For measuring the eye, the instrument should be a bit better—say 17.5 nm RMS. Dividing the maximum expected RMS WFE (=10.1µm) by the desired RMS WFE (17.5nm) gives a required improvement factor of 580X that must be provided by the micromirror array. From Table I we find that approximately 580 hexagonal piston-tip-tilt micromirrors will give this improvement. It is interesting that it would take about 300,000 hexagonal piston-only micromirrors to achieve the same correction.

For this application, a piston-tip-tilt micromirror array is clearly superior. This would probably be true for any application where the WFE contains large Zernike components.

Astronomical imaging through turbulence: Micromirror arrays could be used to correct the seeing of astronomical telescopes. We design a micromirror array to correct the seeing of a 50-cm telescope operating in the visible. We assume "common" seeing conditions with a Fried radius of 5 cm. As would be expected, turbulence has been studied extensively.[2,3,4] The Fried radius[2,3], r_0, is the radius of the average turbulence cell in the atmosphere. A smaller r_0 is associated with more energy in the atmosphere causing greater temperature differences and therefore, more turbulence. From the Fried radius, the Strehl ratio due to turbulence can be calculated[4,5].

$$S_{total} = \exp\{-1.03\ (D/r_0)^{5/3}\}. \qquad (1)$$

where D is the diameter of the aperture of interest. Fortunately this formulation was developed using a Zernike expansion for the WFE with only the piston error (Z1) removed. If the Zernike tilt terms are also removed (Z2 & Z3), the Strehl ratio is dramatically reduced[4,5] as shown below in Eqn. (2):

$$S_{w/o\ tilt} = \exp\{-0.134\ (D/r_0)^{5/3}\}. \qquad (2)$$

Understand that these two expressions describe the time-averaged wavefront error and thus represent the most probable Strehl ratio. This is all of the turbulence theory needed to design a micromirror array for an astronomical telescope. The most probable wavefront quality of an uncorrected aperture is calculated next, which indicates the need for a micromirror array, the design of which will follow.

The Strehl ratio of the 50-cm aperture with no wavefront correction can be calculated using Eqn. (1). It is essentially zero as shown below.

$$S_{total} = \exp\{-1.03\ (50cm/5cm)^{5/3}\} = \exp\{-48\} \approx 0 \qquad (3)$$

If the exponent in (3) were about 0.1, then the Strehl ratio would be about 90%. Thus we see that the uncorrected aperture has a wavefront variance about 500X too large so the RMS WFE is about 22 times larger than it ought to be. This is the motivation for adding a wavefront corrector to the optical system.

For a large telescope ($D >> r_0$) the hills and valleys in the aberration function will (on the average) have a scale size that is roughly equal to the Fried diameter ($2r_0$). Thus the micromirror array needs to have enough micromirrors in each turbulence cell to correct errors of this scale. We choose a Strehl ratio that will give good imagery (e.g. S = 90%) and from this we calculate how many micromirrors are needed in an average turbulence cell to meet the imaging requirement.

The scheme for determining the size of the micromirrors required per turbulence cell is as follows: First assume the telescope aperture is partitioned into an array of equal-sized subapertures. Each subaperture is subject to the same wavefront statistics so they each have the same time-averaged wavefront variance. If the telescope aperture is composed of this array of identical subapertures, then the telescope aperture should have the same variance as each of the subapertures. The micromirror array is imaged on the entrance pupil of the telescope. The images of these micromirrors are chosen to be the above-mentioned subapertures. Further, we assume the micromirrors are hexagonal, which closely approximates a round micromirror.

Thus as we will see, Eqns. (1) & (2) can be used to describe the most probable wavefront variance on each micromirror. Equation (1) was developed assuming that the piston term (Z1) in the Zernike expansion is removed. Thus the expression can be used to compute the Strehl ratio (and most probable wavefront variance) across a piston-only micromirror when it is located at the optimal position to minimize the local turbulence-induced wavefront error. Equation (1) will give us the most probable Strehl ratio for this micromirror. We propose to turn Eqn. (1) inside out, choose a Strehl ratio (S=90%) and from that, calculate the mirror diameter that will give the chosen wavefront quality. The diameter (D_{piston}) for a piston-only micromirror can be calculated as follows:

$S = 90\% = \exp\{-1.03\,(D_{piston}/r_0)^{5/3}\}$ ➔ $D_{piston} = 0.255\,r_0$ ➔ 64 micromirrors/turbulence cell.

We use the same approach and Eqn. (2) to determine the micromirror diameter for a piston-tip-tilt micromirror ($D_{p\text{-}t\text{-}t}$):

$S = 90\% = \exp\{-0.134\,(D_{p\text{-}t\text{-}t}/r_0)^{3/5}\}$ ➔ $D_{p\text{-}t\text{-}t} = 0.866\,r_0$ ➔ 5.33 micromirrors/turbulence cell.

Fortunately the micromirror array can to be imaged onto the telescope primary so the array can be smaller than the primary.

The micromirror arrays can now be specified. If the telescope is 50cm and the Fried diameter ($2r_0$) is 2*5cm = 10 cm, then we have about 25 turbulence cells across the aperture. This wavefront quality and telescope size requires (M_{piston}=)1600 piston-only micromirrors or ($M_{p\text{-}t\text{-}t}$=) 133 piston-tip-tilt micromirrors.

One case has been analyzed in the preceding paragraphs. We can scale the results so we can study worse seeing, a larger telescope, and a different wavelength:

- If the seeing conditions are worse, the Fried radius might be smaller i.e. r_0 = 2.5 cm. The number of micromirrors is then M_{piston}=6400 & $M_{p\text{-}t\text{-}t}$=520
- For a 150-cm primary: M_{piston}=14,400 & $M_{p\text{-}t\text{-}t}$=1200
- A longer wavelength implies a larger Fried radius $r_0 = 5\text{ cm} * [\lambda_{new} / 500\text{ nm}]^{1.2}$. For example, the Fried radius is almost 20 cm for 1.55-µm light.
- A less stringent image quality requirement implies a lower Strehl ratio, thus requiring fewer micromirrors per turbulence cell as calculated in the previous paragraph.

It is interesting that for any atmospheric turbulence wavefront correction problem, using 64 piston-only micromirrors per turbulence cell will always achieve the same correction as will 5.33 piston-tip-tilt micromirrors per cell. This is a ratio of 12:1 piston to p-t-t mirrors implies a 4:1 ratio of actuators. From the fabricators point of view, 4X fewer actuators (and thus pin-outs) is very important. However, there may be other issues (like mirror curvature) that make a larger array of smaller, piston-only micromirror attractive enough to consider...

5. Acknowledgement

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6. Bibliography

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