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Validation of a Viscoplastic Model for Foam Response Over a Wide Temperature Range

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ABSTRACT

Accurate material models are fundamental to predictive structural finite element models. Because potting foams are routinely used to mitigate shock and vibration of encapsulated components in electro/mechanical systems, accurate material models for foams are needed. A viscoplastic foam constitutive model has been developed to represent the large nonlinear and rate dependent crush of a polyurethane foam throughout an application space defined by temperature, strain rate and strain levels. Validation of this viscoplastic model, which is implemented in the transient dynamic Presto finite element code, is being achieved by modeling and testing a series of structural geometries of increasing complexity that have been designed to ensure sensitivity to material parameters. Both experimental and analytical uncertainties are being quantified to ensure fair assessment of model validity. Quantitative model validation metrics are being developed to provide a means of comparing analytical model predictions with experimental observations. This paper focuses on model validation of foam/component behavior over a wide temperature, strain rate, and strain level range using a Presto viscoplastic finite element model. Experiments include simple foam/component test articles crushed in a series of drop table tests. Material variations of density have been included. A double blind validation process is described that brings together test data with model predictions.

Introduction

Polyurethane foams are used to surround sensitive components to mitigate harsh mechanical shocks that can occur during impact events. These foams are designed to absorb energy during shock by undergoing large plastic deformation. Hence, constitutive models that describe foam response to large deformation at various rates and temperatures are needed for use in finite element analyses of impact events. Recently, a new constitutive model, the Viscoplastic Foam Model (VFM), was developed and calibrated by Neilsen et. al. [1] for a rigid polyurethane foam referred to as PMDI. This paper will describe a follow-on process used to validate the VFM for an application with loading rates at 1000+ per second, over a wide temperature range, and with density variations. The VFM was implemented in the PRESTO [2] nonlinear transient dynamic finite element code.

Viscoplastic Foam Model

The Viscoplastic Foam Model in PRESTO was developed from existing plasticity models for porous materials [1]. These existing plasticity models are all very similar but have some differences in the specific forms selected for the evolution equations of the yield function, material state variables, and flow direction. The VFM captures the effects of load path, strain rate, and temperature on mechanical response. Thirteen parameters are required to define the VFM in PRESTO and are given in Table 1. Results from uniaxial compression tests on PMDI foam were used to determine these baseline values for the VFM parameters. The parameters were evaluated on PMDI foam with a density of 17.85 pcf. (lbs/ft^3). Density variations in the material will be discussed later. The parameters were characterized for applications ranging from quasi-static (0.0001 per second) to dynamic (100 per second) strain rates and temperatures between -53.9 ^oC and 73.9 ^oC.

Table 1. Viscoplastic Foam Model [1] parameters for PMDI20 (17.85 pcf).

Parameter	Units	Value	Value	Value
Temperature - □	°C	-53.9	21.1	73.9
Young's Modulus, E_1	psi	27,798.	22,600.	19,879.
Poisson's Ratio	-	0.343	0.343	0.343
Phi - □$_□$	-	0.238		
Flow Rate – ln(h(□))		-10.0	2.32	11.0
Power Exponent – n(□)		15.516	13.45	12.00
Shear Strength, SS_1	psi	513.1	513.1	513.1
Shear Hardening	psi	4629.0		
Shear Exponent	-	2.90		
Hydro. Strength, HS_1	psi	971.0		
Hydro. Hardening	psi	7377.5	7377.5	7377.5
Hydro. Exponent	psi	4.89		
Beta	-	0.95		

Density Dependence

The validation experiments that will be discussed later had a widespread distribution in density values for the test samples evaluated. The density of the PMDI foam in the 56 test samples was measured and found to have densities that ranged from 17.73 pcf to 23.02 pcf and are shown in Figure 1. A beta distribution was fit to these data resulting in an estimated 95% confidence interval from 17.61 pcf to 22.12 pcf [3]. This range will be referenced later in association with upper and lower bound model validation predictions.

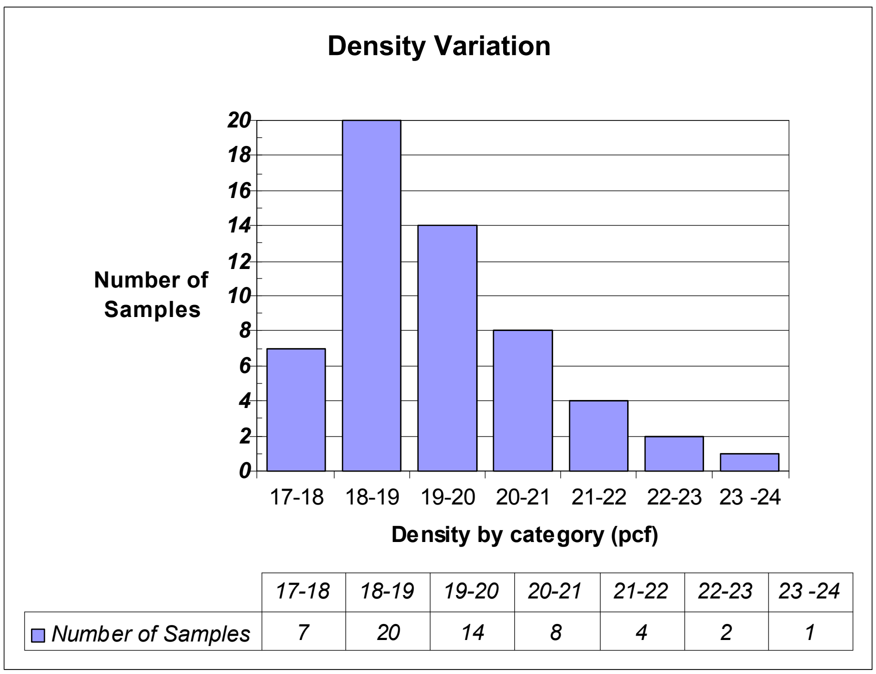


Figure 1. Bulk density distribution for nominal 20 pcf PMDI foam.

To quantify the influence of density variations, constitutive tests were conducted at several different densities. Exponential fits to the test data were then used to scale the material parameters for PMDI foam. Figure 2 shows plots of the modulus of elasticity and yield stress vs. density, respectively. The exponential curve fits from Figure 2 were used to scale the VFM material parameters based on density.

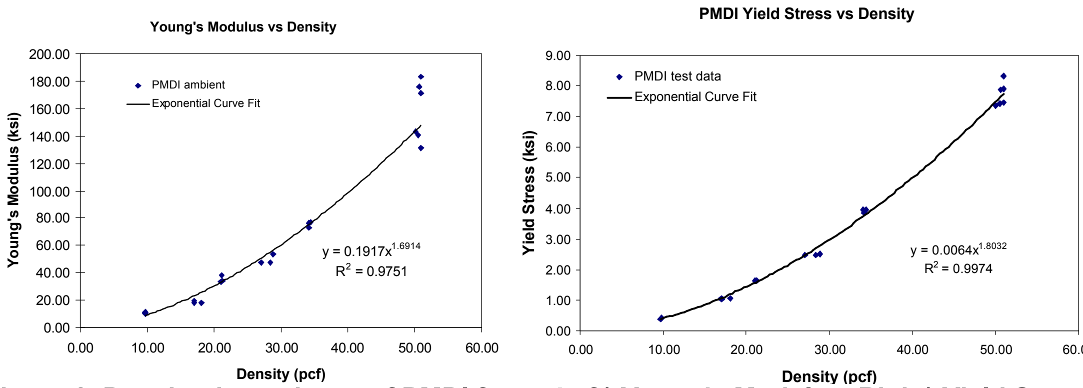


Figure 2. Density dependence of PMDI foam: Left) Young's Modulus, Right) Yield Stress.

The equation for scaling Young's Modulus is defined as:

$$E_2 = E_1 \left(\frac{\rho_2}{\rho_1} \right)^{1.6914} \tag{1}$$

where E_1 is the original modulus of the material and E_2 is the scaled modulus. The density, ρ_1 is the original density and ρ_2 is the scaling density.

Next, the density scaling equation for the yield strength is:

$$\sigma_{y2} = \sigma_{y1} \left(\frac{\rho_2}{\rho_1} \right)^{1.8032} \tag{2}$$

where σ_{y1} is the original yield strength and σ_{y2} is the scaled yield strength of the material. This exponential fit was applied to scaling the shear and hydro strength parameters as follows.

The Shear Strength parameter is scaled as:

$$SS_2 = SS_1 \left(\frac{\rho_2}{\rho_1} \right)^{1.8032} \tag{3}$$

And the Hydro Strength is scaled as:

$$HS_2 = HS_1 \left(\frac{\rho_2}{\rho_1} \right)^{1.8032} \tag{4}$$

The VFM material parameters in Table 1 for the density of 17.85 pcf were scaled to other densities using Equations 1 to 4. These equations were left as deterministic due to the small amount of test data for each density so no uncertainty was included for the modulus or strength values at a given density.

Validation Process

The definition of the term *validation* is the "process of determining the degree to which a computer model is an accurate representation of the real world from the perspective of the intended model applications" [4,5]. This requires comparing model predictions with an experiment and quantifying as many significant uncertainties and variations as possible that are present within the experimental and the model development procedures. Uncertainties that were examined will be discussed later.

The intended model application involves shock mitigation during impact of components surrounded by PMDI foam. Experiments used to validate a model must be sensitive to and measure with a known accuracy the important physics that the model is intended to predict. In this work validation experiments were done with a drop table that could produce relevant shock levels in the test.

Validation Experiments

Validation experiments were designed to quantify the accuracy of the VFM for predicting the behavior of PMDI foam in high strain rate shock mitigation applications. These tests involved loading a test article referred to as the uni-stack assembly in a drop table test machine.
The uni-stack assembly consisted of three main parts as shown on the left in Figure 3. A cylindrical foam pad or test sample was placed between an aluminum base and a 15.5 lb steel mass. The steel mass was designed to simulate the weight of a component (component simulator) acting on the foam. The PMDI foam part was 2.75 in. in diameter and had one of three different axial dimensions; 0.5", 1.0", and 2.0". The right side of Figure 3 shows a photograph of the uni-stack with a foam pad that is 0.5" thick.

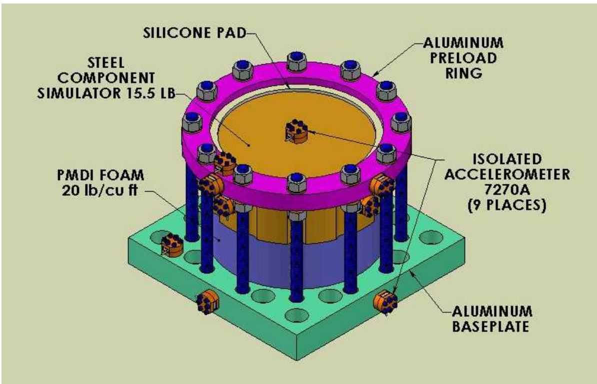


Figure 3. Uni-stack Test Assembly: Left) schematic drawing, Right) actual photograph.

A loading ring was placed over the component simulator and bolted down to the steel base. This ring was used to apply and maintain an approximate 30 psi preload on the uni-stack assembly to hold the component simulator in contact with the foam during the drop test until the initial impact occurs. The steel base of the uni-stack assembly was bolted to the carriage of a drop table as shown in Figure 4.

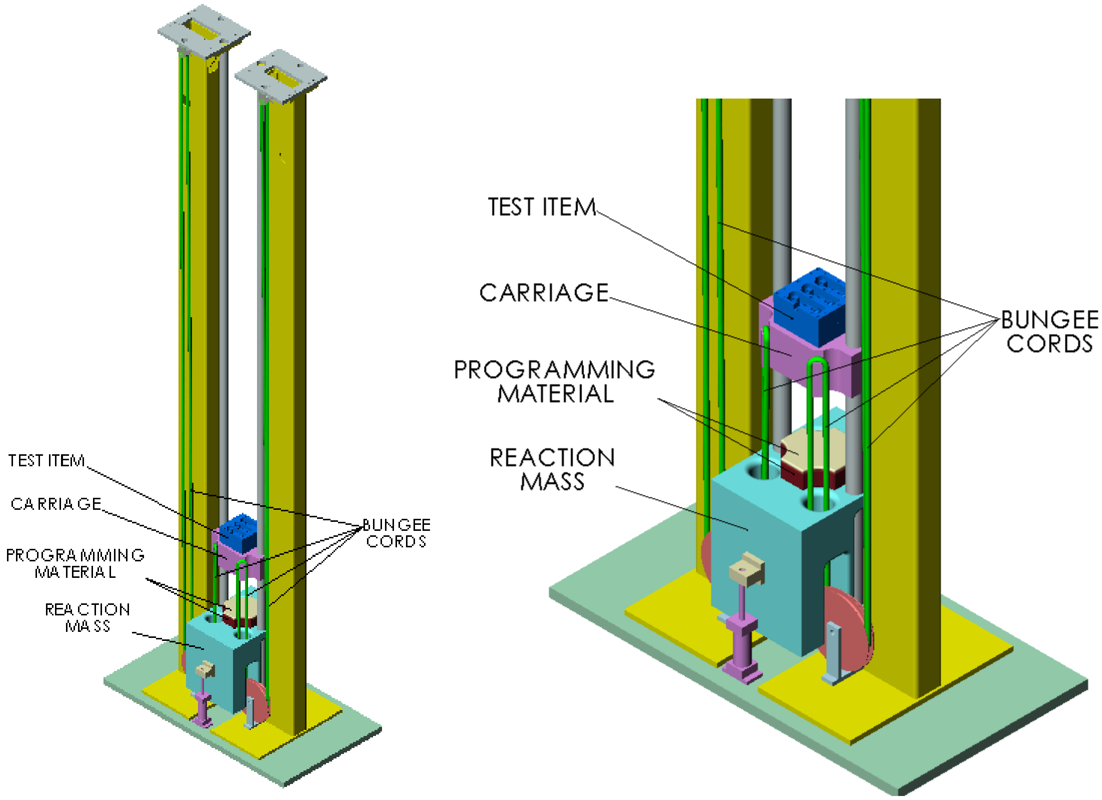


Figure 4. Drop Table drawings.

During the drop test, the carriage accelerates from an initial position on the drop table tower. The actual test begins when the carriage impacts the reaction mass of the table and a mechanical shock is transmitted into the uni-stack base. Peak acceleration and duration of the impulse are selected by the drop height which controls the impact velocity and the choice of programming material that cushions the impact. Acceleration is measured on the uni-stack's aluminum base plate for the input and the component simulator for the response of a foam encapsulated component.

Validation Test Matrix

A matrix of 56 drop table tests was conducted to provide data to validate the VFM as described in Table 2, Table 3, and Table 4. The tests were identified to encompass varying strain rates, temperatures, and foam thicknesses. Foam sample thicknesses were based on the maximum and minimum thicknesses used in the system models. Also, test temperature, peak acceleration, and duration were identified based on system environment requirements.

Scoping tests were conducted on the half inch and two inch thick foam samples at -53.9 °C, 21.1 °C and 73.9 °C where a haversine shaped impulse was applied to the samples with the drop table. The input peak acceleration was varied in order to identify lower and upper bound values. The lower bound peak input acceleration of 3400 G's was chosen to cause near yield stress levels of the foam but still behave primarily in an elastic manner with negligible plastic deformation. The upper bound values of 4000 to 4400 G's were chosen to cause significant plastic deformation without cracking. Also, a one millisecond duration haversine pulse was specified as a compromise between application requirements and the drop table capability.

Table 2. Half inch PMDI foam test matrix.

umber of Tes	ample Siz	Temperature	jet G @ Duration (
3	0.5 in	21.1 □C	3400 @ 1.0
3 (2)	0.5 in	21.1 □C	4400 @ 1.0
3	0.5 in	-53.9 □C	3400 @ 1.0
3	0.5 in	-53.9 □C	4400 @ 1.0
3	0.5 in	73.9 □C	3400 @ 1.0
3 (2)	0.5 in	73.9 □C	4400 @ 1.0

Table 3. One inch PMDI foam test matrix.

umber of Tes	ample Siz	Temperature	jet G @ Duration (
3	1 in	21.1 □C	3400 @ 1.0
3	1 in	21.1 □c	4200 @ 1.0
3 (2)	1 in	-53.9 □C	3400 @ 1.0
3	1 in	-53.9 □C	4200 @ 1.0
3	1 in	73.9 □C	3400 @ 1.0
3	1 in	73.9 □C	4200 @ 1.0

Table 4. Two inch PMDI foam test matrix.

umber of Tes	ample Siz	Temperature	jet G @ Duration (
3	2 in	21.1 □C	3400 @ 1.0
3 (2)	2 in	21.1 □C	4400 @ 1.0
3	2 in	-53.9 □C	3400 @ 1.0
3 (2)	2 in	-53.9 □C	4000 @ 1.0
3	2 in	73.9 □C	3400 @ 1.0
3	2 in	73.9 □C	4000 @ 1.0

(#) actual number of good tests

Experimental Uncertainty

To fairly assess the level of confidence in the model's predictions the validation process must quantify and factor in all significant uncertainties and variations in the model and experiment. Experimental uncertainty will first be discussed.

Initial measurements of the motion of the uni-stack during a drop table test were recorded with triaxial accelerometers. Axial response was the primary interest in these tests. The results indicated that the transverse loading was less than 10% of the axial loading of the uni-stack. Transverse loading was therefore assumed to have a negligible affect on the axial response of the foam and uni-stack response. A 3 KHz low pass filter was used to filter the data.

The uncertainty in the accelerometer measurement results is attributed to: the uncertainty due to the data acquisition system, uncertainty in the accelerometer sensitivity, and the uncertainty in the Mechanical Shock Drop Table Machine. The uncertainty is considered random, so they may be combined in an uncertainty analysis with a 95% confidence level as [6].

$$w_T = \sqrt{w_d^2 + w_s^2} \quad (5)$$

where: w_T = total uncertainty,
w_d = the drop table uncertainty, $\pm$10% and
w_s = accelerometer sensitivity uncertainty, $\pm$5%.

The value of the total uncertainty in acceleration experimental data, w_T, is $\pm$ 11% and is typical for the measurements made in the SNL Mechanical Shock Laboratory.

Model Uncertainty and Variability

The most significant variability in this validation process was estimated to be the range of foam bulk densities given in Figure 1. The associated influence of this density variation on material properties has been deterministically accounted for as given in Equations 1 to 4. A probabilistic approach to handle the foam bulk density variation will be discussed later. Spatial density gradients within the foam samples are being quantified but for this model validation they were assumed negligible

Additionally, other variations in material properties must be checked and evaluated. Anisotropy within foam materials is often a key consideration with polymer foams. They may exhibit significantly different properties between the rise and lateral foam directions. However, PMDI foams with a nominal 20 pcf density do not exhibit significant variations in properties when loaded in different directions [7]. As a result, the foam material was assumed to be isotropic for this study.

The effect of friction is an additional uncertainty within the experiment that must be accounted for in the simulation. Model sensitivity to friction is presented later.

Finite Element Model

A finite element model was constructed using 8-node hexahedral elements. Utilizing symmetry in the model, a quarter of the test set up was analyzed as shown in Figure 5. Three parts were modeled to include the base, foam, and component simulator. The thickness of the foam varied based on the foam thickness used in the tests.

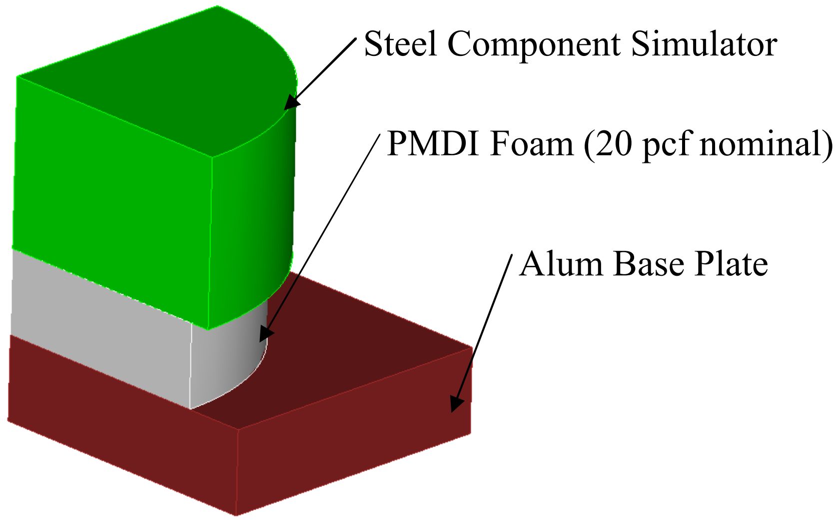


Figure 5 – Analysis Model Geometry with Quarter Symmetry.

The foam in the model was constrained in the transverse directions by friction on the upper and lower surfaces.

Convergence Studies

Prior to using the finite element model for the validation simulations, three mesh convergence studies were performed on models having a foam thickness of half inch, one inch, and two inches, respectively. Three different mesh sizes were used for each model as given Figure 6. A typical input haversine pulse was used to load the model axially at its base and the response variable used to assess convergence was the axial acceleration time history of the component simulator mass.

The element sizes that were considered sufficiently converged were 0.125” for the half inch model and 0.25” for the one and two inch thick foam pads. In each case the acceleration time history curve predicted from the selected element size model varied around 1% from the finer element model and greater than 5% from the larger element

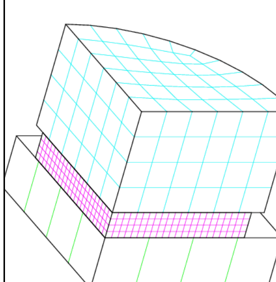
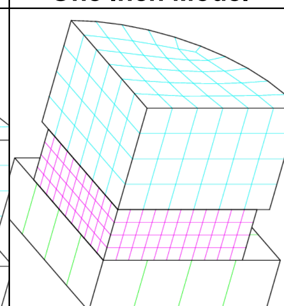
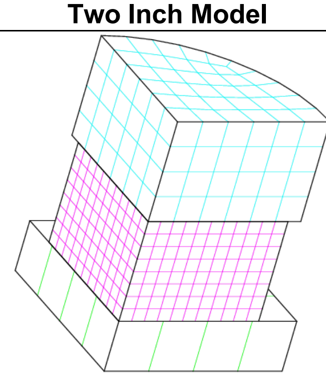
	Half Inch Model	One Inch Model	Two Inch Model
Model			
Element Size	0.25 inch	0.5 inch	0.5 inch
Element Size	0.125 inch	0.25 inch	0.25 inch
Element Size	0.0625 inch	0.125 inch	0.125 inch

Figure 6. Finite element models used for the three foam thicknesses within the Uni-stack assembly plus element sizes examined for convergence.

model. For example, Figure 7 shows the input and response acceleration curves for the three meshes associated with the one inch thick foam pad.

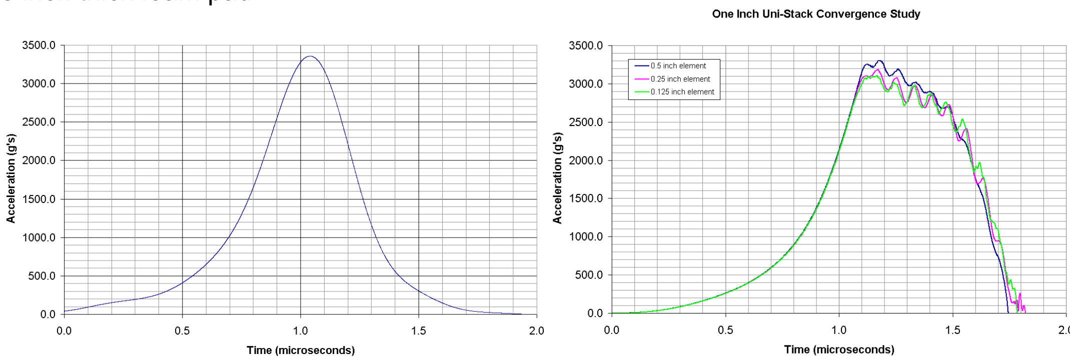


Figure 7. Input acceleration (left) and response curves(right) for the mesh convergence study with 1" thick foam pad.

Friction Sensitivity Studies

Simulations were completed on the three dimensional, uni-stack simulation model to determine the effects of friction. Simple static friction tests conducted on PMDI foam sliding on the steel component simulator indicated the foam had a static coefficient of friction value of 0.26. For the friction sensitivity simulations, the friction value was varied from 0.05 to 0.50. Additionally, due to the range of densities in the foam material, high, average, and low density values were also evaluated to determine the effects of friction based on density. Sensitivity to friction only appeared in the model between values of zero and 0.20 implying that no slippage occurred above the value of 0.20 in the model. Based on data from the simple static friction tests and these uni-stack simulations, a friction coefficient of 0.20 was assigned to the PRESTO input deck for the validation simulations.

Preload Sensitivity

Simulations of the drop table loading on the uni-stack were completed using a PRESTO model with the preload ring and one without. Peak accelerations from the component simulator were compared as shown in Figure 8. The overall shape and magnitude of the acceleration responses were nearly identical so the effects of preload were considered negligible. As a result, the ring was not included in the final uni-stack model.

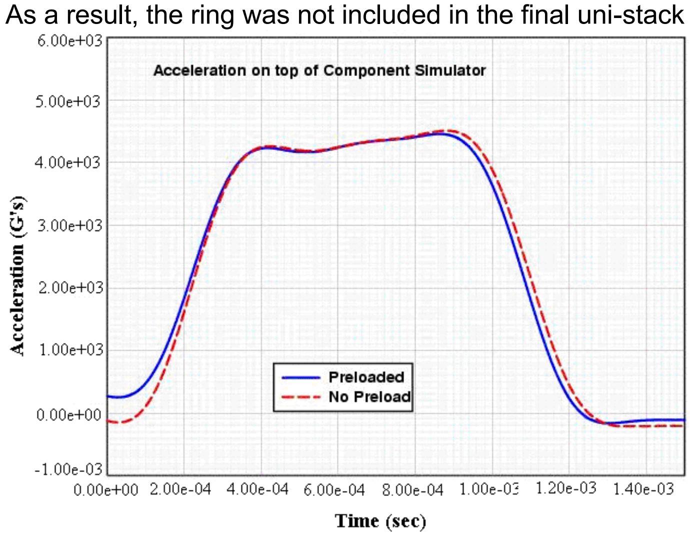


Figure 8. Predicted accelerations with Bolt Preload vs. No Preload.

Validation Metrics

Two metrics were used for the validation of the VFM. The response variable of interest is the shock induced on the component simulator mass acceleration so both metrics involve the acceleration of the component simulator mass. The first metric compares the peak acceleration between model prediction and test. The second metric compares the response impulse which is the area under the acceleration time history curve for both model and test.

Model Predictions with Uncertainty

Variation in density of the PMDI foam that was specified to be 20 pcf was given in Figure 1. The actual density of the foam used in a system may not be known any better than the distribution given in Figure 1 so a probabilistic and a deterministic approach were used to validate the VFM.

The probabilistic approach treated density as a random variable in the model and predictions of the validation tests were completed using an upper bound and a lower bound density value. These density values were estimated in the model based on the density extremes from the test data distribution shown in Figure 1 which are 17.73 pcf and 23.02 pcf, respectively. These bounds are somewhat larger than the 95% confidence interval discussed earlier. The VFM is a nonlinear model but for a given input acceleration impulse, the model predictions increase monotonically with increasing density. So probabilistically, more than 19 out of 20 test results should fall within the upper/lower band of predictions for the model to be valid.

The deterministic approach assumed that the density of the test article was known and so the actual measured density was incorporated into the model with this approach. Consequently, simulations at each density used in the test were completed to predict the individual test responses. With the density uncertainty removed, the model predictions were expected to fall within +/-20% of the test data for it to be valid.

Probabilistic Model Validation Results

Figure 9 shows model predictions versus test data for two example cases of the upper/lower bounding validation approach. Here the upper/lower bound model predictions of acceleration for the component simulator mass are given along with the three test response curves for an ambient temperature case on the left and a cold temperature case on the right. The foam densities from Tests 47A, 48A, and 49A on the left are 17.73, 17.8, 18.53 pcf, respectively, and thus hover around the lower bound model prediction. Whereas, in the right side of Figure 9 the bounding curves from the model very nicely envelope the peak response of the three test curves for the cold temperature case.

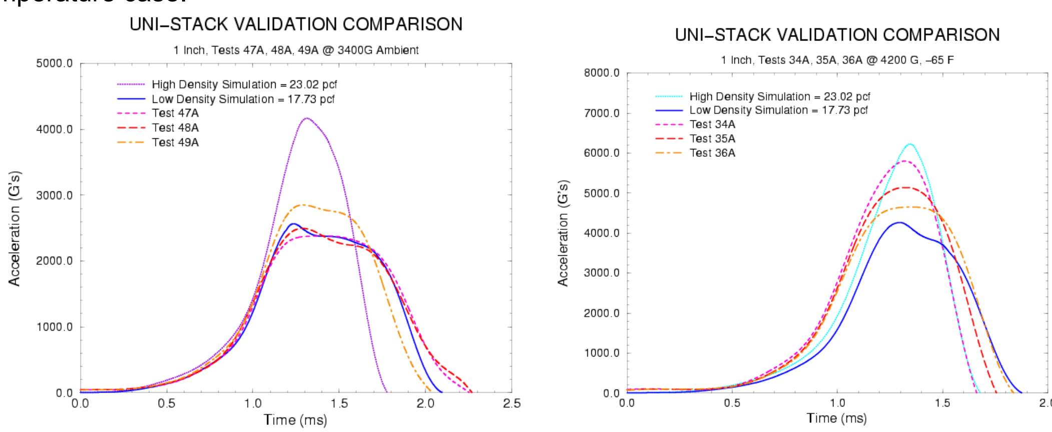


Figure 9. Validation results at ambient with 3400 G input on left side and 4200 G input right side: response with upper/lower bounding density model versus test data.

Comparisons of model and test results for the 1 inch foam tests with upper and lower bound densities in the model are given in Figure 10 and Figure 11. The data from these 17 tests, which include all three temperatures and the two acceleration levels, are contained very well by the model's upper and lower bounds. Thirteen out of seventeen

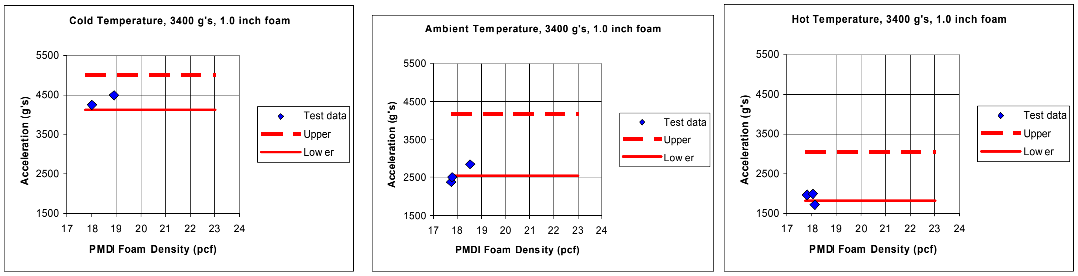


Figure 10. **Bounding peak acceleration predictions versus test data for 3400 G and one inch foam (left to right is cold, ambient and hot temperature data).**

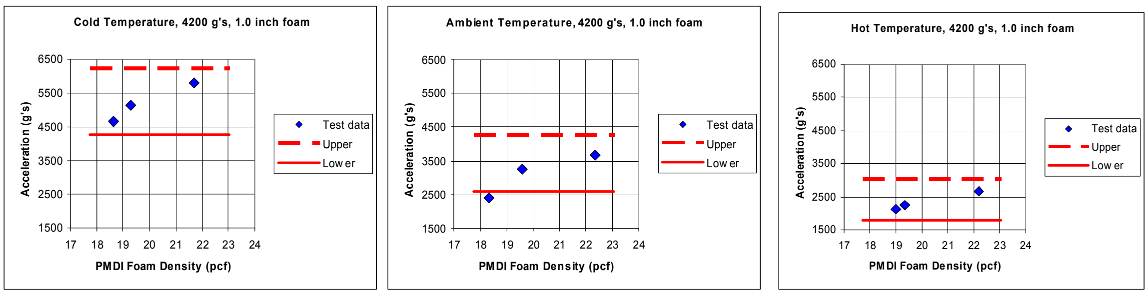


Figure 11. **Bounding peak acceleration predictions versus test data for 4200 G and one inch foam (left to right is cold, ambient and hot temperature data).**

test data, or 76%, fell between the upper and lower bounds and the other four test data that fell outside the bounds were all less than 10% below the lower bound. After factoring in the 11% of experimental uncertainty all of the test points can be concluded to overlap with the upper/lower bounds. Model predictions and test data for the other test articles having the 0.5 and 2.0 inch foam thicknesses displayed similar agreement. Therefore, based on the peak acceleration metric and the upper/lower bound model validation process the VFM model is judged to be valid for the shock loading and strain levels tested.

Deterministic Model Validation Results

Figure 12 shows the input pulse measured in drop Test 48A and the experimental response from Test 48A compared with the deterministic model prediction. Here the same density as measured in the test article (17.8 pcf) is used in the model. The model prediction acceleration or simulation curve looks very similar to the test curve with a slightly higher peak value.

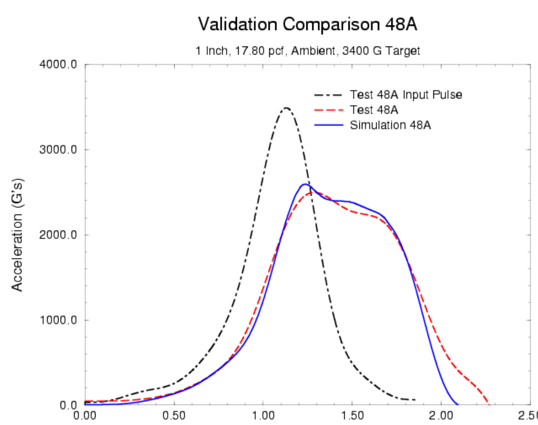


Figure 12. **Validation results at ambient with 3400 G haversine input and model predictions using actual density of test article.**

Figure 13 shows the percent difference between the response impulse predicted by the VFM and the data from all the validation tests. Notice that a segregated cloud of data points form for each temperature. The results corresponding to the 73.9 °C temperature have the largest positive percent difference between model and test, the results corresponding to the -53.9 °C temperature have the largest negative percent difference between model and test data, and the ambient results have the least difference. So there appears to be a systematic offset between model and test that is sensitive to both temperature and density. These results suggest that the model accuracy could be improved by modifying the temperature dependence of some model parameters. However, with the existing set of material paparameters all model predictions did fall within 20% of the test data even without factoring in the experimental uncertainty. Therefore, based on the impulse metric and the deterministic model validation process, the VFM model is judged to be valid for the shock levels tested.

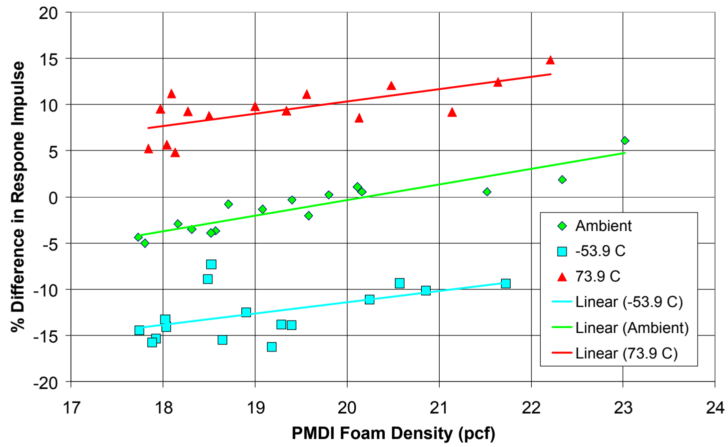


Figure 13. Percent difference in response impulse between the deterministic model and test for the Component Simulator Mass.

Figure 14 shows the percent difference in peak acceleration for the deterministic VFM and all of the validation tests conducted. The data points are color coded for temperature. In contrast to the temperature sensitivities shown in Figure 13, no clear temperature or density trends are evident here. The peak accelerations appear to be randomly distributed with temperature and density and do fall within a +/- 20% envelope of the test data required to validate the model; again without factoring in the experimental uncertainty of 11%.

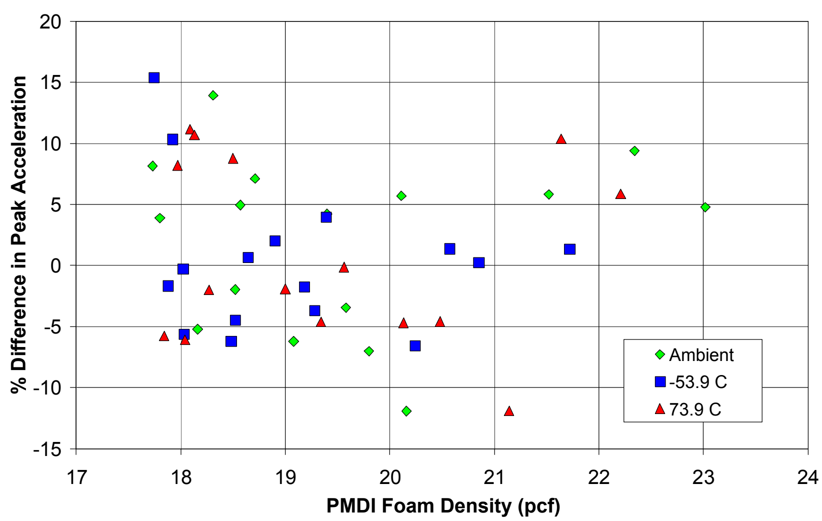


Figure 14. Percent difference between the deterministic model and test for peak acceleration of the Component Simulator Mass.

Summary and Conclusions

This paper presents a validation process for the Viscoplastic Foam Model (VFM) in the PRESTO finite element code for PMDI foam. This process involved a double blind procedure where the analyst did not see the test data nor did the test engineer see the model predictions before each engineer completed their part of the process. The VFM predictive capability was examined in a shock environment over a range of temperatures from -53.9 ^{o}C to 73.9 ^{o}C and with density variations from 17.73 to 23.02 pcf. Model validation was accomplished using a deterministic method and a probabilistic method. The probabilistic method used upper and lower bound estimates for the foam density in the VFM. The deterministic method used measured densities from the test articles in the model predictions. Two metrics were used for comparing model predictions with test data; peak acceleration of the component simulator mass and the impulse of the component. The model predictions satisfied the validation criteria in all cases and the VFM was judged to be valid for PMDI foam under the shock and strain levels tested where no significant cracking occurred.

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