

# Updating a User Friendly Combined Lifetime Failure Distribution

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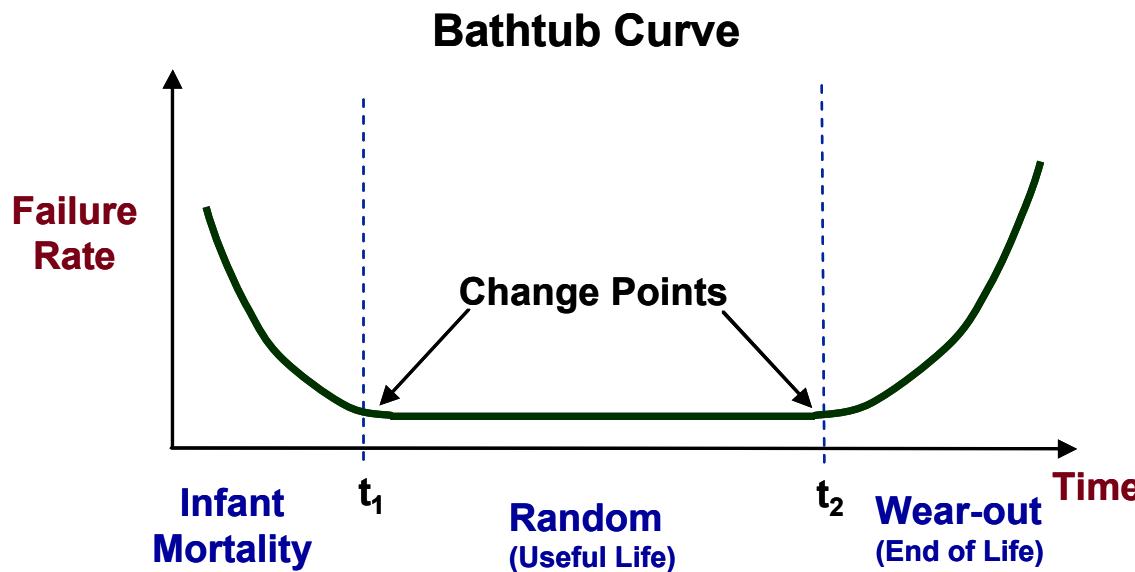
# Outline

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- **Objective: Updating time-to-failure distributions**
- **Introduction**
  - Bathtub Curve
- **Combined Lifecycle (CMBL) Distribution**
  - Advantage
  - Description
- **Updating Methods**
  - 4 Possible
- **Results of Method 1**
  - Convergence to Data
  - Additional challenges
- **Summary**
  - Future Direction

# Bathtub Curve

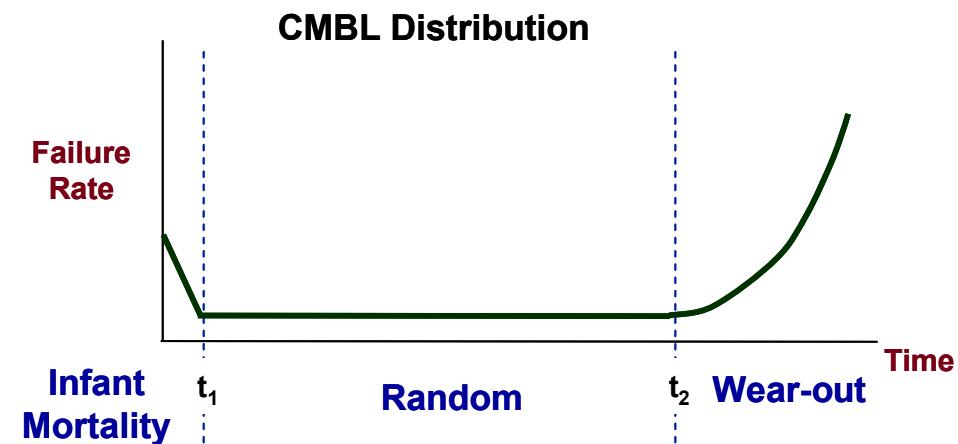
- Probability of failure over lifetime of component
  - Usually modeled as Exponential
- Useful for enterprise level and prognostic focused modeling
  - Optimize supply/repair chain processes
  - Baseline for characterizing component health trends



# CMBL Distribution

- **Advantages**

- Common terms and approach
- Easy to solicit expert opinion

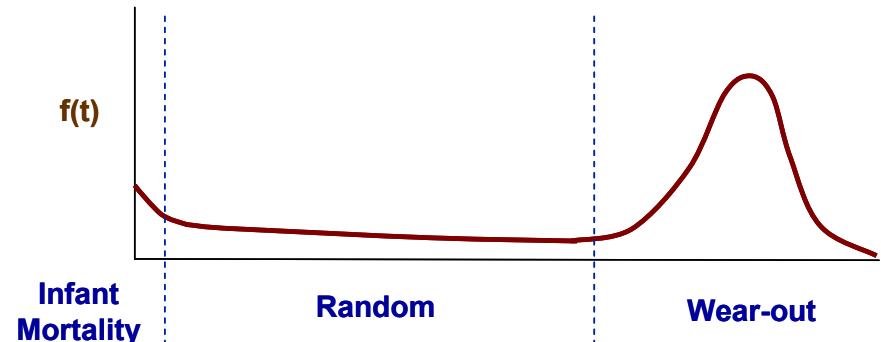


- **Input Parameters**

- Probability that the component will fail during infant mortality
- Duration of infant mortality
- Probability that failure will occur randomly
- Mean of the normally distributed portion
- Standard deviation of normally distributed portion

# CMBL Distribution

$$f(t) = \begin{cases} \lambda_d e^{-\lambda_d t} & 0 \leq t \leq t_1 \quad \lambda_d = (mt + b) \\ \lambda_c e^{-\lambda_c t} & t_1 \leq t \leq t_2 \\ \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(t-u)^2} & t_2 \leq t \leq \infty \end{cases}$$



- **Definitions**

$\mu$  = mean of the normally distributed portion of the TTF distribution

$\sigma$  = standard deviation of the normally distributed portion

$\lambda_d$  = failure rate for the linearly decreasing failure rate portion

$\lambda_c$  = failure rate for the constant failure rate portion

$t_1$  = burn-in duration (BID)

$t_2$  = transition from constant failure rate to the normal TTF portion

$F_1$  = fraction of failures occurring in the infant mortality portion

$F_2$  = fraction of failures occurring in the random failure portion



# Updating Methods

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- **Method 1**
  - Treat each section separately, use appropriate prior based on section
  - Modify the CMBL iteration procedure to get back input parameters
- **Method 2**
  - Model as a single distribution using Markov chain simulation
  - Use regression and curve fitting techniques to get back input parameters
- **Method 3**
  - Use Method 1 or 2 to get empirical distribution
  - Use empirical distribution with Markov chain simulation as next prior
- **Method 4**
  - Investigate possible closed form solution
  - Evaluate possible prior distributions to obtain closed form posterior



# Method 1 Setup

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- **Assumptions**

- Treat each section separately, use appropriate prior based on section
- Only random and wear out portions of distribution were evaluated
- Used a Bayesian updating methodology, one data point at a time
- Introduced a “weighting scheme” to reduce extreme values in data
- The standard deviation was assumed known and held constant

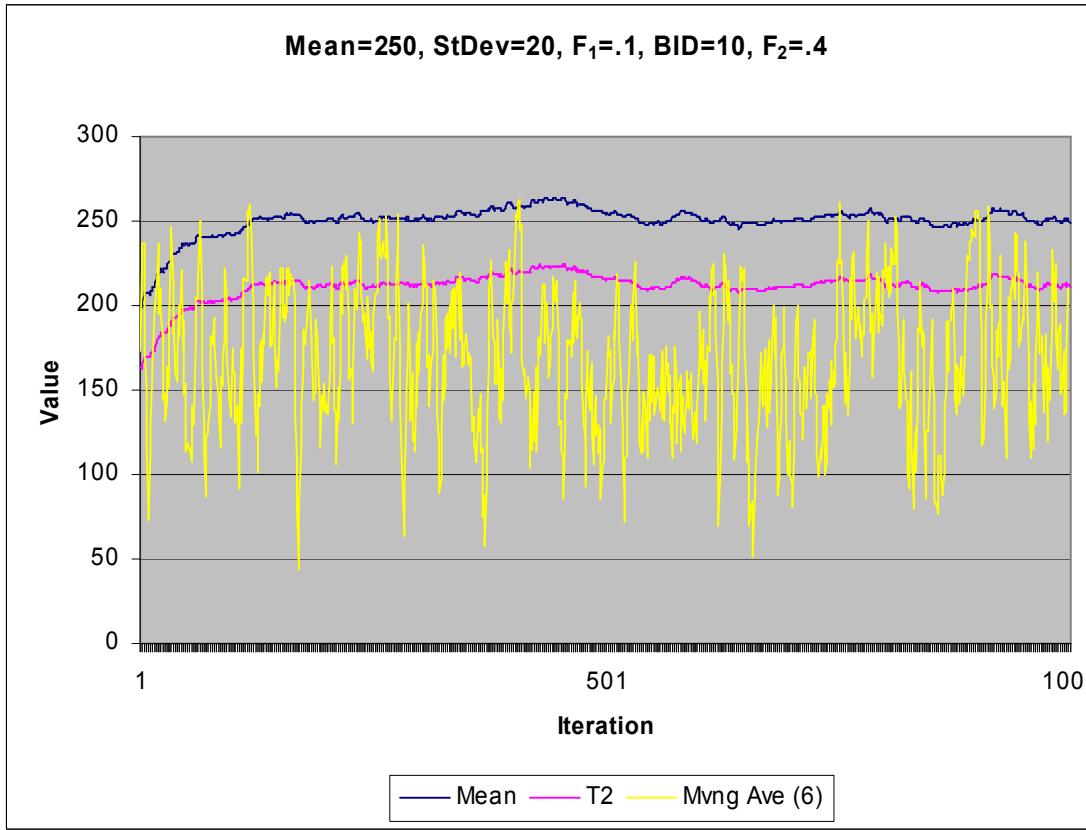
- **Example Input Parameters**

	Baseline	Normal	Random
$\mu$	200	250	150
$\sigma$	20	20	20
$F_1$	.1	.1	.1
$F_2$	.4	.4	.4
$t_1$	10	10	10



# Method 1 Results

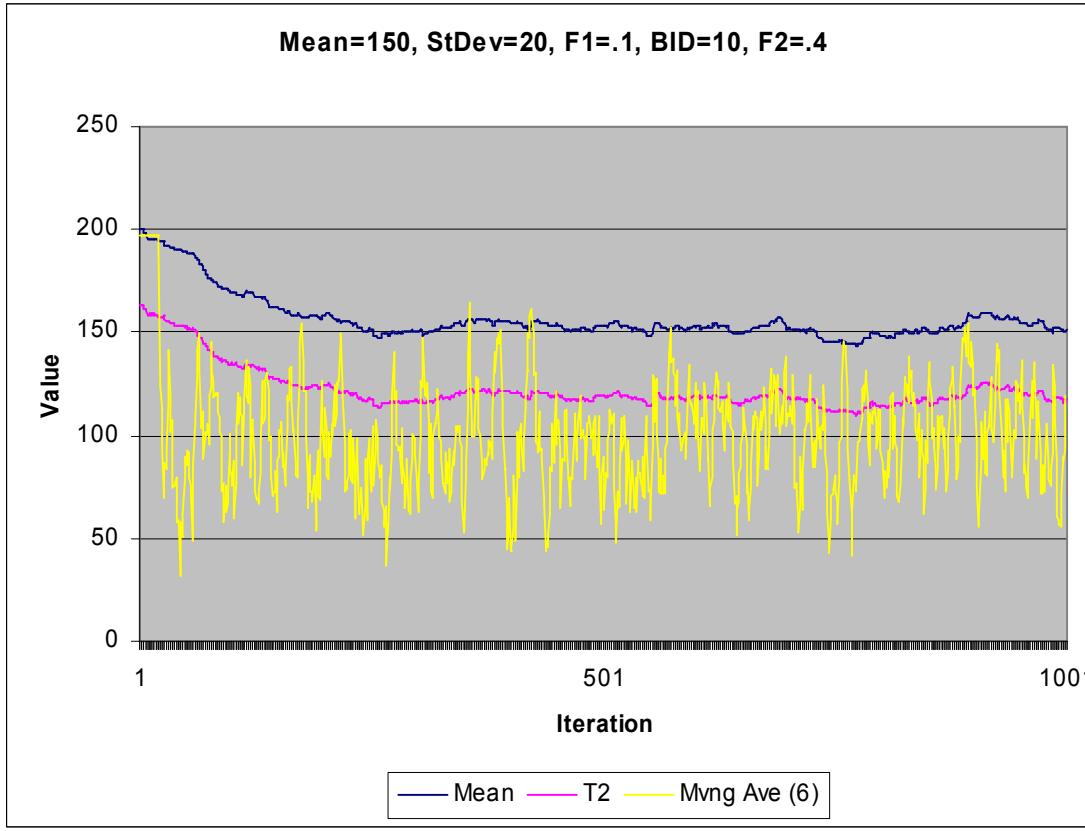
- Mean increased to 250
  - Mean converged in about 200 iterations
  - $t_2$  converged to 212.3 vs calculated 210.9





# Method 1 Results

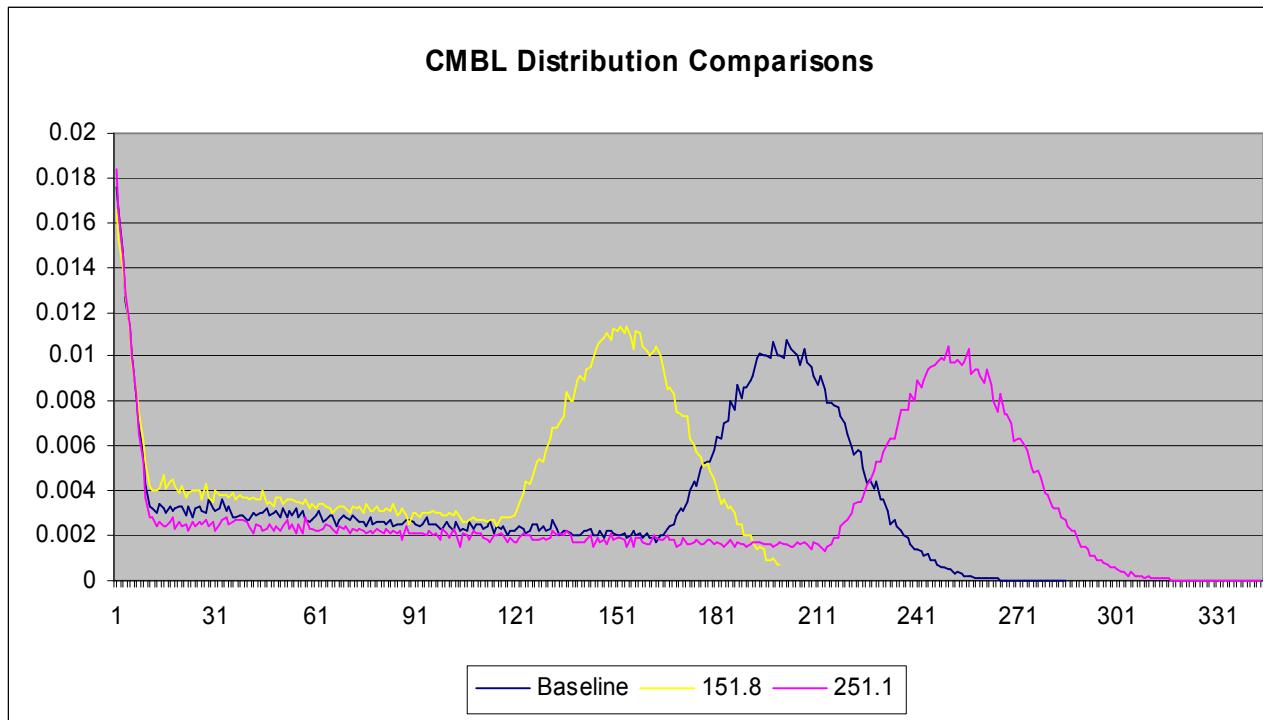
- Mean decreased to 150
  - Mean converged in about 250 iterations
  - $t_2$  converged to 117.9 vs calculated 117.5)





# Method 1 Results

- Results Comparison
  - Expected shift in distribution





# Summary

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- **CMBL distribution useful for logistic support and PHM modeling**
  - Uses expert opinion, data from similar systems, scarce data
  - Updating methods will allow use throughout a components lifecycle
- **Method 1 looks promising**
  - Changes in the mean resulted in convergence to the distribution of the data
  - May apply to other sectional models
- **Future work**
  - Evaluate changes in  $F_2$  and  $\sigma$  for Method 1
  - Include infant mortality in the updating process for Method 1
  - Evaluate Methods 2, 3, and 4
  - Investigate application to other types of lifetime failure models