Stochastic Control of Energy Efficient Buildings: A Semidefinite Programming Approach

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***Abstract*—The key goal in energy efficient buildings is to reduce energy consumption of Heating, Ventilation, and Air-Conditioning (HVAC) systems while maintaining a comfortable temperature and humidity in the building. This paper proposes a novel stochastic control approach for achieving joint performance and power control of HVAC. We employ a constrained Stochastic Linear Quadratic Control (cSLQC) by minimizing a quadratic cost function with a disturbance assumed to be Gaussian. The problem is formulated to minimize the expected cost subject to a linear constraint and a probabilistic constraint. By using cSLQC, the problem is reduced to a semidefinite optimization problem, where the optimal control can be computed efficiently by Semidefinite programming (SDP). Simulation results are provided to demonstrate the effectiveness and power efficiency by utilizing the proposed control approach.**

I. Introduction

Buildings consume up to 40% of the energy produced in the US [1]. Advanced sensors and controls have the potential to reduce the energy consumption of buildings by 20-40% [2], [3]. Heating, Ventilation and Air Conditioning (HVAC) systems play a fundamental role in maintaining a comfortable temperature environment in buildings and account for 50% of building energy consumption [1]. Significant potential for energy savings exist by optimally controlling HVAC systems to reduce consumption while maintaining comfort constraints. Typical building controls are set-point based, where zone-level temperature measurement is used for taking control action to keep the zone temperature in a comfortable range. To make buildings more energy and cost efficient, intelligent predictive automation can be used instead of conventional automation. For instance, the predictive automation controllers can operate the buildings passive thermal storage, based on predicted future disturbances (e.g. weather forecast), by making use of low cost energy sources [4]. The goal is to design an optimal controller that can realize the temperature requirement and minimize energy consumptions.

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The accuracy of the controller heavily depends on the assumption that the sensor always provides exact temperature measurement. However, this assumption is not always valid due the measurement error or real-world environment noise. Consequently, an effective controller for HVAC systems should incorporate time-dependent energy costs, bounds on the control actions, noise from the sensors, as well as account for system uncertainties, i.e., weather conditions and occupancy. Compared with the deterministic control approaches, a key advantage of stochastic control approaches is that a noise term is considered in the model, which represents the unknown and uncertain elements in the system.

Building climate control leads naturally to probabilistic constraints as current standards explicitly state, zone temperatures should be kept within a comfort range with a predefined probability [5], [6]. In order to address this issue and explicitly account for system uncertainties, some efforts have been made for studying a stochastic version of MPC (SMPC) including probabilistic constraints. [7] employed stochastic MPC technique to compute the control strategy for a cost function which was linear in the control variable for the thermal dynamics in a linear state-space model, which described thermal energy and temperatures. [8] proposed a tractable approximation method for the problem. Both schemes in [7] and [8] considered chance constraints and solved them by using affine disturbance feedback.

In this paper, we consider the same building climate plant in [9] which was validated and compared to simulations with TRNSYS, a well known simulation tool for building and HVAC systems [4]. We study a quadratic cost function in terms of temperature errors and control inputs, which is subject to several constraints on the room temperature and control input. In particular, we only consider the case where we assume that the disturbance is **Gaussian** and the problem is formulated to minimize the expected cost subject to a linear constraint on control input and a **probabilistic constraint** on the state. The latter constraint can be reduced to a hard constraint on control input exactly [10]. It should be remarked that the power of this proposed control technique could be extended to a more general **norm-bounded** case with distribution unknown and the problem is formulated as a min-max problem. By using the cSLQC approach proposed in

[10], [11], the optimal solutions of problems in both cases may be solved via semidefinite programming exactly. Due to space limit here, we only discuss the case with Gaussian disturbance.

The differences between our paper and previous works are:

- First, unlike [7] and [8], the chance constraint is simplified to a hard constraint exactly without using affine disturbance feedback.
- Second, we consider a stochastic quadratic cost function, which is taken expectation with respect to Gaussian disturbances. Moreover, the cost function includes both quadratic forms of temperature errors and control input, which means the optimal control is designed to find a compromise between them.
- Third, the problems are formulated into semidefinite optimization problems which may be solved through SDP for the optimal solutions efficiently.

The rest of the paper is organized as follows: In section II, the building climate model is described and presented. In section III, we introduce the control techniques used in this work and solve the problems. Section IV gives the simulation results to show the performance of the methods in controlling the building climate. Section V concludes the work and discusses the future direction.

II. Building Climate Plant

It is well known that the HVAC control can be approached using Model Predictive Control (MPC) strategy. In this section, we describe the model used in this work and formulate the problem. The system model was proposed in [4] and employed in [7]. We briefly describe the model in this section.

A. Building Model

Consider the following continuous-time Linear Time Invariant (LTI) system based on the dynamics of the room temperature, interior-wall surface temperature, and exterior-wall core temperature:

$$\begin{aligned}\dot{t}_1 &= \tfrac{1}{C_1}[(K_1+K_2)(t_2-t_1)+K_5(t_3-t_1)+K_3(\delta_1-t_1)+u_h \\ &\quad +u_c+\delta_2+\delta_3] \\ \dot{t}_2 &= \tfrac{1}{C_2}[(K_1+K_2)(t_1-t_2)+\delta_2] \\ \dot{t}_3 &= \tfrac{1}{C_3}[K_5(t_1-t_3)+K_4(\delta_1-t_3)]\end{aligned}$$

where the parameters used in the above model are defined as:

t_1 : room air temperature [$^\circ F$]
t_2 : interior-wall surface temperature [° F]
t_3 : exterior-wall core temperature [° F]
u_h : heating power (≥ 0) [kW]
u_c : cooling power (≤ 0) [kW]
δ_1 : outside air temperature [° F]
δ_2 : solar radiation [kW]
δ_3 : internal heat sources [kW]

$C_1 = 9.356 \cdot 10^5$	$kJ/^\circ F$	$C_2 = 2.970 \cdot 10^6$	$kJ/^\circ$F
$C_w = 6.695 \cdot 10^5$	$kJ/^\circ$F	$K_1 = 16.48$	$kW/^\circ$ F
$K_2 = 108.5$	$kW/^\circ$ F	$K_3 = 5$	$kW/^\circ$F
$K_4 = 30.5$	$kW/^\circ$ F	$K_5 = 23.04$	$kW/^\circ$F

The system states are the room air temperature t_1, interior wall surface temperature t_2, and exterior wall core temperature t_3. The control signals u_h and u_c represent heating and cooling power, and they can be combined as one variable $u = u_h + u_c$ because heating and cooling are not simultaneous. For more details about this model, please refer to [4], [7].

Define the state vector x, the control signal vector u, and the environment stochastic disturbance vector ω as:

$$x := \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix},\ u := \begin{bmatrix} u_h \\ u_c \end{bmatrix},\ \omega := \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

The continuous-time state-space model can then be described compactly as:

$$\dot{x} = A_c x + B_c u + C_c \omega \tag{1}$$

where

$$A_c := \begin{bmatrix} -\frac{1}{C_1}(K_1+K_2+K_3+K_5) & \frac{1}{C_1}(K_1+K_2) & \frac{K_5}{C_1} \\ \frac{K_1+K_2}{C_2} & -\frac{(K_1+K_2)}{C_2} & 0 \\ \frac{K_1}{C_3} & 0 & -\frac{(K_5+K_4)}{C_3} \end{bmatrix}$$

$$B_c := \begin{bmatrix} \frac{1}{C_1}+\frac{1}{C_2} \\ 0 \\ 0 \end{bmatrix}, \qquad C_c := \begin{bmatrix} \frac{K_3}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & 0 \\ \frac{K_4}{C_3} & 0 & 0 \end{bmatrix}. \tag{2}$$

Discretizing system (1) with period h_k and applying a zero-order-hold, one obtains:

$$x_{k+1} = A_d x_k + B_d u_k + C_d \omega_k \tag{3}$$

where the parameters can be computed from the continuous-time model, and $x_k = [t_{1,k}, t_{2,k}, t_{3,k}]^T$.

We assume the following constraints are imposed on the temperatures during a day to satisfy the requirement:

$$68^\circ\text{F} \leq t_{1,k} \leq 80.6^\circ\text{F} \tag{4}$$

Additionally, we also assume that the control input is the critical actuator yielding its own working properties and conditions. It is meaningful to set a reasonable bound for the u_k. Otherwise, it would cost a lot to build and drive the actuator.
Therefore the control constraint is assumed to be written in terms of u_k as:

$$-50 \leq u_k \leq 200 \tag{5}$$

where $u_k > 0$ means heating and the opposite means cooling.

From above constraints, we can observe that both the room air temperature and control signal are constrained. In the next section, the control problem is formulated.

B. Problem Formulation

We apply the cSLQC theory [10] to design the controller. cSLQC is a tractable control technique that can deal with stochastic discrete-time linear systems in the presence of control and state constraints. This characteristic makes the cSLQC well suited for building climate control.

1) Cost Function: We consider the problem where the temperature t_1 is required to remain within certain bounds of a constant in the presence of the disturbance vector d. Moreover, we can assign setpoints for t_1, t_2 and t_3, but without any other constraints on t_2 and t_3. Thus, we can regulate the output error $e_k := x_k - x_r$ at time k, where x_r is the setpoint vector of x. We hope to minimize the error e to keep the temperature t_1 close to the desired value. Meanwhile, we also hope to use as less power as we can to save energy. Thus, our objective is to find for the system (1) discretized, the M-control sequence $\{u_0, \cdots, u_{M-1}\}$, where $u_i := u(t_i)$, $i = 0, \cdots, M$; M is an integer large enough, $t_i = i\Delta T$, where $\triangle T$ is the sampling period; and corresponding state sequence $\{x_0, \cdots, x_{M-1}\}$ and error sequence $\{e_0, \cdots, e_{M-1}\}$, that minimize the finite horizon objective function:

$$V_N(e_0, \mathbf{u}, \omega) := \frac{1}{2}[(x_N - x_r)^T P (x_N - x_r) + \sum_{k=1}^{N-1} e_k^T Q e_k + \sum_{k=0}^{N-1} {u_k}^T R u_k] \quad (6)$$

where $P \geq 0$, $Q \geq 0$ (i.e., semi-definite positive matrices), $R > 0$ (i.e., positive definite matrix), N is the prediction horizon, and

$$\begin{aligned} \mathbf{x} &:= [x_0^T, ..., x_N^T]^T \\ \mathbf{u} &:= [u_0^T, ..., u_{N-1}^T]^T \\ \omega &:= [\omega_0^T, ..., \omega_{N-1}^T]^T \end{aligned} \quad (7)$$

The differences between the cost function above and those considered in [7] and [8] are that [7] used a linear cost function in the control input and [8] assumed the disturbance was 0 in the cost function which simplified the problem.

2) Constraints on the Temperature: In this paper, we consider the case where the disturbance is assumed to be **Gaussian** distribution.

Due to the unknown disturbances d_k, the state x_k is not exactly known. It is more reasonable to utilize the soft constraints on the state, i.e. we do not require constraints on the response time to be satisfied at all time, but only with a predefined probability. Hence, instead of using hard constraints on the state or no constraint, we use the uncertain linear constraints in a probabilistic sense [10]. Thus, similar as [7] and [8], the constraint on x_k can be described by the so-called chance constraint as follows:

$$P[\mathbf{G}_i \mathbf{x} > \mathbf{g}_i] \quad \leq \quad \alpha_i \quad (8)$$

The above constraint is non-convex and hard to resolve directly. In the first case when the disturbance is Gaussian, as shown in [7] and [8], the authors took u_k as affine disturbance feedback to approximate and simplify this constraint. However, if we do not assume any form of the control input, we can still simplify the chance constraint to a hard constraint exactly as already shown in [10].

Assume the ω are independent and normally distributed, i.e., $\omega \sim \mathcal{N}(\mu, \Sigma)$, where $\Sigma > 0$. Then, we have the following theorem from [10].

Theorem 2.1: [10] Consider a linear system with the state written as

$$\mathbf{x} = \tilde{\mathbf{A}} x_0 + \tilde{\mathbf{B}} \mathbf{u} + \tilde{\mathbf{C}} \omega \quad (9)$$

Then, the constraint

$$\mathbf{p}^T \mathbf{u} \leq q \quad (10)$$

where $\mathbf{p} = \tilde{\mathbf{B}}^T \mathbf{G}_i$, $q = \mathbf{g}_i - \mathbf{G}_i \tilde{\mathbf{A}} x_0 - \mathbf{G}_i \tilde{\mathbf{C}} \mu - \parallel \Sigma^{\frac{1}{2}} \tilde{\mathbf{C}}^T \mathbf{G}_i \parallel_2 \Phi^{-1}(1 - \alpha_i)$ implies the chance constraint (8).

Then, the problem corresponding to the first case can be formulated as follows:

Problem 2.2: Find

$$\mathbf{u}(x_0) := arg\ min_{\mathbf{u}}\ \mathbb{E}_\omega V_N \quad (11)$$

subject to (5), (10), and discretized version of (1).

where $\mathbb{E}_\omega(\bullet)$ denotes the expectation operator with respect to the Gaussian disturbance ω.

In the next section, we employ the technique developed in [10] to transform the problem to a semidefinite optimization problems, where they can be solved efficiently.

III. Control Strategies

If there is no constraint, the optimization problem under Gaussian disturbance can be solved by linear quadratic regulator (LQR) through Bellman's recursion. However, with constraints, this approach involves a huge amount of computation to find the optimal solution. To find the optimal values for each problem, we employ the SLQC to find the solution by formulating the problems as semidefinite optimization problems.

A. SDP Approach for Problem 2.2

In this section, we applied the technique in [10] to formulate Problem 2.2 as a semidefinite optimization problem. Unlike the MPC method, which is quadratic programming, the problem will be converted to an SDP optimization problem. An obvious result about the cost function is given in the following proposition.

Proposition 3.1: The cost function (6) can be written as:

$$\begin{aligned} V_N(e_0, \mathbf{u}, \omega) &= 2\mathbf{a}^T e_0 + e_0^T \mathbf{A} e_0 + 2\mathbf{b}^T \mathbf{u} + \mathbf{u}^T \mathbf{B} \mathbf{u} + 2\mathbf{c}^T \omega \\ &+ \omega^T \mathbf{C} \omega + 2\mathbf{u}^T \mathbf{D} \omega + \hat{l} \end{aligned} \quad (12)$$

for vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ with appropriate dimensions, and where $\mathbf{B} > 0, \mathbf{C} \geq 0$

Proof: The original system can be written in terms of error dynamics, at time k,

$$e_k = \tilde{\mathbf{A}}_{k-1} e_0 + \tilde{\mathbf{B}}_{k-1} \mathbf{u} + \tilde{\mathbf{C}}_{k-1} \omega + L_{k-1}$$

where $L_{k-1} = A_d^{k-1} x_r - x_r$ and from (1), x_k, ω_k are 3×1 and u_k is a scalar so that

$$\begin{aligned}
\tilde{\mathbf{A}}_{k-1} &= A_d^k \\
\tilde{\mathbf{B}}_{k-1} &= [A_d^{k-1} B_d \ \cdots \ B_d \ \mathbf{0}_{3\times(N-k)}] \\
\tilde{\mathbf{C}}_{k-1} &= [A_d^{k-1} C_d \ \cdots \ C_d \ \mathbf{0}_{3\times 3(N-k)}]
\end{aligned} \tag{13}$$

Then, after some manipulations, the error state term in cost function becomes:

$$\begin{aligned}
e_k^T Q e_k = \ & e_0^T \tilde{\mathbf{A}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1} e_0 + 2 e_0^T \tilde{\mathbf{A}}_{k-1}^T Q (\tilde{\mathbf{B}}_{k-1}\mathbf{u} + \tilde{\mathbf{C}}_{k-1}\omega) \\
& + \mathbf{u}^T \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1} \mathbf{u} + \omega^T \tilde{\mathbf{C}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1} \omega \\
& + 2\mathbf{u}^T \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1} \omega + 2 L_{k-1}^T Q (\tilde{\mathbf{A}}_{k-1} e_0 \\
& + \tilde{\mathbf{B}}_{k-1}\mathbf{u} + \tilde{\mathbf{C}}_{k-1}\omega) + L_{k-1}^T Q L_{k-1}
\end{aligned} \tag{14}$$

Thus, we reach the formula of the cost function stated above with

$$\mathbf{A} = \sum_{k=1}^{N} \tilde{\mathbf{A}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1}$$

$$\mathbf{B} = \mathbf{diag}(R, \cdots, R) + \sum_{k=1}^{N} \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{B}}_{k-1}$$

$$\mathbf{C} = \sum_{k=1}^{N} \tilde{\mathbf{C}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1}, \ \mathbf{D} = \sum_{k=1}^{N} \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{C}}_{k-1}$$

$$\mathbf{c} = \left(\sum_{k=1}^{N} \tilde{\mathbf{C}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1}\right) e_0 + \sum_{k=1}^{N} \tilde{\mathbf{C}}_{k-1}^T Q L_{k-1}$$

$$\mathbf{a} = \sum_{k=1}^{N} \tilde{\mathbf{A}}_{k-1}^T Q L_{k-1}, \ \hat{l} = \sum_{k=1}^{N} L_{k-1}^T Q L_{k-1}$$

$$\mathbf{b} = \left(\sum_{k=1}^{N} \tilde{\mathbf{B}}_{k-1}^T Q \tilde{\mathbf{A}}_{k-1}\right) e_0 + \sum_{k=1}^{N} \tilde{\mathbf{B}}_{k-1}^T Q L_{k-1} \tag{15}$$

■

Similarly as [10], let $\mathbf{h} = \mathbf{c} - \mathbf{D}^T \mathbf{B}^{-1} \mathbf{b}$ and $\mathbf{F} = \mathbf{B}^{-1/2} \mathbf{D}$, then by eliminating the constant terms and take $\mathbf{u} = \mathbf{B}^{-1/2}\mathbf{y} - \mathbf{B}^{-1}\mathbf{b}$, the cost function above can be further reduced to be:

$$\tilde{V}_N(e_0, \mathbf{y}, \mathbf{w}) = \mathbf{y}^T \mathbf{y} + 2\mathbf{h}^T \mathbf{w} + 2\mathbf{y}^T \mathbf{F} \omega + \omega^T \mathbf{C} \omega \tag{16}$$

Taking the expectation of the above cost, we have

$$\hat{V}_N(e_0, \mathbf{y}, \omega) = \mathbf{y}^T \mathbf{y} + 2\mathbf{h}^T \mu + 2\mathbf{y}^T \mathbf{F} \mu + trace(\mathbf{C}\Sigma) \tag{17}$$

Again, taking away constant terms, the cost to be minimized is $\hat{V}_N(e_0, \mathbf{y}, \omega) = \mathbf{y}^T \mathbf{y} + 2\mathbf{y}^T \mathbf{F}\mu$. Then, the problem 2.2 is equivalent to find $\mathbf{u}(x_0) := arg\ min_{\mathbf{u}}\ \hat{V}_N$. This problem can be solved through SDP to obtain the optimal solution, as shown in the next theorem.

Theorem 3.2: Problem 2.2 may be solved by the following semidefinite optimization problem:

$$\begin{aligned}
& minimize\ z \\
& subject\ to\ (5), (10) \\
& \begin{bmatrix} \mathbf{I}_N & \mathbf{y} + \mathbf{F}\mu \\ \mathbf{y}^T + \mu^T \mathbf{F}^T & z + (\mathbf{F}\mu)^T \mathbf{F}\mu \end{bmatrix} \geq 0
\end{aligned} \tag{18}$$

in decision variables $\mathbf{y}$ and z.

Proof: The proof is given below by following the technique in *Theorem 3* in [10].

First, note the minimization of $\hat{V}_N(e_0, \mathbf{y}, \mathbf{w})$ can be rewritten as

$$\begin{aligned}
& minimize\ z \\
& subject\ to\ z - \mathbf{y}^T \mathbf{y} - 2\mathbf{y}^T \mathbf{F}\mu \geq 0
\end{aligned} \tag{19}$$

The constraint (19) can be further written as

$$\begin{aligned}
& z - \mathbf{y}^T \mathbf{y} - 2\mathbf{y}^T \mathbf{F}\mu - (\mathbf{F}\mu)^T \mathbf{F}\mu + (\mathbf{F}\mu)^T \mathbf{F}\mu \geq 0 \\
\Updownarrow \\
& z + (\mathbf{F}\mu)^T \mathbf{F}\mu - (\mathbf{y} + \mathbf{F}\mu)^T (\mathbf{y} + \mathbf{F}\mu) \geq 0
\end{aligned} \tag{20}$$

Then, by Schur complement lemma, (20) can be formulated as (18). Moreover, note that (5) and (10) are linear constraints on the control input, which can be added without increasing the complexity type. Thus, we obtain the statement. ■

B. Chance Constraints on the Performance

Another interesting requirement is the performance guarantee. The work in [10] has demonstrated that the probability

$$\mathbf{P}(V_N(e_0, \mathbf{u}, \omega) > v) \leq \varepsilon$$

may be implied by a convex quadratic constraint, which can be added to either problem without raising the complexity type.

C. Hysteresis band

It should be noted that the AC continues to cool the building for a few minutes even after it is turned off because of the dynamics of the heat pump. Specifically, it takes a while for the evaporator that cools the air to warm up, and so it keeps cooling the air for some time after the heat pump is turned off [12]. Therefore, there exists a "delay" in the system model. For the purpose of approximating the real energy consumption of HVAC, we have to consider a 0.5°F hysteresis band ($k =$ 0.5°F) in the control scheme.

Practically, we use this hysteresis band to represent the delay of the cooling system for example. In order to cool the room to reach the setting temperature, i.e. x_r, we need to turn off the cooling unit k°F before reaching x_r. This also helps save energy consumption as shown in Sec. IV-B2.

IV. Simulation Results

A. Description of the experimental setup

In this section, we present simulation results which demonstrate validity of the SLQC method in the above problems. The system model is described in Section II. As mentioned before, this model was proposed in [4] and employed in [7]. The desired temperature or reference temperature of the room is set as $22°C = 71.6°F$. The temperature is sampled every 10 minutes, and we plot t_1 and the control input for each method during a period of 10 days in the sequel. The disturbances corresponding to different states are shown in Fig. 1.

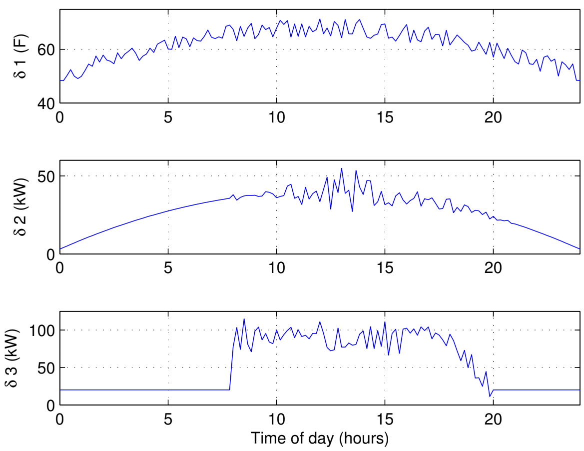

Fig. 1. Disturbance to the building climate system.

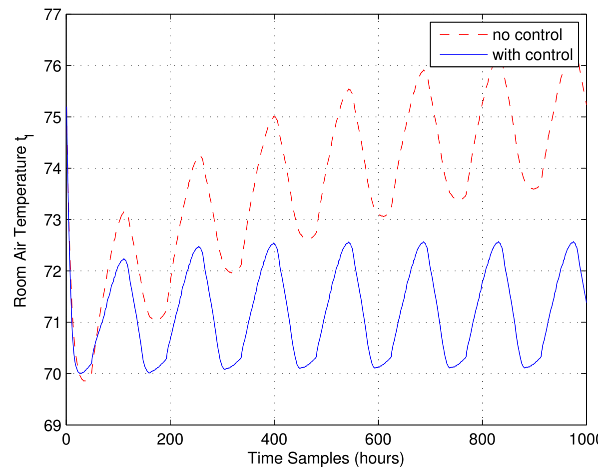

Fig. 2. Room temperature t_1 comparison with/without control using LQR.

B. Summary of the results

1) The Disturbance Distribution is Gaussian: First, we plot the trend of t_1 using LQR control in Fig. 2, and using SDP through (18) in Fig. 3. It is obvious that both LQR and cSLQC techniques can keep room temperature t_1 in the desired range and close to the reference temperature. Notice that the control input is also bounded below by -50, which is as desired.

To evaluate the control performance for both of the controllers, we compare the room temperature t_1 with and without enabling the controller as shown in Fig. 2 and Fig. 4.

Simulation results show that without the controller, the temperature will depart far away from the reference temperature.

To further illustrate the difference in energy saving between the two control principles, we compute the total input energy, i.e. the energy cost in all the 10 days denoted by $\| \mathbf{u} \|_2$ (2-norm) for both methods.
For LQR controller, $\| \mathbf{u} \|_2 = 1665.3$; while for cSLQC, $\| \mathbf{u} \|_2 = 1268.3$, which show that the proposed cSLQC technique achieves a more efficient control policy which contributes to reducing energy consumption.

2) Control with hysteresis band: As discussed in Sec.III-C, here we incorporate the proposed cSLQC with a 0.5°F hysteresis band to approximate the real working condition of HVAC as well as pursuing more energy efficiency. In the stage of the experiments we set the control signal to be zero as long as the temperature t_1 reaches the hysteresis band. Basically, we compare the energy consumption, thereby the energy cost, of the three control strategies running for 10 days and 30 days separately. It should be remarked that the disturbance is not exactly the same value for each day. In stead, the disturbance follows the same distribution, while the values may not be equal for different days.

The results shown in Fig. 5 and Fig. 6 clearly indicate that both our controllers outperform the current LQR controller in terms of both energy use and violations of the thermal comfort range. It can be seen from Fig. 5 that the cSLQC controller never breaks the desired comfort band constraints, while the LQR controller tends to have violations of the lower bound on the temperature.

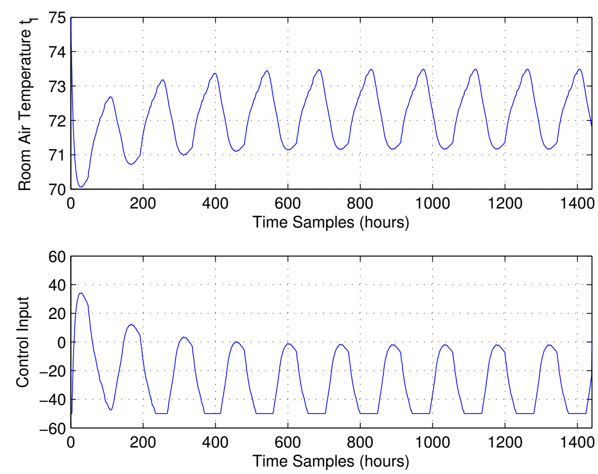

Fig. 3. Room temperature t_1 and control power in 10 days using cSLQC in Theorem 3.2.

Moreover, the temperature variations are smaller with cSLQC, which is a more favorable behavior in terms of comfort. The improvements in energy saving for both short-term and long-term can be explained by Fig. 6, where the hysteresis effect is also considered.

V. Conclusion

This study presents an investigation of constrained quadratic control of room temperature on a dynamic building climate model. In this paper, we propose a cSLQC controller for HVAC systems, aiming at reducing the energy required to maintain indoor thermal comfort. The constrained SLQC approach is employed to solve stochastic optimization problems with chance constraints by SDP.
The mechanism to account for the probabilistic nature of

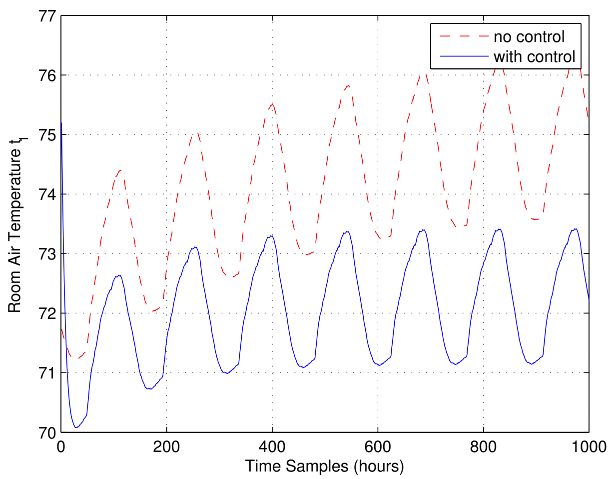

Fig. 4. Room temperature t_1 comparison with/without control using cSLQC in Theorem 3.2.

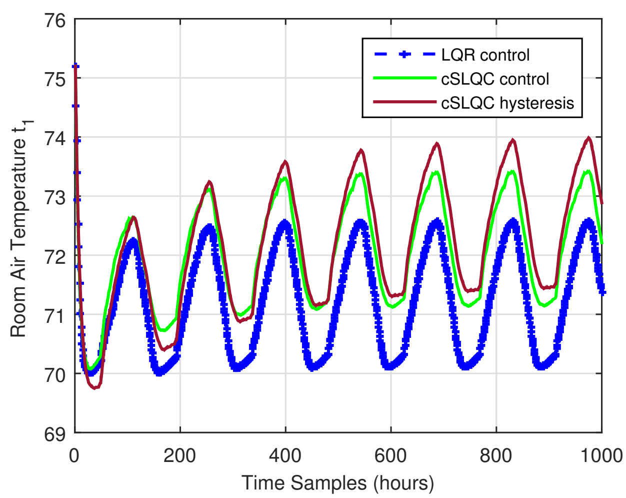

Fig. 5. Room temperature t_1 comparison.

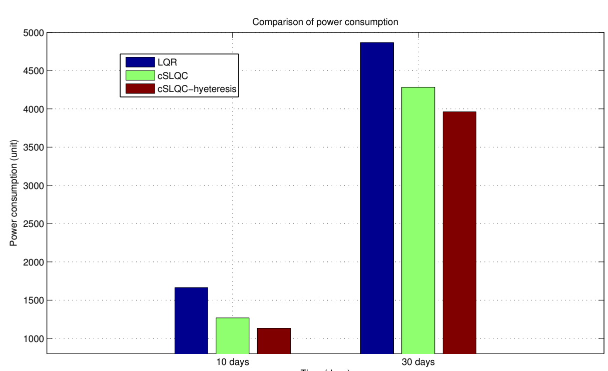

Fig. 6. Comparison of power consumption for control inputs in 10 days / 30 days.

the disturbances affecting the comfort indicators is simplified to a hard constraint exactly without using affine disturbance feedback. Moreover, we consider a stochastic quadratic cost function, which is taken expectation with respect to Gaussian disturbances in this paper. An upcoming paper which deals with more general non-Gaussian noise (maximization over the bounded set of disturbances) is under preparation.

In this paper, it is assumed that all the probabilistic constraints and stochastic disturbance are only associated with states which are three different temperatures in the model. A more practical consideration is towards the generalization of the control scheme to the case of whole buildings, which leads to increased complexity for both the models and the costs.

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