

# Evaluating Risk from Acts of Terrorism with Belief and Fuzzy Sets

SAND XXXX

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**John Darby**  
**Sandia National Laboratories**

[jdarby@sandia.gov](mailto:jdarby@sandia.gov) 505-284-7862





# Presentation Topics

- Risk
  - Intentional acts are different from Random acts
- Uncertainty
  - Epistemic uncertainty
  - Ambiguity and Vagueness
- Adversary/Defender Model for Evaluation of Risk from Intentional Acts of Terrorism



# Risk of Terrorist Attack

- Risk =  $f \times P \times C$ 
  - Risk of a successful terrorist attack
  - $f$  is frequency of attack
  - $P$  is probability adversaries succeed given attack
  - $C$  is consequence given adversaries succeed
- Risk depends on the Adversary Scenario
  - Scenario: Adversary Resources, Target, Attack Plan
- Why Evaluate Risk?
  - Allocation of Resources for Protection

# Risk of Terrorist Attack

- What is frequency  $f_i$  for a Scenario “i”?
- Which “i” scenarios?

$$Risk_i = f_i \times P_i \times C_i$$

**Adversary: Select scenario “i” from choice of *all* scenarios**

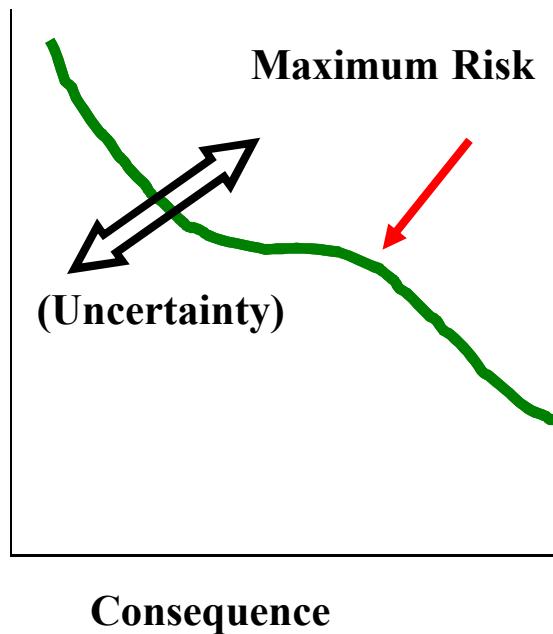
**Defender: Evaluate the *single* scenario “i”**



# Safety Risk vs. Terrorist Risk

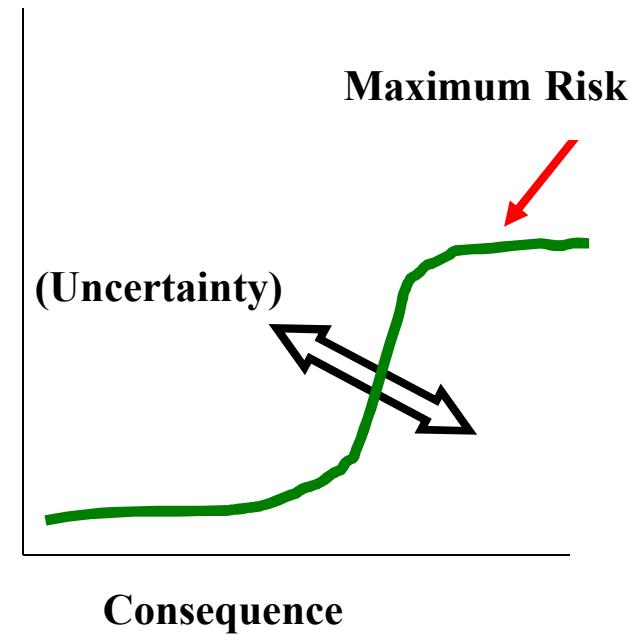
## Safety Risk (Random)

Likelihood



## Terrorist Risk (Intentional)

Likelihood





# Uncertainty

- Ambiguity
  - Uncertainty as to what will occur **in the future**
    - Dow Jones Industrial Average Close on Dec. 31, 2006
      - Will be one value
      - Ambiguity as to what that value will be
- Vagueness
  - Uncertainty as how to categorize a **known** outcome
    - Dow Jones close is 11,300 on Dec. 31, 2006
      - Is this “High” ?
      - What do you mean by “High”?
  - Vagueness can be expressed Linguistically (Words)



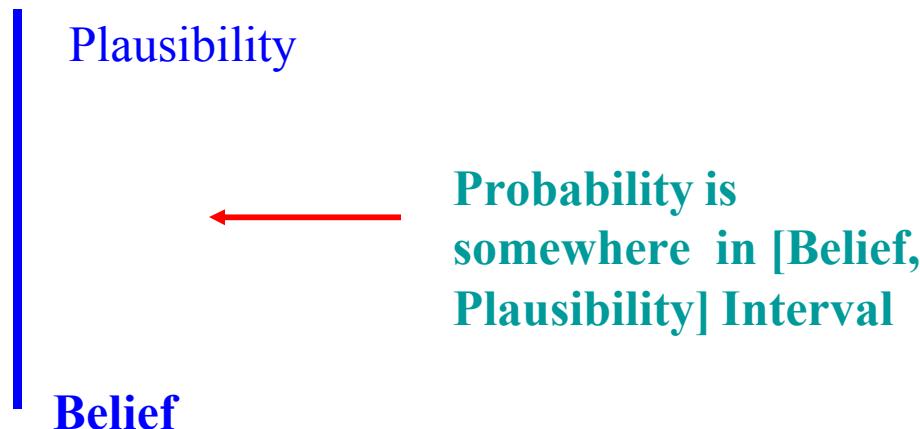
# Belief/Plausibility for Epistemic Uncertainty

- For a Fair Coin
  - Uncertainty is **Aleatory** (random)
  - Probability Heads is  $\frac{1}{2}$
  - Probability Tails is  $\frac{1}{2}$
- But if we cannot toss coin, we do not know coin is fair, we do not even know if coin has Heads and Tails
  - May not be Fair Coin (may be Weighted for Tails)
  - May be Two-Headed or Two-Tailed Coin
  - **Epistemic** (state of knowledge) uncertainty
  - Insufficient information to assign Probability to Heads and Tails
  - For Total Ignorance
    - Belief/Plausibility for Heads is 0/1
    - Belief/Plausibility for Tails is 0/1
- With more information (actually tossing the coin) we can reduce Epistemic Uncertainty
  - If at least one Heads and one Tails occur in a series of tosses, we know coin has Heads and Tails
  - Many tosses needed to assess if coin is fair
- For Fair Coin we cannot reduce aleatory uncertainty



# Belief and Plausibility

- Belief / Plausibility form a Lower / Upper Bound for Probability
- Belief is what probability **will** be
- Plausibility is what probability **could** be

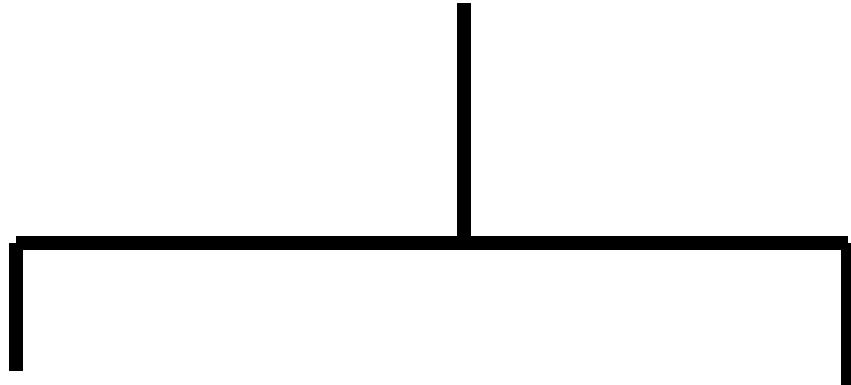


- Similar to a Confidence Interval for a Parameter of a probability distribution; a confidence measure that parameter is in interval, but exactly where in interval is not known
- Belief/Plausibility both reduce to Probability if Evidence is Specific



# Measures of Ambiguity

## Belief/Plausibility



### Probability

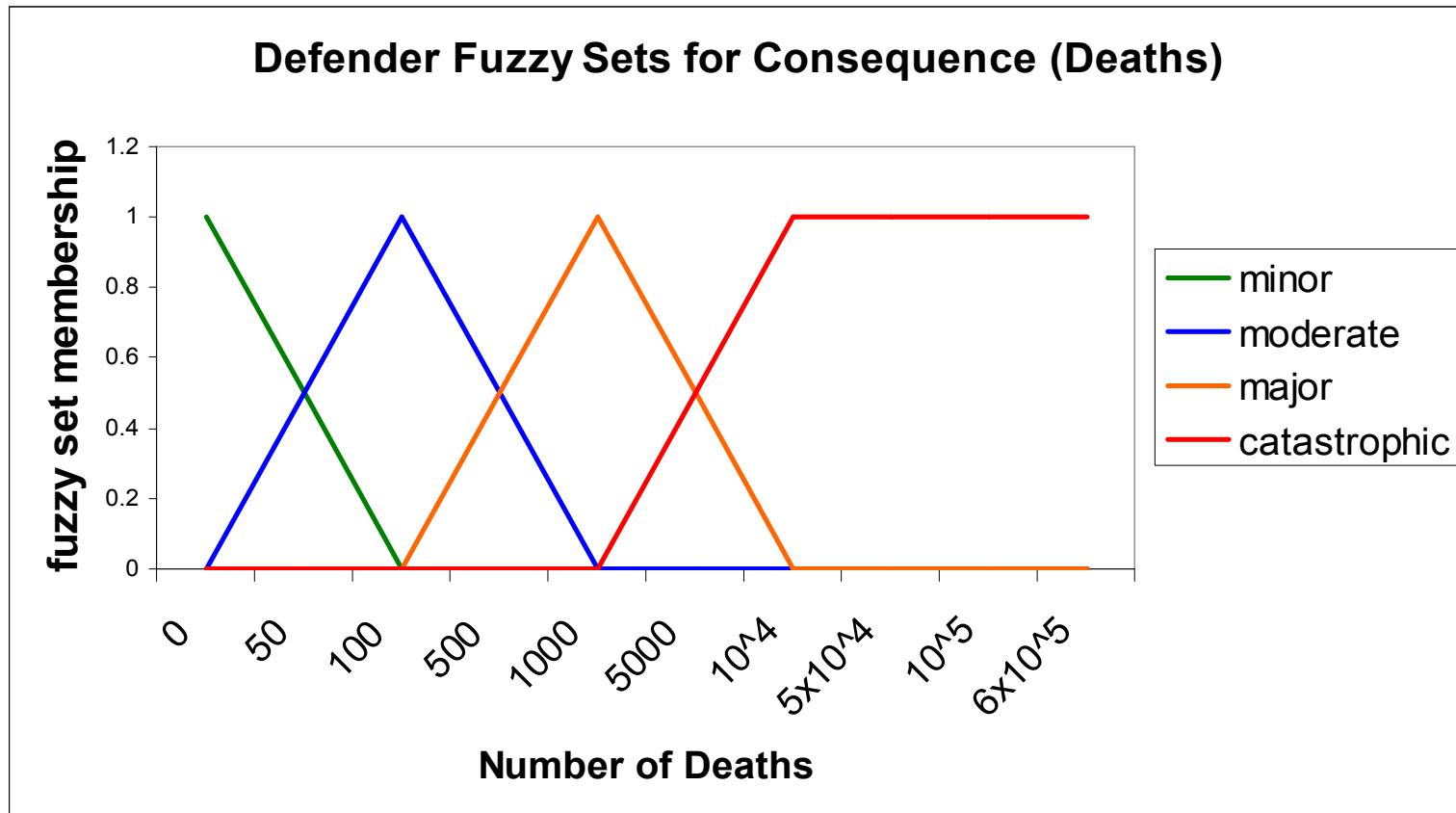
- Evidence is Specific

### Necessity/Possibility

- Evidence is Coherent

# Fuzzy Sets represent Vagueness

- Consequence (Deaths) are “Major”
  - “Major” is fuzzy: between *about* 500 and *about* 5000 deaths





# Adversary/Defender Model

- Adversary (them)
- Defender (us)
- Adversary and Defender each have different goals and different states of knowledge
- Risk = Threat x Vulnerability x Consequence
  - Defender goal: Minimize Risk with available resources
  - Adversary goal: Maximize Consequence with available resources (working assumption)
- Adversary is the Threat
  - Epistemic Uncertainty for Vulnerability and Consequence
- Defender knows Vulnerability and Consequence
  - Epistemic Uncertainty for Threat



# Defender Model for a Scenario

- Risk =  $f \times P \times C = f \times (1 - P_E) \times C$ 
  - f is frequency of attack
  - P is probability of success given attack
    - $P_E$  is effectiveness of security system in defeating attack
  - C is consequence given success of attack
- f, P, and C are random variables with uncertainty
- Degrees of Evidence to f, P, C based on state of knowledge
- Numerical Convolution using Belief/Plausibility Measure of Uncertainty



# Example of Defender Model

- $f$  and  $P$  have considerable epistemic uncertainty
- $C$  has only aleatory uncertainty (for this example)
- $C$  is Deaths



# Example of Defender Model

- Evidence for frequency of attack,  $f$ , per year
  - 0.1 to interval  $[1 \times 10^{-4}, 0.1]$
  - 0.9 to the interval  $[1 \times 10^{-3}, 0.01]$
- Evidence for probability of adversary success,  $P$ 
  - 0.3 to interval  $[0.1, 0.9]$
  - 0.7 to interval  $[0.3, 0.5]$

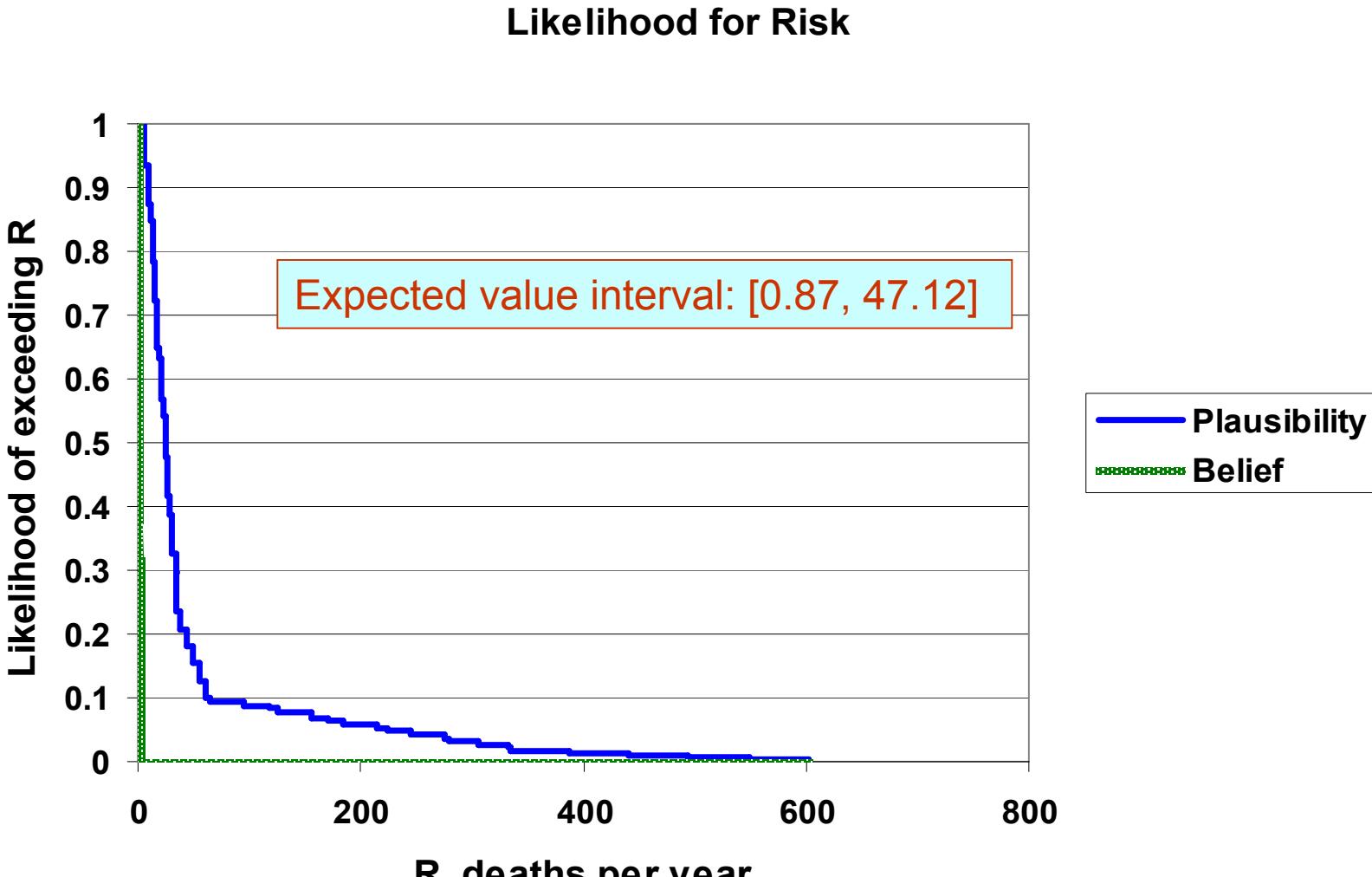


# Example of Defender Model

- Evidence for consequence, C, deaths
  - Uniform probability distribution
    - Minimum 1000
    - Maximum 7000
    - Mean 4000

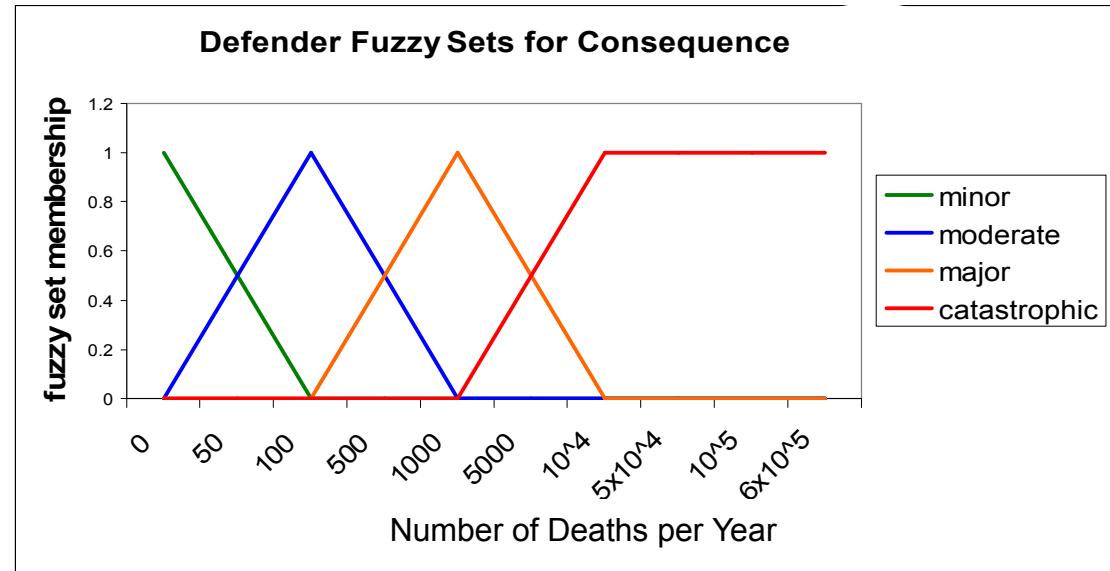


# Example of Defender Model

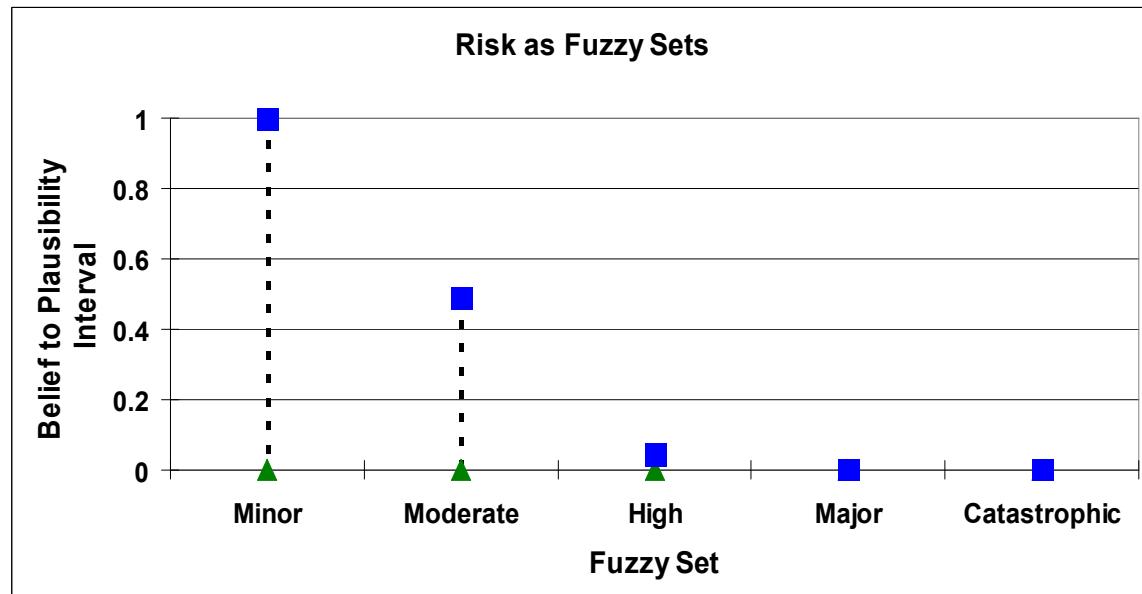


# Likelihood for Risk using Fuzzy Sets

Define  
Fuzzy Sets  
For Risk:

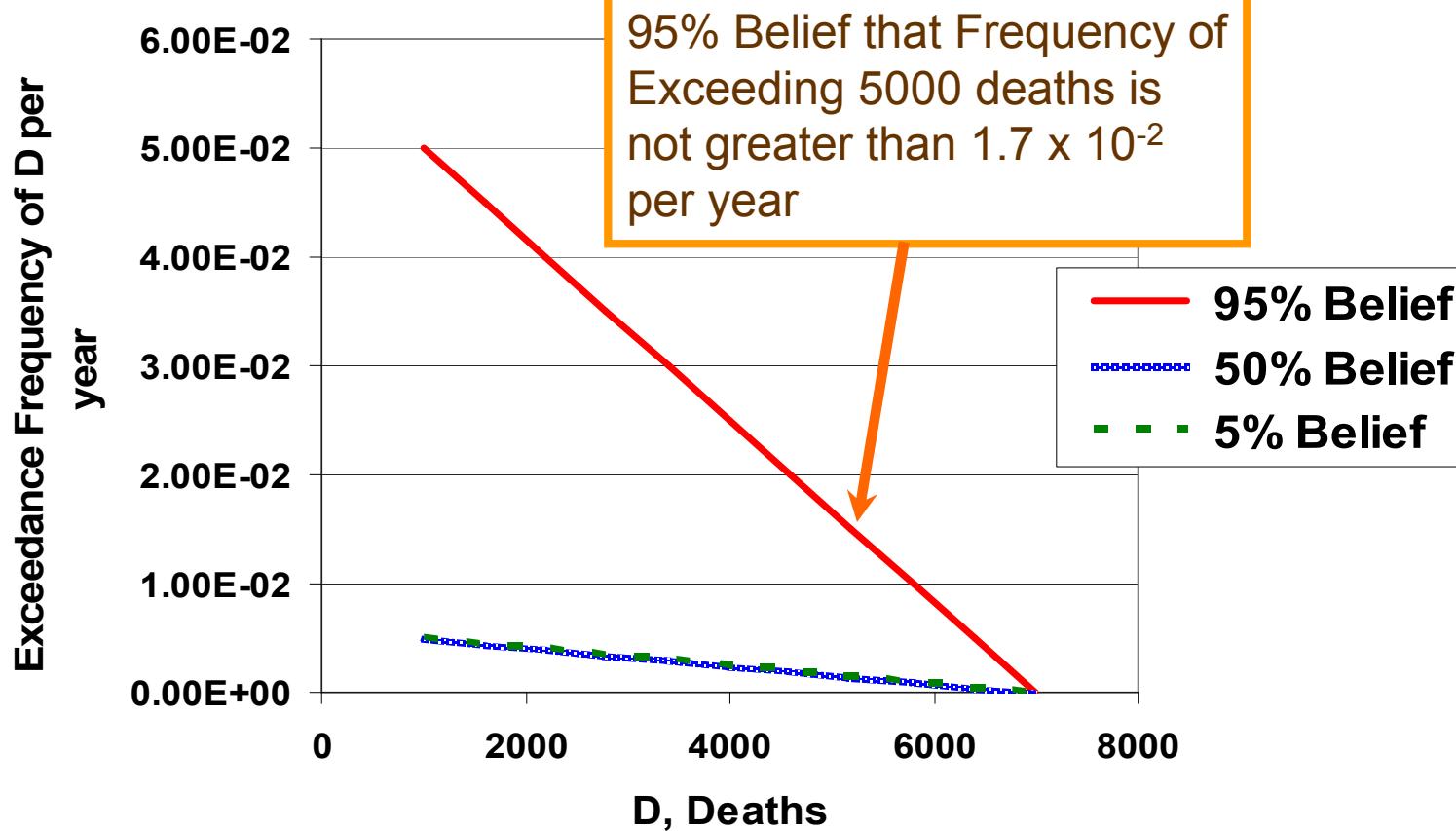


Likelihood of  
Fuzzy Sets,  
[Belief, Plausibility]:



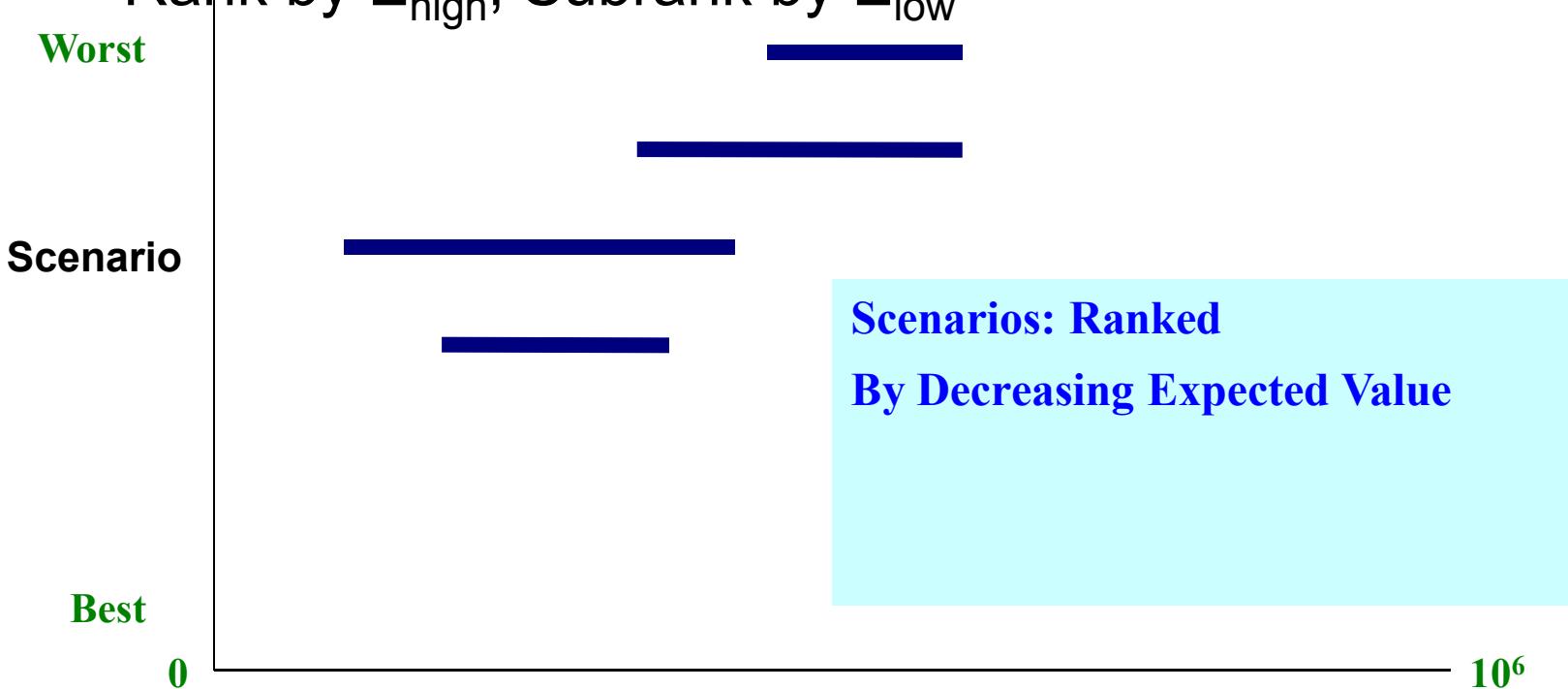
# Exceedance Frequency of Consequence

## Exceedance Frequency of Consequence: Upper Bound



# Defender Ranking of Scenarios

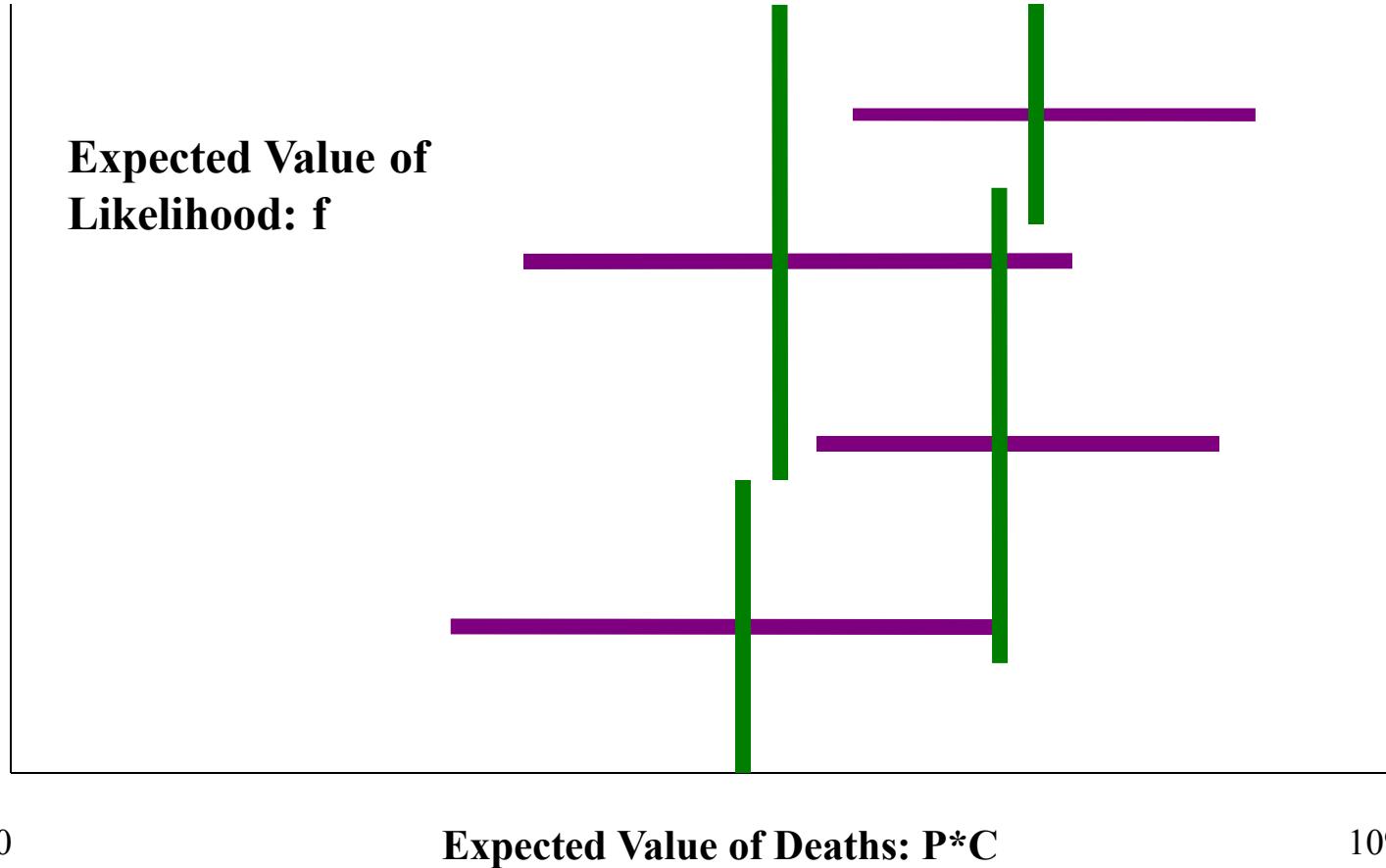
- For Belief/Plausibility Expected Value is an Interval  $[E_{\text{low}}, E_{\text{high}}]$ . Reduces to point (Mean) for Probability
- Rank by  $E_{\text{high}}$ , Subrank by  $E_{\text{low}}$



Expected Value of Deaths per Year:  $f^*P^*C$   
[jdarby@sandia.gov](mailto:jdarby@sandia.gov) 505-284-7862



# Next Level Of Detail for Defender Ranking





# Adversary Model

- Use Surrogate Adversary (Special Forces)
- Adversary has Choice
  - All Variables of concern must be “OK” or Adversary will pick another scenario
    - Recruit Insider? Not unless already placed
    - Large Team? Concern about being detected by Intelligence
    - Uncertainty?
      - Door was **green** yesterday, is **red** today...What else changed?
- Variables for Adversary Decision are Not all Numeric
  - Consequence = Deaths x Economic Damage x Fear in Populace x Damage to National Security x Religious Significance x .....
  - Deaths and Economic Damage are numeric
  - Fear in Populace, Damage to National Security, and Religious Significance are not numeric



# Adversary Model

- Purely Linguistic Model
- Develop Fuzzy Sets for Each Variable
- Develop Approximate Reasoning Rule Base for Linguistic Convolution of Variables to Reflect Scenario Selection Decision Process
- We are not the Adversary, we try to think like the Adversary
  - Considerable Epistemic Uncertainty
  - Use Belief/Plausibility Measure of Uncertainty Propagated up the Rule Base



# Adversary Model

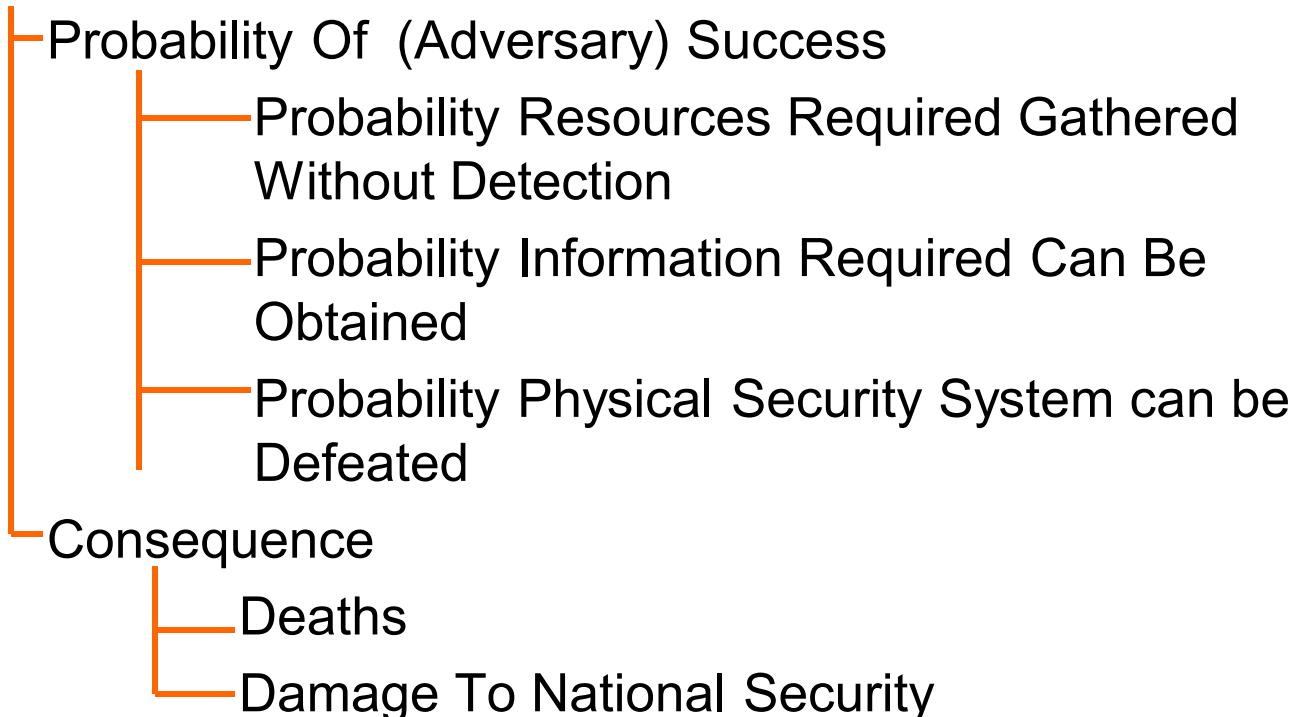
- Assume Adversary Goal is Maximize Expected Consequence
  - Expected Consequence  $\equiv P \times C$
  - Expected Consequence is Adversary estimate of Consequence, C, weighted by Adversary estimate of Probability of Success, P



# Example of Adversary Model

- Rule Base and Variables

Expected Consequence





# Example of Adversary Model

- **Linguistics: Fuzzy Sets**

- Expected Consequence = {No, Maybe, Yes}
- Probability Of Success = {Low, Medium, High}
- Consequence = {Small, Medium, Large}
- Probability Resources Required Gathered Without Detection = {Low, Medium, High}
- Probability Information Required Can Be Obtained = {Low, Medium, High}
- Probability Physical Security System can be Defeated = {Low, Medium, High}
- Deaths = {Minor, Moderate, Major, Catastrophic}
- Damage To National Security = {Insignificant, Significant, Very Significant}

# Example of Adversary Model

## Rule Base for Expected Consequence

Probability Of Success	Low	Medium	High
Consequence			
Small	<i>No</i>	<i>No</i>	<i>No</i>
Medium	<i>No</i>	<i>No</i>	<i>Maybe</i>
Large	<i>No</i>	<i>Maybe</i>	<i>Yes</i>

# Example of Adversary Model

Portion of Rule Base for Probability Of Success  
**Probability Physical Security System can be Defeated = High**

Probability Resources Required Gathered Without Detection	Low	Medium	High
Probability Information Required can Be Obtained	<i>Probability Of Success</i>		
Low	<i>Low</i>	<i>Low</i>	<i>Low</i>
Medium	<i>Low</i>	<i>Medium</i>	<i>Medium</i>
High	<i>Low</i>	<i>Medium</i>	<i>High</i>

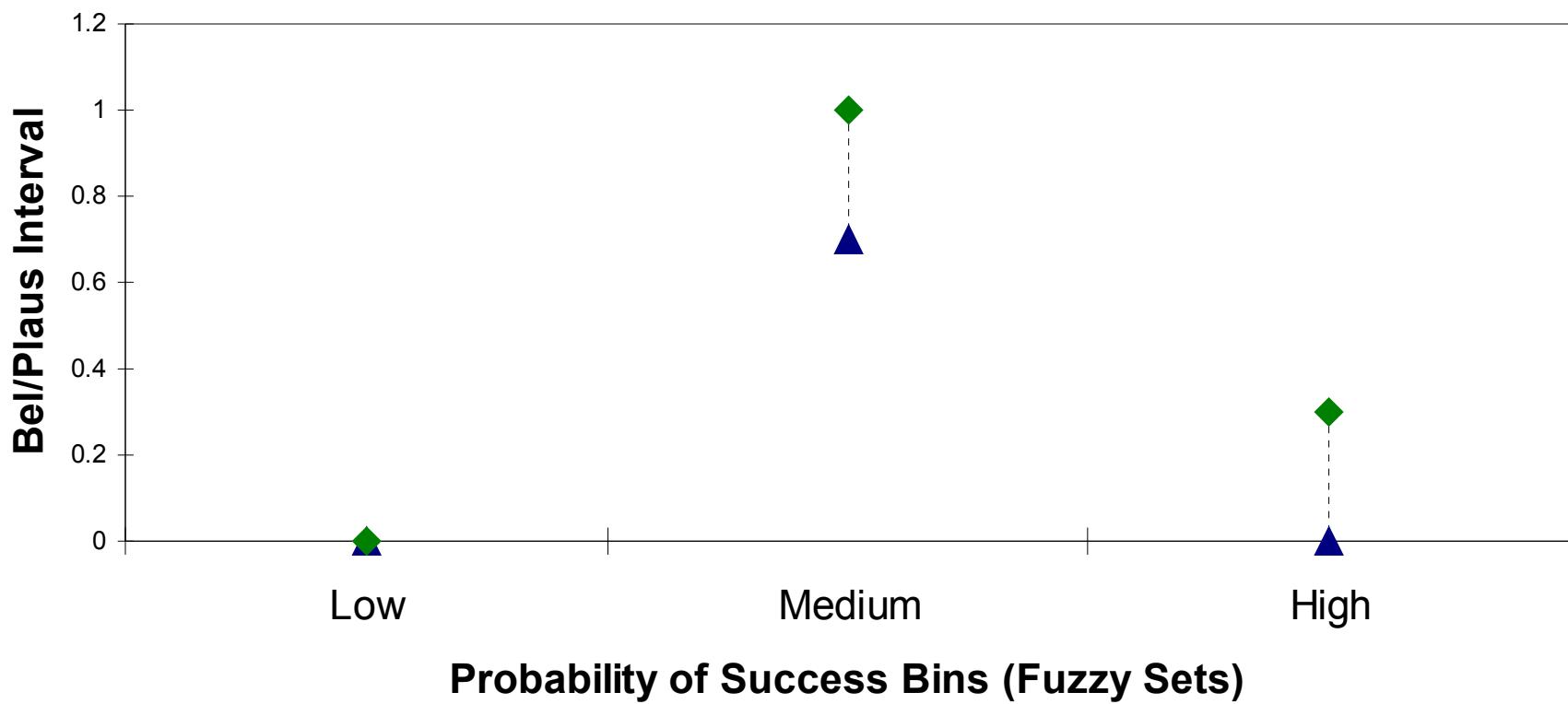


# Example of Adversary Model

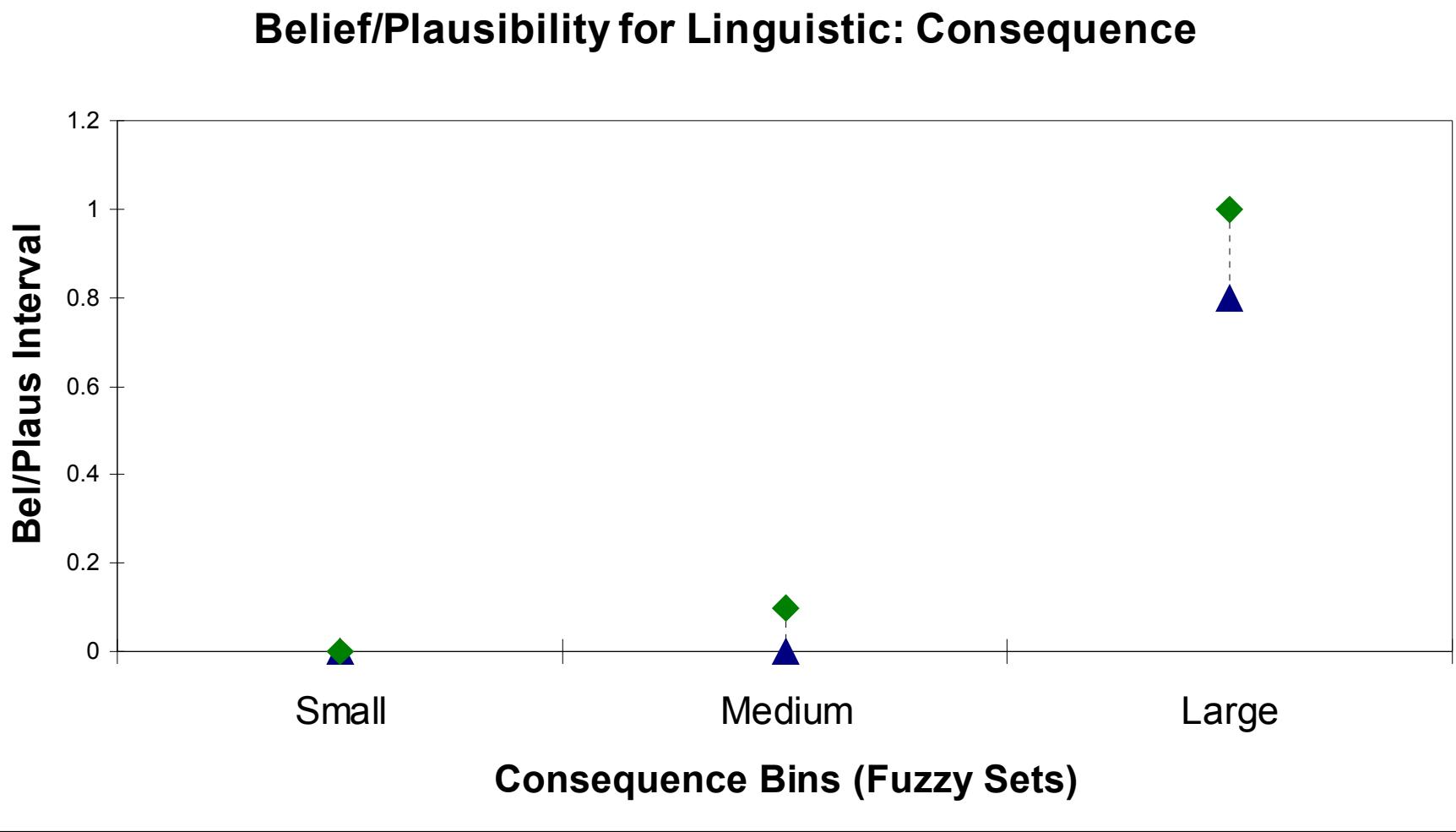
- Focal Elements (Evidence) for A Particular Scenario
  - Deaths
    - 0.8 for {Major, Catastrophic}
    - 0.2 for {Moderate, Major}
  - Damage To National Security
    - 0.1 to {Insignificant, Significant}
    - 0.9 to {Significant, Very Significant}
  - Probability Resources Required Obtained Without Detection
    - 0.7 to {Medium}
    - 0.3 to {Medium, High}
  - Probability Information Required can Be Obtained
    - 0.15 to {Medium}
    - 0.85 to {Medium, High}
  - Probability Physical Security System can be Defeated
    - 1.0 to {Medium, High}

# Example of Adversary Model

## Belief/Plausibility for Linguistic: Probability of Success (Adversary)

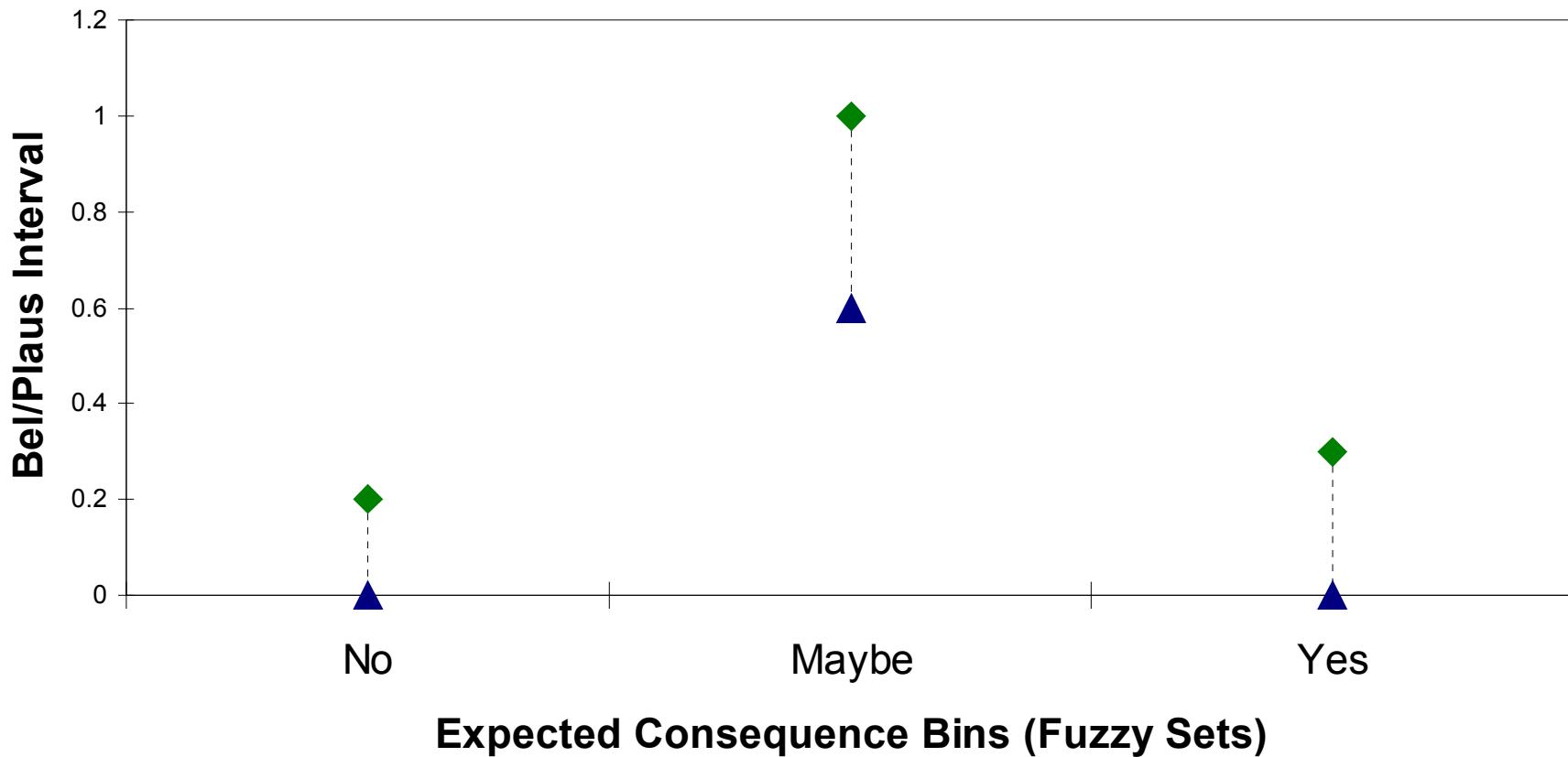


# Example of Adversary Model



# Example of Adversary Model

## Belief/Plausibility for Linguistic: Expected Consequence





# Ranking of Scenarios from Adversary Model

- Defender (thinking like Adversary) Ranks by **Plausibility**
  - Rank scenarios based on the plausibility for the worst fuzzy set for expected consequence, “Yes” in the prior example, sub-ranked by plausibility of the next-worst fuzzy sets, “Maybe” and “No” in the prior example
- Note: Actual Adversary using the Model would Rank by **Belief**
  - “We will not attempt a scenario unless we *believe* it will succeed”... Osama



# Software Tools

- Numerical Evaluation of Risk for Defender
  - BeliefConvolution Java code (written by author)
  - RAMAS RiskCalc
- Linguistic Evaluation for Adversary
  - LinguisticBelief Java code (written by author)
  - LANL LEDTools

# LinguisticBelief Code

LinguisticBelief Application

File Utilities Help

New Open Save EXIT! Exit

Current Analysis: SAND report section 5 example

Rule Linguistics

- probabilityOfSuccess
- consequence
- expected consequence

Basic Linguistics

- deaths
- damageToNationalSecurity
- probabilityInformationRequiredCanBeObtained
- probabilityResourcesRequiredGatheredWithoutDetection
- probabilityPhysicalSecuritySystemCanBeDefeated

Information for Selected Tree Node

For the Rule Linguistic expected consequence the Fuzzy Sets are as follows: No, Maybe, Yes,. The input linguistics are as follows: consequence, probabilityOfSuccess, . The Rules are as follows: \* Rule input fuzzy sets: small, low, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: small, medium, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: small, high, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: medium, low, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: medium, medium, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: medium, high, -- Rule output fuzzy set: Maybe\*, \* Rule input fuzzy sets: large, low, -- Rule output fuzzy set: No\*, \* Rule input fuzzy sets: large, medium, -- Rule output fuzzy set: Maybe\*, \* Rule input fuzzy sets: large, high, -- Rule output fuzzy set: Yes\*, [Belief, Plausibility] Intervals for fuzzy sets are as follows: Fuzzy Set: No [0.000, 0.200], Fuzzy Set: Maybe [0.596, 1.000], Fuzzy Set: Yes [0.000, 0.255].

Belief and Plausibility and Focal Elements

Linguistic is: expected consequence

BELIEF AND PLAUSIBILITY FOR FUZZY SETS:

No has [Belief, Plausibility] interval of: [0.000, 0.200]  
Maybe has [Belief, Plausibility] interval of: [0.596, 1.000]  
Yes has [Belief, Plausibility] interval of: [0.000, 0.255]

FOCAL ELEMENTS:

Focal Element has Fuzzy Sets: Maybe. Evidence is: 5.960e-01.  
Focal Element has Fuzzy Sets: Maybe & Yes. Evidence is: 2.040e-01.  
Focal Element has Fuzzy Sets: No & Maybe. Evidence is: 1.490e-01.  
Focal Element has Fuzzy Sets: No & Maybe & Yes. Evidence is: 5.100e-02.

Select Family of Fuzzy Sets for [Belief, Plausibility] Interval Calculation

No  
Maybe  
Yes

Calculate [Belief, Plausibility] interval for Selected Family of Fuzzy Sets [0.000e-01, 1.000e+00]

Cancel

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LinguisticBelief Applic...

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