Effect of Air Pressure on Vibrations of Micro Plates

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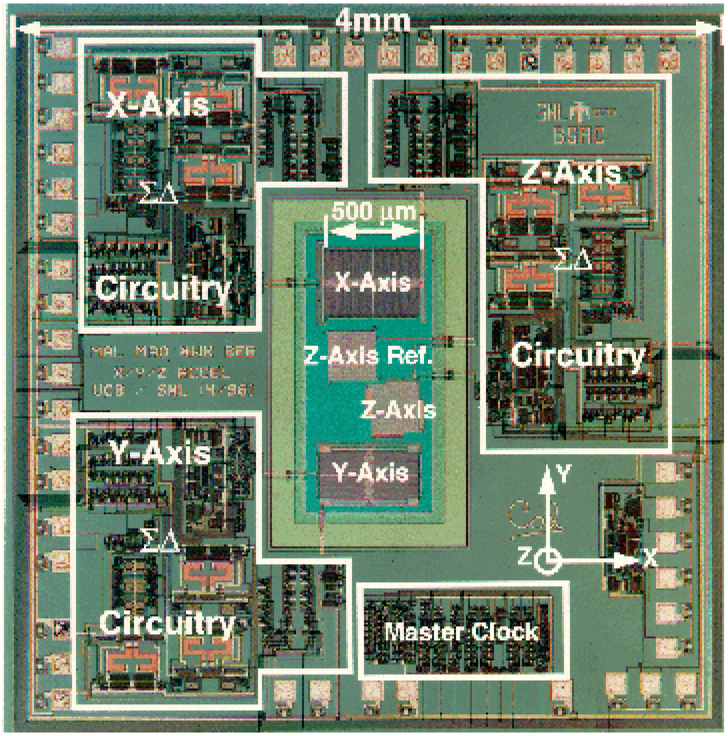
Motivation and Objective

Motivation:

- Electrostatic, parallel-plate actuation is very important in many microsystems applications.
- Squeezed-film damping determines the dynamics of plates moving a few microns above the substrate. Examples abound in
  - MEMS accelerometers.
  - MEMS switches.
  - MEMS gyroscopes.
- Models for predicting squeezed-film damping have not been validated experimentally the in high-frequency/low-pressure regime.

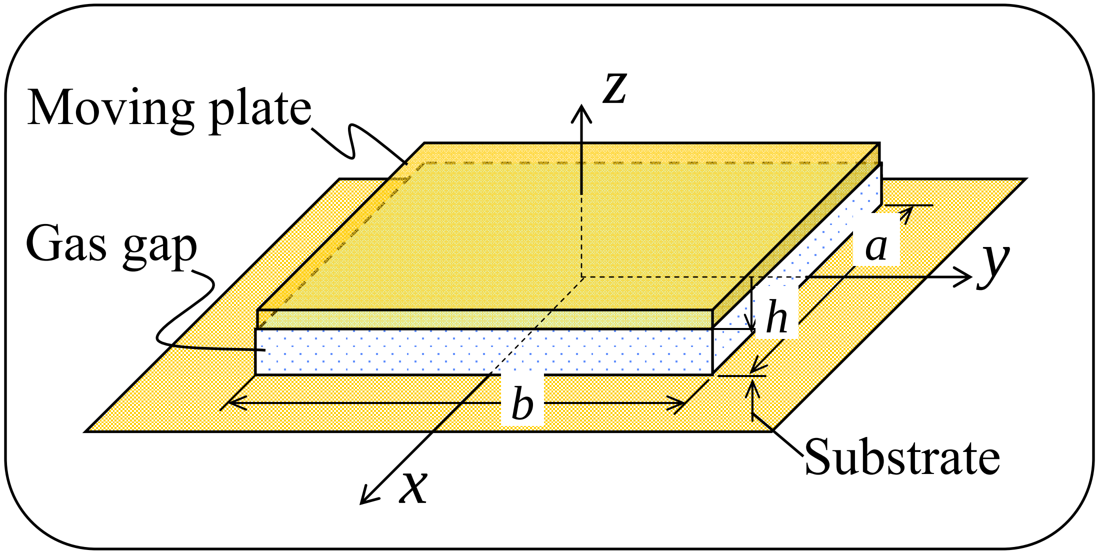
Objective:

- Provide experimental validation to widely used squeezed-film damping models for plates.



http://www.sandia.gov/mstc/images/fig3.gif

Squeezed-Film Damping



Forces on moving plate from gas layer can be obtained from the linearized **Reynold's equation**

$$\frac{Ph^2}{12\mu}\nabla^2\left(\frac{p}{P}\right)-\frac{\partial}{\partial t}\left(\frac{p}{P}\right)=\frac{\partial}{\partial t}\left(\frac{z}{h}\right)$$

P = ambient pressure, Pa
h = gap size, m
μ = viscosity, Pa s
p = pressure at (x,y), Pa
t = time, s

Assumptions:

1. Rigid plate
2. Small gap $h/a \ll 1$
3. Small displacement z/h
4. Small pressure variation p/P
5. Isothermal process
6. Pressure on edges = P
7. Small molecular mean free path
8. No inertial effects of gas moving in and out of the gap

Molecular mean free path λ is a mean distance a molecule travels before colliding.

$$\lambda=\frac{\mu}{P}\sqrt{\frac{2RT}{m_m}}$$

R = universal gas constant,
T = temperature, K
m_m=molecular mass, kg/mol

Harmonically Oscillating Plate

- This presentation discusses a plate oscillating with a displacement

$$\varepsilon(t) = e_0\, exp(j\omega t)$$

- Remember assumption 8: No inertial effects of gas moving in and out of the gap.
- The above Reynold's equation may not be valid if the gas is pushed in and out of the gap at high frequency ω.
  - Inertia and viscosity trap and compress the gas in the gap.

- Three models will be compared with measurement:
  - Blech's model
  - Andrews et al.'s limit
  - Veijola's model

Blech Model

- A measure of gas compression is the **squeeze number**

$$\sigma = 12\mu\left(\frac{a}{h_m}\right)^2\left(\frac{\omega}{P}\right)$$

- The force from the squeezed film is, from Blech's theory,

$$F_{damp}^{Blech} = \frac{64}{\pi^6}\frac{ab}{h}12\mu\left(\frac{a}{h}\right)^2 e_0\omega \sum_{m,n\ \text{odd}} \frac{m^2+n^2(a/b)^2}{m^2n^2\left[m^2+n^2(a/b)^2\right]^2+\sigma^2/\pi^4}$$

- Can be decomposed into

Damping/velocity

Spring force/velocity

velocity

$$F^{Blech}(\omega) = \left[c^{Blech}(\omega)+\kappa^{Blech}(\omega)\right] j\omega e_0(\omega)$$

$$c^{Blech}(\omega) = \frac{768}{\pi^6}\frac{a^3b}{h^3}\mu\sum_{m,n\ \text{odd}}\frac{m^2+n^2(a/b)^2}{m^2n^2\left[m^2+n^2(a/b)^2\right]^2+\sigma^2/\pi^4}$$

$$\kappa^{Blech}(\omega) = \frac{9216}{\pi^8}\frac{a^5b}{h^5P}\mu^2\sum_{m,n\ \text{odd}}\frac{m^2+n^2(a/b)^2}{m^2n^2\left[m^2+n^2(a/b)^2\right]^2+\sigma^2/\pi^4}\frac{\omega}{j}$$

a = width, m
b = length, m
e_0 = amplitude, m
h_m = mean gap size, m
h_{plate} = plate thickness, m
j = √-1
P = ambient pressure, Pa
t = time, s
μ = viscosity, Pa s
ω = frequency, rad/s
ω_n = natural frequency, rad/s
ρ_{plate} = plate mass density, kg/m^3

Blech, J.J., 1983, "On Isothermal Squeeze Films", *Journal of Lubrication Technology*, 105, p 615-620.

Effect of Low Pressure

- Remember assumption 7: Small molecular mean free path.
- The above Reynold's equation is not valid if the molecular mean free path λ is of comparable magnitude with the gap size *h*. i.e., the gas is rarefied.
- A measure of gas rarefaction is the (modified) **Knudsen number**

$$K_s = 1.016\ \lambda/h$$

- Large Knudsen number means:
  - The mean free path becomes much longer than the gap.
  - The continuum assumption of the Reynold's equation may break down.
  - Reynold's equation must be modified.

Effect of Inertia

- Taking into account the inertia of the gas flowing in and out of the gap, Veijola (2004) modified Reynold's equation into

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}Q_{pr}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\mu}Q_{pr}\frac{\partial p}{\partial y}\right)=\frac{\partial \rho h}{\partial t}$$

- Q_{pr} is a flow-rate coefficient that relates flow rates to pressure gradients.
  - Time-dependent
  - Derivation is tedious but crucial in the accuracy of the model.
  - Solved for in Veijola (2004) in frequency domain

- Veijola et al. (1995) also takes into account the gas rarefaction in low pressures by developing an effective viscosity

$$\mu_{eff} = \mu / \left(1 + 9.638 K_n^{\ 1.159}\right)$$

h = gap size, m
p = pressure at (x,y), Pa
t = time, s
μ = viscosity, Pa s
ρ = density, kg/m^3

Knudsen number

$K_n = \lambda / h$

λ = mean free path, m

Veijola, T., Kuisma, H., Lahdenpera, J., and Ryhanen, T., 1995, "Equivalent-circuit model of the squeezed gas film in a silicon accelerometer", *Sensors and Actuators A*, 48, p 239-248.

Veijola, T., 2004, "Compact models for squeezed-film dampers with inertial and rarefied gas effects", *Journal of Micromechanics and Microengineering*, 14, p 1109-1118.

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Veijola Model

If the gap oscillation is $\varepsilon(t) = e_0 \exp(j\omega t)$, then the damping force complex amplitude is

$$F^{Veij}(\omega) = j\omega e_0 \Phi(\omega)$$

$$\boxed{\Phi(\omega) = \sum_{m=1,3,\ldots}^{M} \sum_{n=1,3,\ldots}^{N} \frac{1}{Q_{\mathrm{pr}} G_{mn} + j\omega C_{mn}}}$$

$$C_{mn} = \frac{\pi^4 h (mn)^2}{64 abn_\gamma P}$$

$$G_{mn} = \frac{\pi^6 h^3 (mn)^2}{768 \mu ab} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$$

$$Q_{\mathrm{pr}} = \sum_{k=1,3,\ldots} \frac{1+6K_s}{\frac{k^4 \pi^4}{96} + j\omega \frac{k^2 \pi^2 \rho h^2 \left(1+10K_s+30K_s^2\right)}{96\mu(1+6K_s)}}$$

a = width, m
b = length, m
e_0 = amplitude, m
h = gap size, m
j = √-1
n_χ = 1 for isothermal, (= c_p/c_v for adiabatic).
P = ambient pressure, Pa
t = time, s
μ = viscosity, Pa s
ω = frequency, rad/s
ρ = gas mass density, kg/m³

Modified Knudsen number
$K_s = 1.016\ \lambda/h$

λ = mean free path, m

Veijola, T., 2004, "Compact models for squeezed-film dampers with inertial and rarefied gas effects", *J. Micromech. Microeng.*, 14, p 1109-1118.

Model of Test Structure

Test structure modeled as a Single-Degree-of-Freedom (SDOF) system

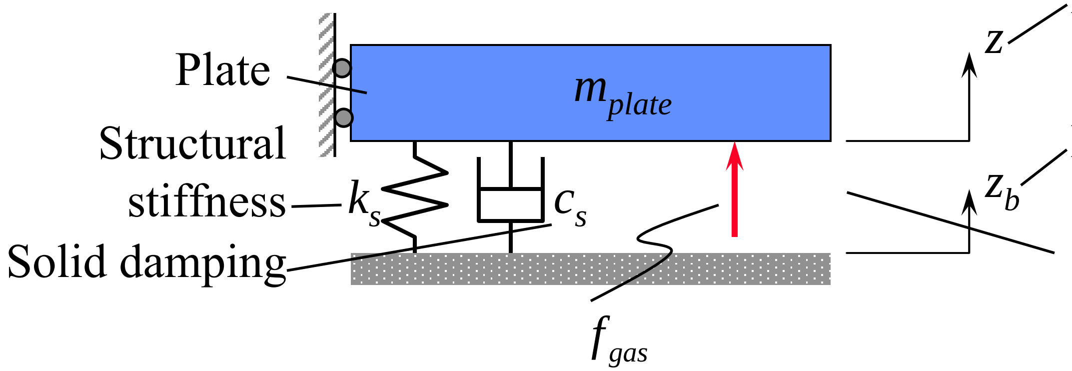


Plate displacement $z(t) = Z\, exp(j\omega t)$

Base displacement $z_b(t) = Z_b\, exp(j\omega t)$

Oscillation of air gap thickness is

$$\tilde{h}(t) = (z(t) - z_b(t)) - mean(z(t) - z_b(t))$$
$$= H \exp(j\omega t)$$

Excitation is harmonic. System is linear(ized). So response is harmonic

Equation of motion $\ddot{m} = k(z_b - z) + c_s(\dot{z}_b - \dot{z}) + f_{gas}$

Frequency response function from base displacement to gap oscillation:

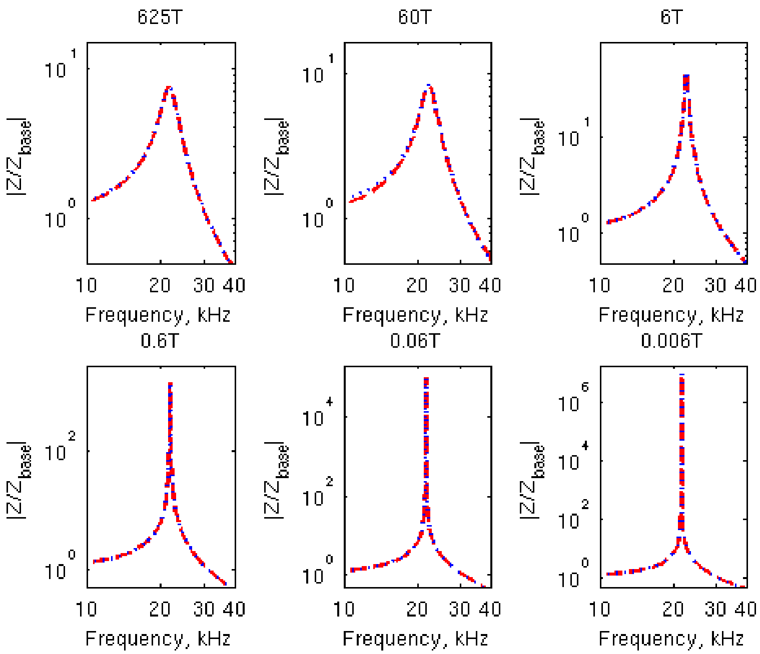
$$\frac{H(\omega)}{Z_b(\omega)} = \frac{m\omega^2}{-m\omega^2 + j\omega(c_s + \Phi(\omega)) + k_s}$$

from gas damping models $\Phi(\omega) = F_{gas}(\omega)/(j\omega H)$ gap velocity

Constant-coefficient Model from Curve-Fitting

slide 10

1. Synthesize frequency response function with damping models:
   - Assume structural stiffness k_s and mass m are known.
   - Assume solid damping $c_s = 0$.
   - Compute transmissibility from base displacement to plate displacement.



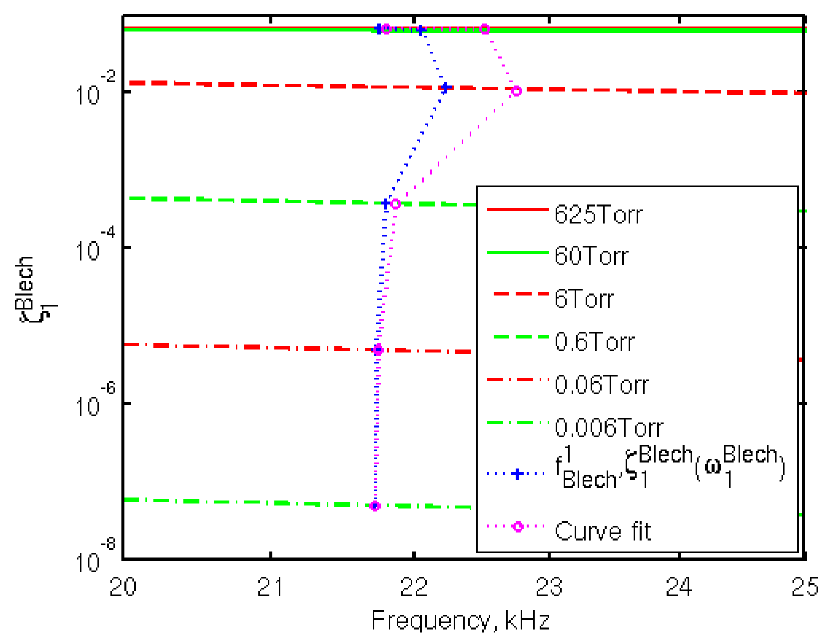
2. Curve-fit around resonant peaks to obtain damping ratio ζ and resonant frequency.

3. Compare the resulting damping ratio ζ from
   a) curve fitting
   b) using the resonant frequency ω_d in Blech's model, i.e.,

$$F_{damp}^{Blech} = \frac{64}{\pi^6}\frac{ab}{h}12\mu\left(\frac{a}{h}\right)^2 e_0\omega_d \sum_{m,n\ \mathrm{odd}} \frac{m^2+n^2(a/b)^2}{m^2n^2\left[m^2+n^2(a/b)^2\right]^2+\sigma_d^{\,2}/\pi^4}$$

and

$$\zeta_{Blech} = \frac{c_{Blech}}{2m_{plate}\omega_n} = \frac{F_{Blech}^{damp}}{e_0\omega_d\, 2m_{plate}\omega_n}$$



Result: 3a) and 3b) are almost identical.

Constant-coefficient Model Using Resonant Frequency

- If the equation of motion $m_{plate}\ddot{z} + c\dot{z} + kz = f_{ext}(t)$ is re-written into the modal form

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n{}^2 z = \hat{f}_{ext}(t)$$

- Then the squeezed-film damping force can be recast into an equivalent viscous damping ratio

- For Blech's model, $$\zeta_{Blech} = \frac{c_{Blech}}{2m_{plate}\omega_n} = \frac{F_{Blech}^{damp}}{e_0\omega_d 2m_{plate}\omega_n}$$

- For Veijola's model, $$\zeta_{Veij} = \mathrm{Re}\left(\frac{F_{Veij}}{j\omega_d e_0 2m_{plate}\omega_n}\right)$$

- The mass of the plate is $m_{plate} = \rho_{plate}\, h_{plate}\, ab$

a = width, m
b = length, m
e_0 = amplitude, m
h = gap size, m
h_{plate} = plate thickness, m
j = $\sqrt{-1}$
m_{plate} = plate mass, kg
P = ambient pressure, Pa
t = time, s
μ = viscosity, Pa s
ω_d = resonant freq., rad/s
ω_n = natural frequency, rad/s
ρ_{plate} = plate density, kg/m^3

NNSA National Nuclear Security Administration

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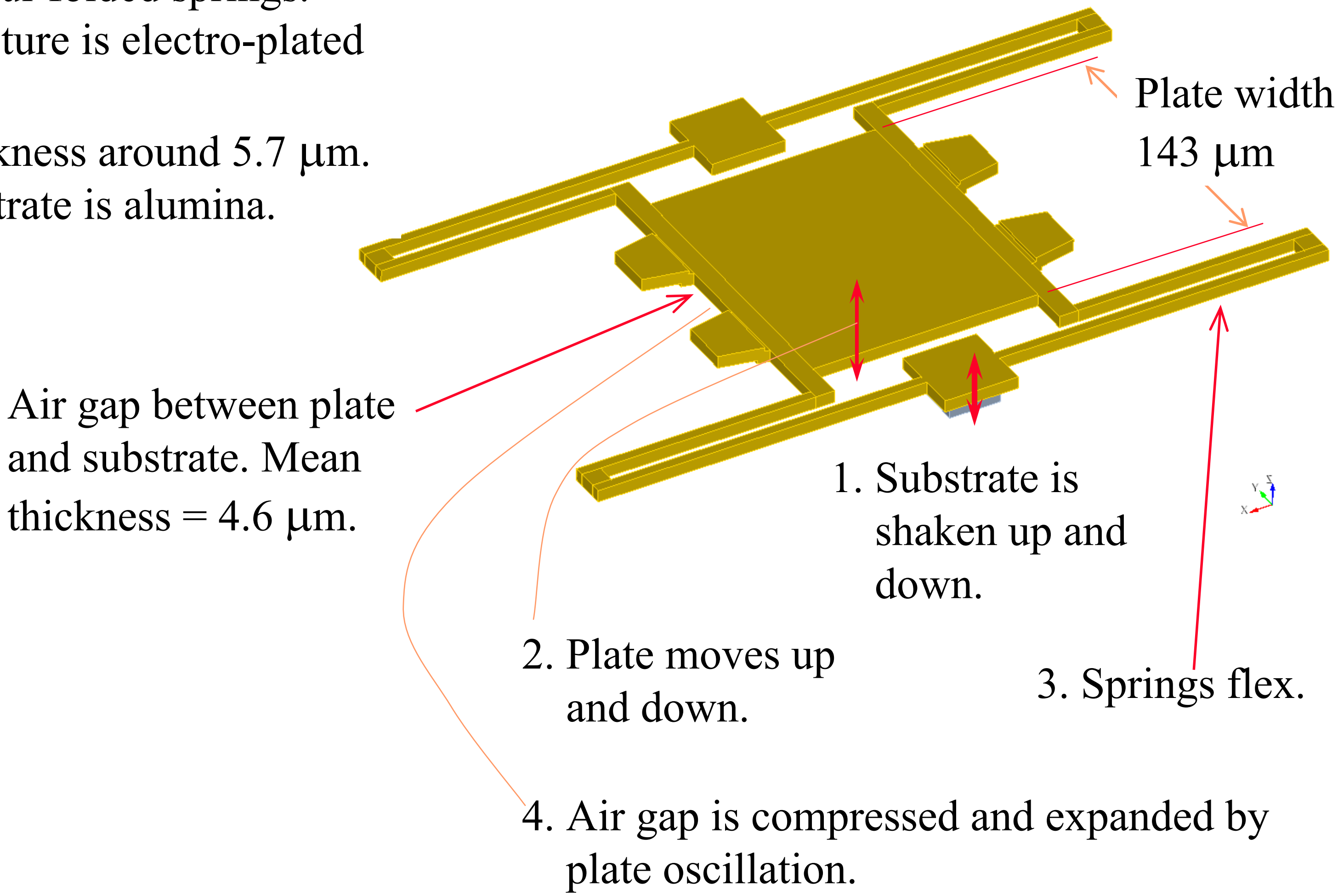
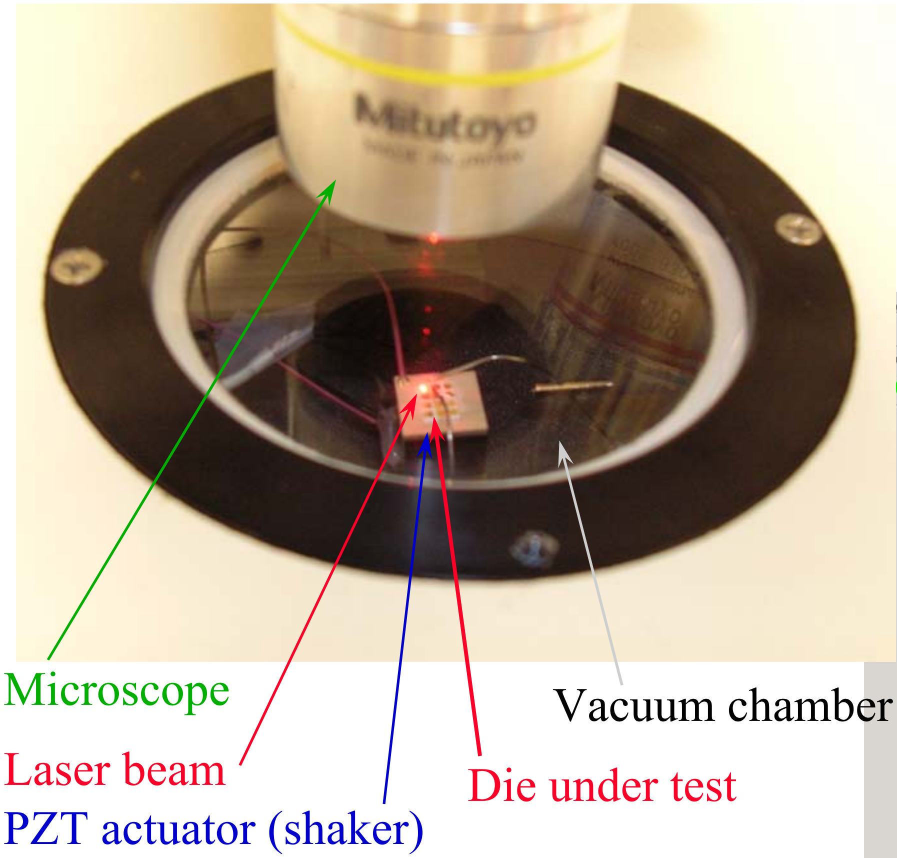
Test Device

- Oscillator plate suspended by four folded springs.
- Structure is electro-plated gold.
- Thickness around 5.7 μm.
- Substrate is alumina.

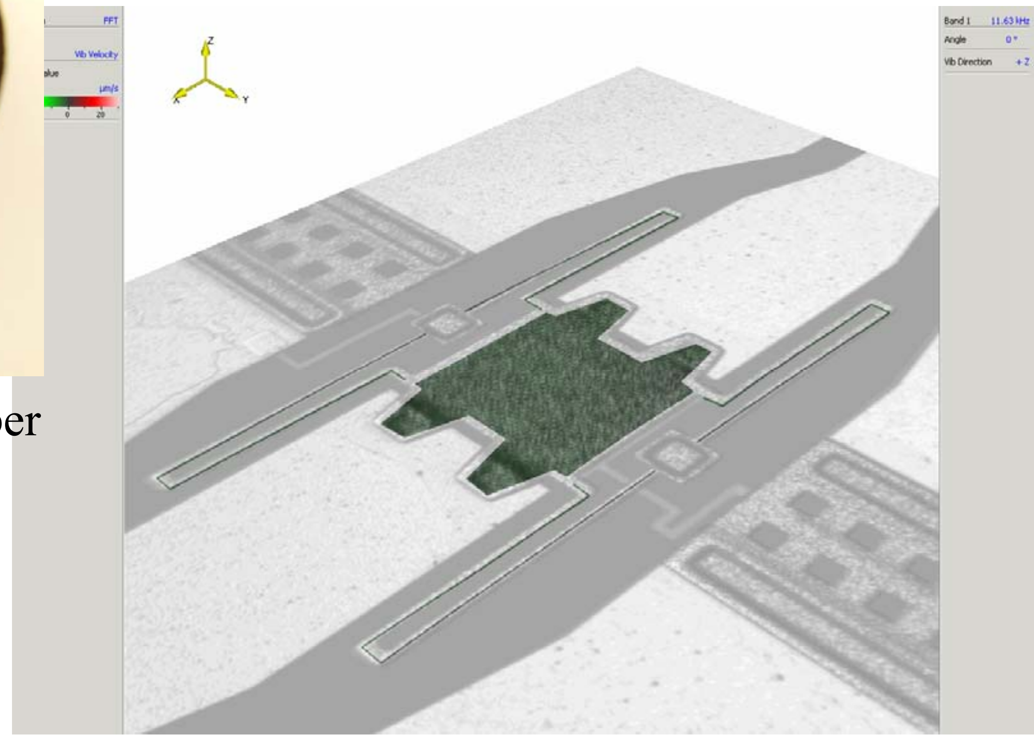


Test Setup



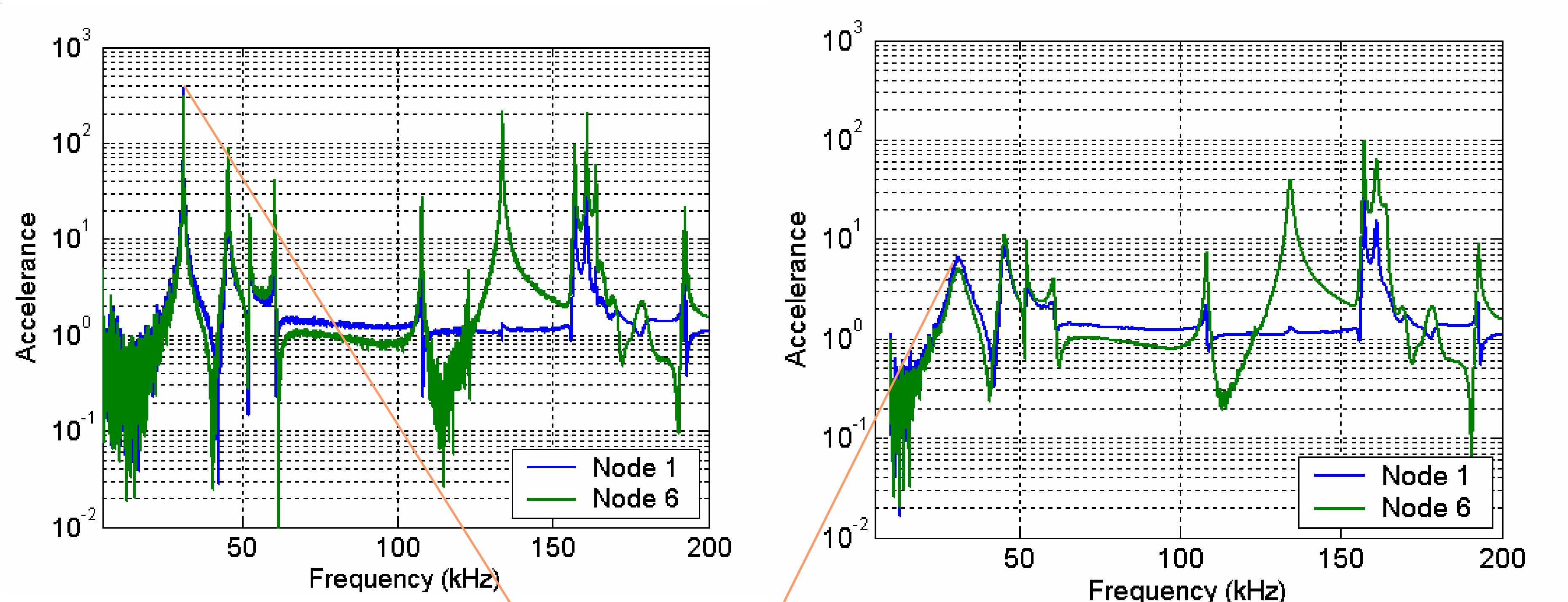
- Excitation: Base displacement with piezoelectric actuator.
- Sensing: Scanning Laser Doppler Vibrometer.

- Modal analysis was done in 2k-50 kHz.
- Tests were repeated at five different air pressures from atmospheric (640 Torr) to near-vacuum (6 milliTorr).



Measured deflection shape, first mode.

Effect of Air Pressure on FRF



Near vacuum (0.06 Torr) **Atmospheric (625 Torr)**

- From Frequency Response Functions (FRFs), natural frequencies and mode shapes were computed with I-DEAS.
- Only the lowest-frequency mode will be discussed here.
- Atmospheric air damped the first resonant response by two orders of magnitude.
- Atmospheric air did not damp spring bending (higher) modes as much.

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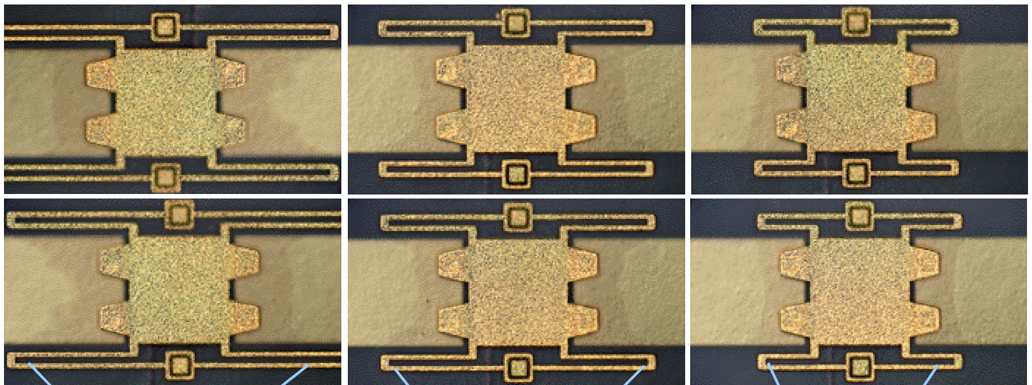
Array of Test Devices

- A 2 x 3 part of a larger array: *Des. and Fab. Chris Dyck, SNL*

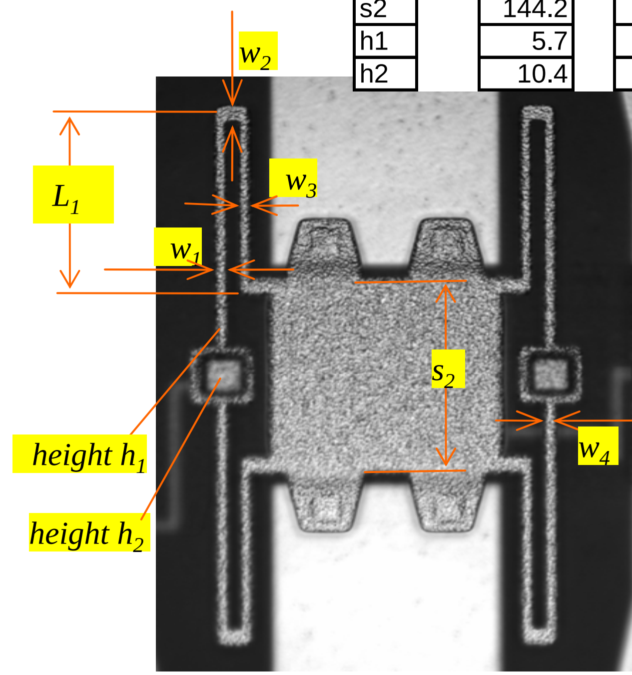


Long springs | Medium springs | Short springs

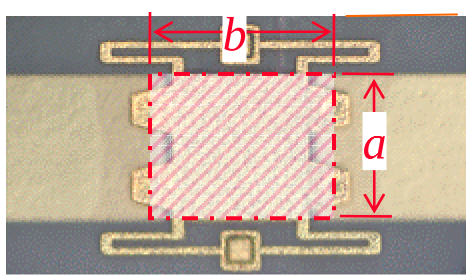
- Two rows nominally identical.
- Measured dimensions, μm:

w2	14.6	13.6	13.6
w3	9.7	8.8	8.8
w4	8.8	9.7	9.7
L1	225.1	175.4	126.7
s1	0	0	0
s2	143.2	144.2	144.2
h1	5.7	5.7	5.7
h2	10.4	10.3	10.2

w1	9.7	10.7	10.7
w2	14.6	13.6	13.6
w3	8.8	9.7	8.8
w4	9.7	9.7	9.7
L1	226.1	175.4	126.7
s1	0	0	0
s2	144.2	144.2	144.2
h1	5.7	5.7	5.7
h2	10.4	10.6	10



- Natural frequency and damping were obtained from experimental modal analysis.
- Squeezed-film damping ratios were predicted using measured dimensions, measured natural frequency.
- Assumed width *a* and length *b*, where *ab* = true plate area.



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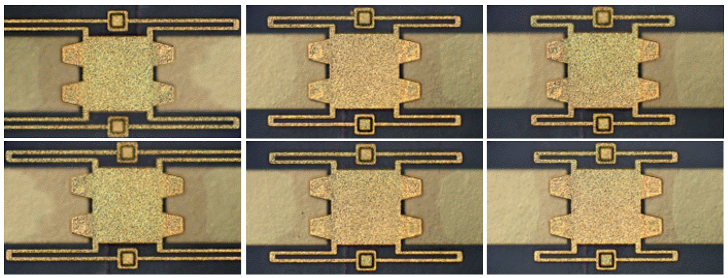
Result: Natural Frequency and Damping

Numbers in tables correspond to position in array



Natural frequency, Hz

10509	17350	21721
12003	18507	29071

•Shorter springs result in higher natural frequencies, as expected.

- The two rows were significantly different.
  - Fabrication variation
  - Much more common in MEMS than in macro world

Damping ratios ζ, % of critical

Atm P=625Torr		
10.05	7.61	5.35
10.36	7.72	5.93
P = 60 Torr		
5.58	3.57	2.26
4.93	3.56	2.58
P = 6 Torr		
1.03	0.59	0.49
1	0.79	0.41
P = 60 mTorr		
0.49	0.12	0.2
0.3	0.34	0.09
P = 6 mTorr		
0.54	0.07	0.1
0.12	0.28	0.09

•Lower pressure results in lower damping, as expected.

•Curve-fitting was not reliable at 6 mT.

- $\zeta < 0.1\%$.
- Window was needed to reduce leakage, but distorted damping.

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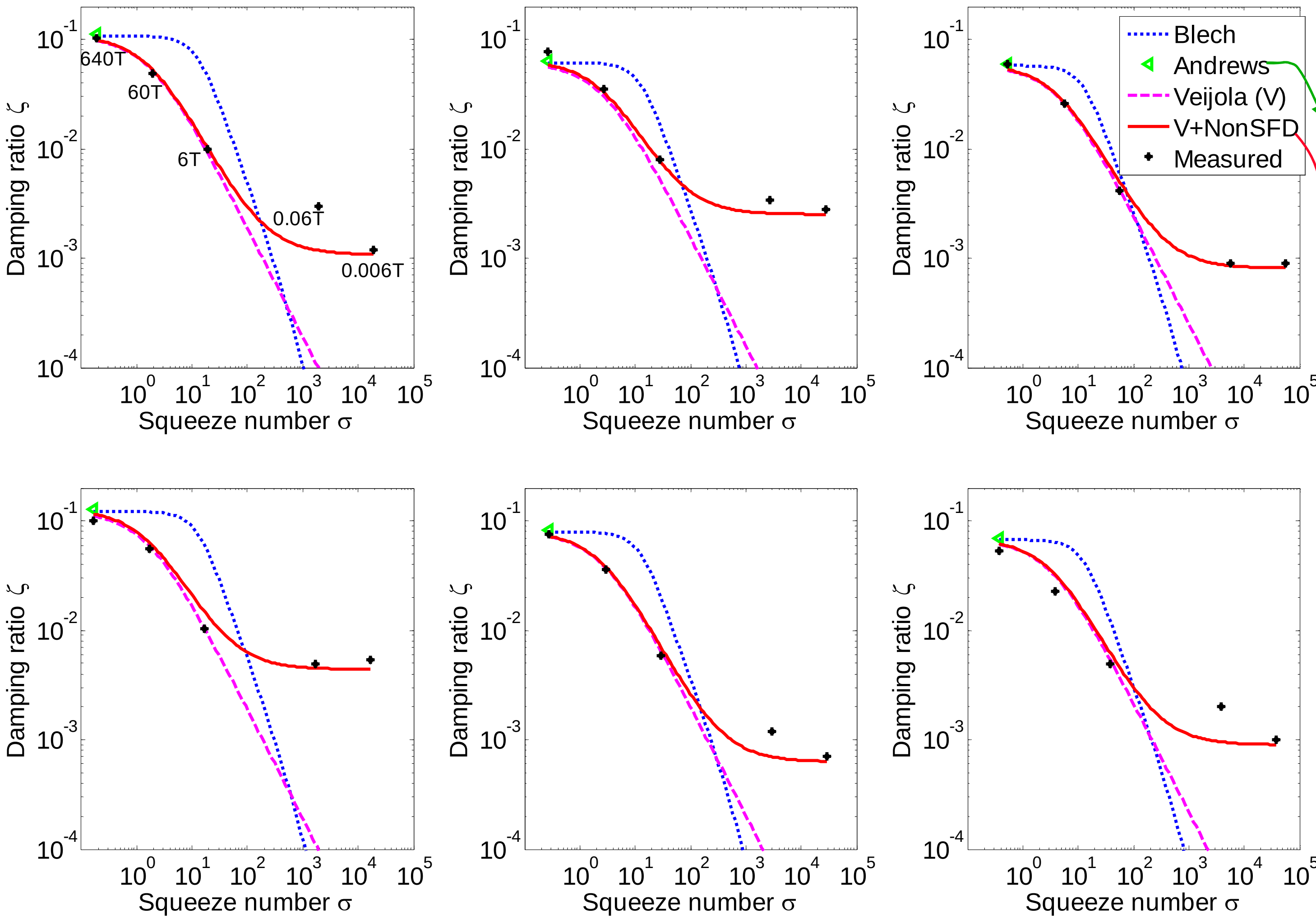
Result: Predictions versus Measured



Squeeze number

$$\sigma = 12\mu\left(\frac{a}{h}\right)^2\left(\frac{\omega}{P}\right)$$

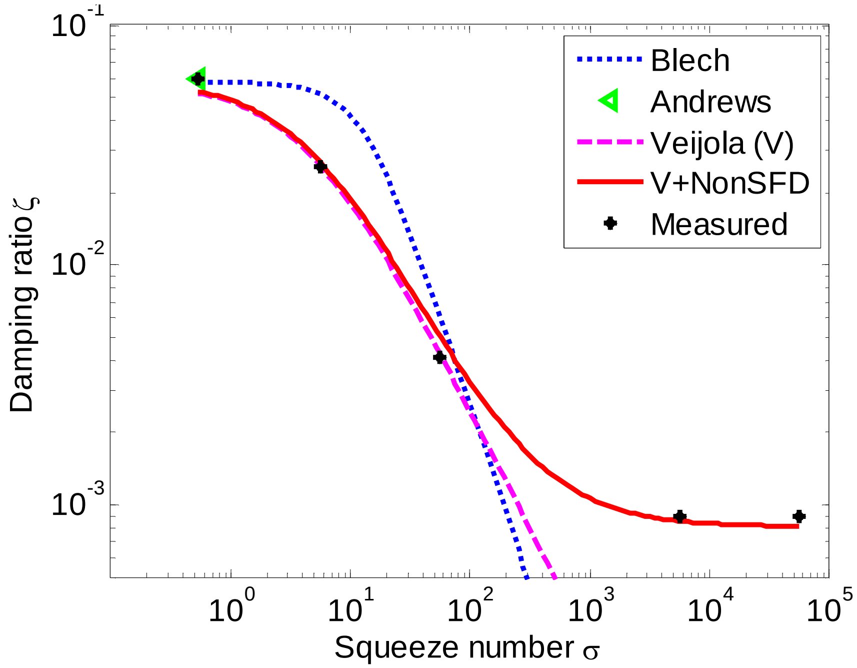
For low squeeze numbers,

$$\boxed{\zeta = 0.42\frac{(ab)^2\mu}{h^3 2m\omega_n}}$$

- Non-squeeze-film damping accounts for solid structural and other unknown damping.
- At high pressure, nonSFD does not contribute much.
- At near-vacuum:
  - Gas damping is negligible.
  - Assume 90% of damping is nonSFD.

Andrews, M., Harris, I., Turner, G., 1993, “A comparison of squeeze-film theory with measurements on a microstructure”, *Sensors and Actuators A*, 36, p 79-87.

Conclusions



- On rigid plates with dimensions about 140 µm, oscillating a few µm above the substrate, squeezed air film can cause large damping.
- This work has compared the Blech model, Andrews et al. model, and Veijola model.
- For the conditions tested here, in atmospheric air the simplest model mentioned by Andrews et al. is as good as any more sophisticated models.
- In the high squeeze number regime (low pressures or high frequencies), the Veijola model is shown to match experimental data accurately.

•The experiments did not allow strong conclusions for the **very high** squeeze-number regime.

- •Gas behaves as colliding particles. Continuum theories break down.
- •Experiments must use structures with much lower structural damping (e.g. Si with Q ~100,000).

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Questions ??

Thank you!

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