

Development and Validation of a Viscoelastic Foam Model for Encapsulated Components

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for the United States Department of Energy under contract DE-AC04-94AL85000.



Overview

- Introduction
- Viscoelastic Model Description
- Model Parameter Identification with UQ
- Model Predictions compared to Test Data
- Conclusions
- Future Work



Foam Model Development and Validation Team

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Experimental Modal Data

Mark Stavig

Constitutive Testing

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Team Manager

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Model Validation

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Uncertainty Quantification

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Foam Development

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Foam Development

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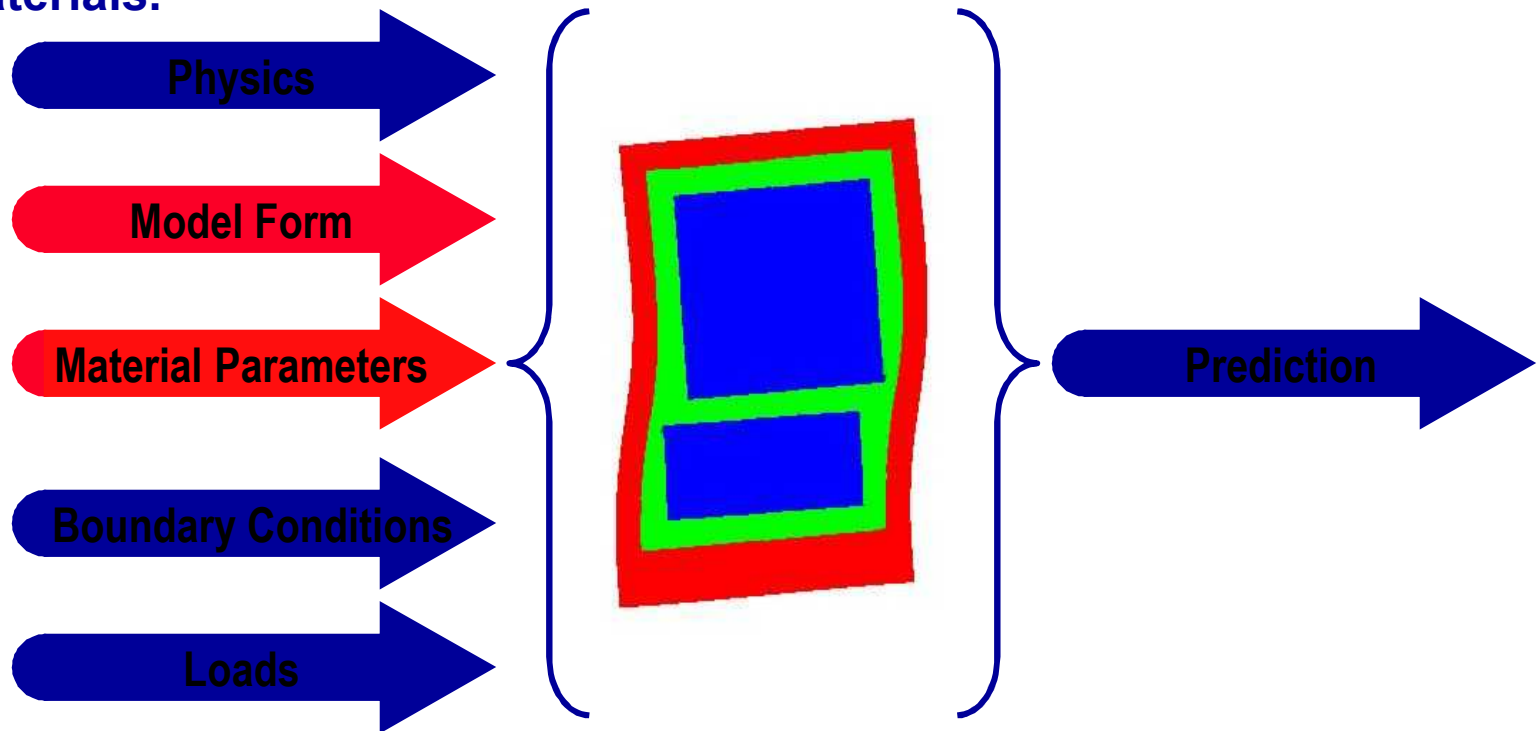
Viscoelastic Modeling

Dan Segalman

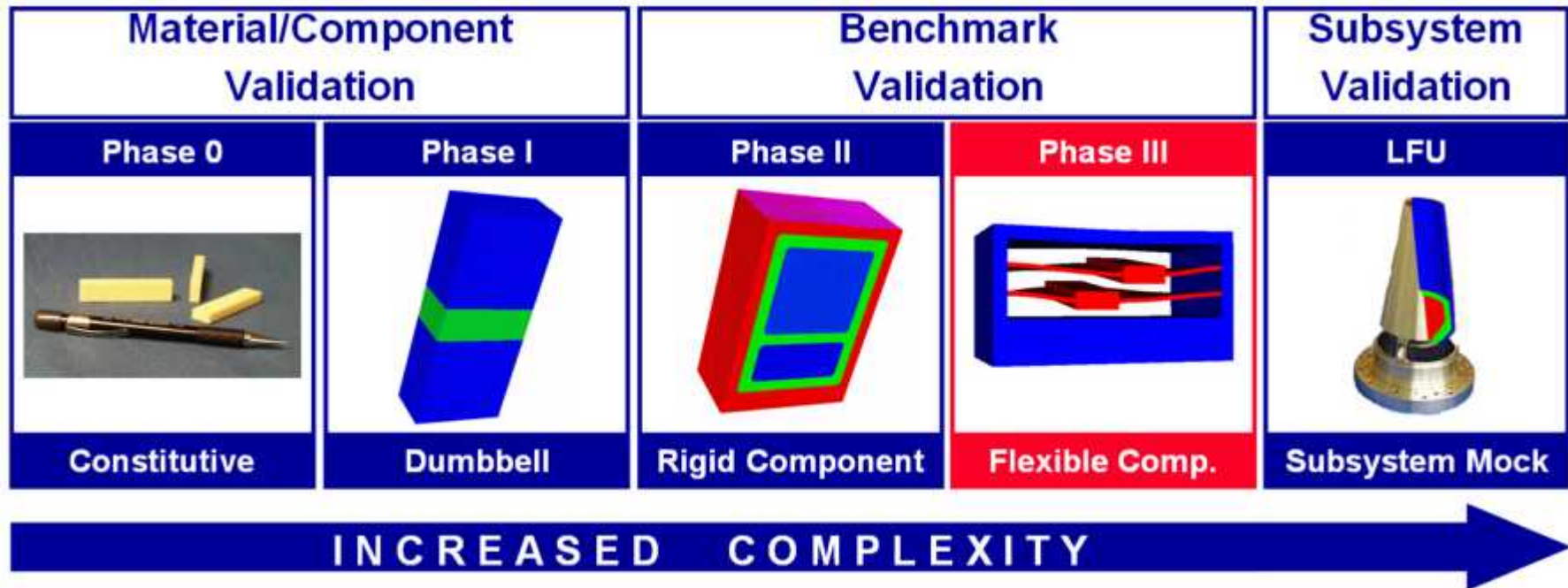
Viscoelastic Modeling

Project Objective

The objective of this project is to improve the predictive modeling capability of components and systems using foam encapsulants. Specifically, this project seeks to support the development, calibration and validation of finite element models using foam encapsulant materials.

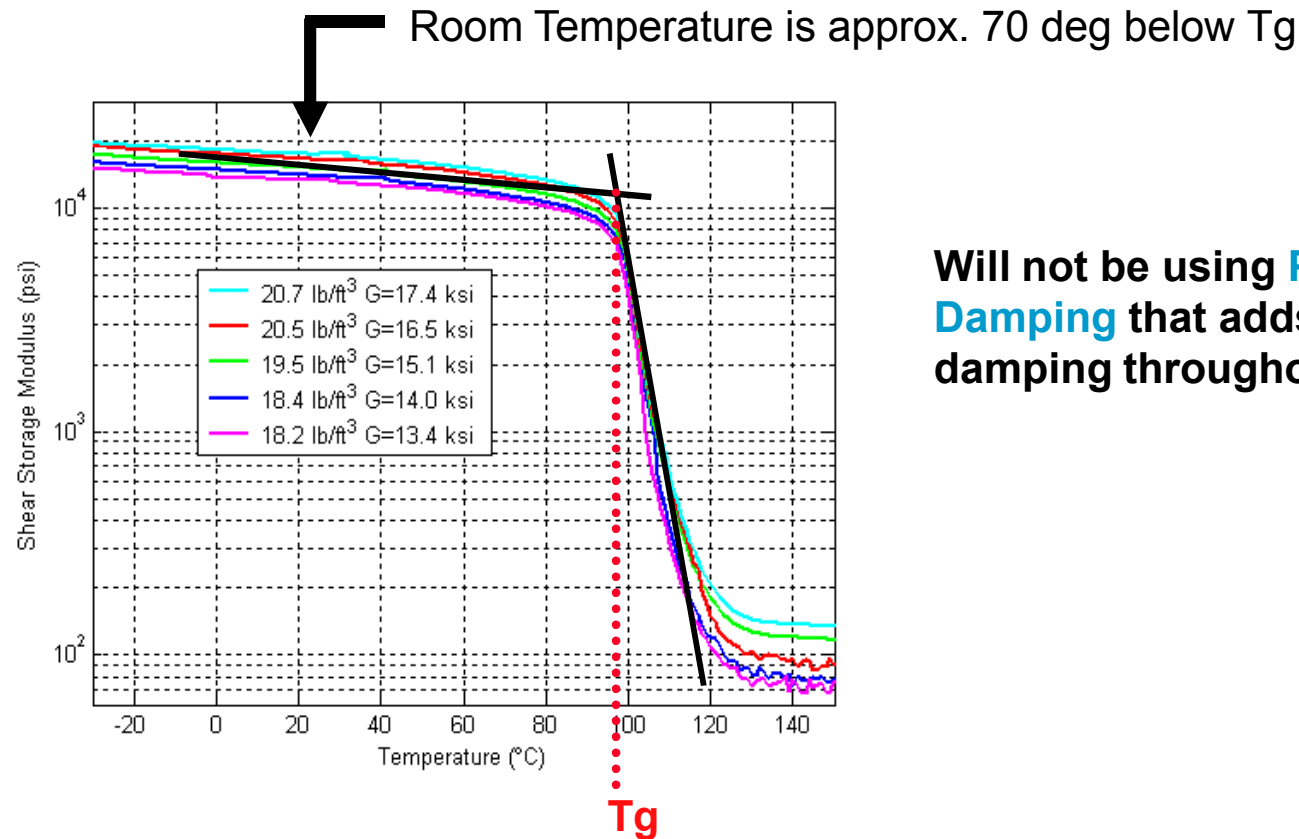


Foam/Component Development Process



- Project experiments will emphasize the fundamental physics associated of foam-confined components with geometries of increased complexity
- The physical parameter space is significantly simplified, but the **environmental parameter space** involving temperature, frequency, strain and density **will be maintained**

Goal of Viscoelastic Model Development



Will not be using **Proportional Damping** that adds unwanted damping throughout the structure

Goal of Viscoelastic Model Development: Accurately represent the elasticity and predict the **small amount of damping** that is present in the foam at room temperature

Calibration Procedure for the Viscoelastic Model in the Salinas Code

Dynamic Mechanical Analysis (DMA)

Tests provides:

- estimates trend of shear modulus vs. frequency and temperature
- basis for fitting Prony Series
- estimates of material loss factor
- still need second elastic constant
- works best near the glass transition temperature (95 deg C)

Prony Series

Viscoelastic Model

Phase I Modal Tests

- provides modal frequencies and damping
- analytically back out E and G with Salinas by matching test modes

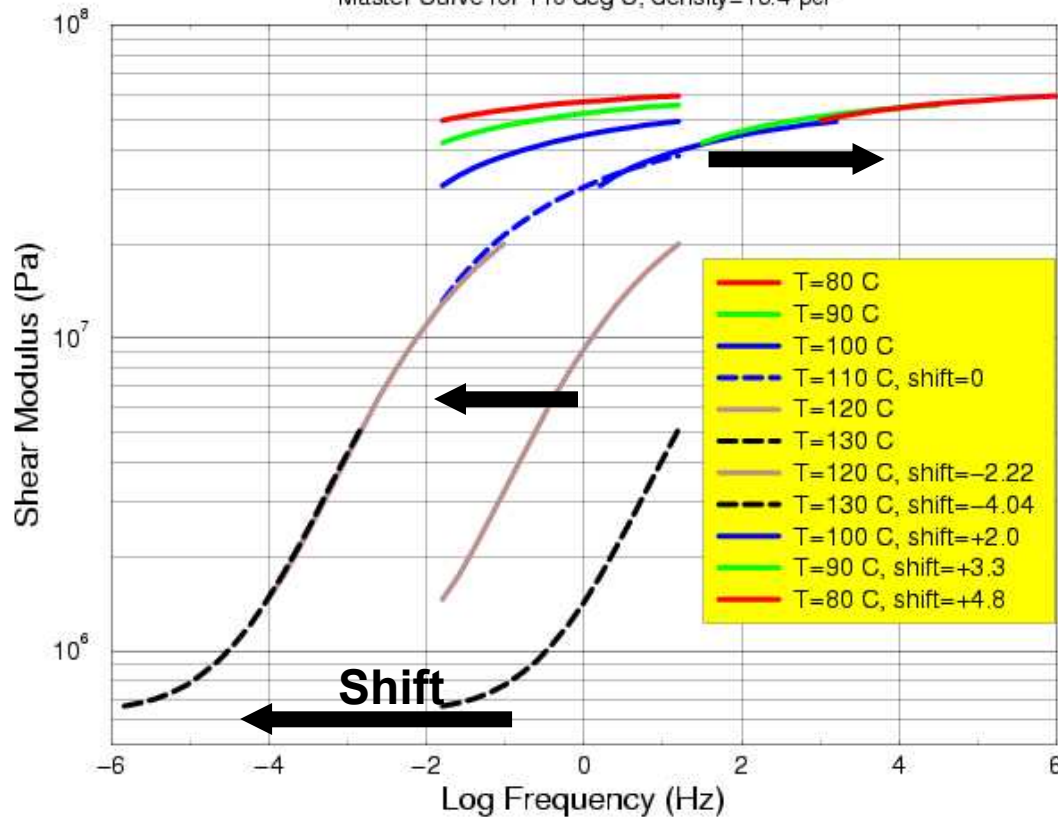
Young's Modulus, Shear Modulus and damping

Constitutive Experiments

DMA Temperature/Frequency Shifts

EF-AR20 Epoxy Foam – Sample 5

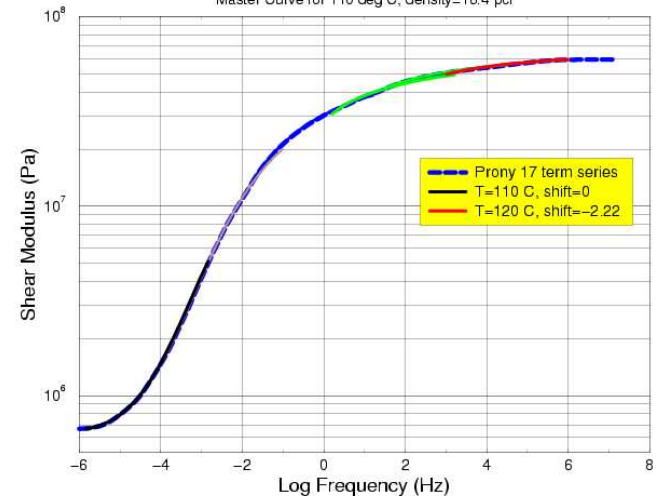
Master Curve for 110 deg C, density=16.4 pcf



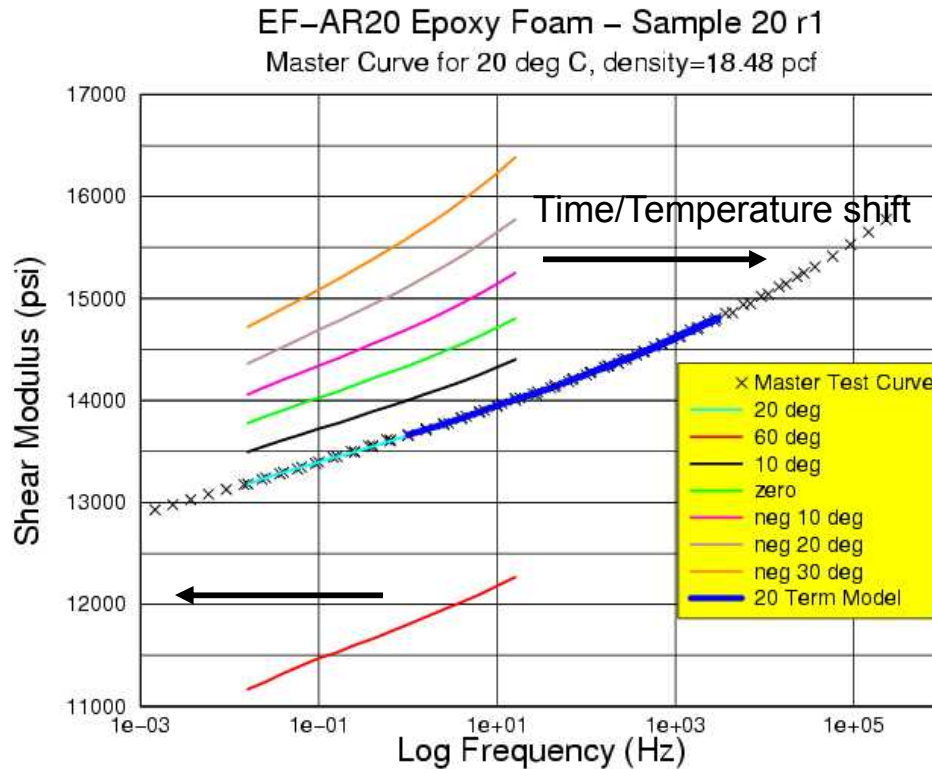
17 Term Prony Series Fit

EF-AR20 Epoxy Foam – Sample 5

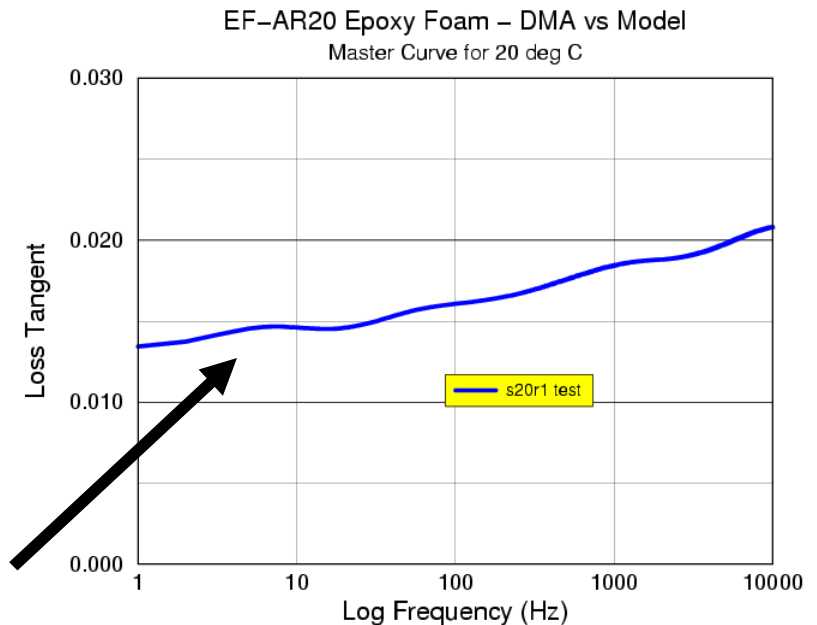
Master Curve for 110 deg C, density=16.4 pcf



Prony Model of Master Curve at Room Temperature



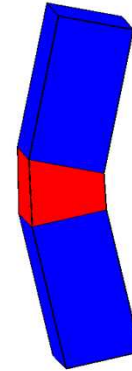
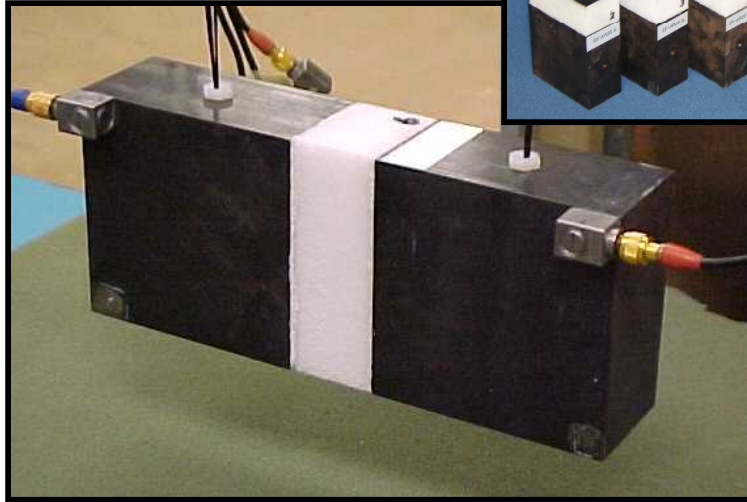
- Individual DMA curves shifted to form Master Curve
- 20 Term Prony Series fit to Master Curve (blue curve)



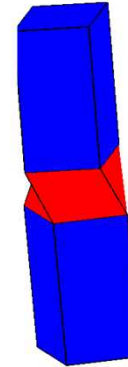
Resulting master curve for Loss Tangent
(damping = 0.5 LT for Phase I)



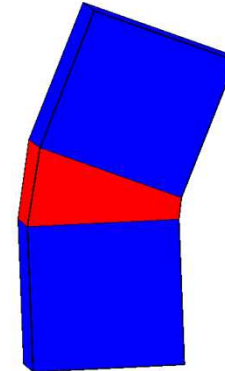
EF-AR20 Phase I Calibration Data



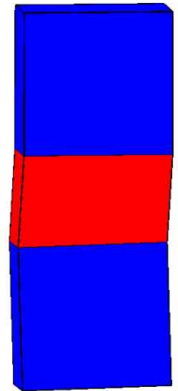
1st Bending X



Torsion



1st Bending Y

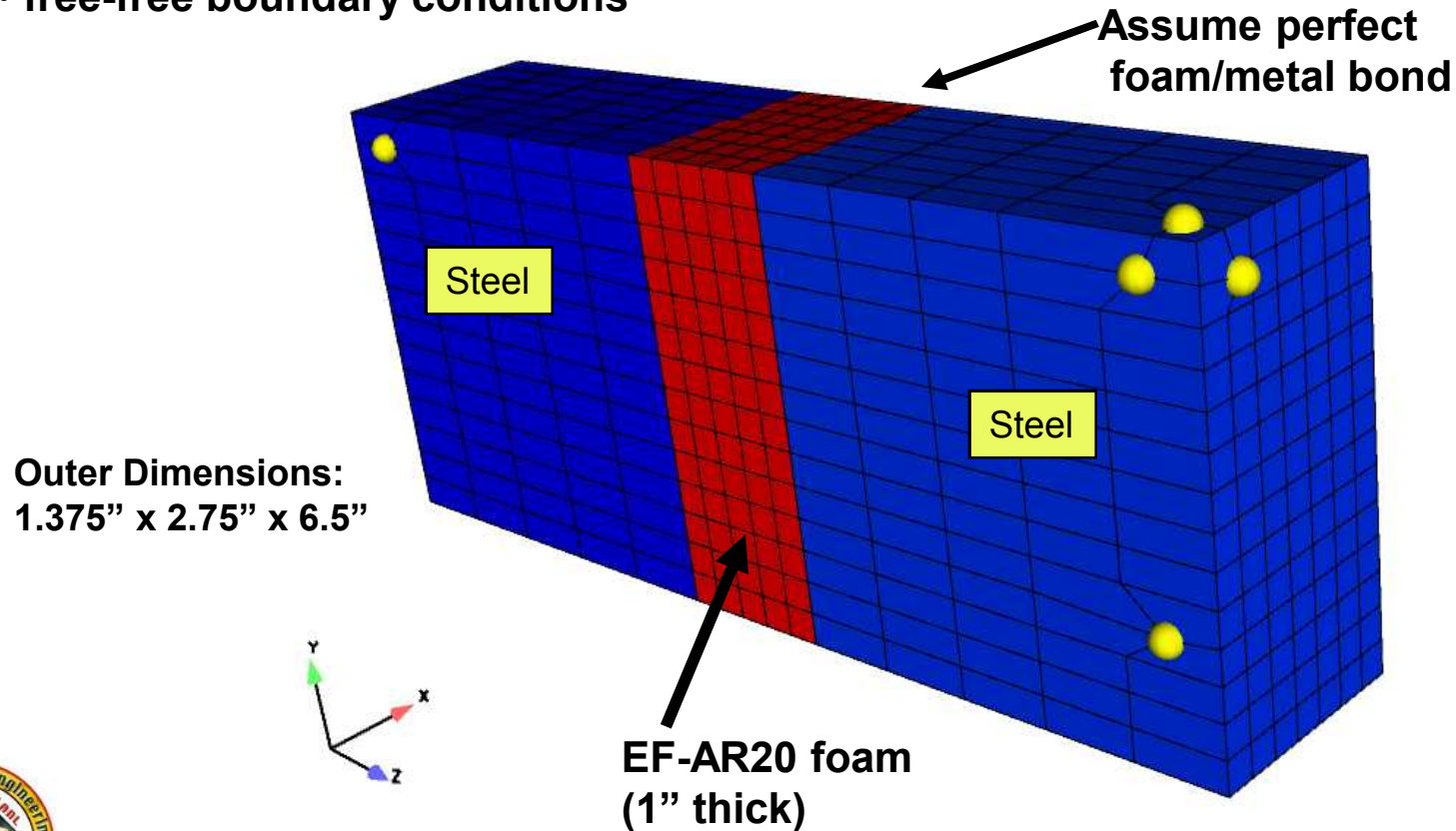


Axial

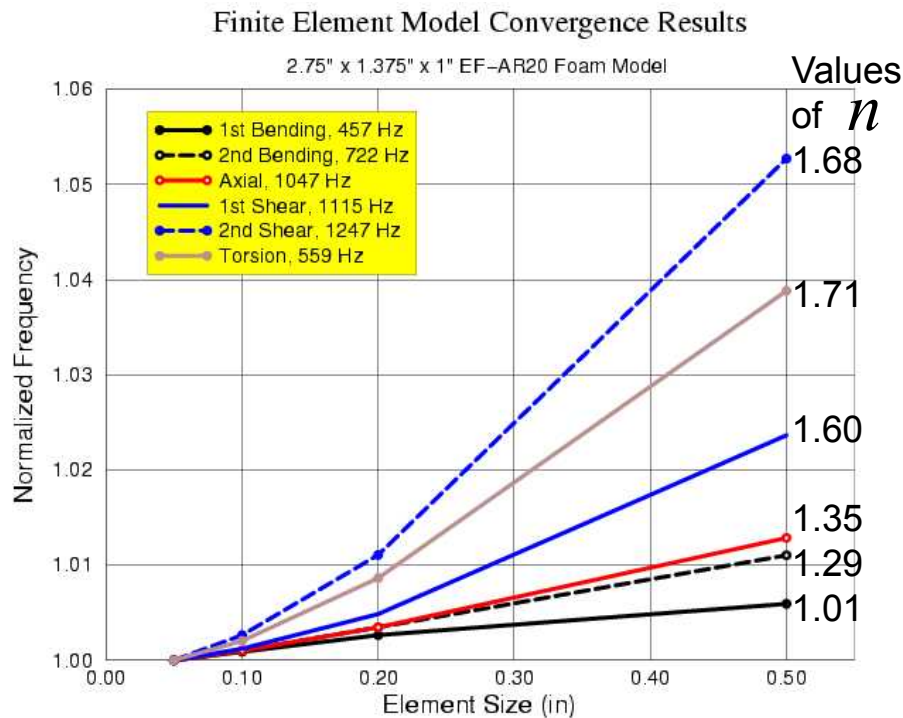
- Phase I samples used to supplement constitutive data
- Mechanism to test various frequencies at room temperature
- Low strain helps minimize uncertainties caused by cellular interaction
- 27 Phase I baseline samples
- 6 Phase I Calibration samples
- 5 Phase I Validation samples

Phase I Calibration Finite Element Model

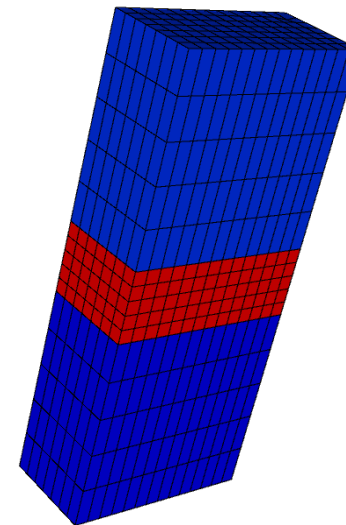
- 1470 eight node hex elements
- 1920 nodes
- yellow spheres designate input/output node set locations
- free-free boundary conditions



Phase 1 Model Convergence using Modal Frequencies



0.2" element selected for computational efficiency



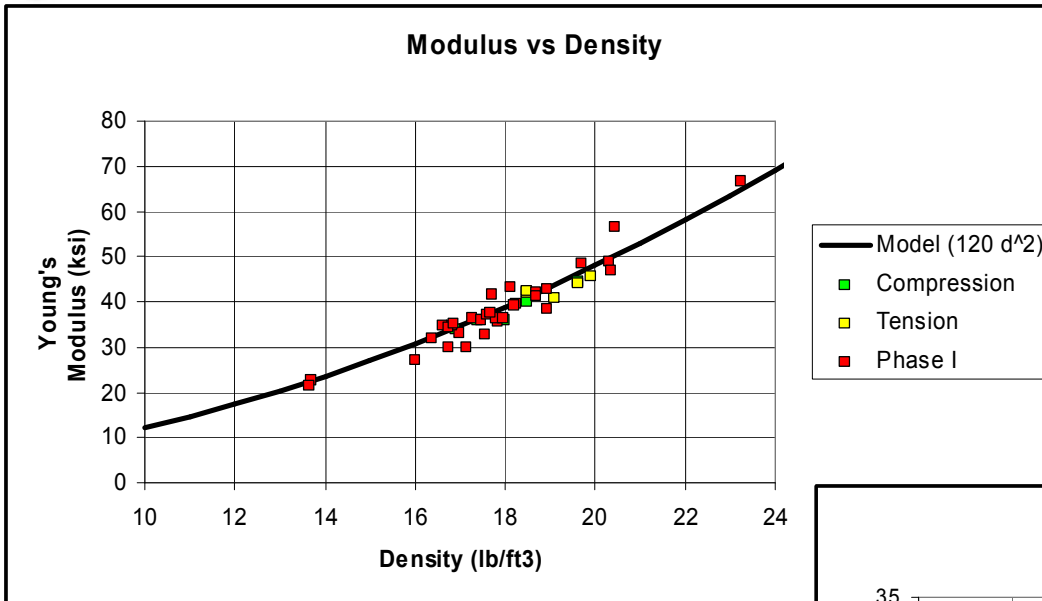
8 node hexahedral elements used

Richardson Extrapolation: $E(h) = E' + Ch^n$

Exponent n , has values given above, ranging from 1.01 to 1.71

Normalized error, $(E - E') / E'$, same as shown in plot at $h = 0.2$, less than 1.3% error

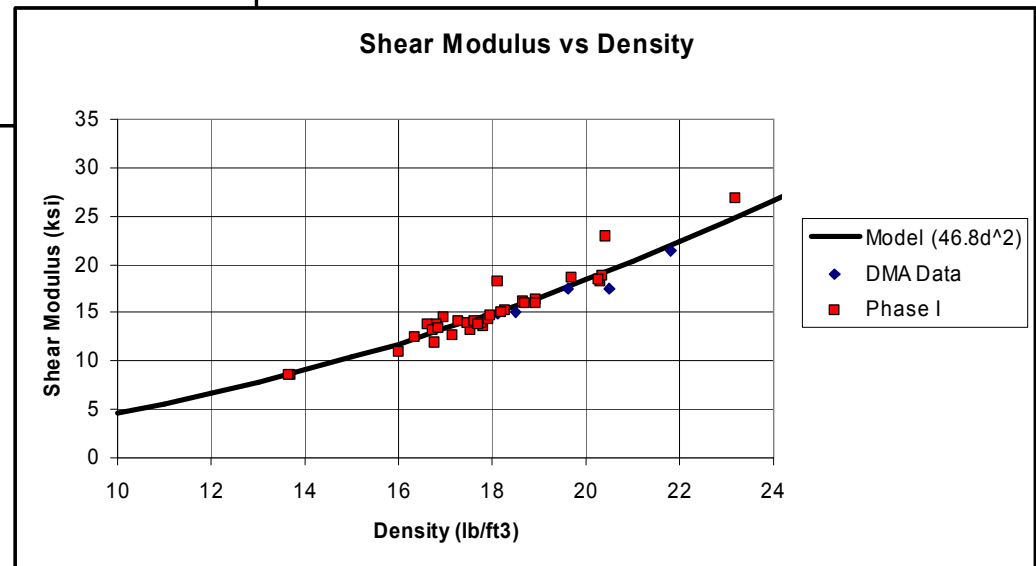
Experimental/Analytical Estimating Results for EF-AR20 Modulus vs Density



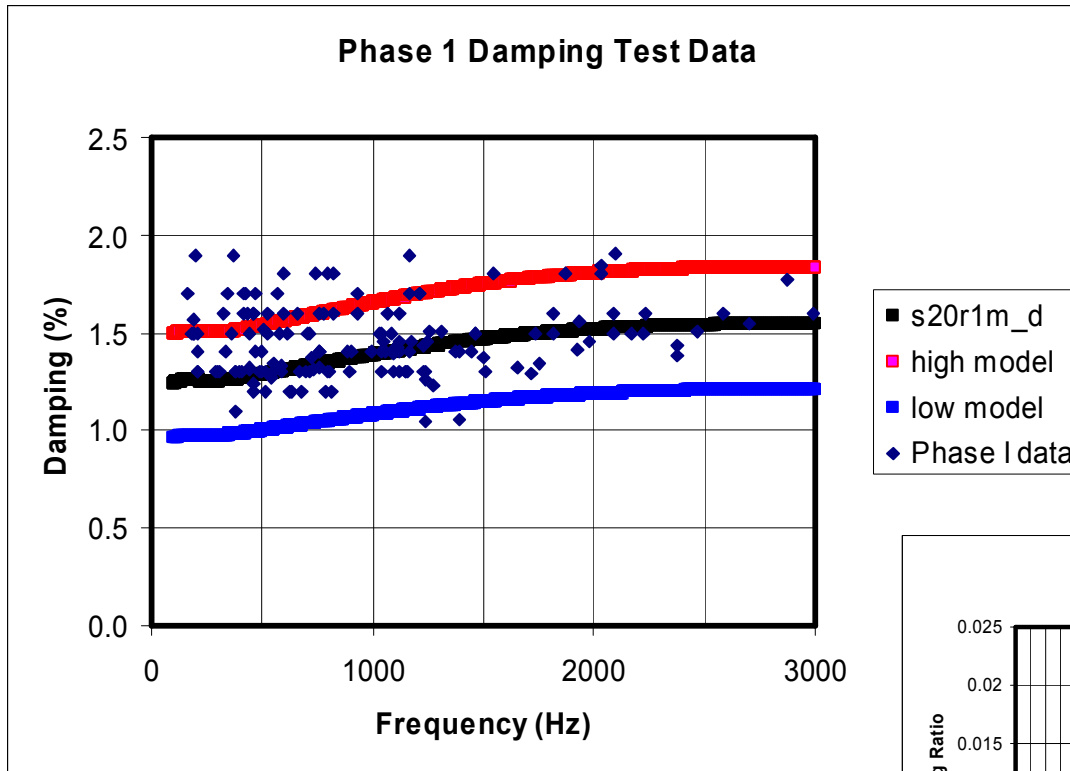
- Plot of experimentally inferred Young's and Shear modulus as a function of density (dots)
- Perform regression to estimate parameter of squared function fit
- Correlation of E and G residuals will be matched also

- Modulus proportional to density squared
- Results compare well with models from Gibson & Ashby's "Cellular Solids:

$$E = E_s \left(\frac{\rho}{\rho_s} \right)^2$$

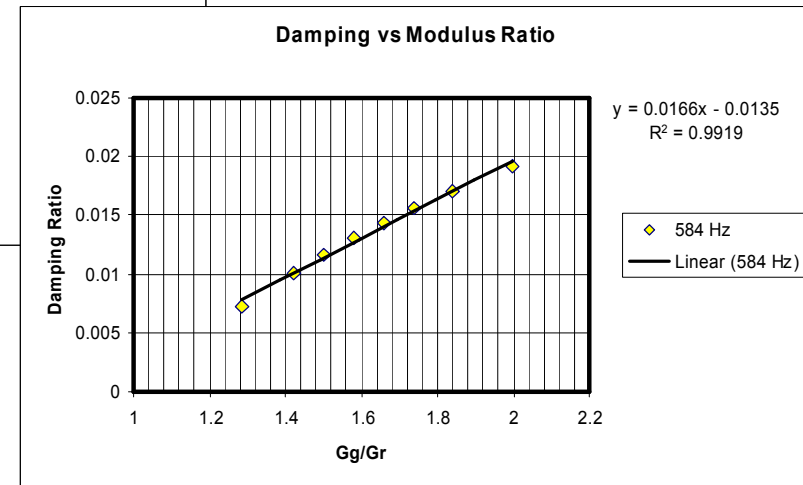


Phase I Experimental Damping versus Viscoelastic Model Damping

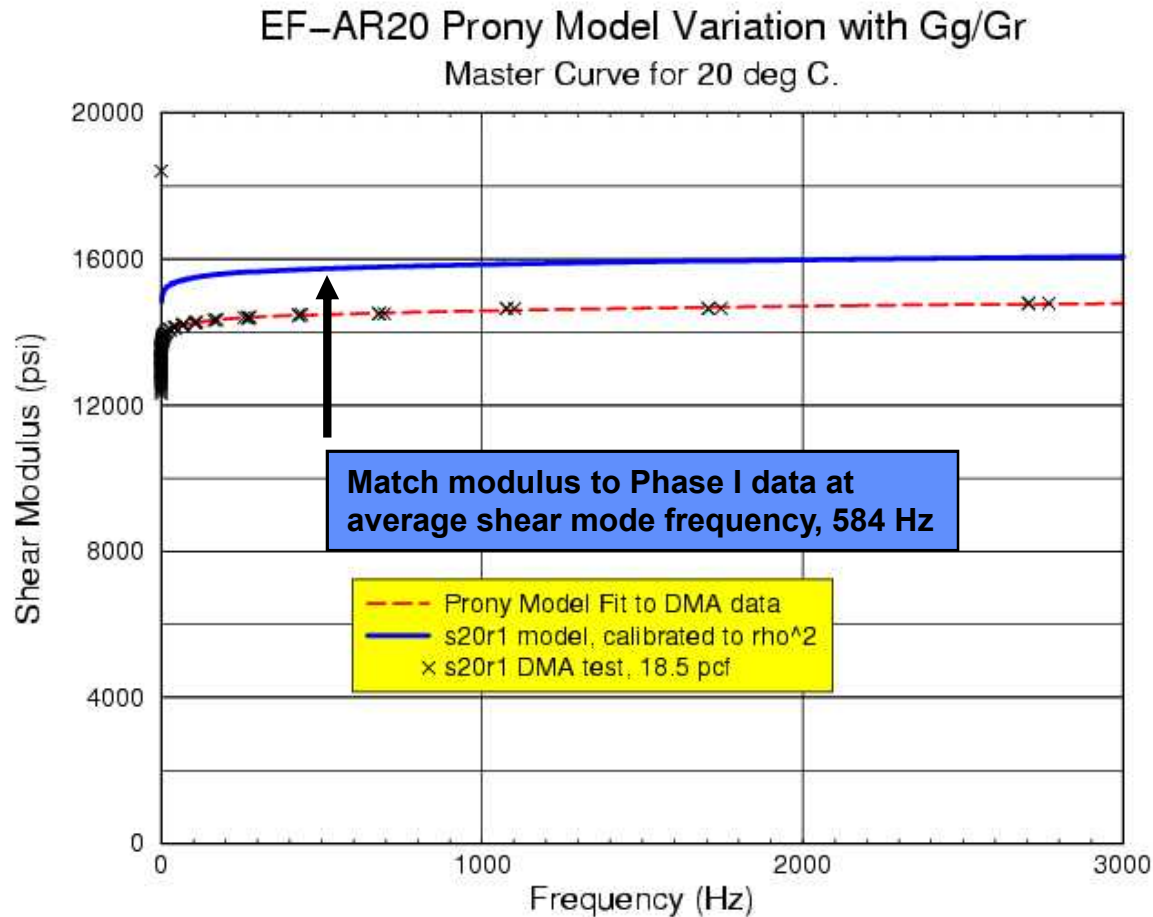


Will treat damping as a random variable

- Three Prony series realizations shown with colored curves
- damping will be specified via modulus ratios as shown below



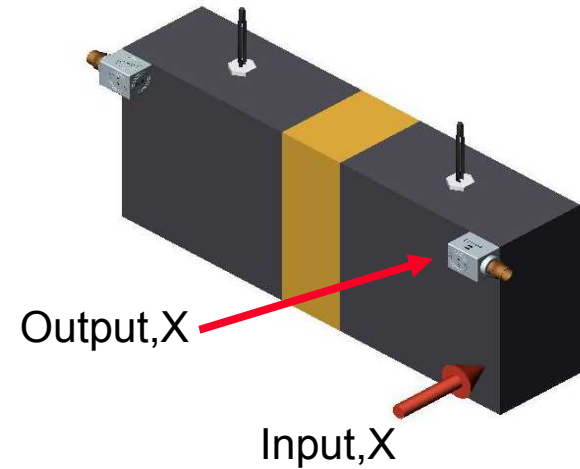
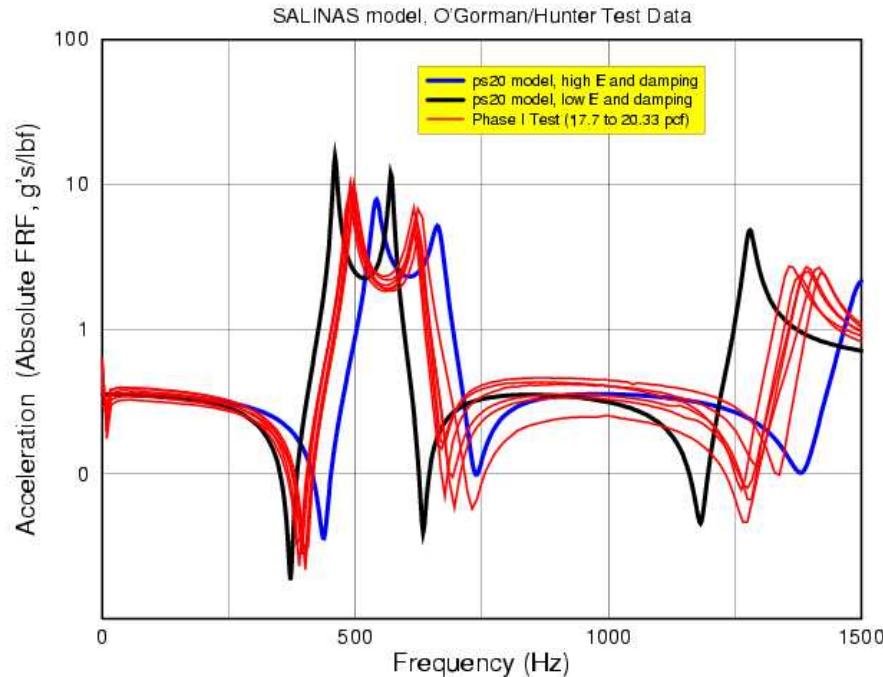
Matching Prony Series Model with Phase I Test



Prony Series model is matched to Phase I derived modulus values by shifting the curve with modified G_g and G_r values

SALINAS Viscoelastic Model Predictions of Phase I Calibration Tests with EF-AR20 Foam

Phase I FRF for 6XX Test, Shift to 20 PCF

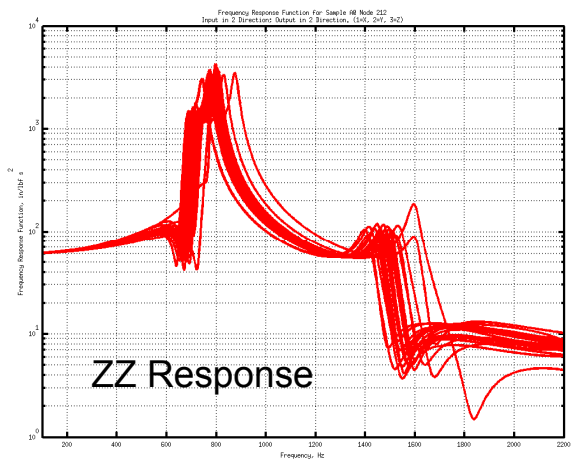
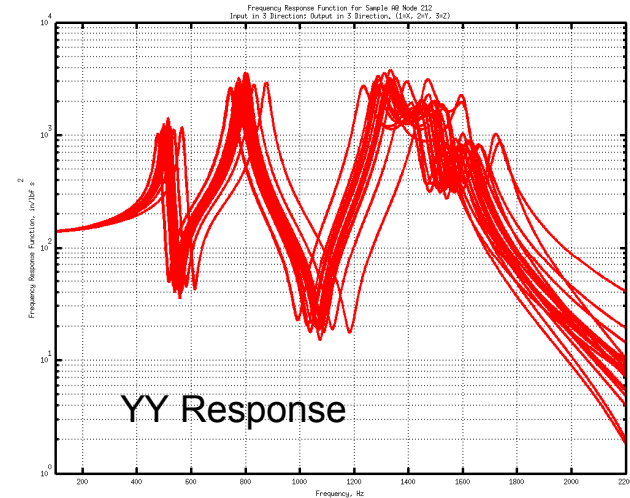
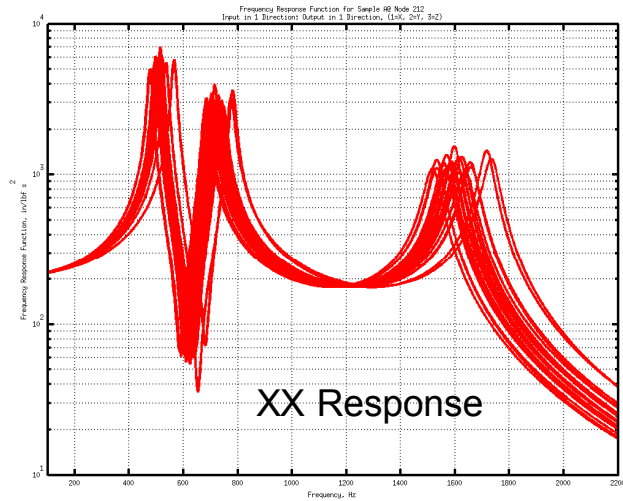


6 sets of test data
shifted to common
density of 20 pcf

Model brackets composite of test data between high
modulus/damping and low modulus/damping predictions

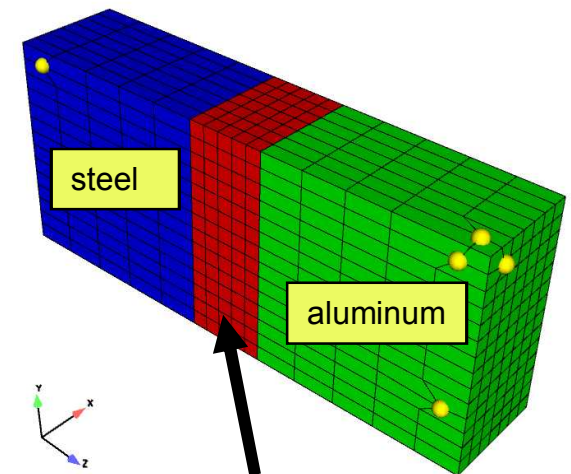
$$f_2 / f_1 = \sqrt{E_2 / E_1} = \rho_2 / \rho_1$$

Phase I Validation Model Predictions For Free-Free Conditions



20 sets of random variables
generated to represent PDF of:

- E - Young's Modulus
- G - Shear Modulus
- Density
- Damping



EF-AR20 foam

Phase I Experiment/Model Prediction

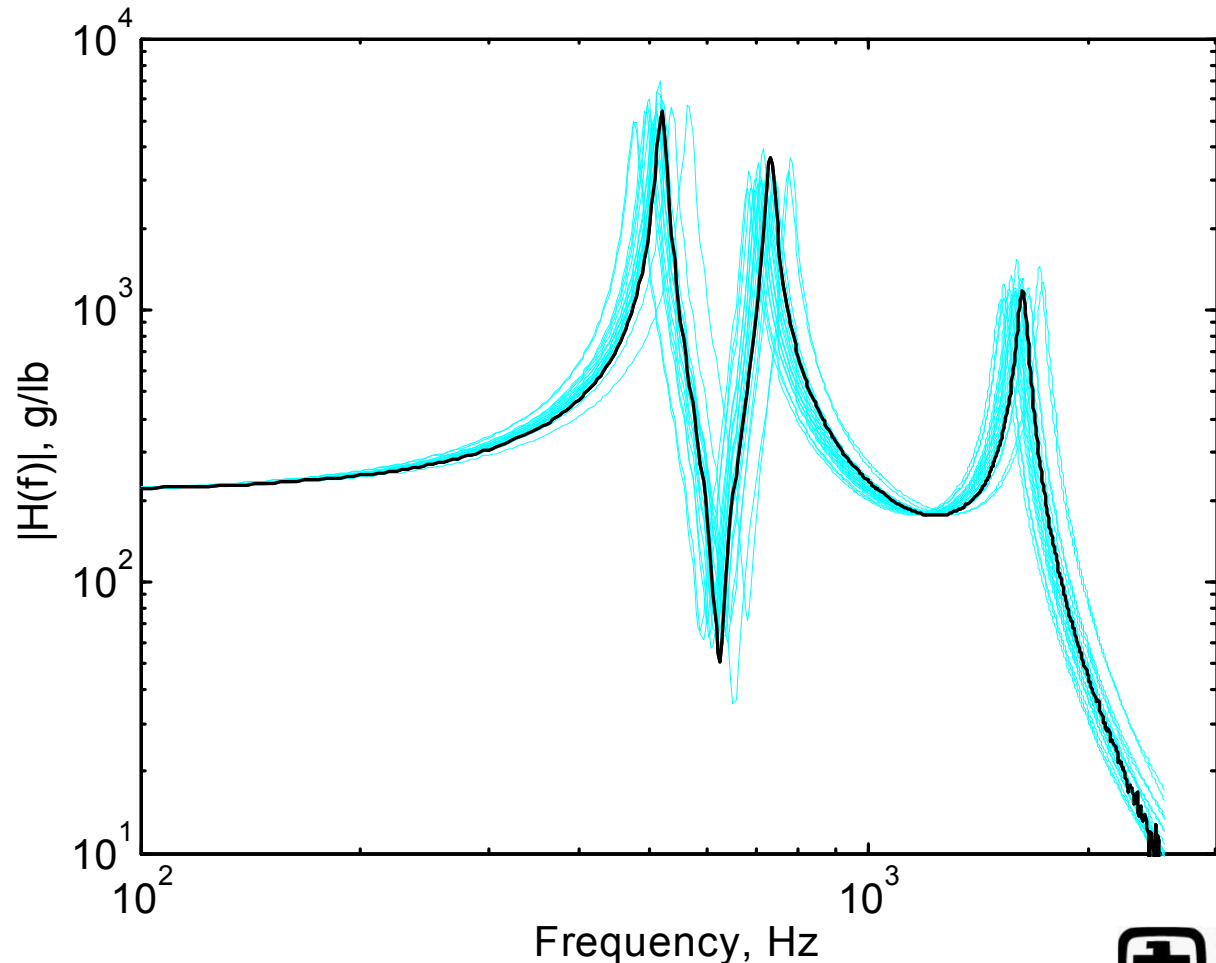
x – direction, $\rho = 16.42 \text{ lb/ft}^3$

Experiment

- One realization
- At each of five densities
- Tested in three directions

Model

- 20 Realizations
- At each of the five densities
- Predicted in three directions





Summary & Conclusions

Foam/Component Mechanical Modeling

- Model convergence adequacy demonstrated
- Salinas viscoelastic model parameters identified
 - Fit Prony series to master shear modulus curve for room temperature based on DMA test data
 - Fine tuned viscoelastic model derived from DMA tests with modulus and damping measured in Phase I Tests
- Random variables modeled:
 - Young's and Shear modulus with correlation
 - Damping and Density
- Phase I model is calibrated and model validation predictions computed

FY06 Foam Modeling Plans

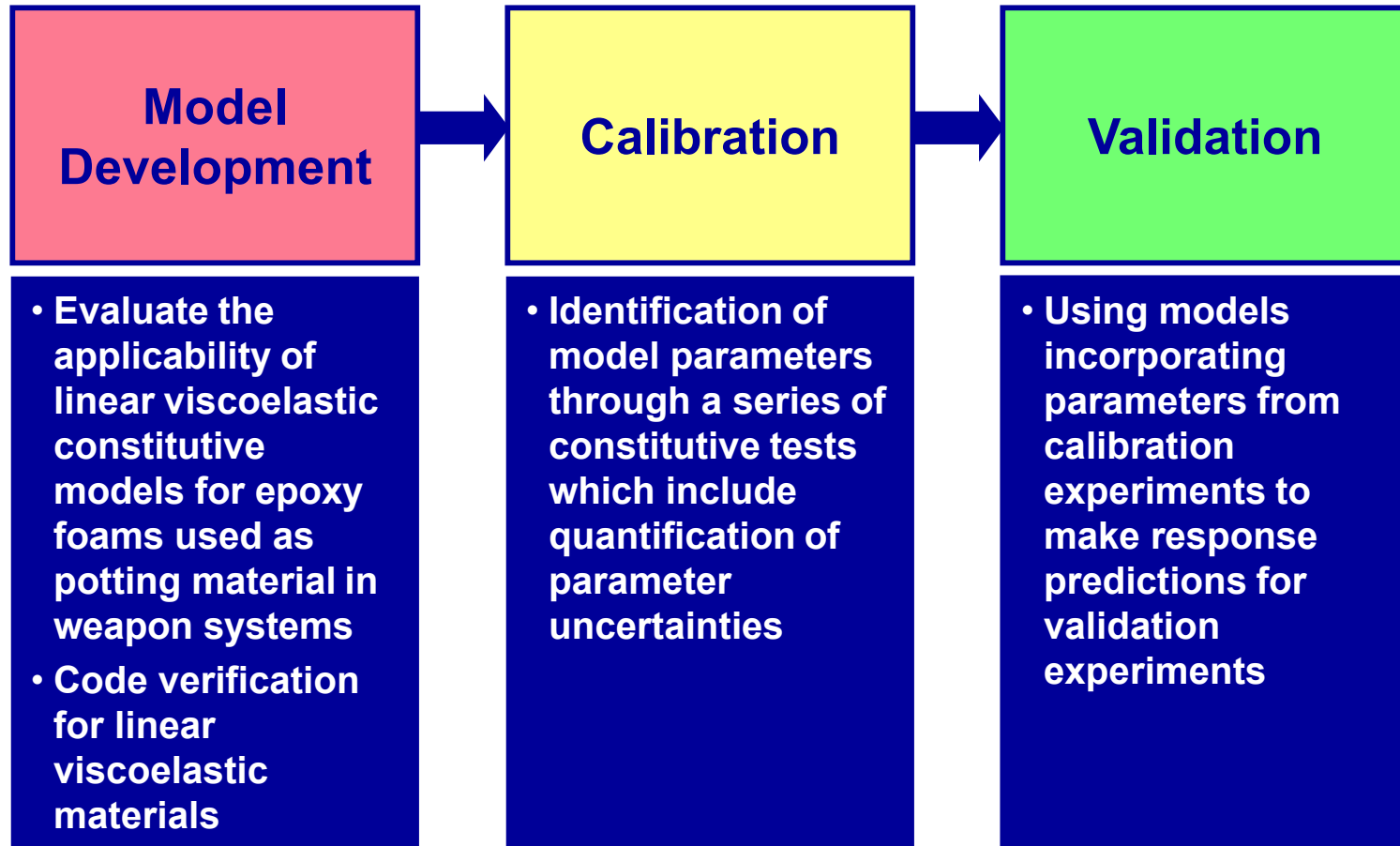
Viscoelastic Model Calibration & Validation Plans, FY06

	Phase 0		Phase 1		Phase 2		Phase 3	
Material	Ambient	L/H Temp	Ambient	L/H Temp	Ambient	L/H Temp	Ambient	L/H Temp
EF-AR20	C	C	V/C	V/C	V	V	V	V
REF 308	C	C	V/C	V/C			V	V
REF 320	C	C	V/C	V/C				
RSF 320	C	C	V/C	V/C				
PMDI 20	C	C	V/C	V/C				
TF6070	C	C	V/C	V/C				

Backup Slides

- Backup
- Slides

Development Process



Linear Viscoelasticity in SALINAS

- Stress is an Integral function of strain: **Convolution Integral**

$$\sigma_{ij}(t) = \int_0^t \hat{G}_{ijkl}(t-s) \frac{d\varepsilon_{kl}}{ds} ds$$

- **Isotropy is assumed:**

$$\sigma_{ij}(t) = \int_0^t 2G(t-s) \frac{d}{ds} \varepsilon_{ij}^d ds + \delta_{ij} \int_0^t K(t-s) \frac{d}{ds} \text{tr}(\varepsilon) ds$$

where $\varepsilon^d = \varepsilon - \delta_{ij} \text{tr}(\varepsilon) / 3$

- **Material functions $G(\)$ and $K(\)$ are selected to reproduce experimental data**

Measure Shear Relaxation Modulus with DMA tests and fit Prony Series:

$$G(t) = G_r + (G_g - G_r) \sum_{j=1}^N m_j \exp(-t / \tau_j)$$

Use same Prony Series for the Bulk modulus and estimate K_g and K_r :

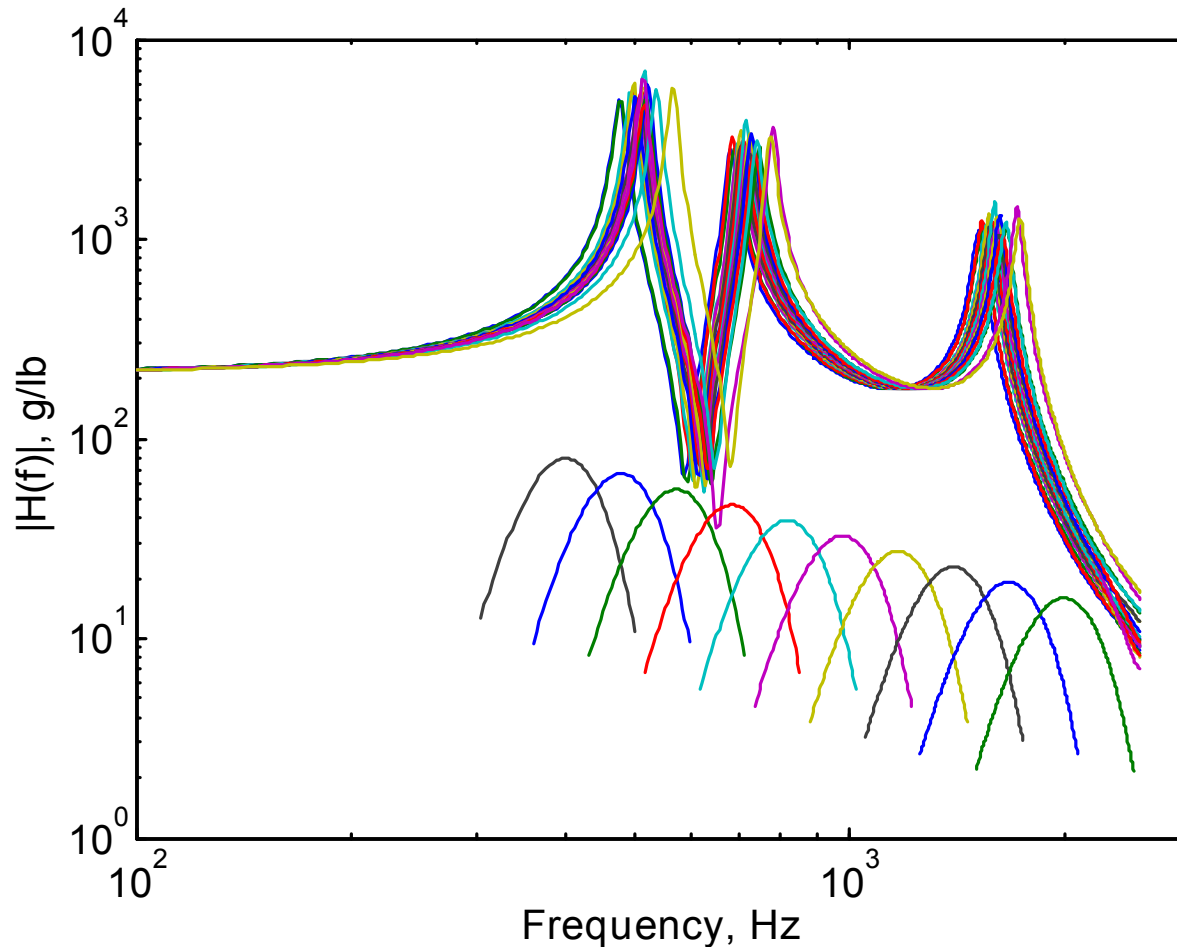
$$K(t) = K_r + (K_g - K_r) \sum_{j=1}^N m_j \exp(-t / \tau_j)$$

A constant value of Poisson's ratio with UQ for this viscoelastic foam will be used based on best estimates from measuring E and G directly in constitutive tests and indirectly from Phase I modal tests.

Assuming Isotropic behavior, Poisson's Ratio is: $\nu = \left(\frac{E}{2G}\right) - 1$

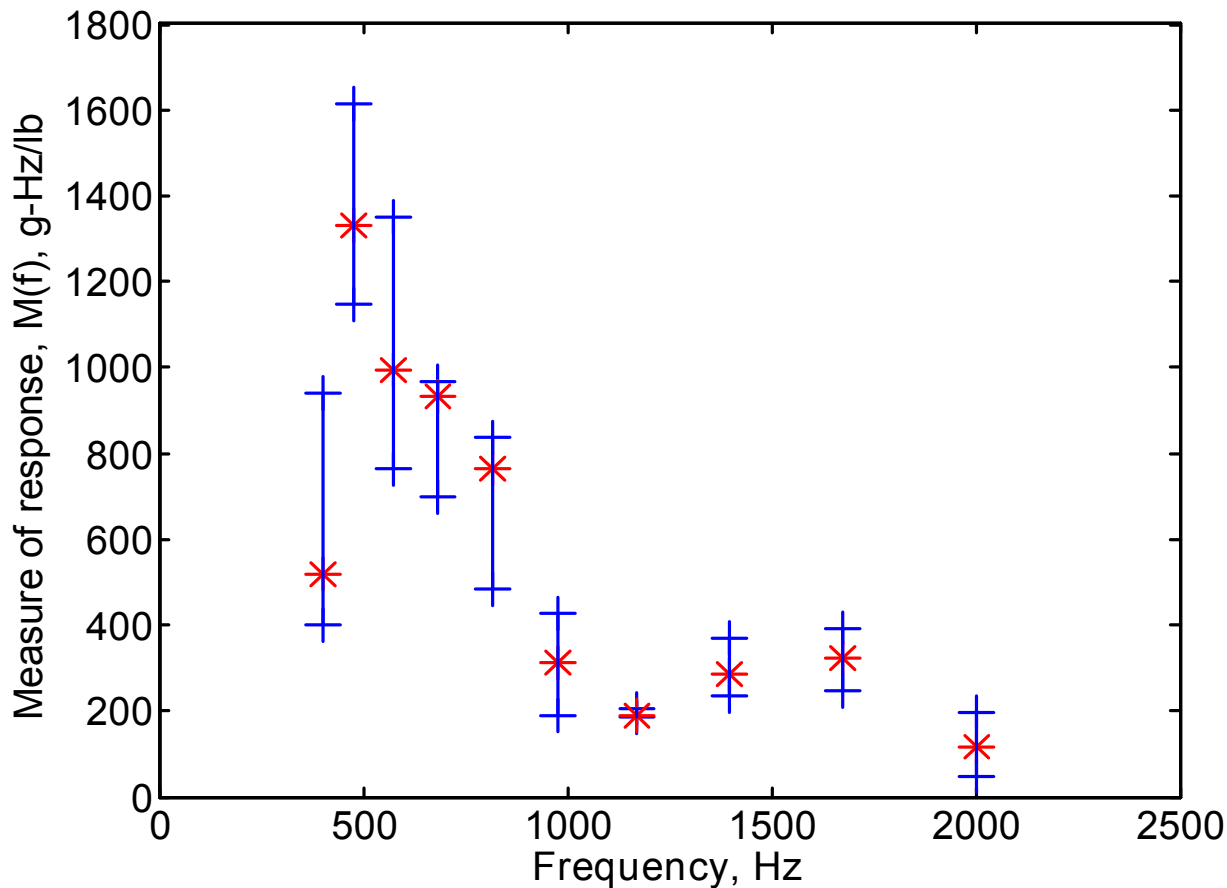
and the Bulk Modulus will be estimated as follows: $K = 2G(1 + \nu) / 3(1 - 2\nu)$

Window Functions for Validation Metric



Example

Phase I Validation Specimen, 16.42 lb/ft³, x – direction, 90% prob intervals $[L_{90}, U_{90}]$ of $M_{mod}(f_c)$ and $M_{exp}(f_c)$



Foam Model Applicability

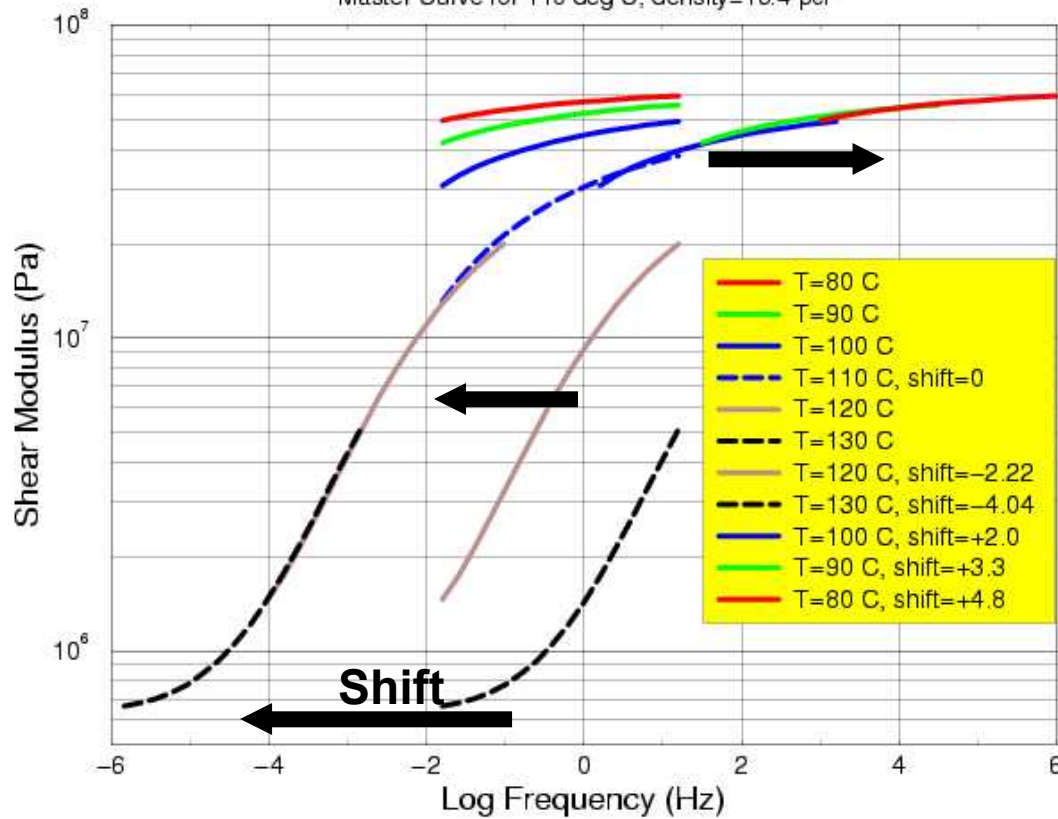
- Stochastic form of foam model suitable for simulation of some measures of response of EF-AR20 foam under some environments at room temperature
- Foam model calibrated via modal analysis and finite element model parameter identification, and validated for hammer impact/decaying response to produce adequate frequency response function predictions in the frequency range [400,2000] Hz, at strain levels up to 0.05 percent
- Model parameters determined throughout density range [14,22] lb/ft³ and suitable for hostile blast level predictions within that range for strains up to 0.05 percent
- No significant evidence found to reject inference that visco-elastic model functions correctly in Salinas code
- Phase I validation success indicates the visco-elastic material model is satisfactorily credible and encourages progressing to next level of validation complexity

Constitutive Experiments

DMA Temperature/Frequency Shifts

EF-AR20 Epoxy Foam – Sample 5

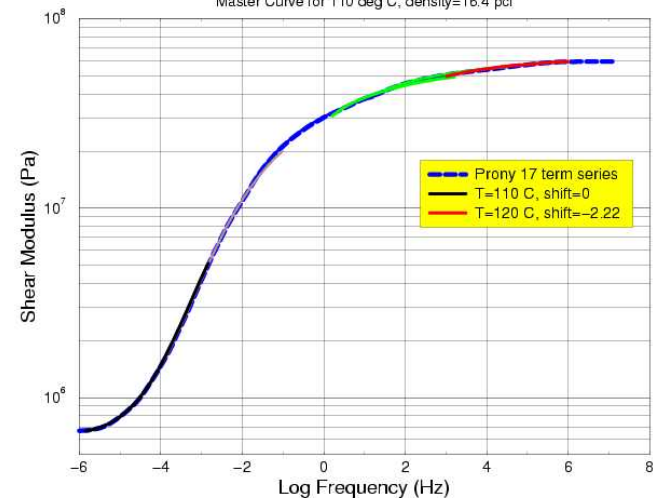
Master Curve for 110 deg C, density=16.4 pcf



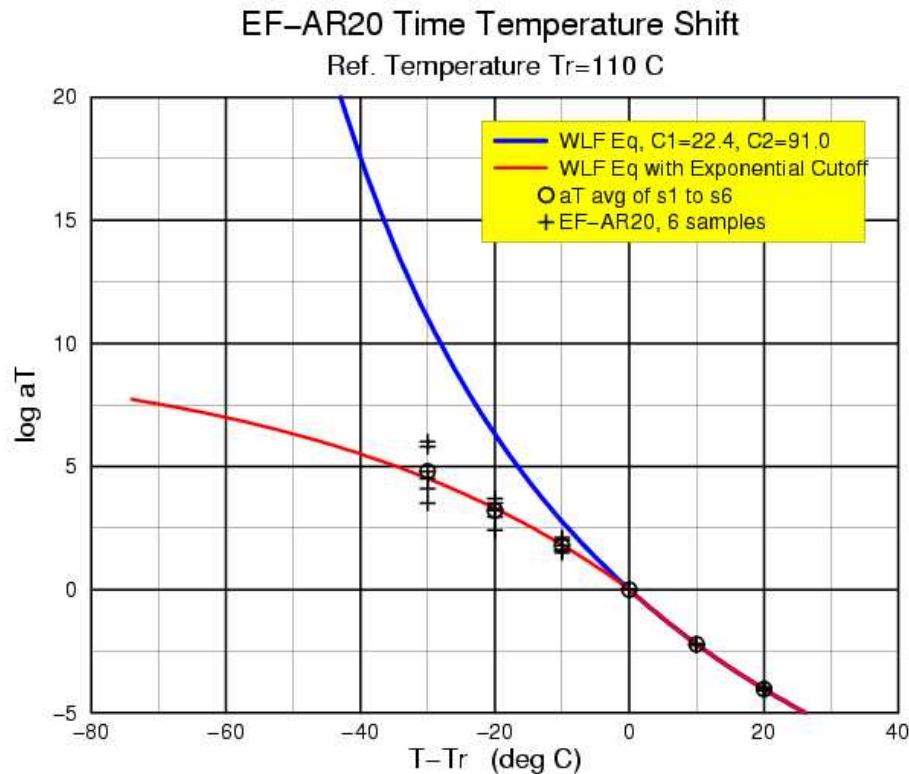
17 Term Prony Series Fit

EF-AR20 Epoxy Foam – Sample 5

Master Curve for 110 deg C, density=16.4 pcf



Time(Frequency)/Temperature Shift Function based on the Master Curve for EF-AR20 Foam



Time domain:

$$G(t) = G_r + (G_g - G_r) \sum_{j=1}^N m_j \exp(-t / a_T \tau_j)$$

Frequency domain:

$$G'(\omega) = G_r + (G_g - G_r) \sum_{j=1}^N \frac{m_j (\omega a_T \tau_j)^2}{(1 + (\omega a_T \tau_j)^2)}$$

where:

$$a_T = t / t_r$$

t_r = reference time

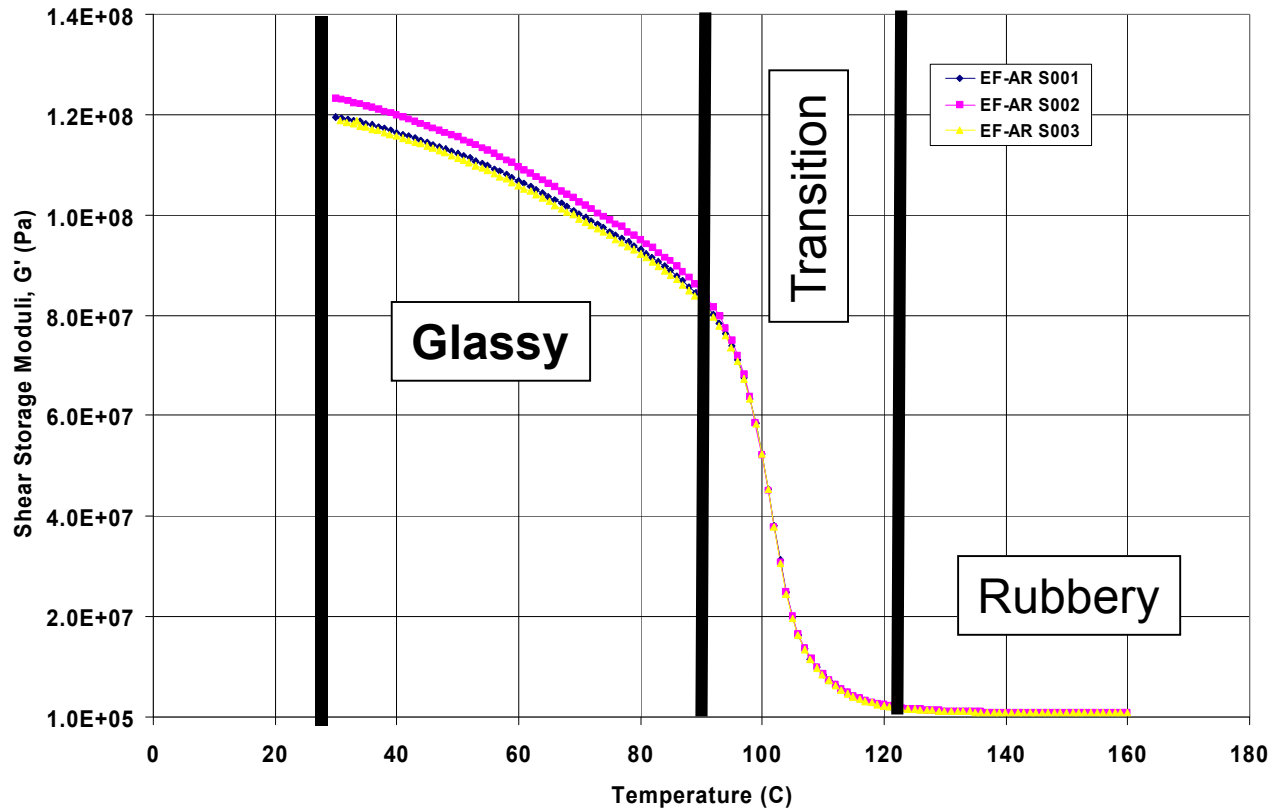
Modified WLF Time-Temperature Shift Function

$$\text{Log}(t / t_r) = (-C_1 * (T - T_r)) / (C_2 + (T - T_r)) \quad T > T_r$$

$$\text{Log}(t / t_r) = a_1 * (1 - \exp(a_2 * (T - T_r))) \quad T < T_r$$

Glassy Modulus depends on Temperature

EF-AR20 Blown Foam DMA Tests (Pre-Test Samples)



Glassy modulus has an approx. linear temperature dependence in the Glassy region that can be combined with the Prony series model as shown below

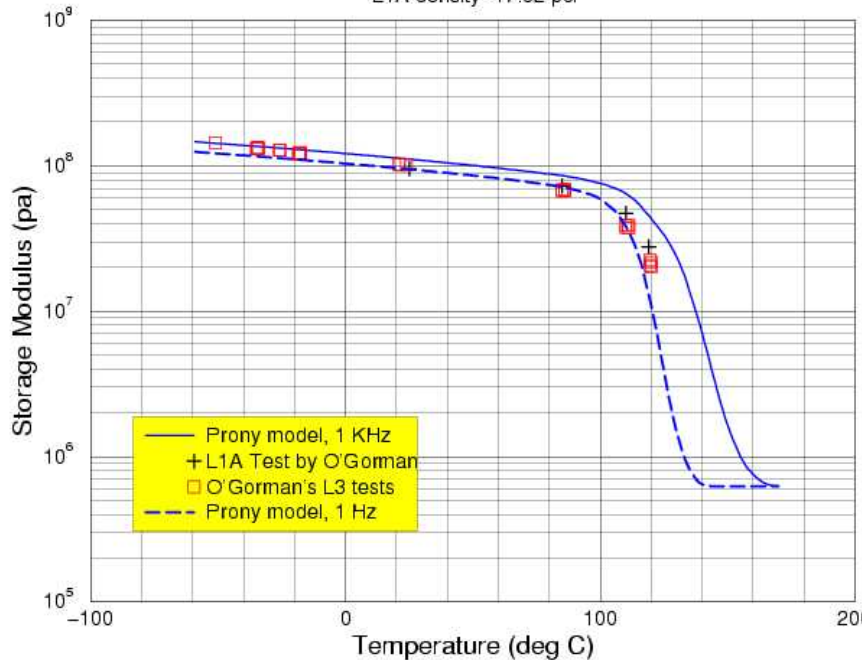
$$G_g = G_{g@T_g} * A_1 * (1 - A_2 * T / T_g)$$

Epoxy Foam Behavior

Modulus & Damping vs Temperature Test Data superimposed on model curves

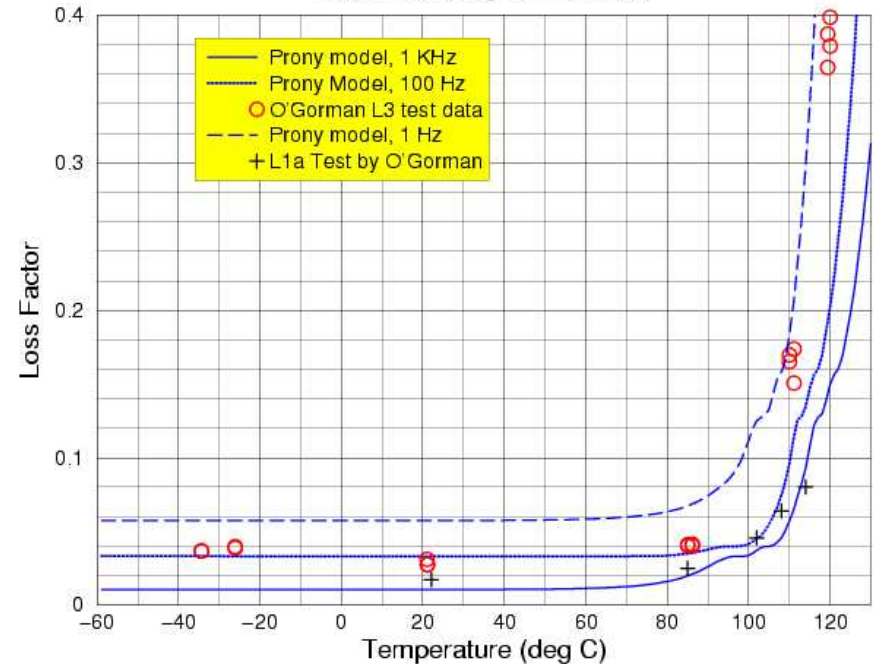
EF-AR20 Model vs Modal Test Data

L1A density=17.82 pcf



Viscoelastic Model for EF-AR Foam

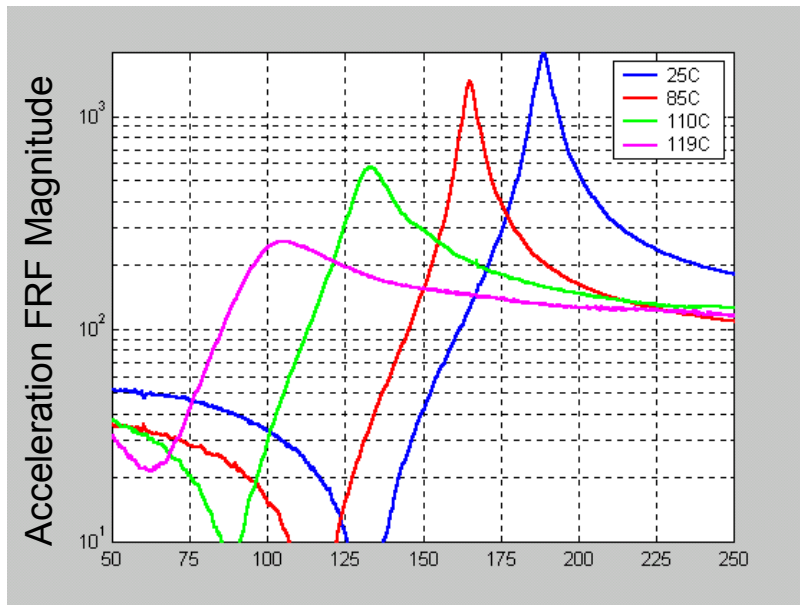
Model Damping vs Test Data





Modal Test Results Compared with SALINAS Viscoelastic Model

Modal Test Data ref. O’Gorman



SALINAS Model Results ref. Hinnerichs

L1A Test Simulation of EF-AR20
Acceleration FRF vs Temperature

