

---

# Development and Validation of a Viscoelastic Foam Model for Encapsulated Components

**Society of Experimental Mechanics Conference, St. Louis**  
**June 5, 2006**

**Terry Hinnerichs, Angel Urbina, Tom Paez, and Chris O'Gorman**  
**Sandia National Laboratories**  
**Albuquerque, NM**

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,  
for the United States Department of Energy under contract DE-AC04-94AL85000.

# Overview

---

- Introduction
- Viscoelastic Model Description
- Model Parameter Identification with UQ
- Model Predictions compared to Test Data
- Conclusions
- Future Work



---

# Foam Model Development and Validation Team

**Chris O'Gorman**

*Lead Experimentalist*

**Patrick Hunter**

*Experimental Modal Data*

**Mark Stavig**

*Constitutive Testing*

**Jim Redmond**

*Team Manager*

**Tom Paez**

*Model Validation*

**Angel Urbina**

*Uncertainty Quantification*

**Brian Rutherford**

*Design of Experiments*

**Fernando Bitsie**

*Model Development*

**Garth Reese**

*Salinas Code Development*

**Tim Walsh**

*Salinas Code Development*

**Terry Hinnerichs**

*Lead Analyst*

**Ed Russick**

*Foam Development*

**Jim Aubert**

*Foam Development*

**Doug Adolf**

*Foam Development*

**Bob Chambers**

*Viscoelastic Modeling*

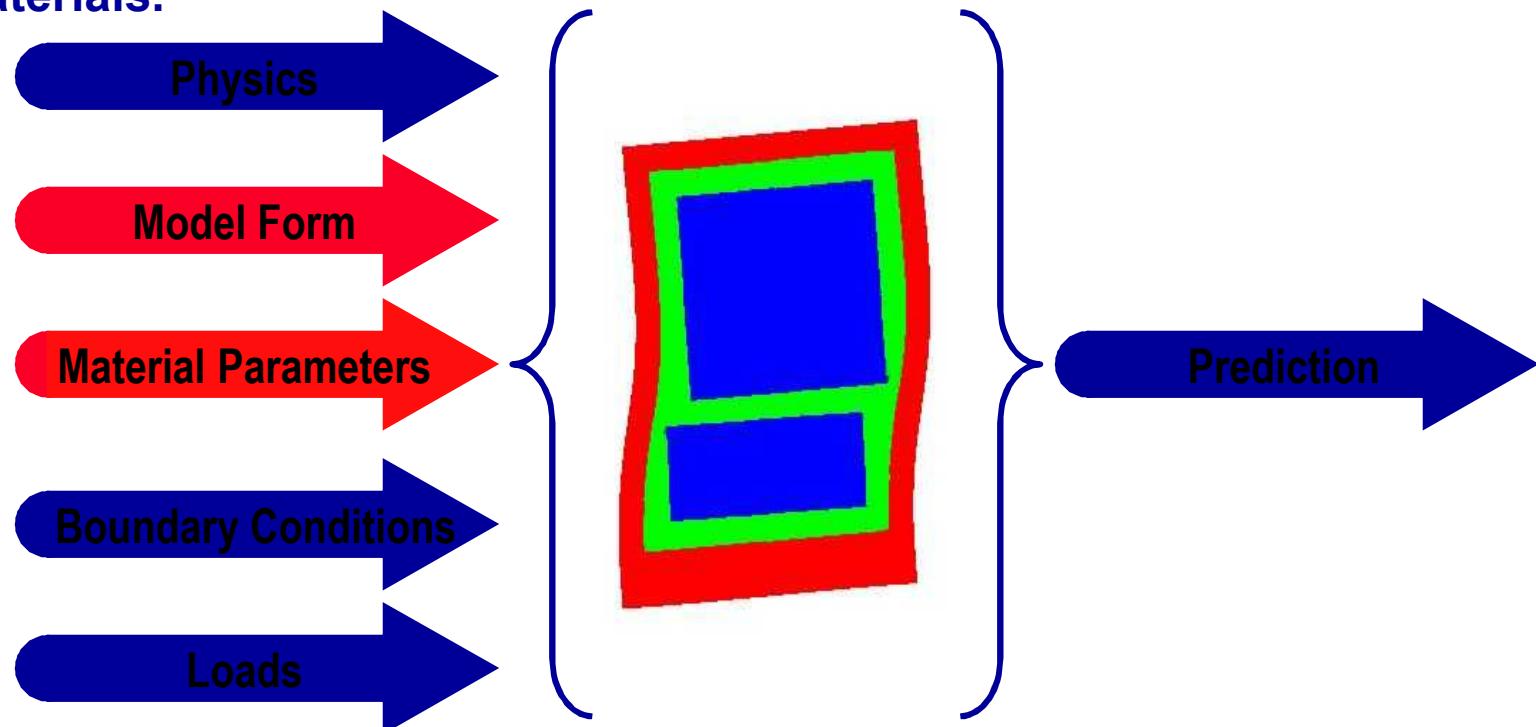
**Dan Segalman**

*Viscoelastic Modeling*

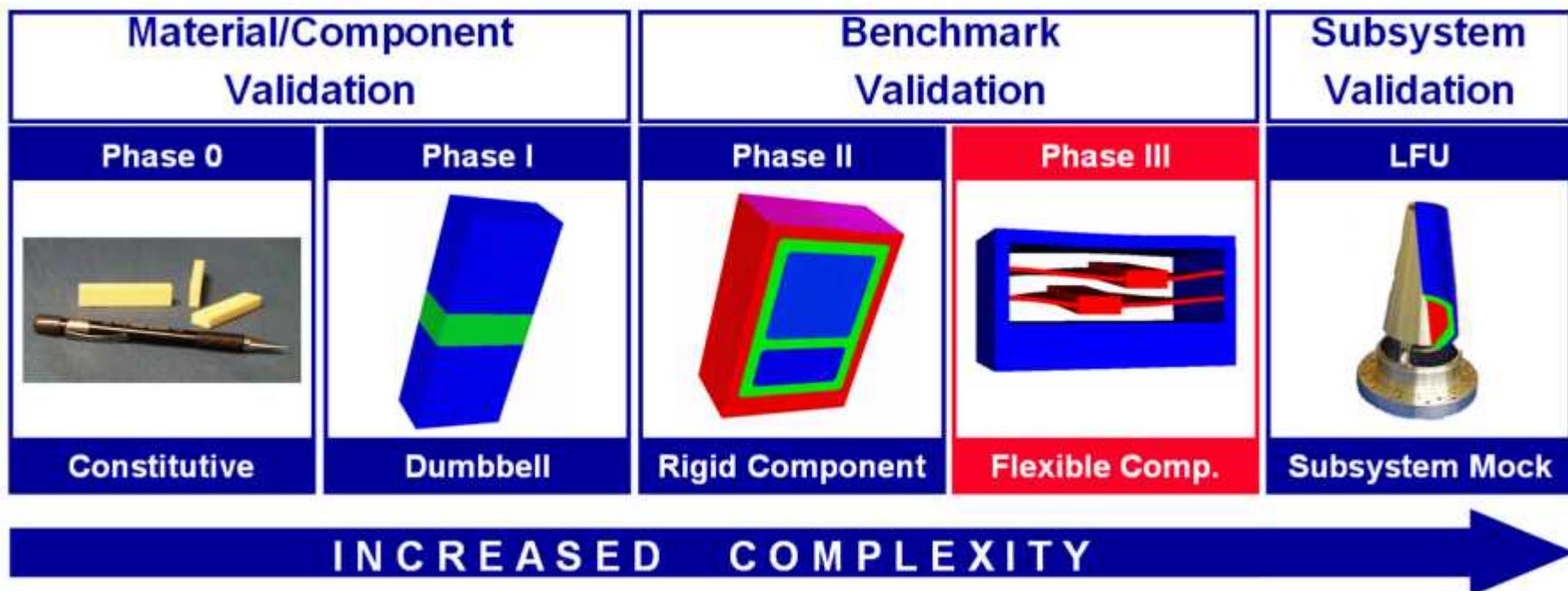


# Project Objective

The objective of this project is to improve the predictive modeling capability of components and systems using foam encapsulants. Specifically, this project seeks to support the development, calibration and validation of finite element models using foam encapsulant materials.

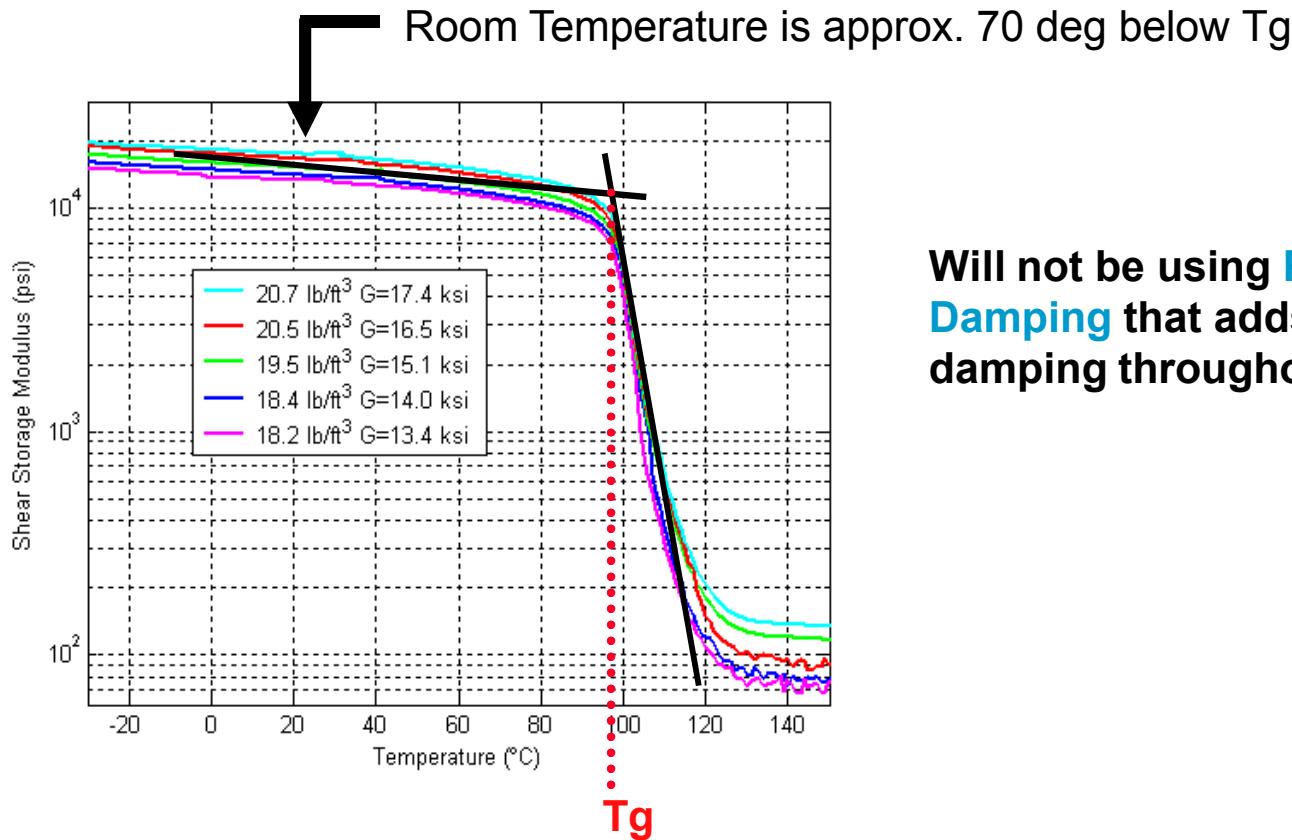


# Foam/Component Development Process



- Project experiments will emphasize the fundamental physics associated of foam-confined components with geometries of increased complexity
- The physical parameter space is significantly simplified, but the **environmental parameter space** involving temperature, frequency, strain and density **will be maintained**

# Goal of Viscoelastic Model Development



Will not be using **Proportional Damping** that adds unwanted damping throughout the structure

**Goal of Viscoelastic Model Development:** Accurately represent the elasticity and predict the **small amount of damping** that is present in the foam at room temperature

# Calibration Procedure for the Viscoelastic Model in the Salinas Code

## Dynamic Mechanical Analysis (DMA)

### Tests provides:

- estimates trend of shear modulus vs. frequency and temperature
- basis for fitting Prony Series
- estimates of material loss factor
- still need second elastic constant
- works best near the glass transition temperature (95 deg C)

## Prony Series

## Viscoelastic Model

## Phase I Modal Tests

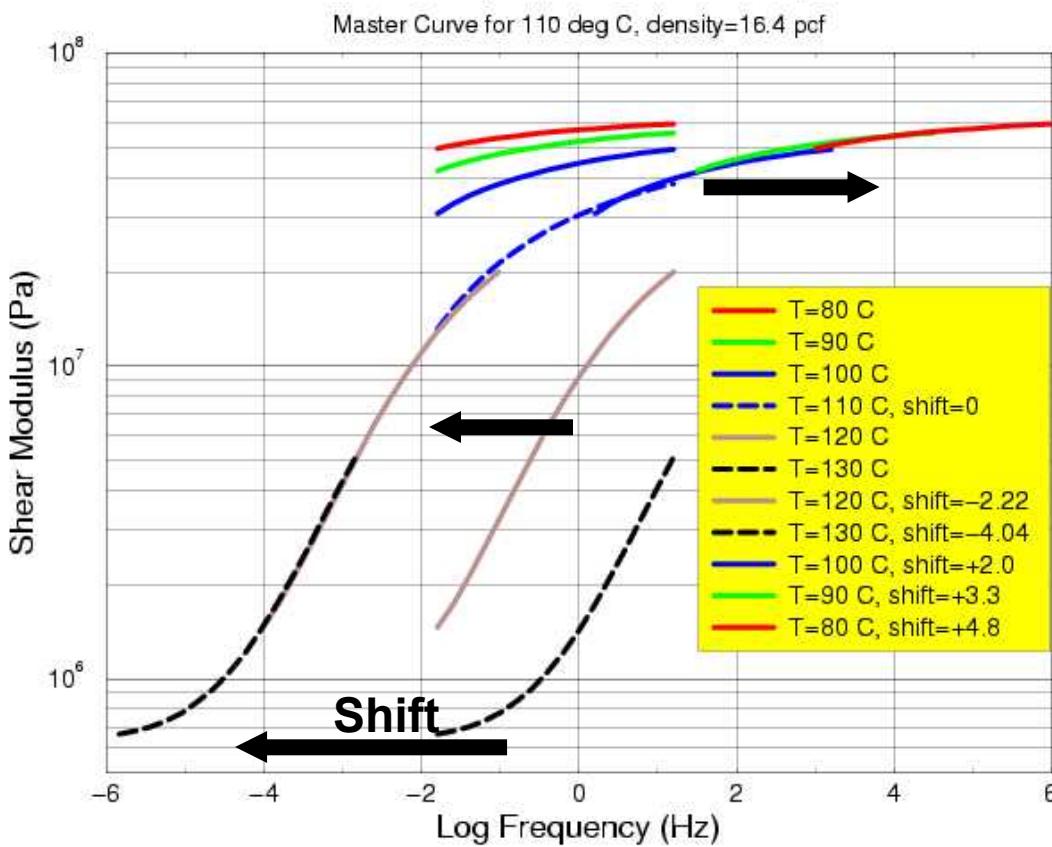
- provides modal frequencies and damping
- analytically back out E and G with Salinas by matching test modes

## Young's Modulus, Shear Modulus and damping

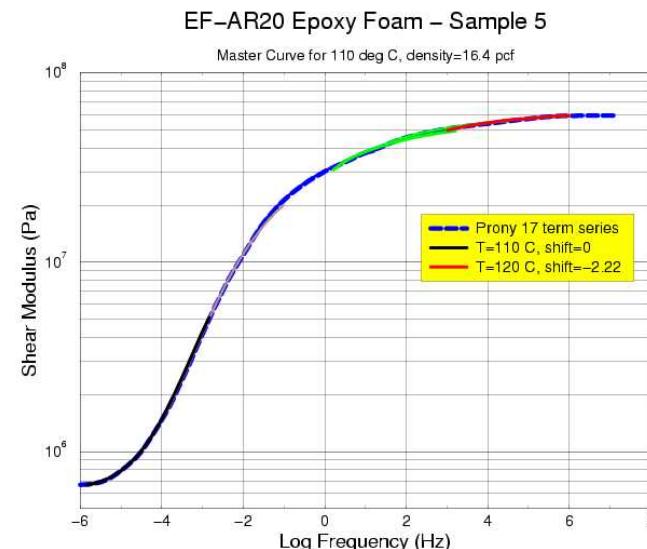
# Constitutive Experiments

## DMA Temperature/Frequency Shifts

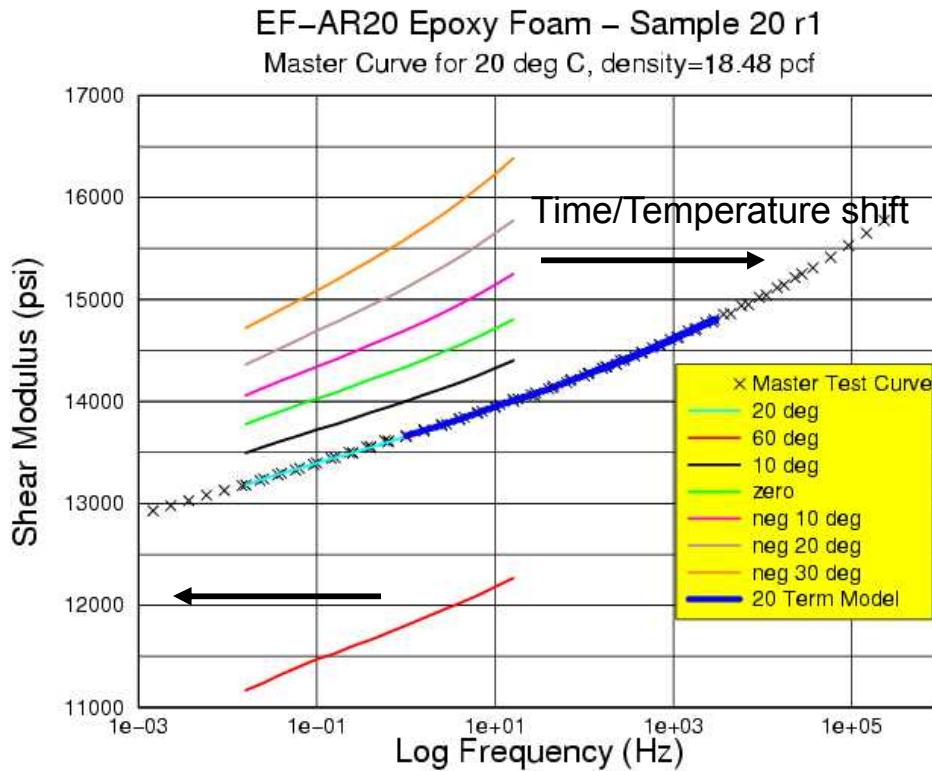
EF-AR20 Epoxy Foam – Sample 5



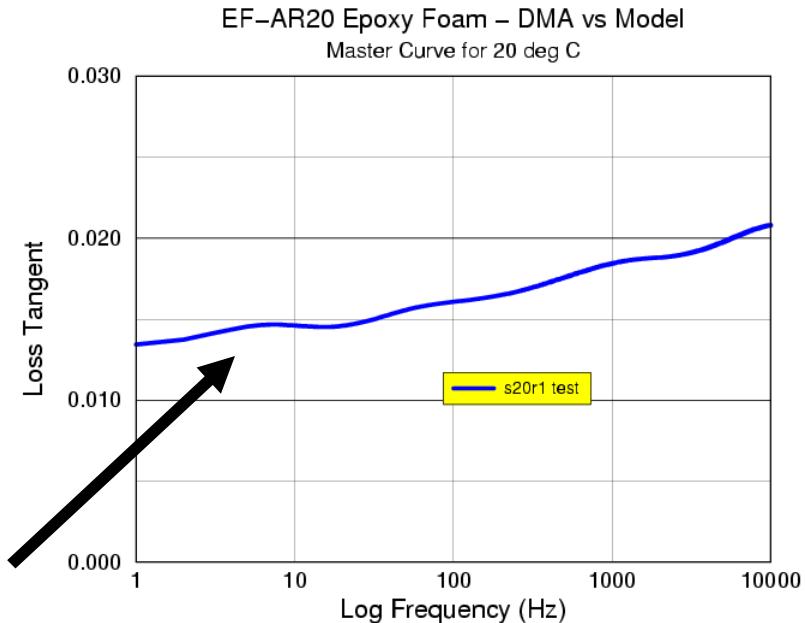
### 17 Term Prony Series Fit



# Prony Model of Master Curve at Room Temperature

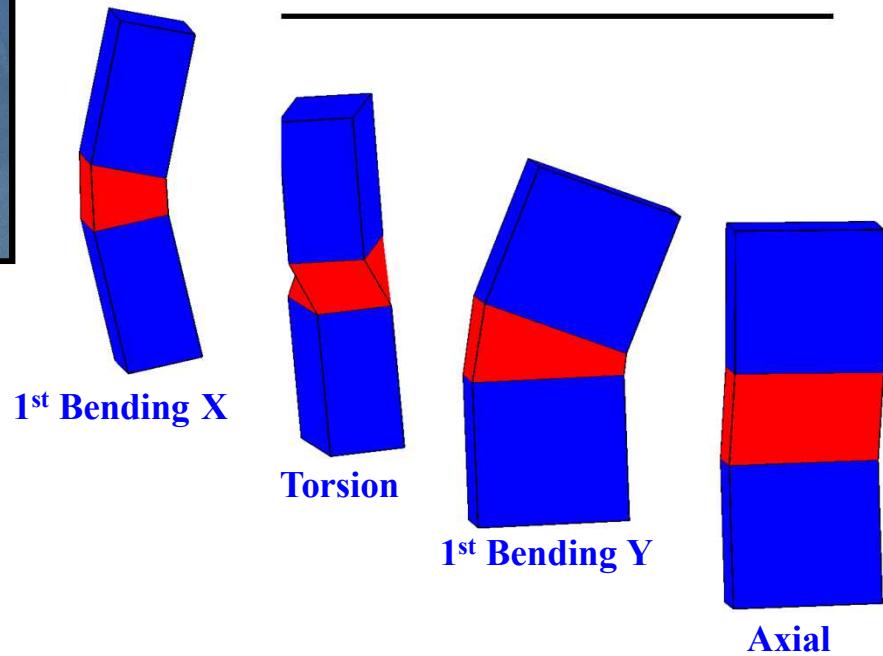
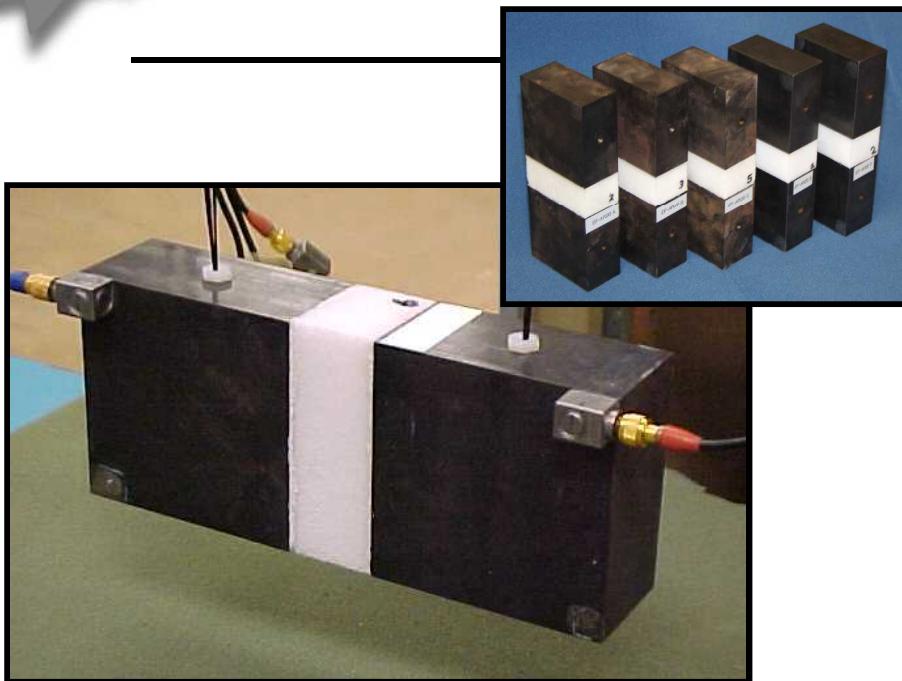


- Individual DMA curves shifted to form Master Curve
- 20 Term Prony Series fit to Master Curve (blue curve)



Resulting master curve for Loss Tangent  
(damping = 0.5 LT for Phase I)

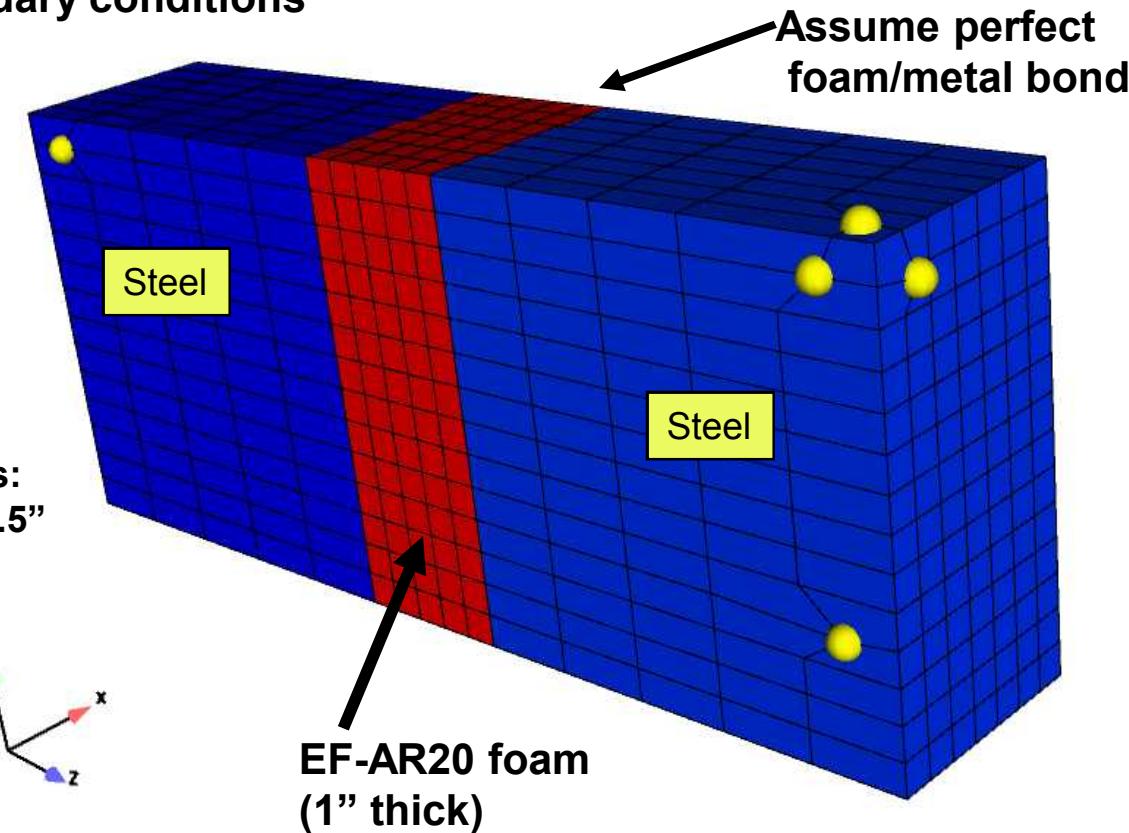
# EF-AR20 Phase I Calibration Data



- Phase I samples used to supplement constitutive data
- Mechanism to test various frequencies at room temperature
- Low strain helps minimize uncertainties caused by cellular interaction
- 27 Phase I baseline samples
- 6 Phase I Calibration samples
- 5 Phase I Validation samples

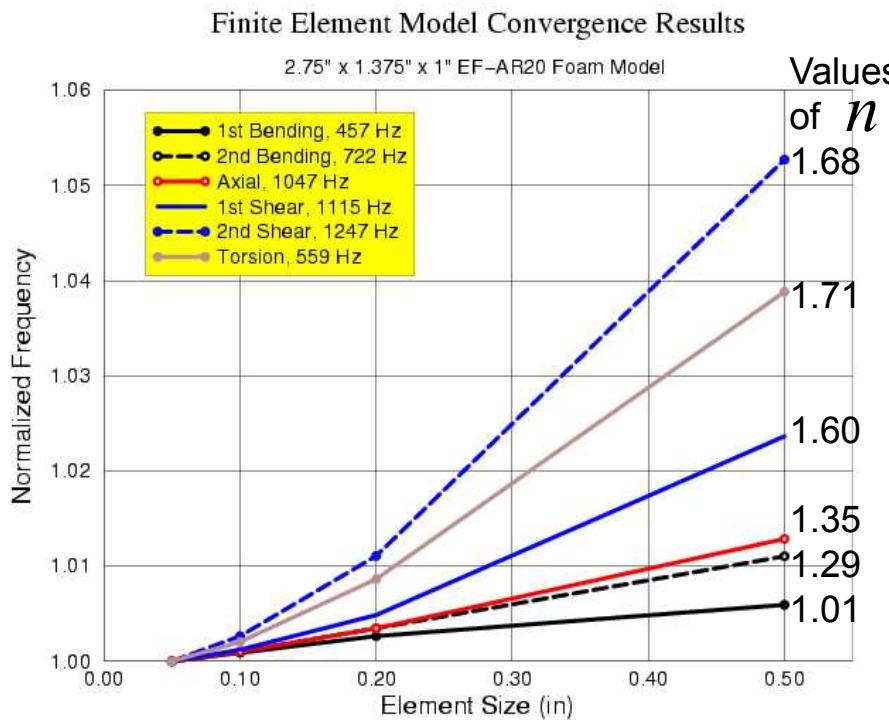
# Phase I Calibration Finite Element Model

- 1470 eight node hex elements
- 1920 nodes
- yellow spheres designate input/output node set locations
- free-free boundary conditions

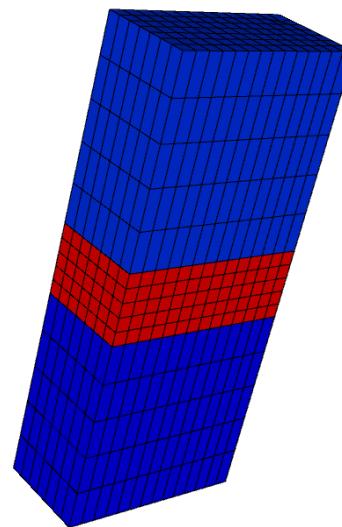


Outer Dimensions:  
1.375" x 2.75" x 6.5"

# Phase 1 Model Convergence using Modal Frequencies



**0.2" element selected  
for computational  
efficiency**



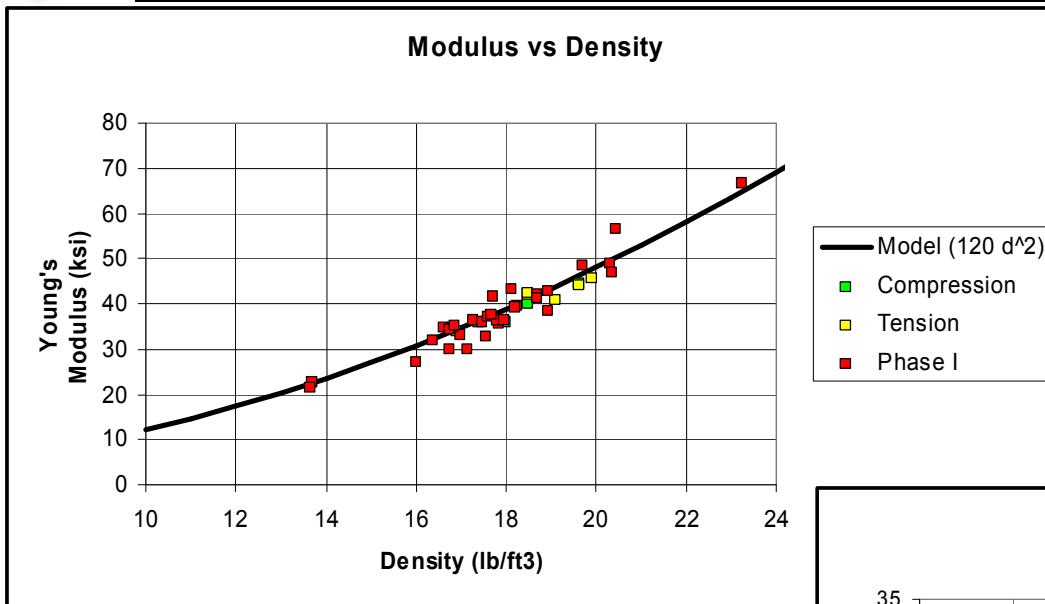
8 node hexahedral  
elements used

Richardson Extrapolation:  $E(h) = E' + Ch^n$

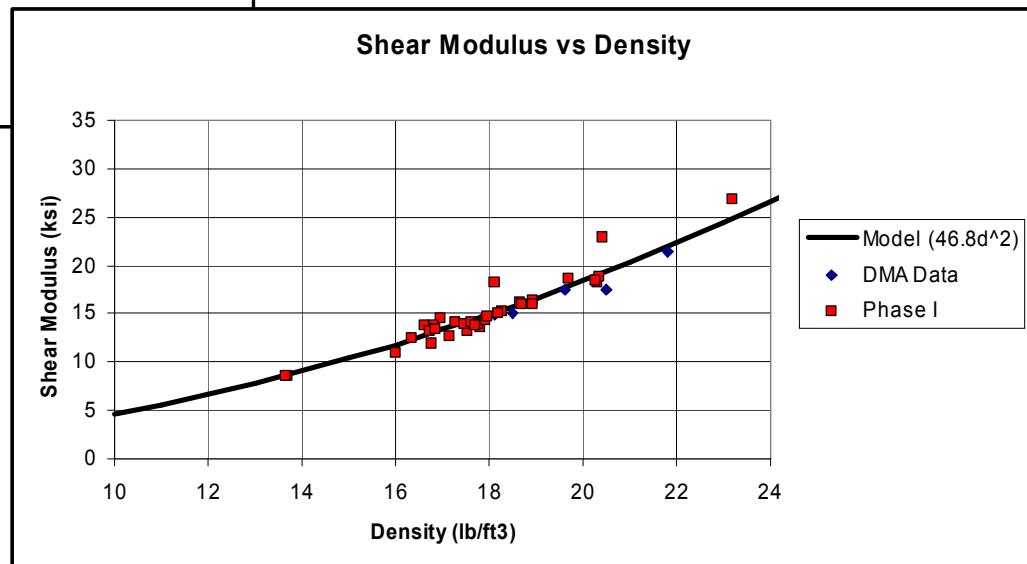
Exponent  $n$ , has values given above, ranging from 1.01 to 1.71

Normalized error,  $(E - E') / E'$ , same as shown in plot at  $h = 0.2$ , **less than 1.3% error**

# Experimental/Analytical Estimating Results for EF-AR20 Modulus vs Density



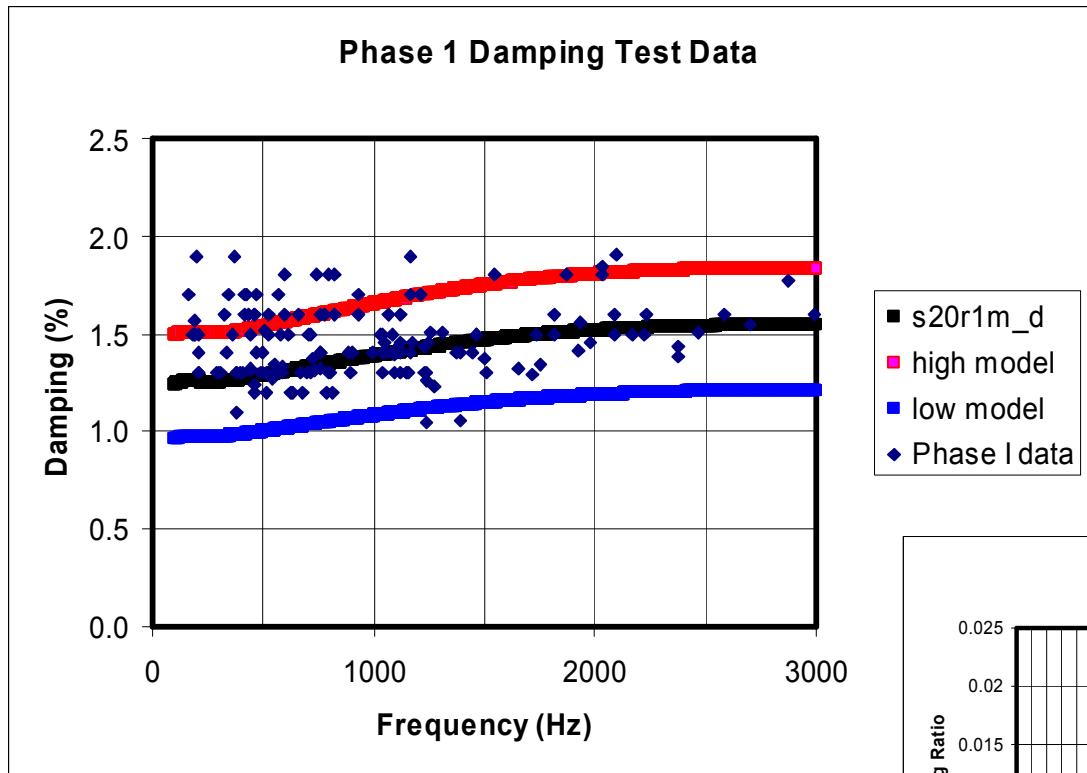
- Plot of experimentally inferred Young's and Shear modulus as a function of density (dots)
- Perform regression to estimate parameter of squared function fit
- Correlation of E and G residuals will be matched also



- Modulus proportional to density squared
- Results compare well with models from Gibson & Ashby's "Cellular Solids":

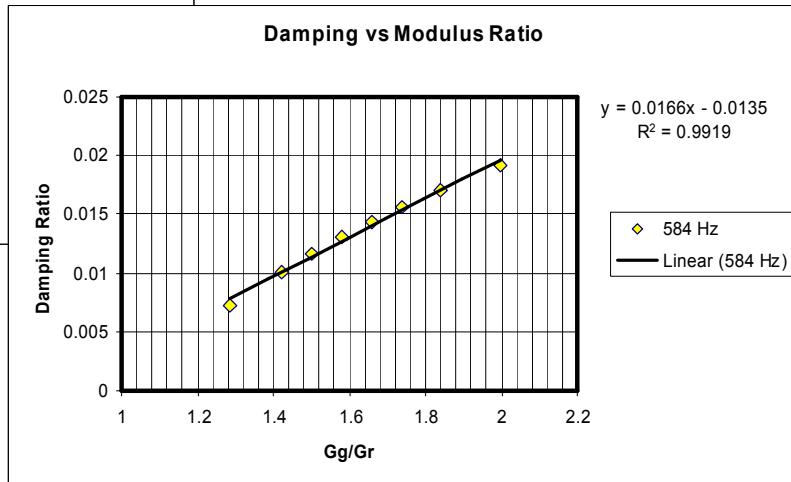
$$E = E_s \left( \frac{\rho}{\rho_s} \right)^2$$

# Phase I Experimental Damping versus Viscoelastic Model Damping

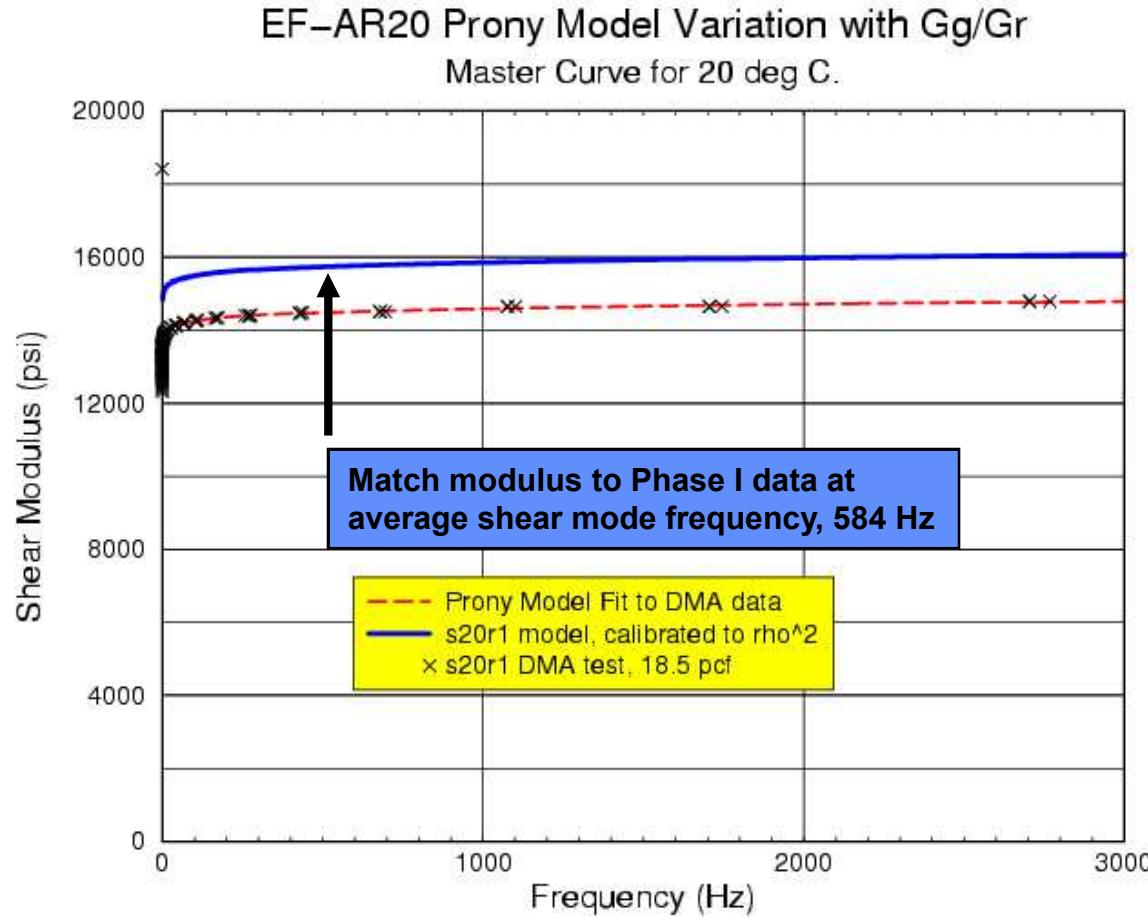


Will treat damping as a random variable

- Three Prony series realizations shown with colored curves
- damping will be specified via modulus ratios as shown below

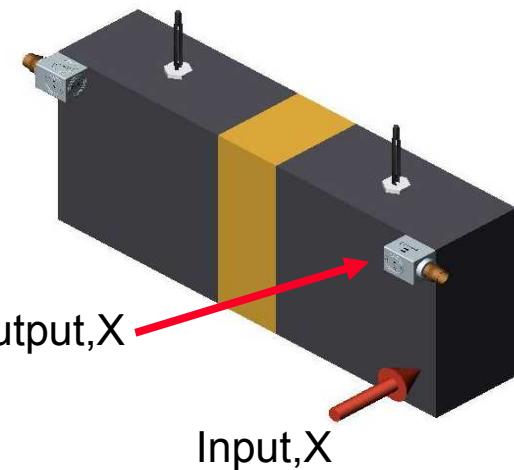
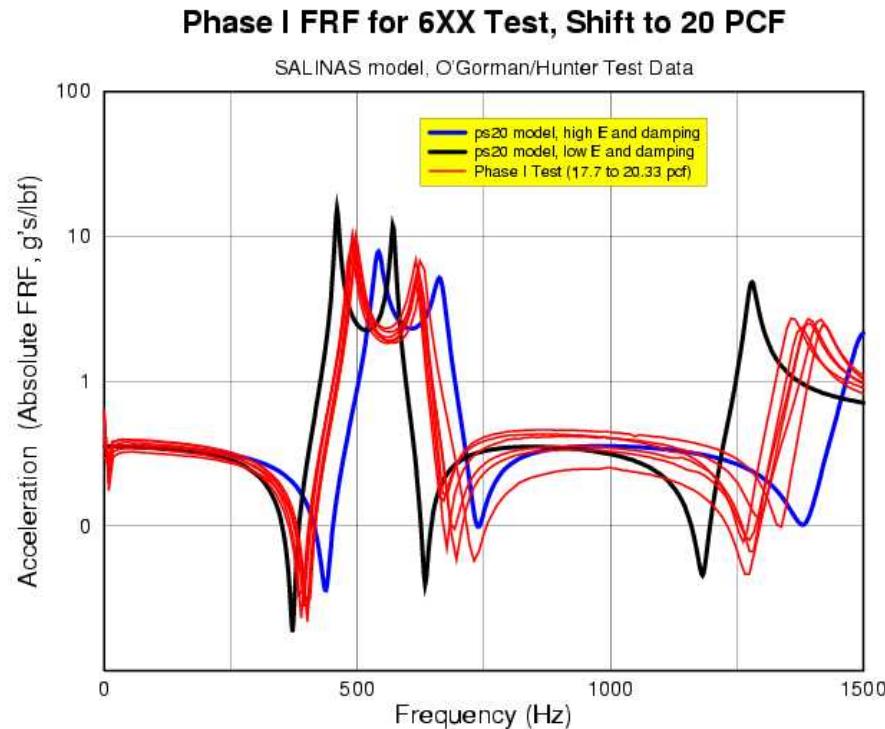


# Matching Prony Series Model with Phase I Test



Prony Series model is matched to Phase I derived modulus values by shifting the curve with modified Gg and Gr values

# SALINAS Viscoelastic Model Predictions of Phase I Calibration Tests with EF-AR20 Foam

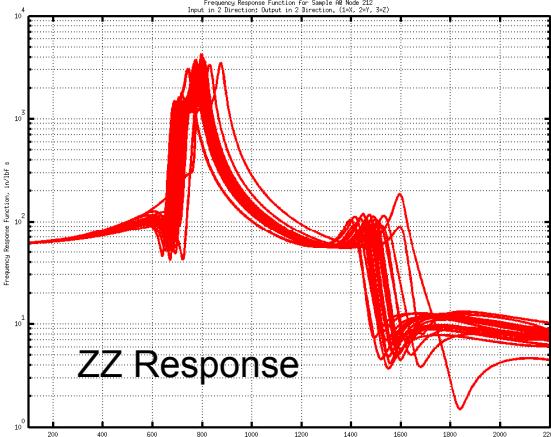
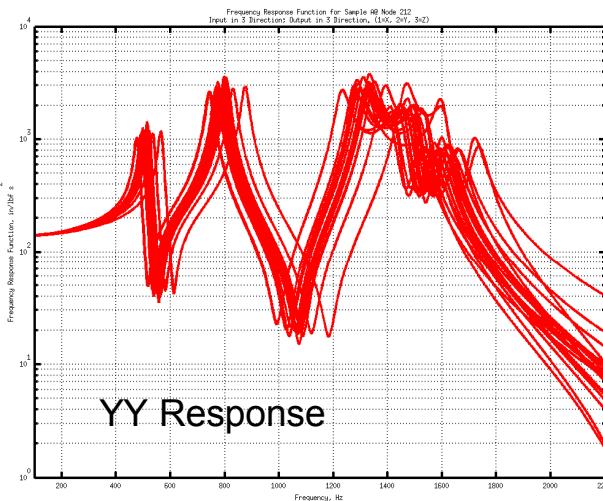
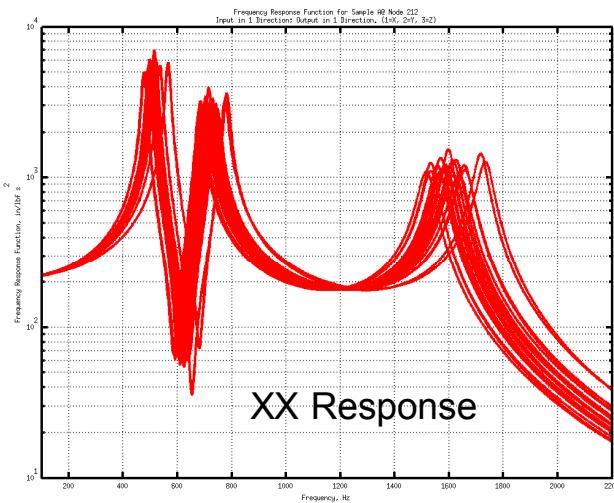


6 sets of test data  
shifted to common  
density of 20 pcf

Model brackets composite of test data between high  
modulus/damping and low modulus/damping predictions

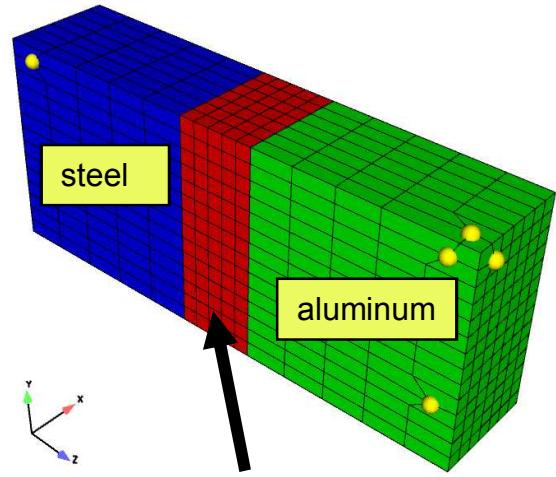
$$f_2 / f_1 = \sqrt{E_2 / E_1} = \rho_2 / \rho_1$$

# Phase I Validation Model Predictions For Free-Free Conditions



20 sets of random variables generated to represent PDF of:

- E - Young's Modulus
- G - Shear Modulus
- Density
- Damping



EF-AR20 foam

# Phase I Experiment/Model Prediction

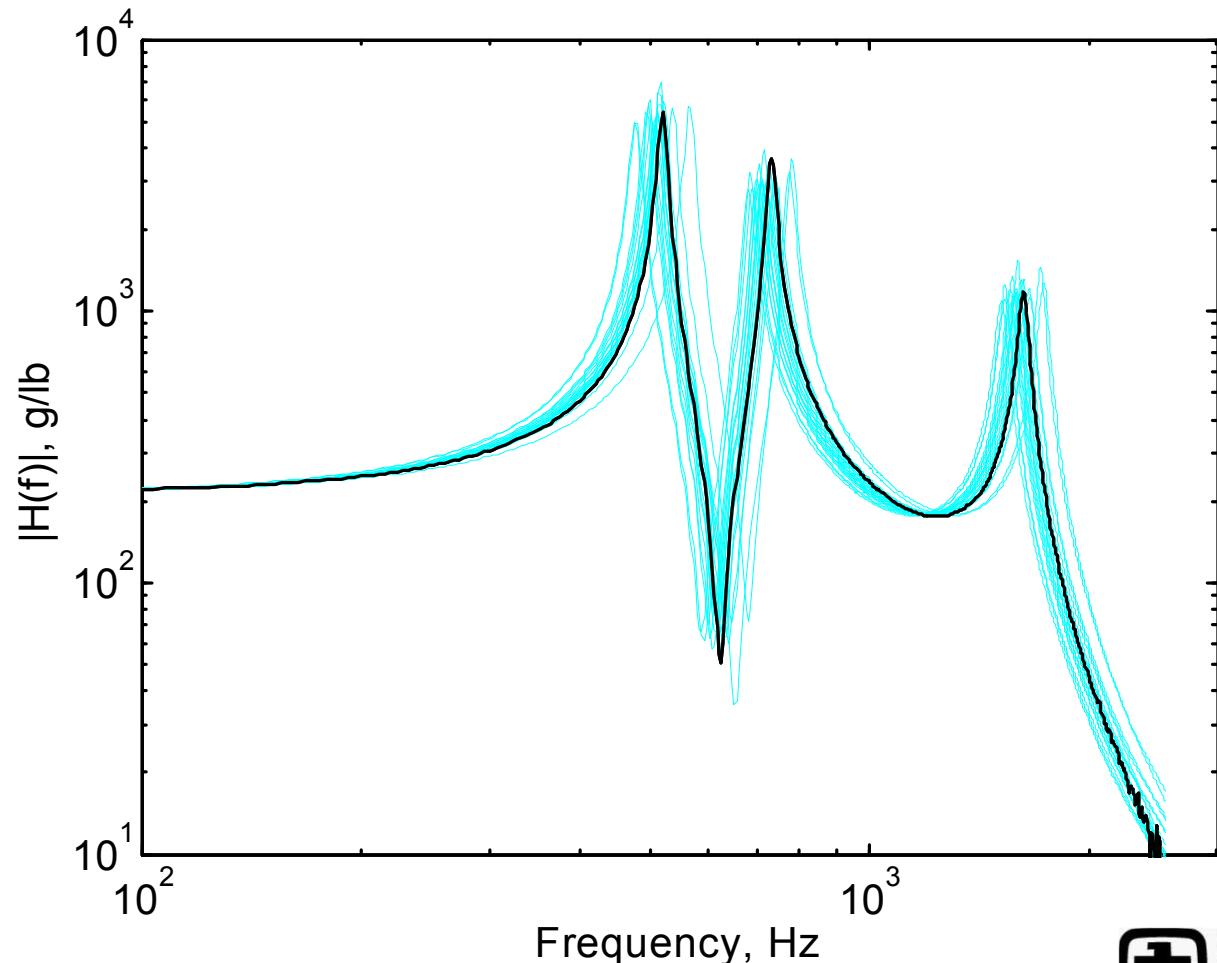
$x$  – direction,  $\rho = 16.42 \text{ lb/ft}^3$

## Experiment

- One realization
- At each of five densities
- Tested in three directions

## Model

- 20 Realizations
- At each of the five densities
- Predicted in three directions



# Summary & Conclusions

## Foam/Component Mechanical Modeling

---

- Model convergence adequacy demonstrated
- Salinas viscoelastic model parameters identified
  - Fit Prony series to master shear modulus curve for room temperature based on DMA test data
  - Fine tuned viscoelastic model derived from DMA tests with modulus and damping measured in Phase I Tests
- Random variables modeled:
  - Young's and Shear modulus with correlation
  - Damping and Density
- Phase I model is calibrated and model validation predictions computed

# FY06 Foam Modeling Plans

---

## Viscoelastic Model Calibration & Validation Plans, FY06

|          | Phase 0 |          | Phase 1 |          | Phase 2 |          | Phase 3 |          |
|----------|---------|----------|---------|----------|---------|----------|---------|----------|
| Material | Ambient | L/H Temp |
| EF-AR20  | C       | C        | V/C     | V/C      | V       | V        | V       | V        |
| REF 308  | C       | C        | V/C     | V/C      |         |          | V       | V        |
| REF 320  | C       | C        | V/C     | V/C      |         |          |         |          |
| RSF 320  | C       | C        | V/C     | V/C      |         |          |         |          |
| PMDI 20  | C       | C        | V/C     | V/C      |         |          |         |          |
| TF6070   | C       | C        | V/C     | V/C      |         |          |         |          |

## Backup Slides

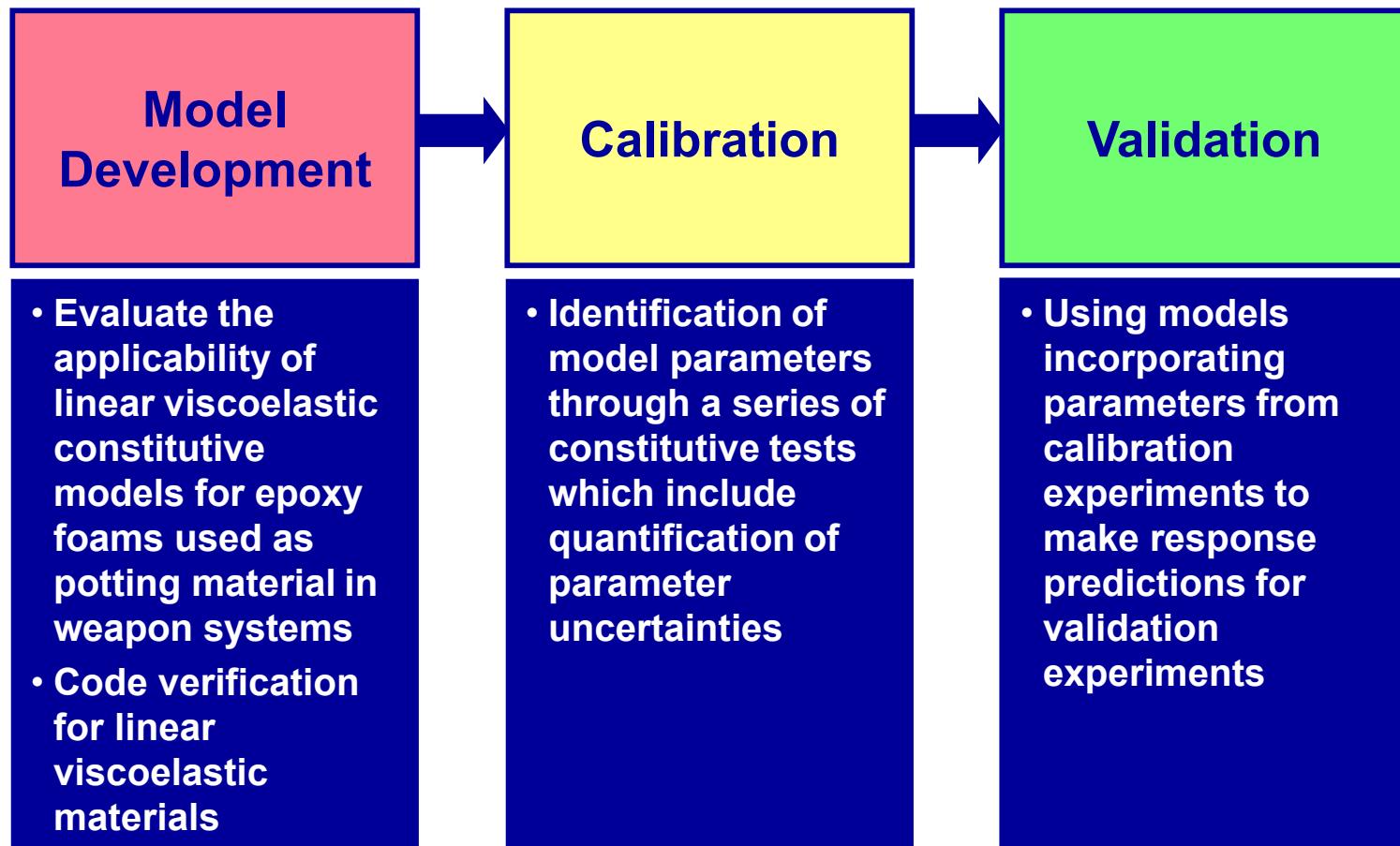
---

- Backup
- Slides



# Development Process

---



# Linear Viscoelasticity in SALINAS

- Stress is an Integral function of strain: Convolution Integral

$$\sigma_{ij}(t) = \int_0^t \hat{G}_{ijkl}(t-s) \frac{d\varepsilon_{kl}}{ds} ds$$

- Isotropy is assumed:

$$\sigma_{ij}(t) = \int_0^t 2G(t-s) \frac{d}{ds} \varepsilon_{ij}^d ds + \delta_{ij} \int_0^t K(t-s) \frac{d}{ds} \text{tr}(\varepsilon) ds$$

where  $\varepsilon^d = \varepsilon - \delta_{ij} \text{tr}(\varepsilon)/3$

- Material functions  $G(\cdot)$  and  $K(\cdot)$  are selected to reproduce experimental data



# Model Form in SALINAS Code

**Measure Shear Relaxation Modulus with DMA tests and fit Prony Series:**

$$G(t) = G_r + (G_g - G_r) \sum_{j=1}^N m_j \exp(-t / \tau_j)$$

**Use same Prony Series for the Bulk modulus and estimate  $K_g$  and  $K_r$ :**

$$K(t) = K_r + (K_g - K_r) \sum_{j=1}^N m_j \exp(-t / \tau_j)$$

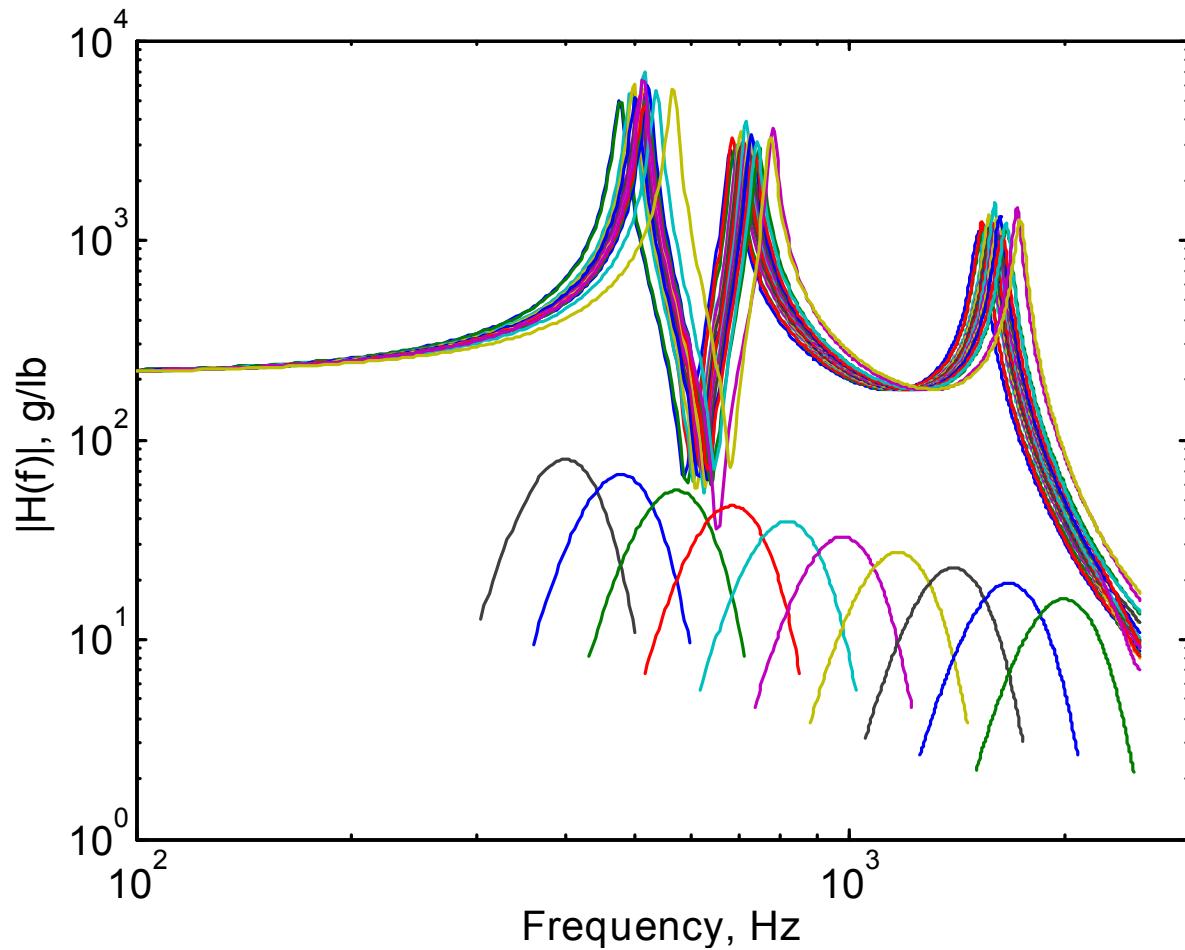
**A constant value of Poisson's ratio with UQ for this viscoelastic foam will be used based on best estimates from measuring E and G directly in constitutive tests and indirectly from Phase I modal tests.**

**Assuming Isotropic behavior, Poisson's Ratio is:**  $\nu = \left(\frac{E}{2G}\right) - 1$

**and the Bulk Modulus will be estimated as follows:**  $K = 2G(1+\nu)/3(1-2\nu)$

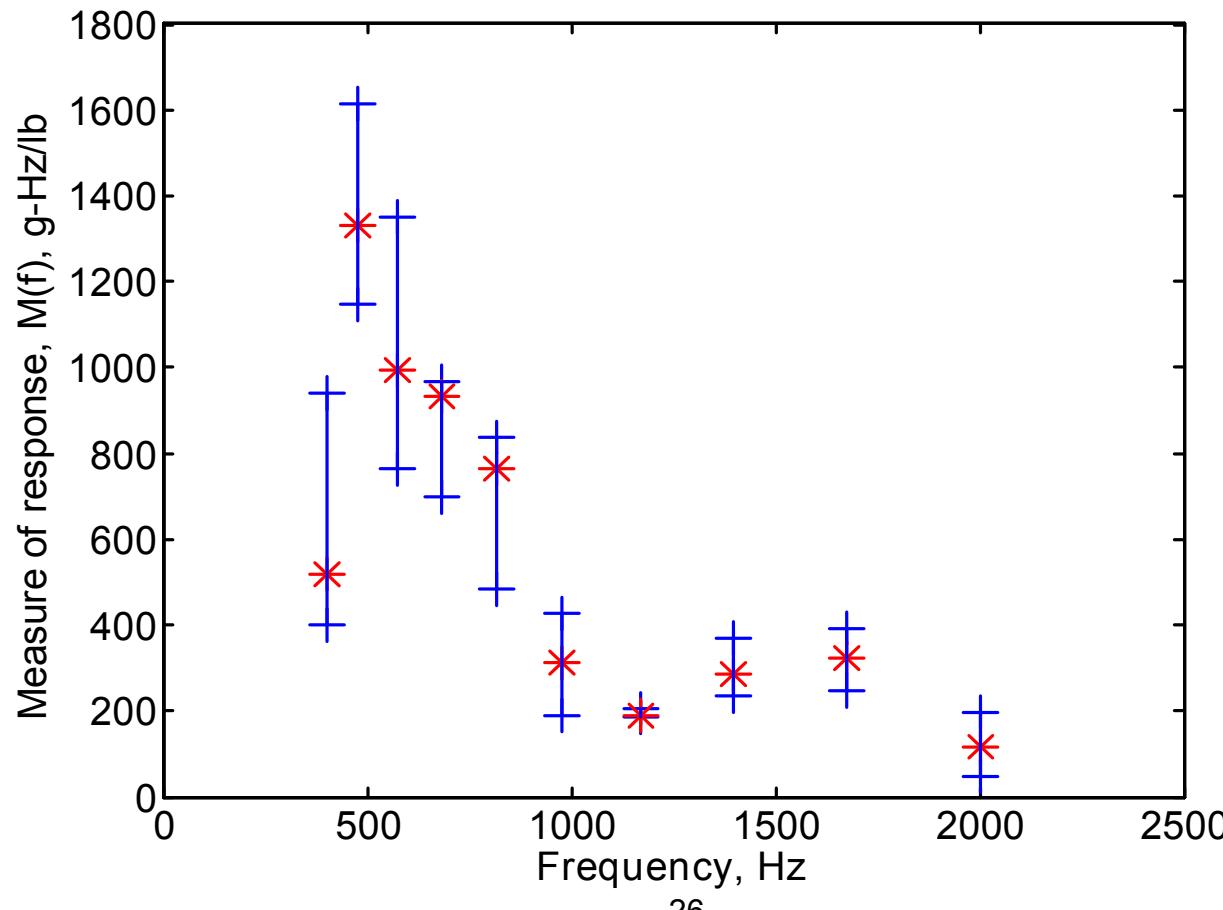
# Window Functions for Validation Metric

---



# Example

Phase I Validation Specimen, 16.42 lb/ft<sup>3</sup>, x – direction, 90% prob intervals [ $L_{90}, U_{90}$ ] of  $M_{mod}(f_c)$  and  $M_{exp}(f_c)$



# Foam Model Applicability

---

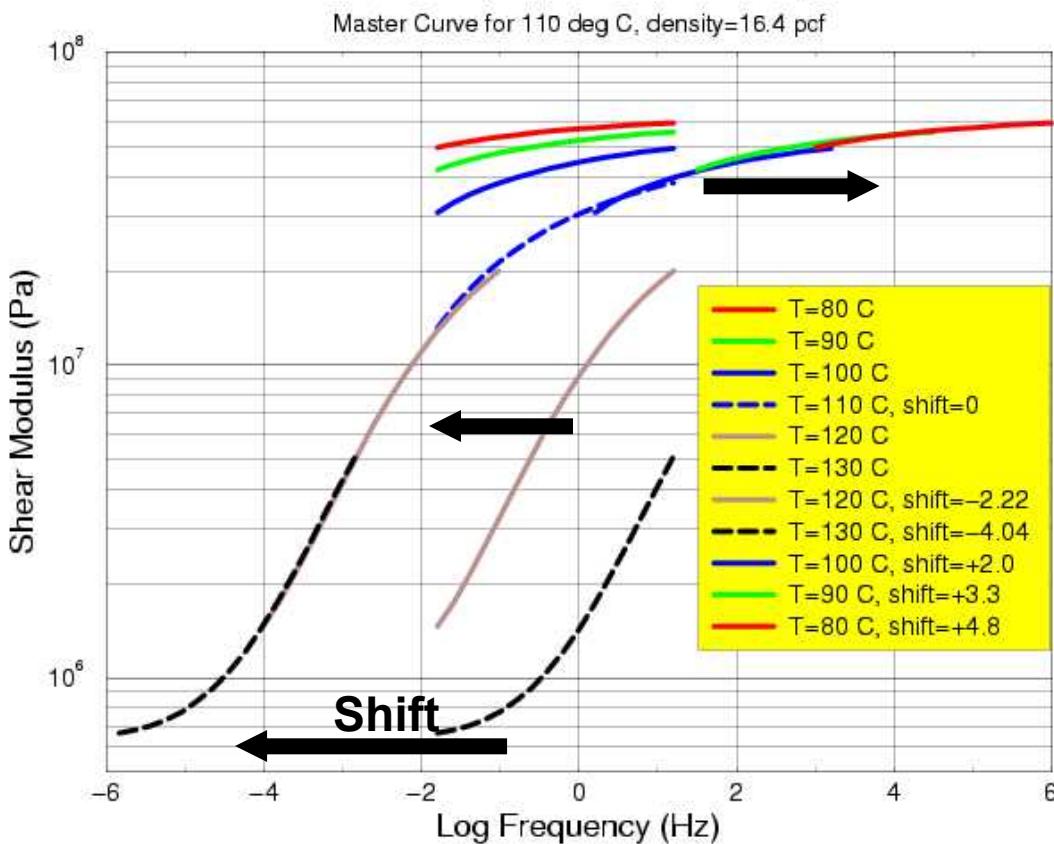
- **Stochastic form of foam model suitable for simulation of some measures of response of EF-AR20 foam under some environments at room temperature**
- **Foam model calibrated via modal analysis and finite element model parameter identification, and validated for hammer impact/decaying response to produce adequate frequency response function predictions in the frequency range [400,2000] Hz, at strain levels up to 0.05 percent**
- **Model parameters determined throughout density range [14,22] lb/ft<sup>3</sup> and suitable for hostile blast level predictions within that range for strains up to 0.05 percent**
- **No significant evidence found to reject inference that visco-elastic model functions correctly in Salinas code**
- **Phase I validation success indicates the visco-elastic material model is satisfactorily credible and encourages progressing to next level of validation complexity**



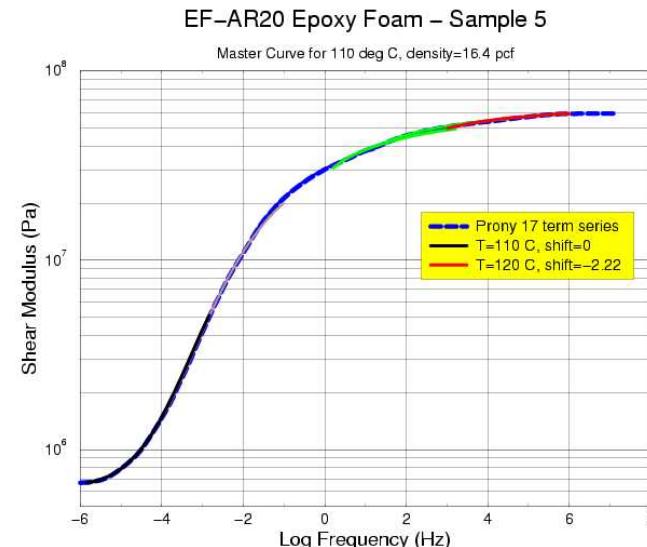
# Constitutive Experiments

## DMA Temperature/Frequency Shifts

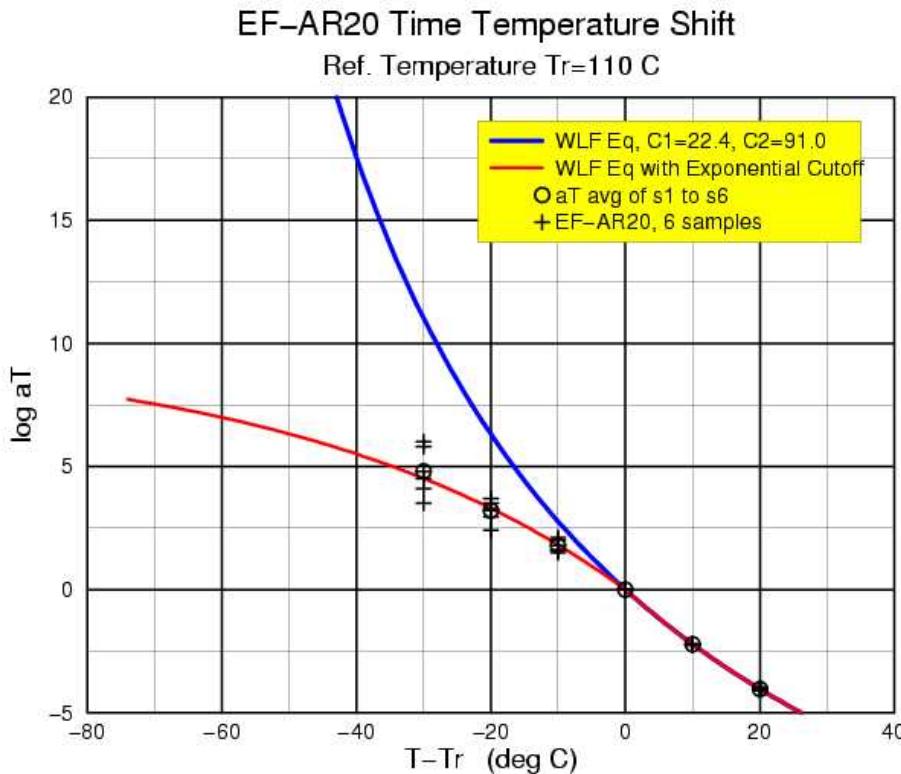
EF-AR20 Epoxy Foam – Sample 5



### 17 Term Prony Series Fit



## Time(Frequency)/Temperature Shift Function based on the Master Curve for EF-AR20 Foam



### Time domain:

$$G(t) = G_r + (G_g - G_r) \sum_{j=1}^N m_j \exp(-t / a_T \tau_j)$$

### Frequency domain:

$$G'(\omega) = G_r + (G_g - G_r) \sum_{j=1}^N \frac{m_j (\omega a_T \tau_j)^2}{(1 + (\omega a_T \tau_j)^2)}$$

where:

$$a_T = \frac{t}{t_r}$$

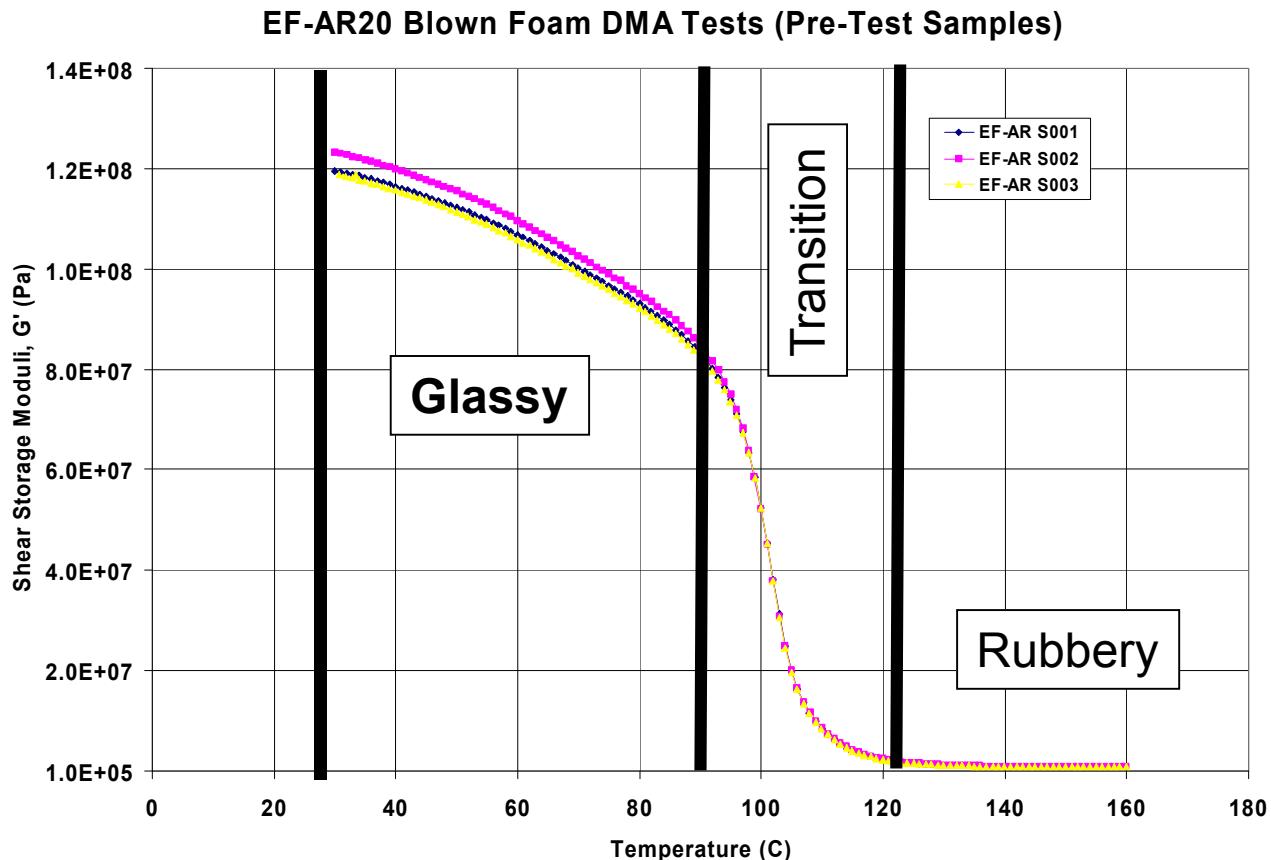
$t_r$  = reference time

### Modified WLF Time-Temperature Shift Function

$$\log(t/t_r) = \left( -C_1 * (T - T_r) \right) / \left( C_2 + (T - T_r) \right) \quad T > T_r$$

$$\log(t/t_r) = a_1 * (1 - \exp(a_2 * (T - T_r))) \quad T < T_r$$

# Glassy Modulus depends on Temperature

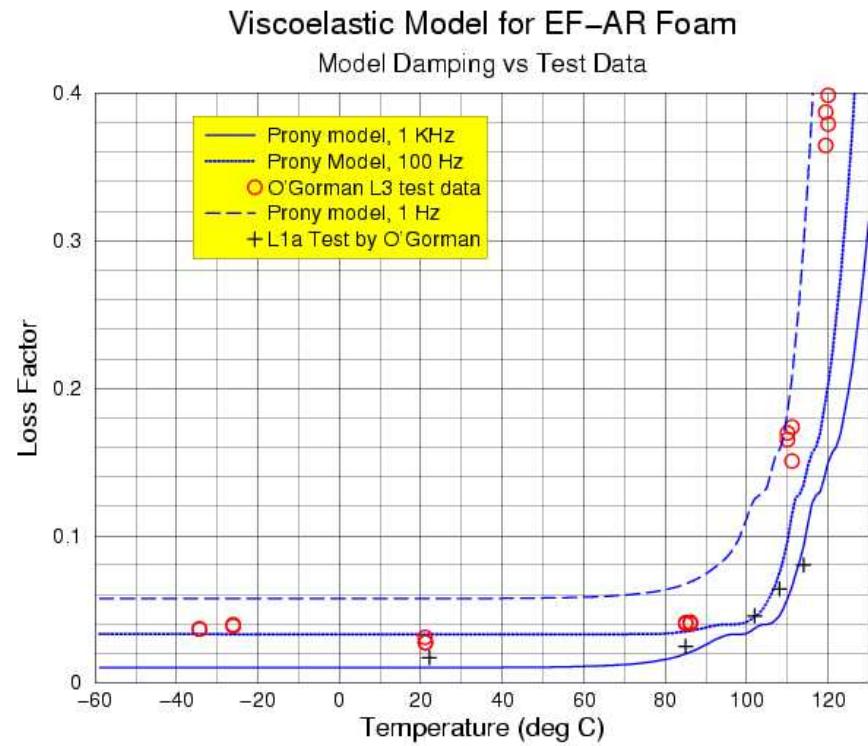
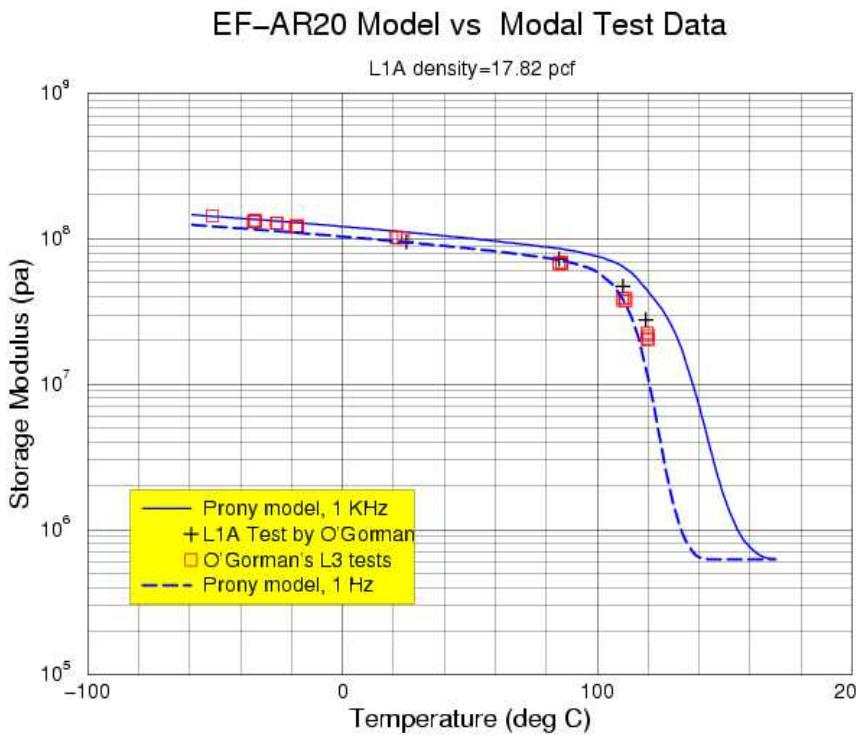


Glassy modulus has an approx. linear temperature dependence in the Glassy region that can be combined with the Prony series model as shown below

$$G_g = G_{g@T_g} * A_1 * (1 - A_2 * T / T_g)$$

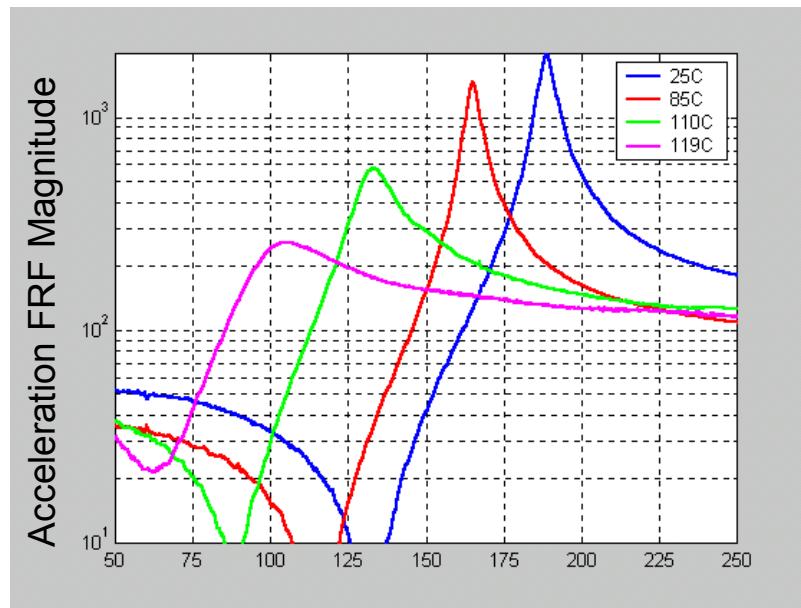
# Epoxy Foam Behavior

## Modulus & Damping vs Temperature Test Data superimposed on model curves



# Modal Test Results Compared with SALINAS Viscoelastic Model

## Modal Test Data ref. O'Gorman



## SALINAS Model Results ref. Hinnerichs

L1A Test Simulation of EF-AR20  
Acceleration FRF vs Temperature

