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Assessing a Continuum Description of Wide Shear Zones in Slow Granular Flow by Discrete Element Simulations

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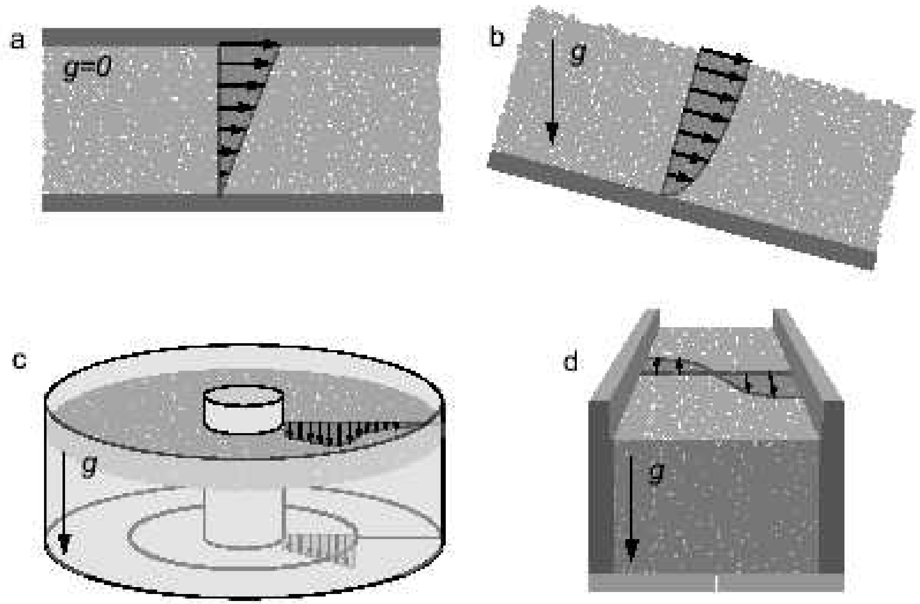
Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

Smooth, Dense, Slow Granular Flow

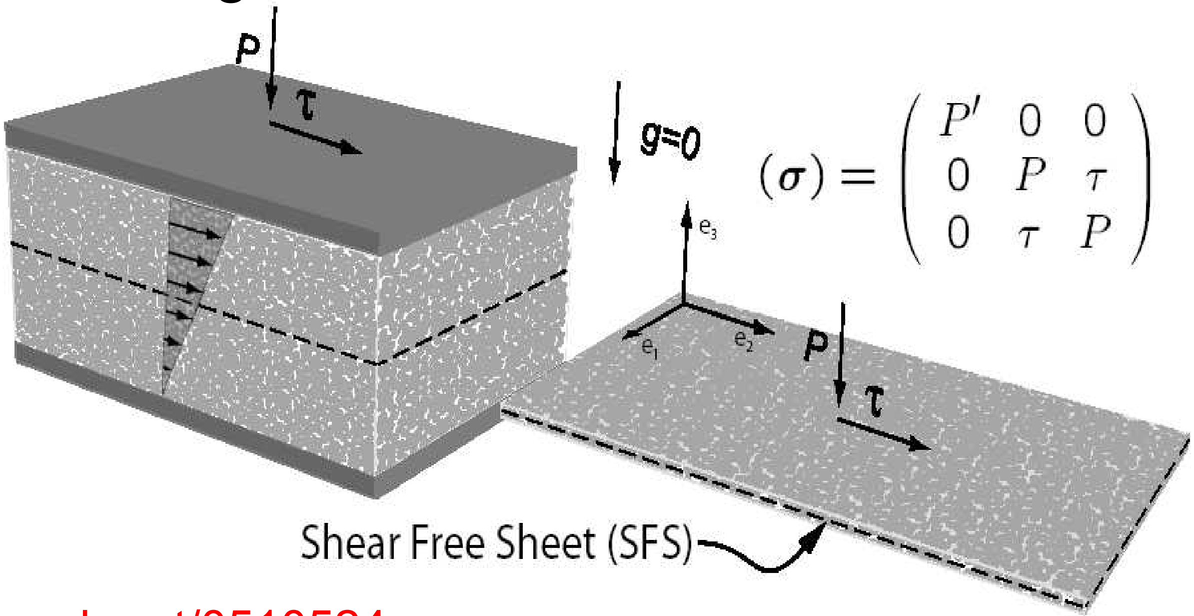
- Quasi-static: no inertial effects
- Multiple, enduring contacts per particle
- Wide shear zones
  - (a) Linear shear
  - (b) Flow down inclined plane
  - (c) Split-bottomed Couette cell
  - (d) Linear shear over split bottom



Depken et al. cond-mat/0510524

Continuum description of Smooth, Dense and Slow Granular Flows

- Fact: The **stresses** and **normalized velocity profiles** are independent of driving rate (c.f. solid friction, $\tau = \mu F_N$)
- Assumption: **Stress fluctuations relax fast** enough for elastic shear stresses between non-shearing granular elements to be ignored



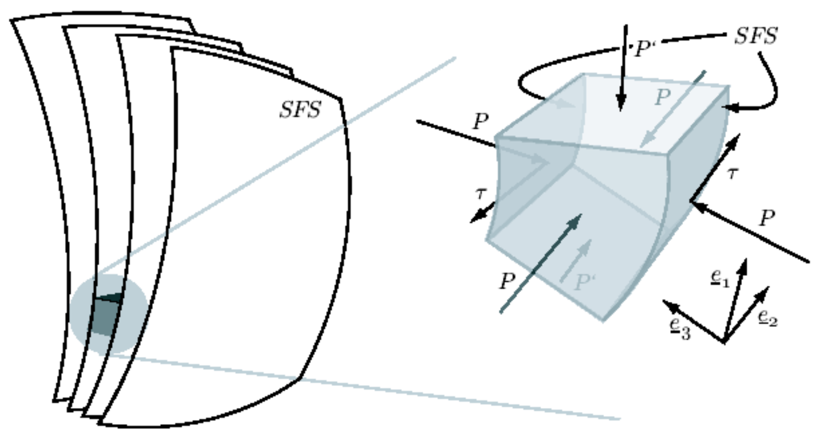
$$(\sigma) = \begin{pmatrix} P' & 0 & 0 \\ 0 & P & \tau \\ 0 & \tau & P \end{pmatrix}$$

Depken et al. cond-mat/0510524

Curved SFS

- Easily generalized to situations where SFS not flat
- No shear, no stress

$$(\underline{\sigma})_{\mathrm{SFS}} = \begin{pmatrix} P' & 0 & 0 \\ 0 & P & \tau \\ 0 & \tau & P \end{pmatrix}$$



- Flow occurs in the plane of maximal shear stress, and not where the ratio between shear and normal stresses is maximal!

Discrete Element Method

- Allows observation of bulk behavior away from influence of side walls without the use special techniques (e.g., MRI)
- Allows detailed measurements of microscopic quantities (e.g., inter-particle forces)

integrate Newton's equations

$$\mathbf{F}_n = f(\delta / d)(k_n \delta \mathbf{n}_{ij} - \frac{m}{2}\gamma_n \mathbf{v}_n)$$

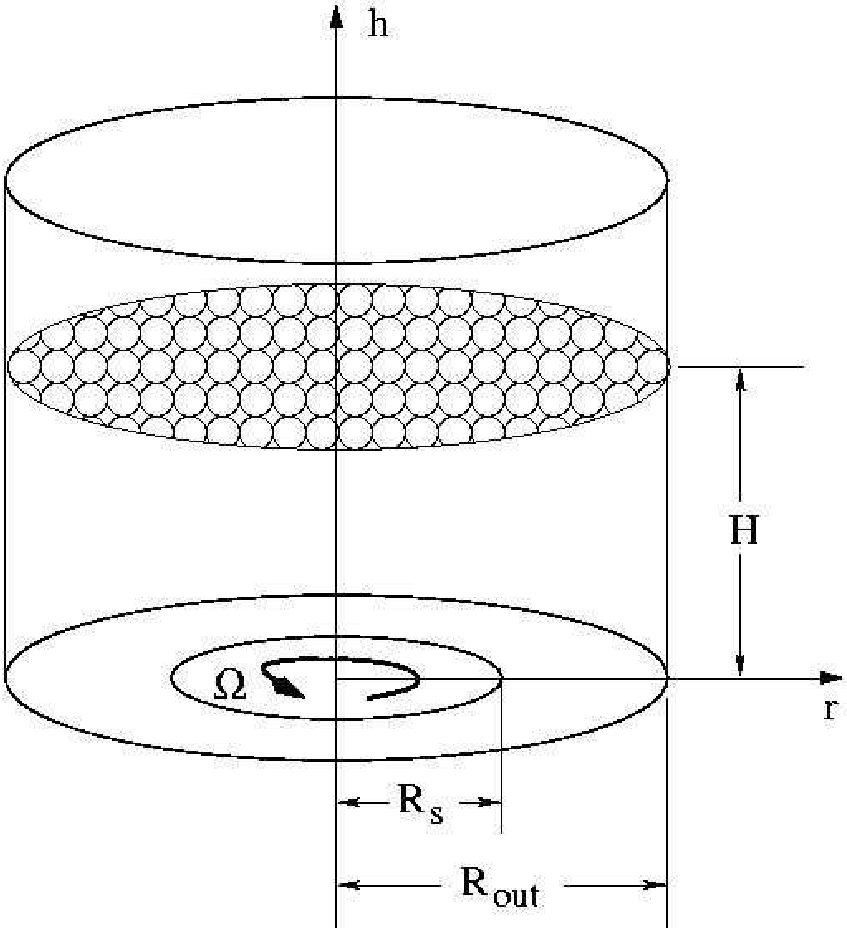
$$\mathbf{F}_t = f(\delta / d)(-k_t \Delta \mathbf{s}_t - \frac{m}{2}\gamma_t \mathbf{v}_t)$$

$f(x) = \sqrt{x}$ **Hertzian springs**

$\Delta \mathbf{s}_t$ **Elastic tangential displacement**

$F_t \le \mu F_n$ **Coulomb Failure Criterion**

Circular Geometry: System Parameters



$R_s = 30.0d$

$R_{out} = 37.8d$

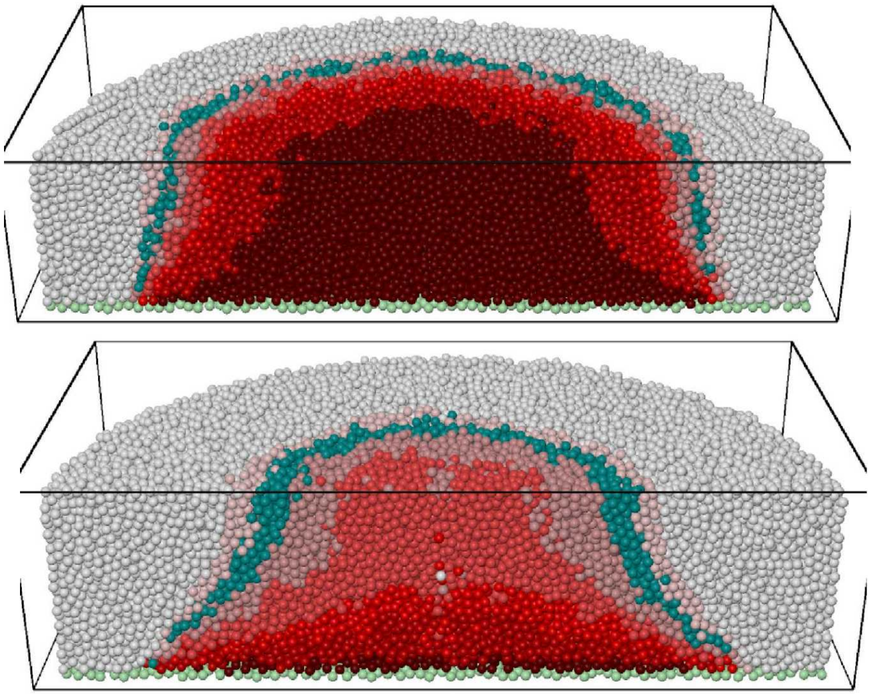
$\Omega = 1.39 rad/\tau$ where $\tau = \sqrt{d/g}$

$H = 12.6d, \quad 19.8d$

60,000 – 100,000 particles

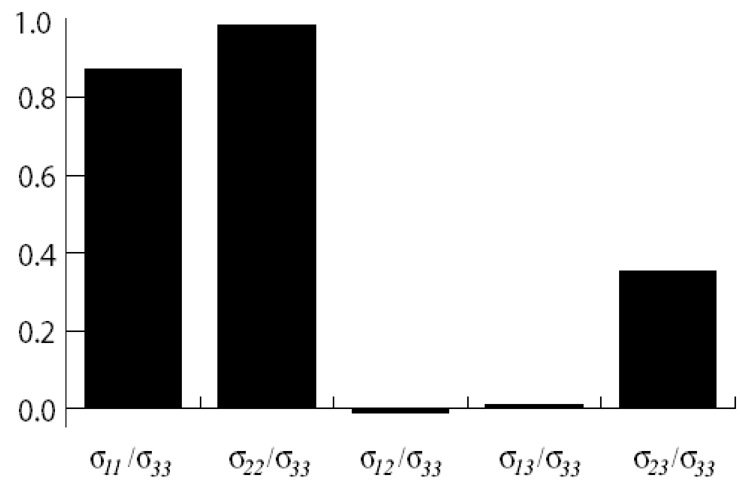
rough bottom composed of layer of 'frozen' particles

Numerical Check of Proposed Form of Stress Tensor



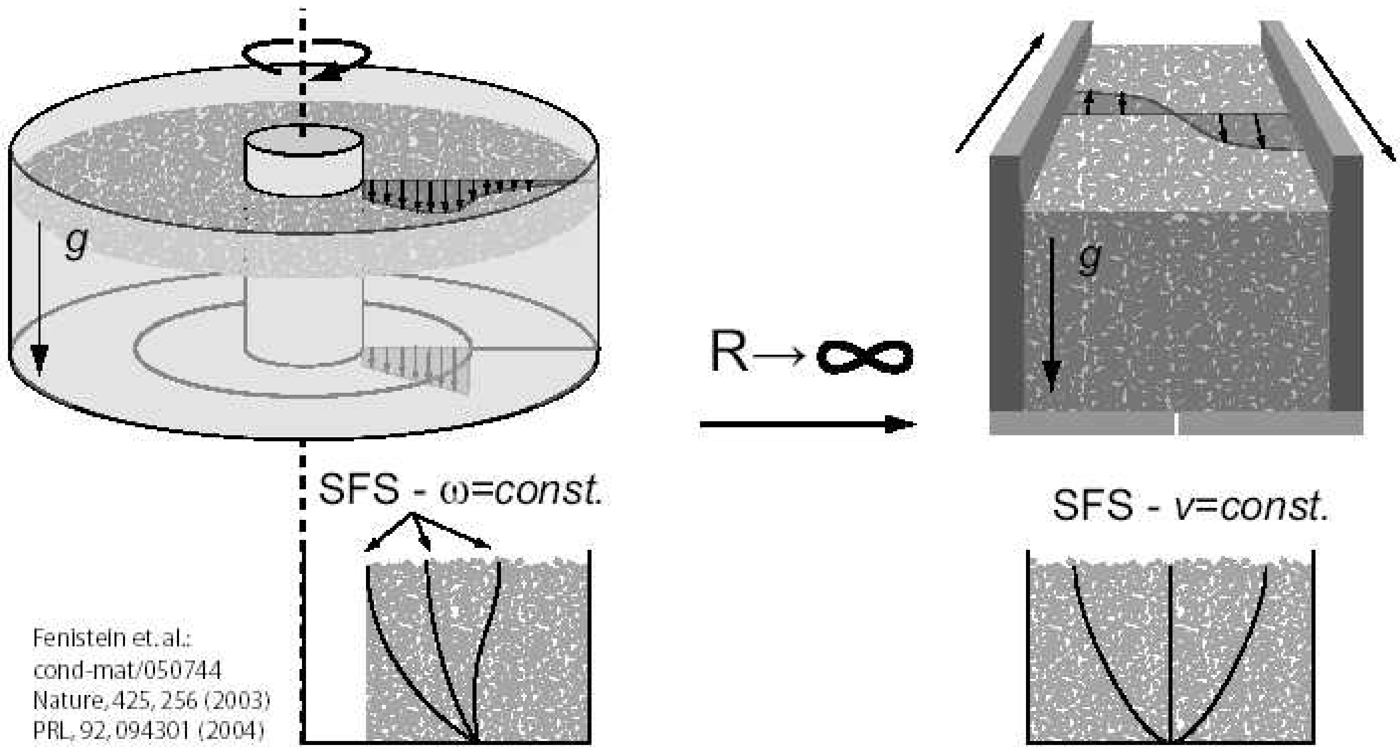
Contours of ω/Ω

$$(\underline{\underline{\sigma}})_{\mathrm{SFS}} = \begin{pmatrix} P' & 0 & 0 \\ 0 & P & \tau \\ 0 & \tau & P \end{pmatrix}$$



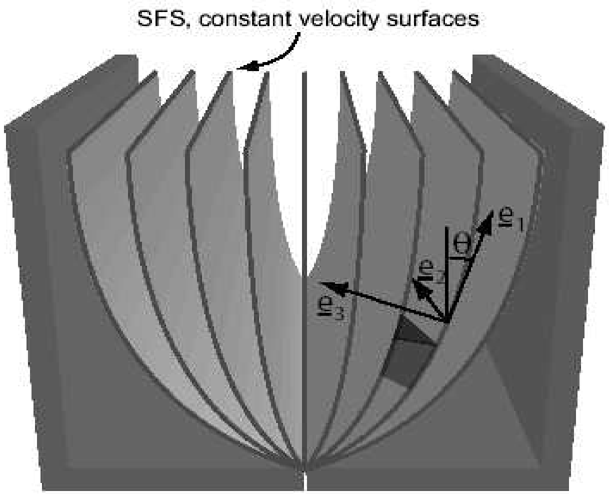
- Orientation of contours of normalized angular velocity give SFS basis
- In SFS basis, values of stress ratios averaged throughout the shear zone are as expected

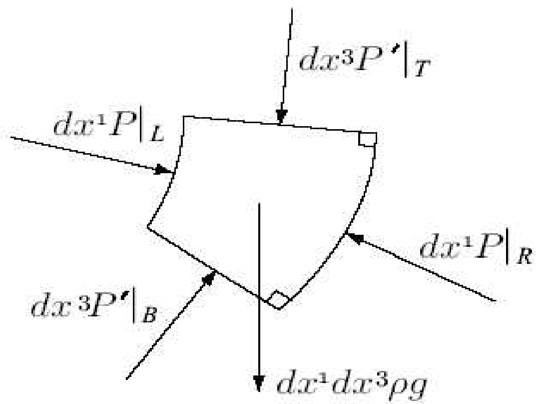
Linear Geometry

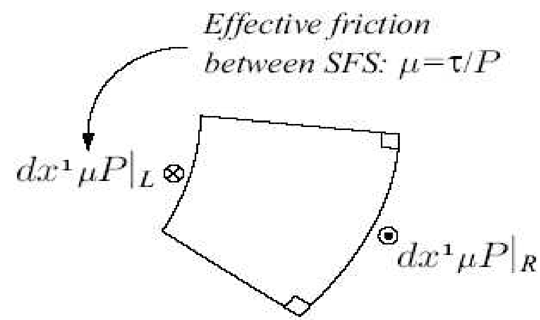


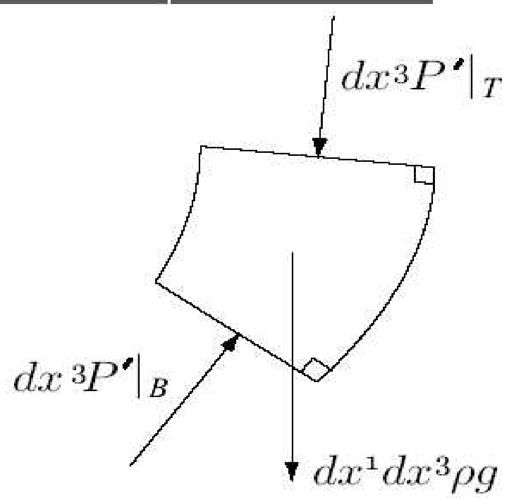
- Leads to simplified analysis and interesting prediction

Force Balance in Linear Split-bottom Cell









Forces cannot be balanced!

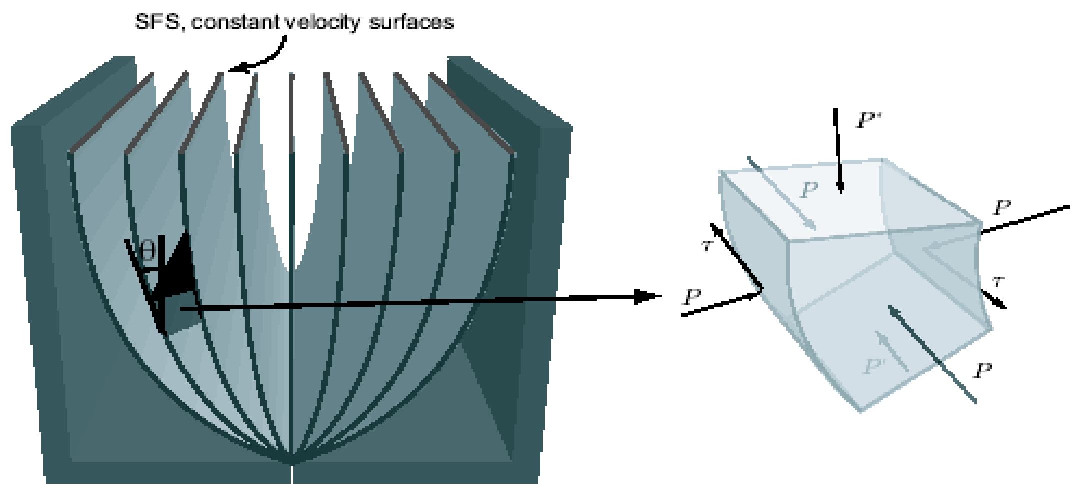
$\Rightarrow$

μ must decrease as we move away from the center.

What can cause μ to change?

- Dimensional Analysis
  - Packing fraction: ϕ
  - Pressure ratio: $\nu = P'/P$
  - Orientation of SFS: θ

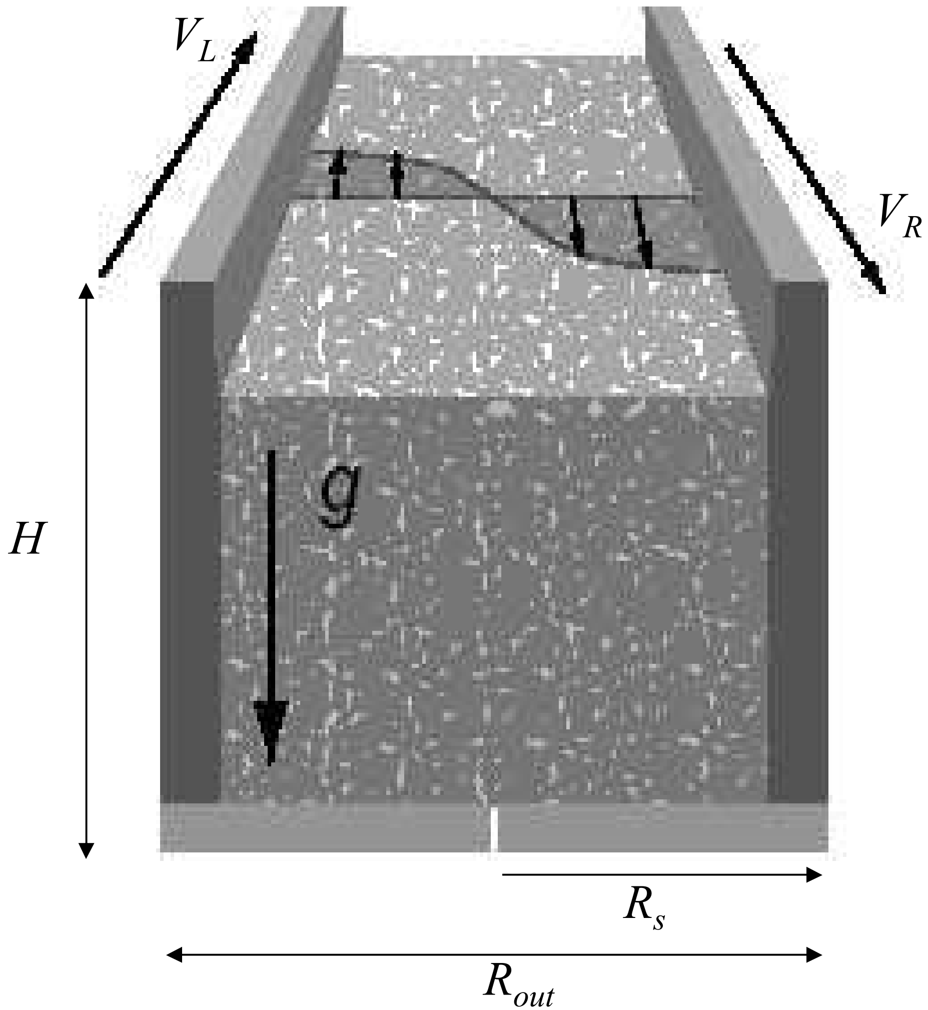
- However, ϕ is approximately constant
  - cf. Silbert et al., PRE **64**, 051302 and GDR midi, Euro. Phys. E **14**, 341



Working hypothesis: $\mu = \mu(\theta)$

- Assume internal stresses do no affect stresses between SFS
  - μ is independent of ν

Linear Geometry: System Parameters



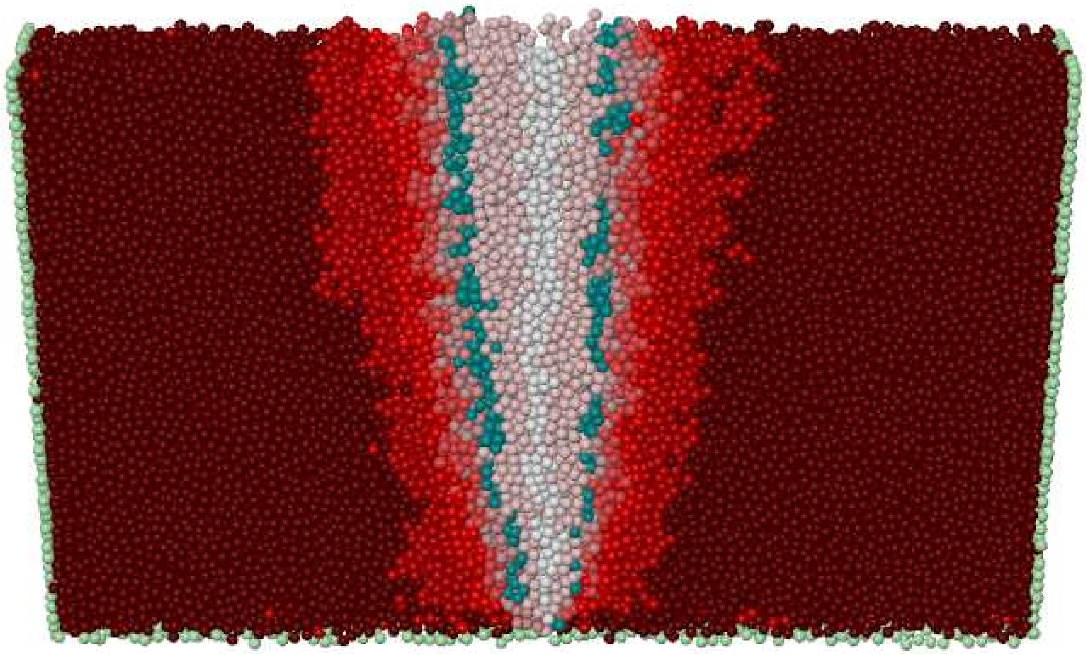
$R_s = 40.0d$
$R_{out} = 80.0d$

$V_{L,R} = \pm 0.05d/\tau, \quad \pm 0.005d/\tau$
where $\tau = \sqrt{d/g}$

$H = 50.0d$
110,000 particles

rough bottom and side walls composed of layer of 'frozen' particles

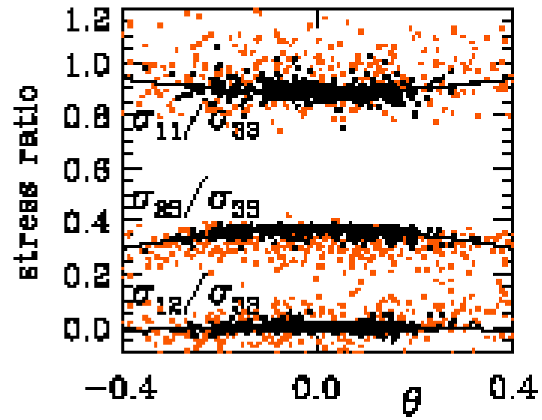
Velocity Contours

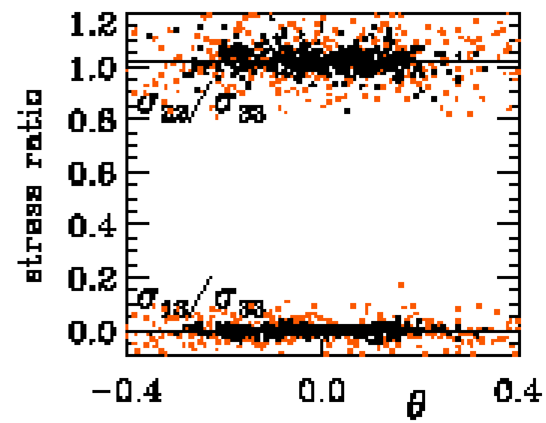


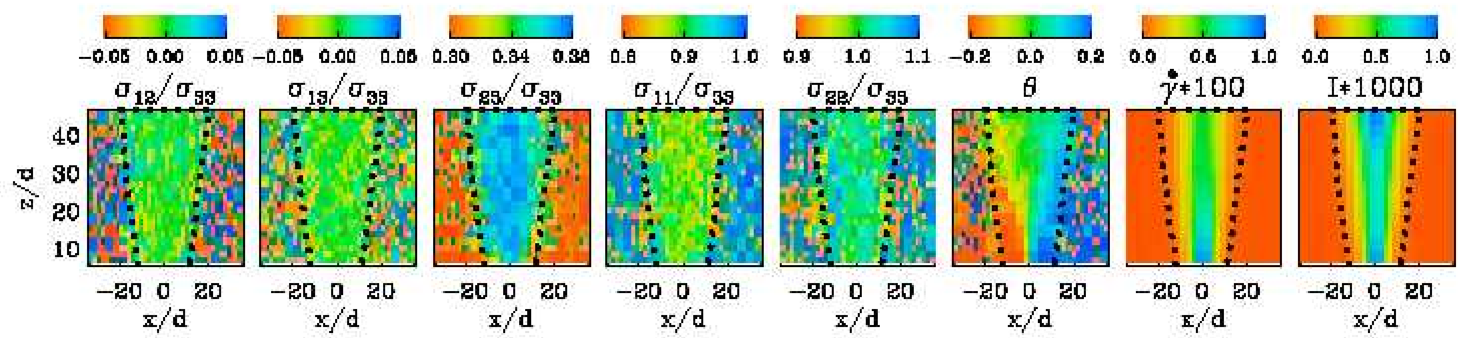
- Again, orientation of velocity contours gives orientation of SFS
- Look at stress tensor components in this basis

Stress Ratios in Linear Geometry

- Stress tensor has expected form as in circular geometry
- μ shows a dependence on orientation of SFS with respect to gravity
- Data for different heights within the bulk fall roughly together







Conclusions

- Simulations confirm fast stress relaxation
  - Simple form of the stress tensor
- Effective friction must vary in order to get curved shear profiles (in linear geometry)
- $\mu = \mu(\theta)$ yields data collapse – bulk MC effective friction not a material constant; depends on σ_2 (c.f. Lade 1977)
- Connection between failure and subsequent “fluidization” can be determined (yield criterion and flow rule)
- Microscopic origins, e.g., of θ dependence of μ and ν still an open, interesting question

Acknowledgements

- D. Fenistein, L. Silbert, X. Cheng, A. F. Barbero, H. M. Jaeger, S. R. Nagel
- Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Corporation, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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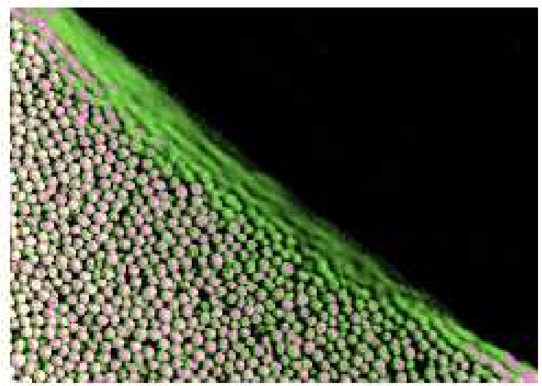
Slow, Dense Granular Flow

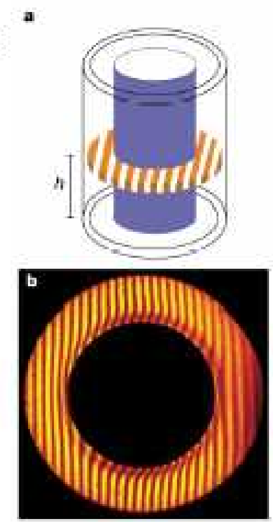
- Shear bands: narrow and distinct bands of high rates of shear deformation (localization of energy dissipation)
  - Phenomenon plays an important role in many applications
    - ballistic impact
    - explosive fragmentation
    - metal forming
    - interfacial friction
    - powder compaction
    - soil failure
    - seismic events
    - **granular flow** →
- What is the role of micro-structure?
- Difficult to access range of a flowing states to test flow theories
- Non-universality

Free surface granular flow:



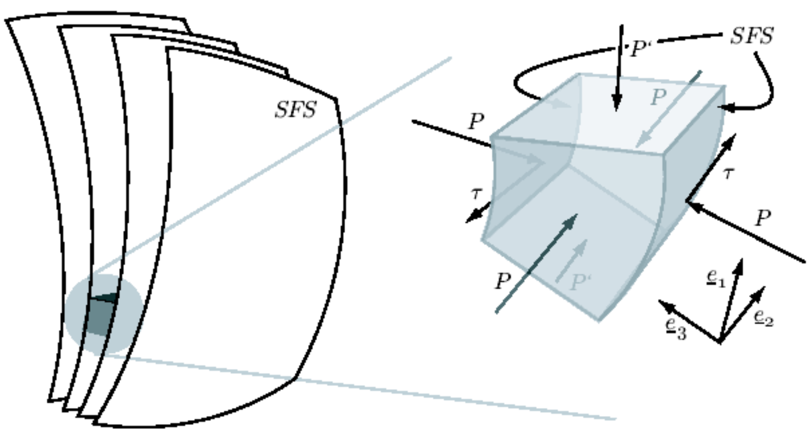
H.M. Jaeger et al, Rev. Mod. Phys. 68, 1259 (1996).

Couette Cell:



D.M. Mueth, et al, Nature 406, 385 (2000).

exponential velocity profiles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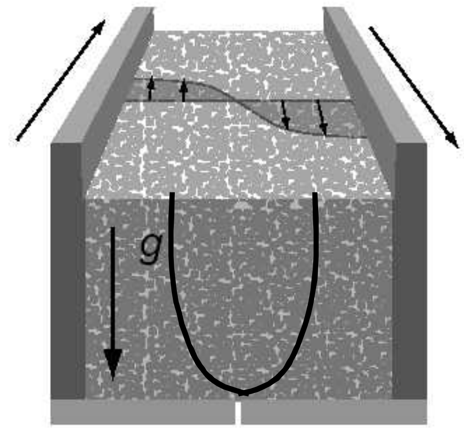
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