SAND2015-5516C

Exceptional service in the national interest

Sandia National Laboratories

Honoring the Career of Prof. P.L.E. Uslenghi

Application of Matrix Compression in the Method of Moments Code EIGER

Dr. J. D. Kotulski

Sandia National Laboratories – Organization 1352

July ,2015

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

Outline

- EIGER Description
- Thin-slot formulation
- Matrix compression
- Results
- Conclusions / Future Work
- Congratulations and Comments to Prof Uslenghi

EIGER

- **Frequency-domain method of moments solution**
  - Steady state solution
  - F90 (95, 2003) code – Object Oriented Design

- **Boundary element formulation**
  - Mesh surfaces of parts – interface between regions

- **Normal formulation results in dense (fully populated) matrix**
  - Galerkin testing - Rao, Wilton, and Glisson bases functions

$$\bar{\bar{Z}}\,\bar{I} = \bar{V}$$

  - Simulations can be limited by available memory

    Entries are double precision complex

EIGER Thin-Slot Formulation

- **This modeling feature enables the incorporation of potential penetration points on a structure that couple fields into a cavity without gridding the slot explicitly.**

- **Based on research by Warne and Chen.**
  - **Slot is modeled by a wire (carrying magnetic current) whose effective radius depends of the depth and width of the slot.**
    - **Note the length of the slot >> depth, width**
  - **Incorporated into EIGER and used by other investigators.**
  - **Validated**
    - **Compared to analytic and experimental results.**

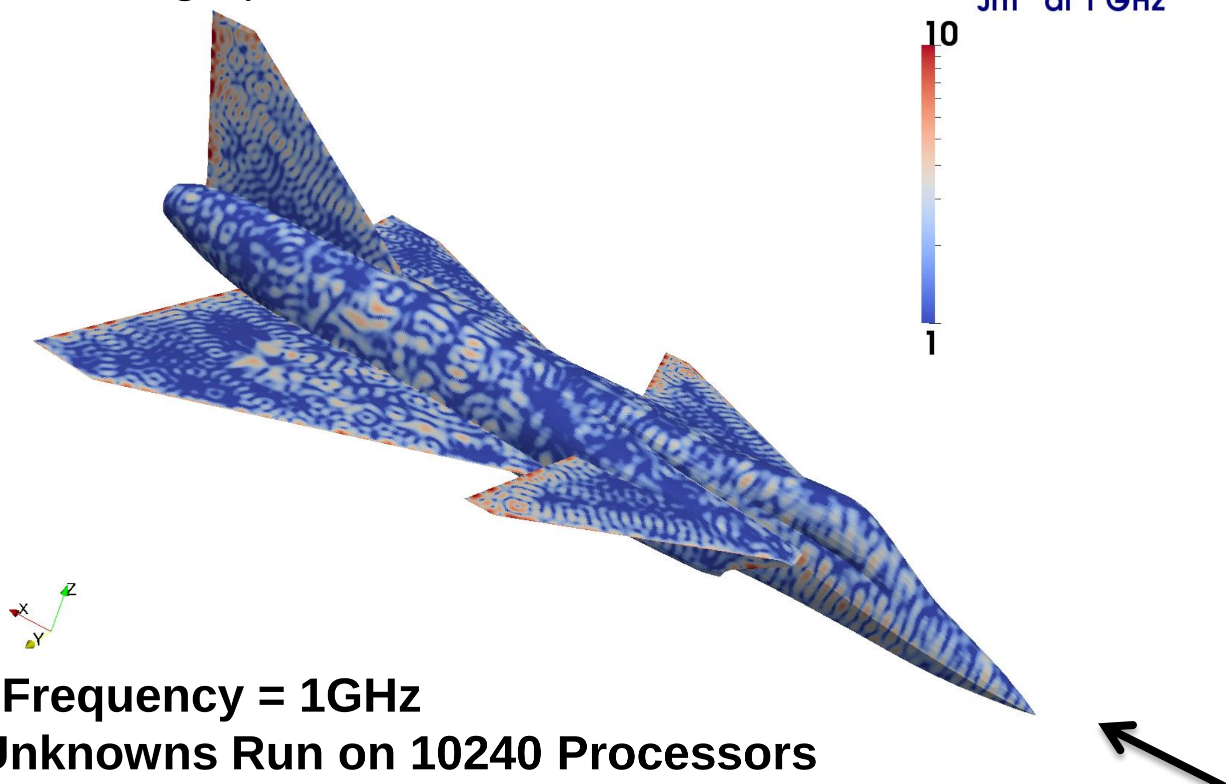
EIGER Thin-Slot Formulation

- **Key features**
  - Integral equation for the exterior surface current and slot current (magnetic current)
  - Integral equation for the interior surface current and slot current (magnetic current).
  - Two contributions
    - Green's function
    - Non-Green's function

- **Implications**
  - The exterior unknowns do not interact with the interior unknowns.
  - Coupling of the exterior to the interior is through the slot contribution.
    - Matrix has blocks with zero elements – no coupling.

Results – External Problem
(Direct Solve on CIELO)

External Problem
VFY 218 (50.6 ft. length)



Frequency = 1GHz
930,000 Unknowns Run on 10240 Processors
Memory Requirement 13.8 TBytes

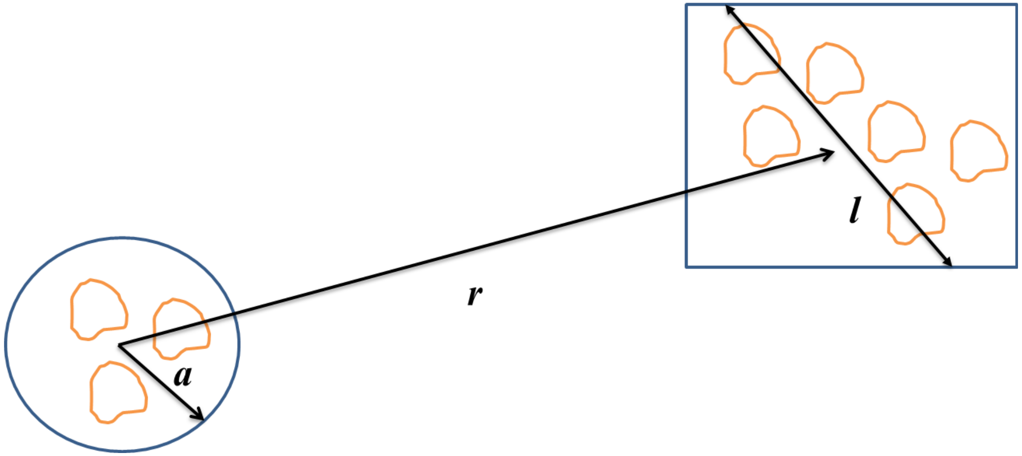
Direction of Incident Field

Compression Techniques

- **These are techniques that no longer store the full matrix but a lower rank version of the matrix.**

- **Based on work by Bucci and Francescetti**
  - **"On the Degrees of Freedom of Scattered Fields" IEEE AP, July 1989**

$$N_{dof} = \frac{4la}{r\lambda}$$



Compression Techniques

- **Fast Multipole Method (FMM)**
  - **Compression achieved through Green's function simplification:**
    - **Factorization**
    - **Use of the addition theorem**
    - **Diagonalization**
    - **Results in low-rank approximation of matrix blocks**

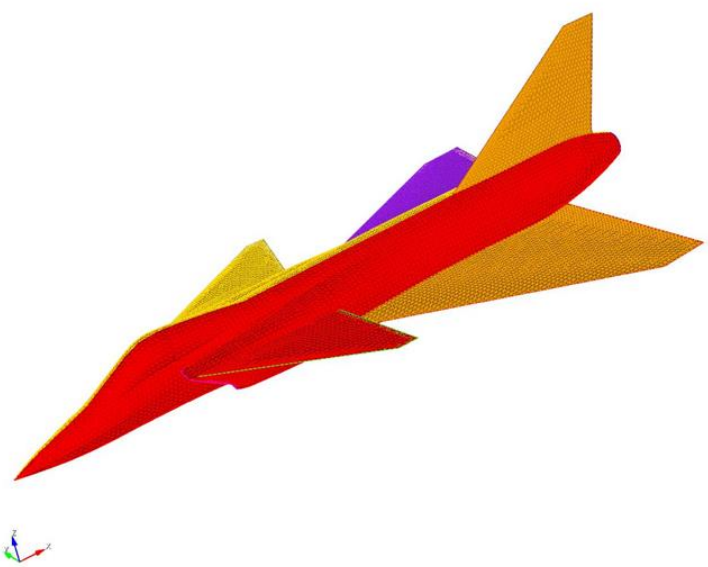
- **Adaptive Cross Approximation (ACA)**
  - **Compression achieved:**
    - **Low-rank approximation of matrix blocks.**
    - **Done on the fly**
      - **Compressed matrix blocks never fully populated.**
    - **Since the process only operates on matrix blocks it is independent of Green's function simplification.**
    - **There are also multilevel variants of this method.**

Compression Techniques

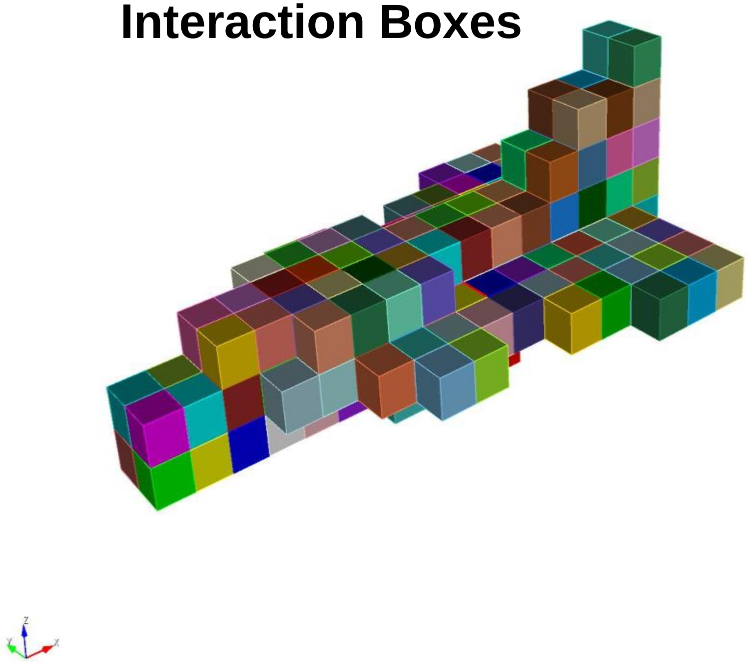
- **Identification of all matrix blocks**
  - **Discretized object (meshed) is encased in a oct-tree structure**

VFY 218

Meshed Object



Interaction Boxes



All compression techniques use this step in the solution process

ACA Matrix Compression

- **Each box contains elements with current unknowns on the elements.**
  - **Can be compared to a 1-level fast multipole algorithm**

- **2 boxes interact to form a matrix block.**

- **The distance between boxes, size of the boxes, and wavelength determine if a reduced or low-rank approximation can be used.**
  - **Not all blocks can be compressed.**
    - **Compression criterion :**
      - **Distance between the center of boxes > 2 * (box radius)**

ACA Matrix Compression

- **The matrix $\bar{\bar{Z}}$ is given by:**

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

MOM_Blocks – Moment method matrix blocks (full matrix blocks)

COM_Blocks – Compressed matrix blocks (low-rank approximation)

Solution of the Compressed System

- **The matrix equation to be solved is :**

$$\bar{\bar{Z}}\,\bar{I} = \bar{V}$$

- **The matrix is not completely available but is stored as:**

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

- **Therefore a iterative solution approach needs to be used.**
  - **Generalized Minimum residual method(GMRES)**
    - **Saad and Schultz 1986**
  - **Transpose Free Quasi Minimum Residual (TFQMR)**
    - **Freund 1993**

Solution of the Compressed System

- **The Iterative solution technique of choice is the TFQMR method.**
  - **Based on heuristic numerical experiments performed on electromagnetic problems.**
  - **Extended for use on parallel platforms.**

- **On a parallel machine each processor does not have all the matrix blocks – they are partitioned on different processors for load balancing and memory balancing.**
  - **No processor can have more or less than one block than any other processor.**
  - **Processors have both MOM and COM blocks.**

Solution of the Compressed System

Using the TFQMR Method

- **In all iterative methods a matrix vector product is needed during the solution process.**
  - **This is performed in parallel (each processor has a portion of the compressed and MOM blocks).**

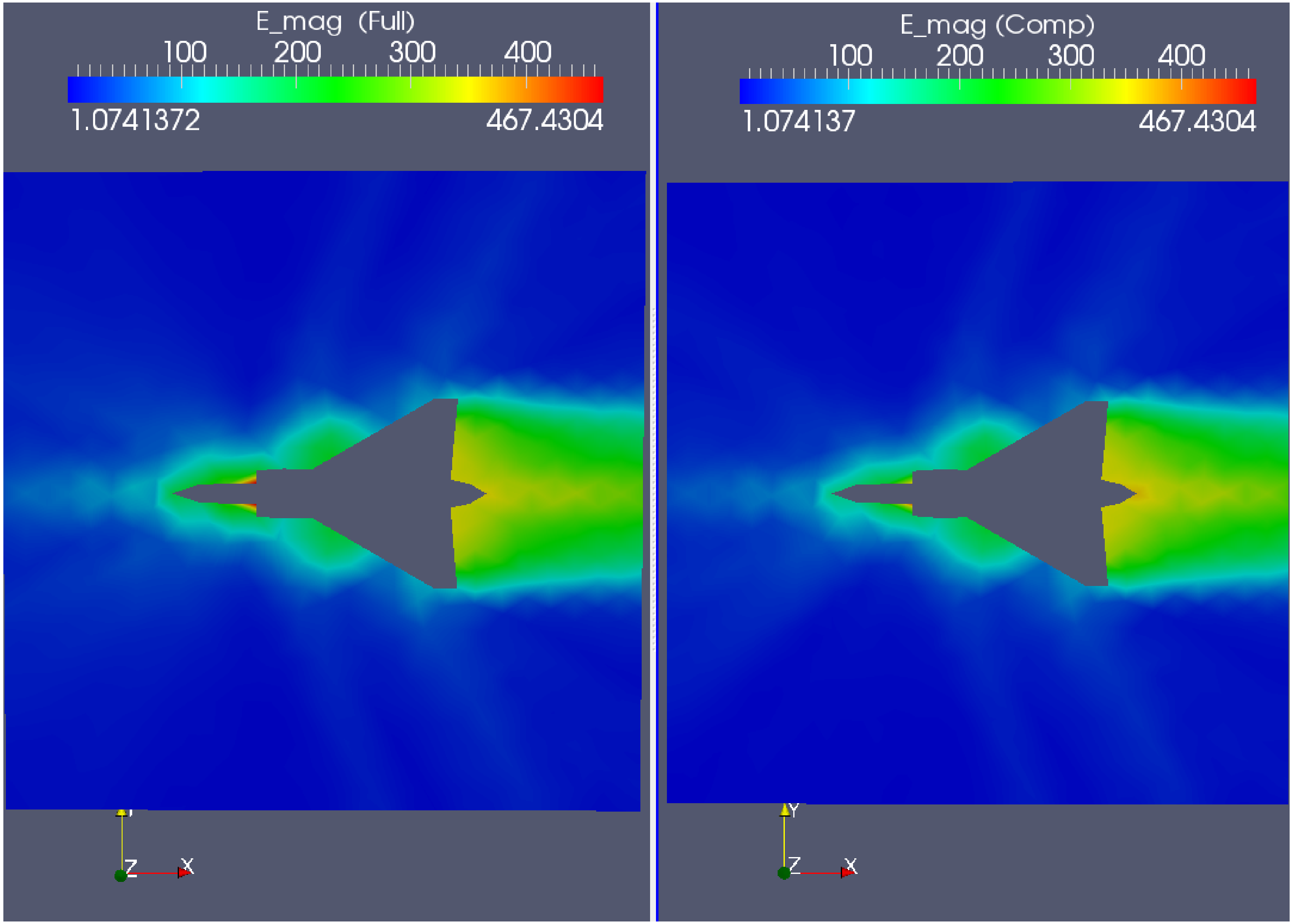
- **In the original algorithm (used here) the residual norm is not available.**
  - **However an estimate is computable.**
  - **The convergence curves show two values**
    - **The normalized initial residual norm**
    - **The estimate to the norm.**
- **A solution tolerance of 5 e-3 was used in all problems.**
  - **Will affect accuracy.**

VFY-218 Compression Results

- 15 meter long aircraft.
- Frequency 1 GHz
- Number of unknowns 934128
  - 2500 iterations
  - 256 Processors
  - 70,826 sec.
- Epsilon 4.e-02
- Memory
  - Full matrix 16 *(872) GBytes
  - Compressed 16*(19 + 7.7) GBytes
  - ~ 97 % compressed.

Compression Results VFY-218

Magnitude of the near field full and compressed matrix solution.



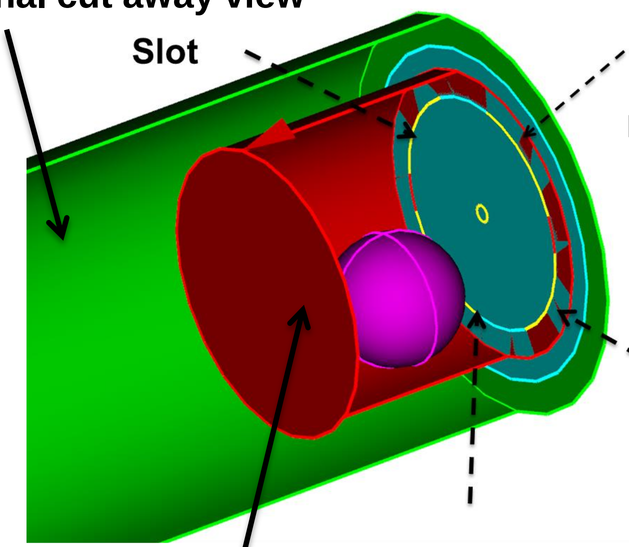
Compression applied to an object with slots

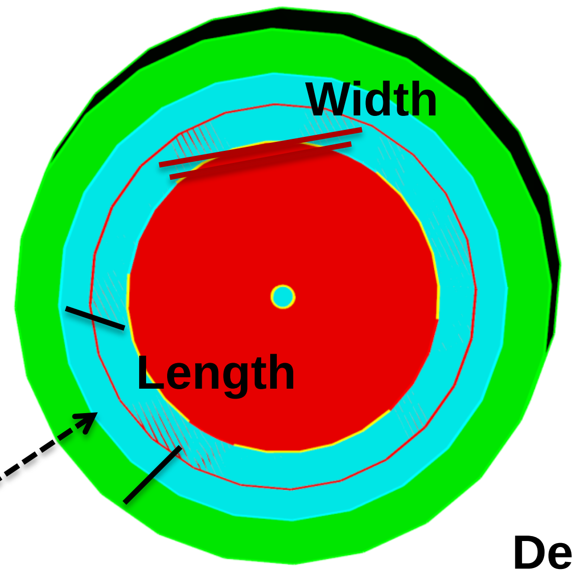
- Referred to as D_cavity.

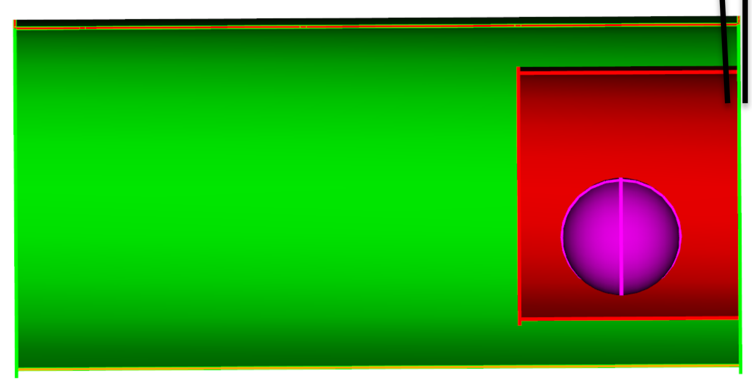
- A number of different mesh densities considered.
  - Increases the useful upper frequency limit for the model.

- Contains essential features to exercise the compression algorithm on an problem with slots.

Thin-Slot Parameters

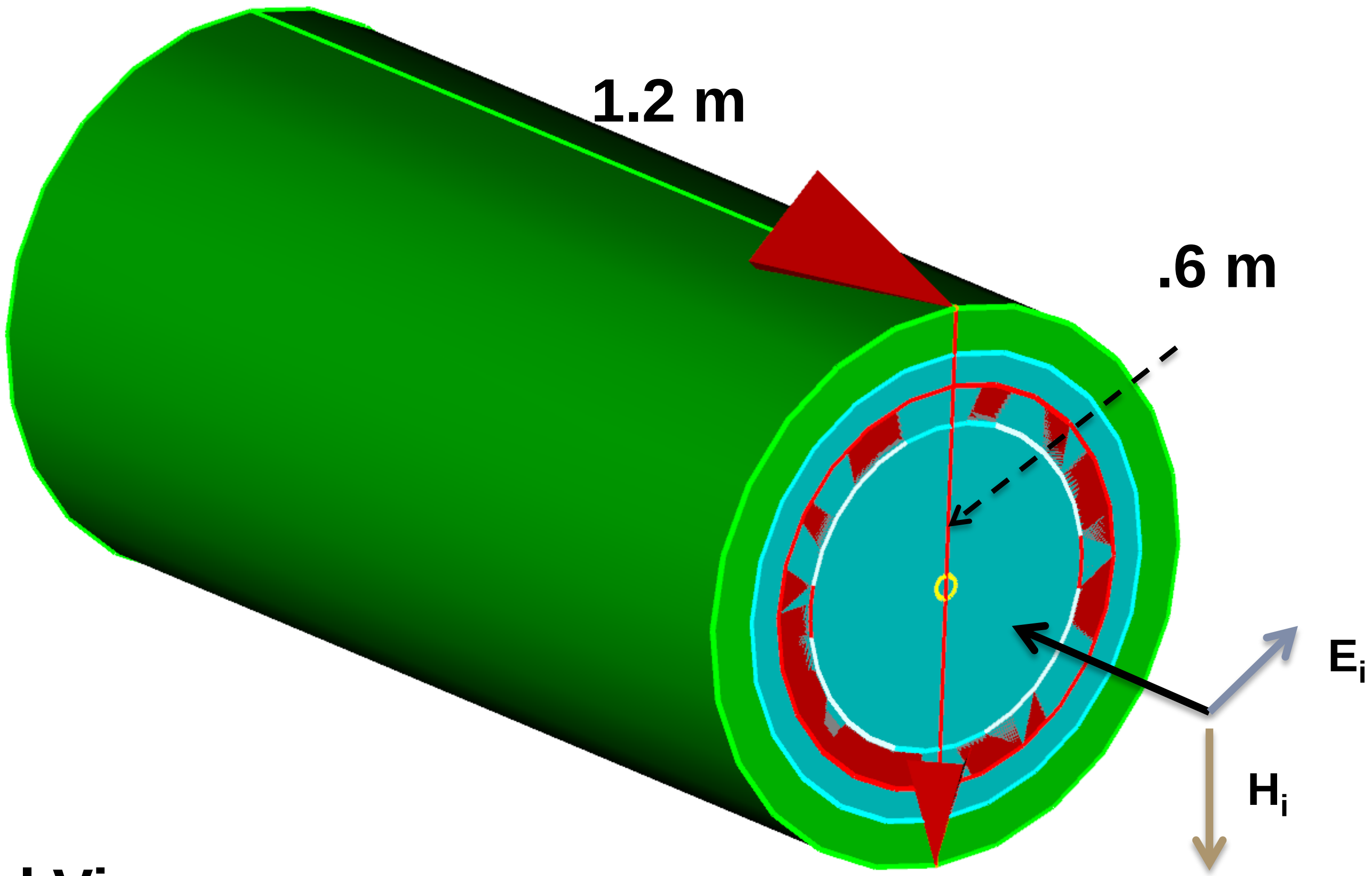






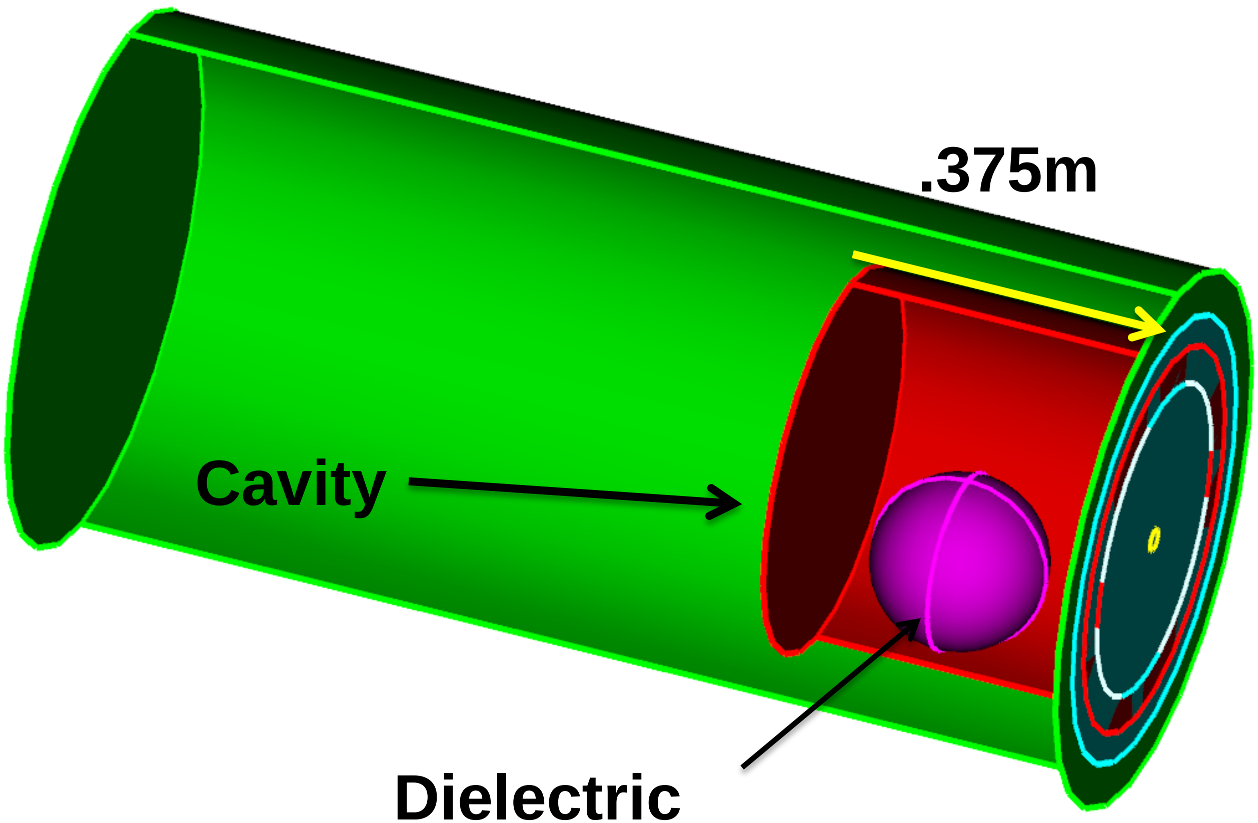
For most problems
Width, depth vary from .5 to 3mm

Geometry D_cavity



External View

Geometry D_cavity

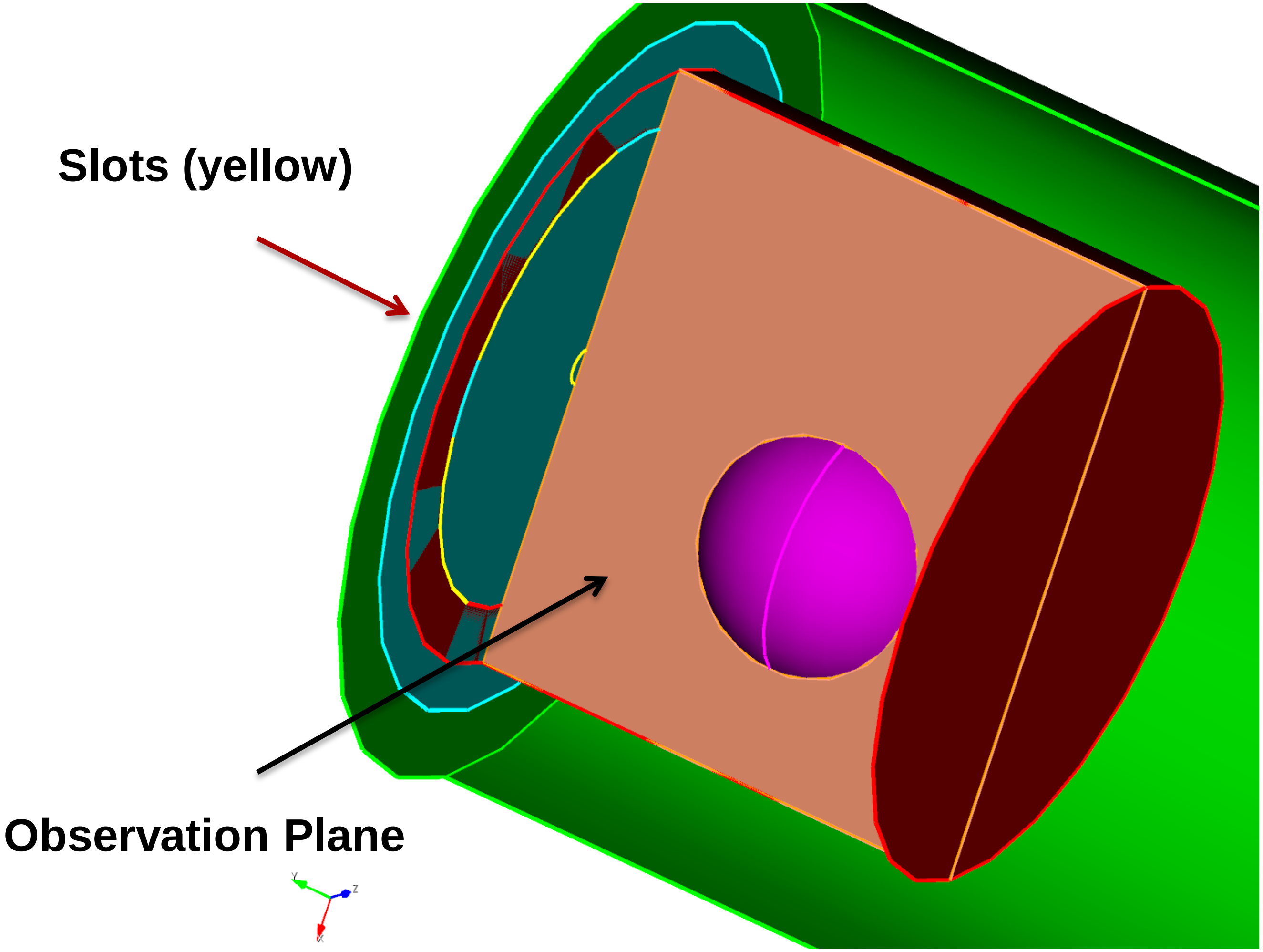


Internal View

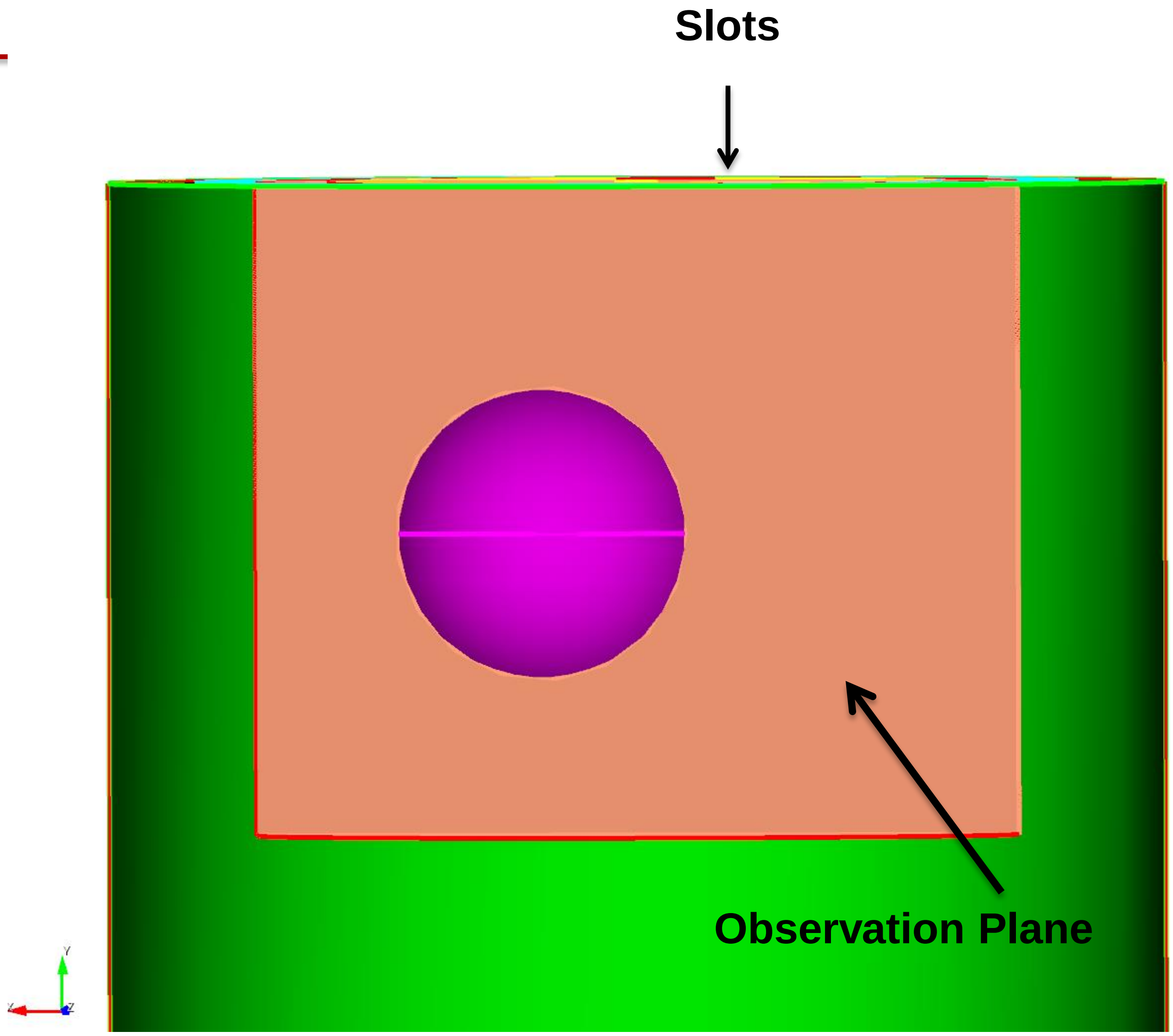
Results

- **The magnitude of the scattered electric field will be considered.**
- **This field value will be calculated on planes both inside the cavity and outside the cavity.**
- **Because of the proximity of these observation points to the object these are near field quantities.**

Data Results Observation Plane



Data Results Observation Plane

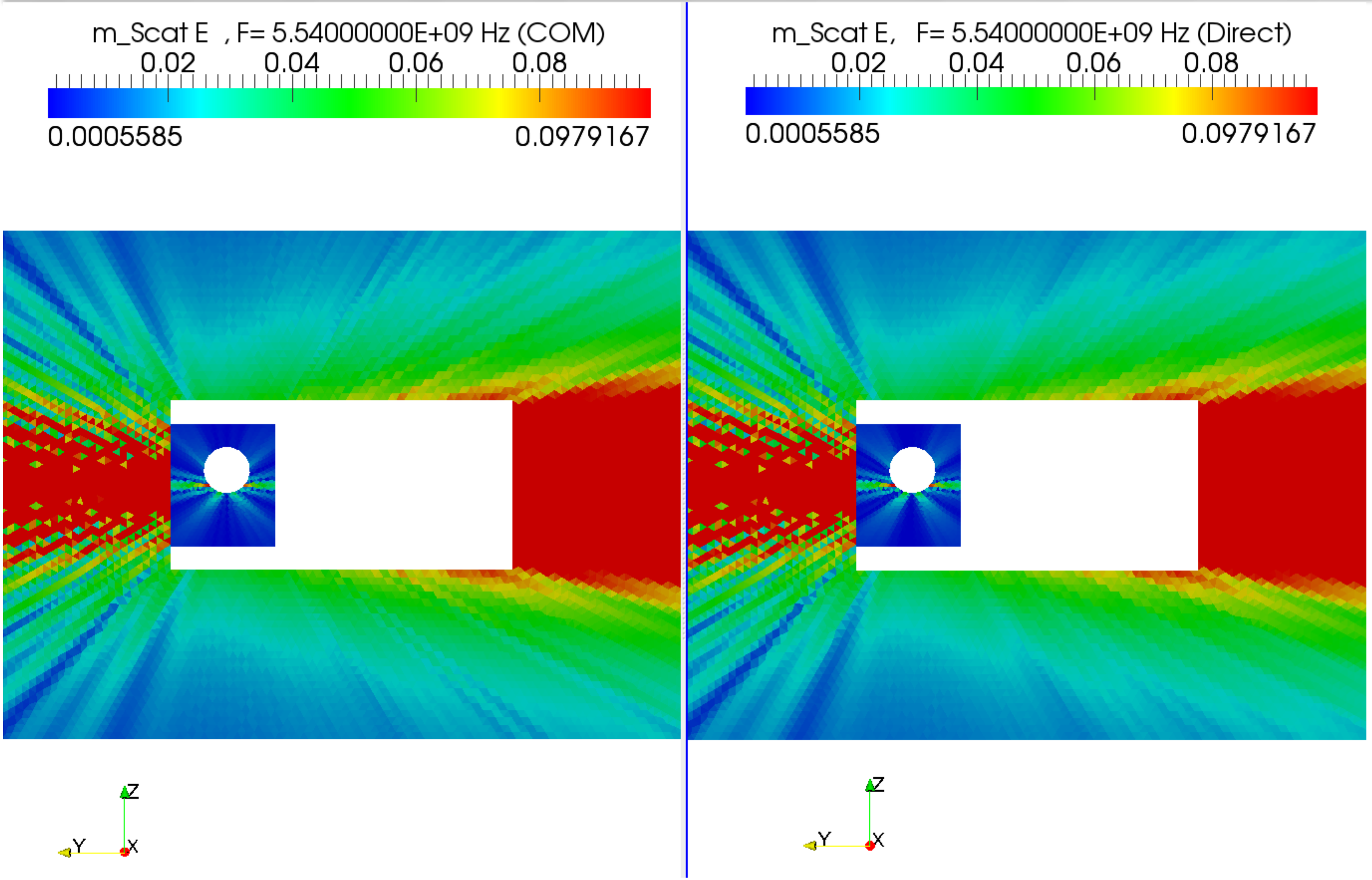


Results - Mesh 3 D_cavity

- Object 1.2 m in length
- Frequency 5.5 GHz
- Number of Unknowns 247604
- Epsilon 3.1e-03
- Memory
  - Full matrix 16*(61) GBytes
  - Compressed 16*(36.8 + .2) Gbytes
  - ~40% compressed.

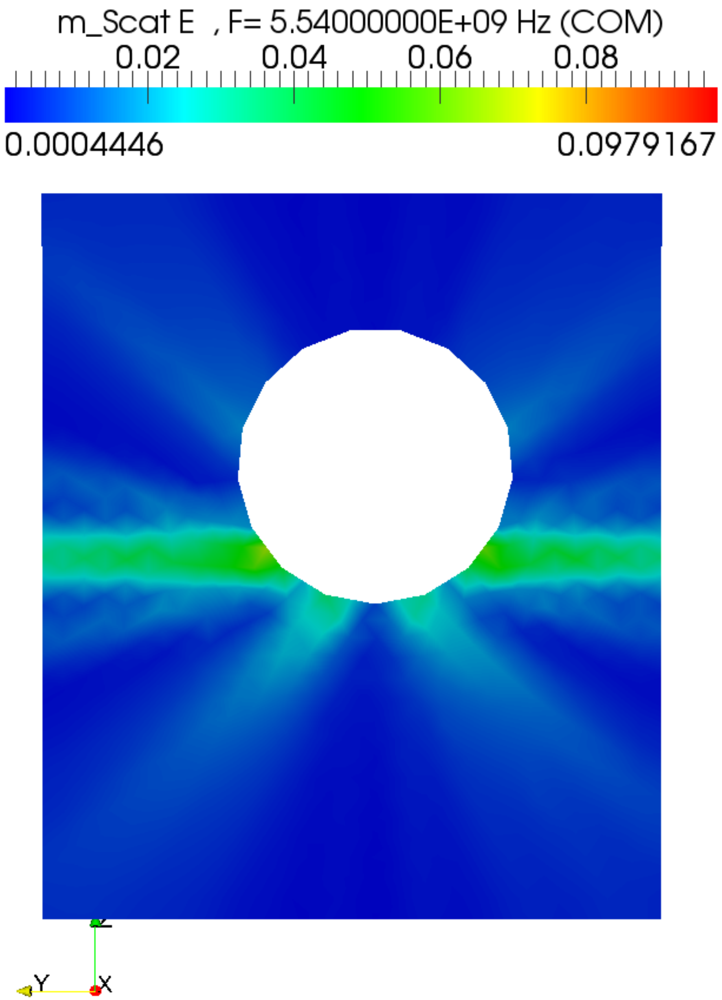
Results - Mesh 3 D_cavity

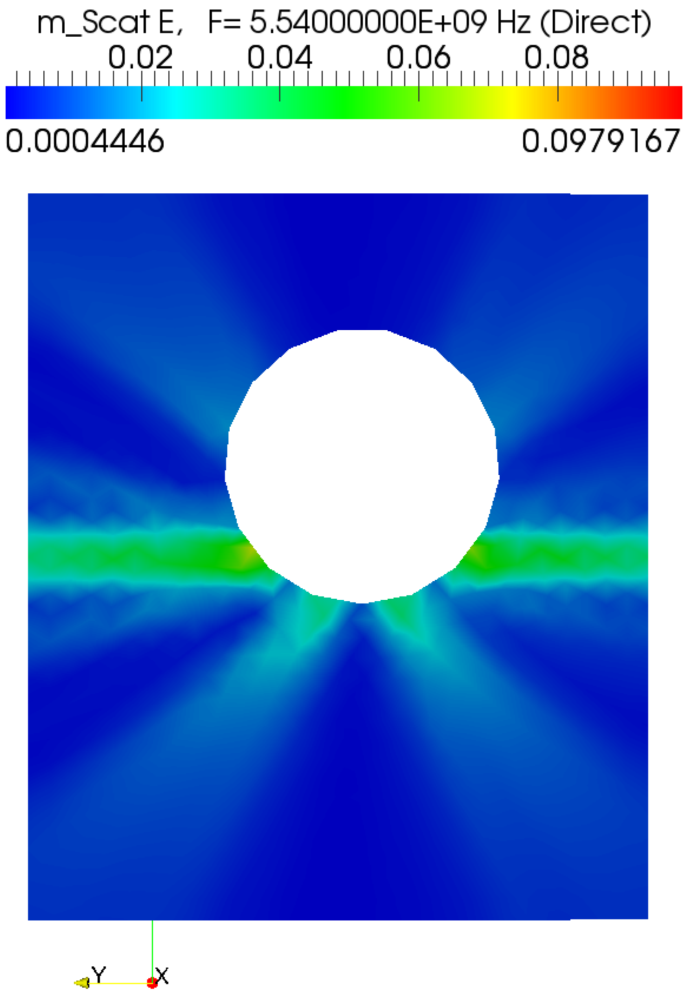
Magnitude of Scattered Field



Results - Mesh 3   D_cavity

Magnitude of Scattered Field





Conclusions

- The matrix compression has been successfully integrated in EIGER.
  - For parallel machines
  - With iterative solver
- The viability of the technique has been demonstrated on a diverse group of problems.
  - Exterior problems
  - Problems with external geometry connected through slots.
    - Uses the thin-slot formulation already integrated in EIGER

Future Work - Compression

- **Improve the load balancing of the matrix:**
  - **For the MOM blocks, by the block size not just by block number.**
  - **Use preprocessing to generate block matrix structure.**

- **Improve solution time by reducing the iteration count**
  - **Preliminary work performed by Matt Bettencourt on preconditioning revealed:**
    - **Standard methods ILU, Diagonal preconditioning will fail**
    - **Use Sparse Approximate Inverse (SAI)**
    - **Applied it to the two smaller problems discussed earlier.**
    - **Defined the algorithm to implement and tested it in MATLAB.**

- **Continue testing on problems of interest to Sandia.**
  - **Verify and quantify errors for a robust implementation.**

Congratulations to Prof. Uslenghi

"George"

G -- gracious

E -- encouraging

O -- outgoing

R -- respectful and respected

G -- generous

E -- energetic

Final Comments

- Thanks for allowing me to be one of your students.
- Thanks for your example over the years:
  - Technical expertise
  - Relational wisdom
- Thanks for being a friend.
- Please slow down – “I can’t keep up with you”