SAND2015-5367C

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Constrained Interpolation Remap for Interface-Capturing Finite Element Methods Applied to Multi-Material Electromagnetics

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USNCCM13, July 2015

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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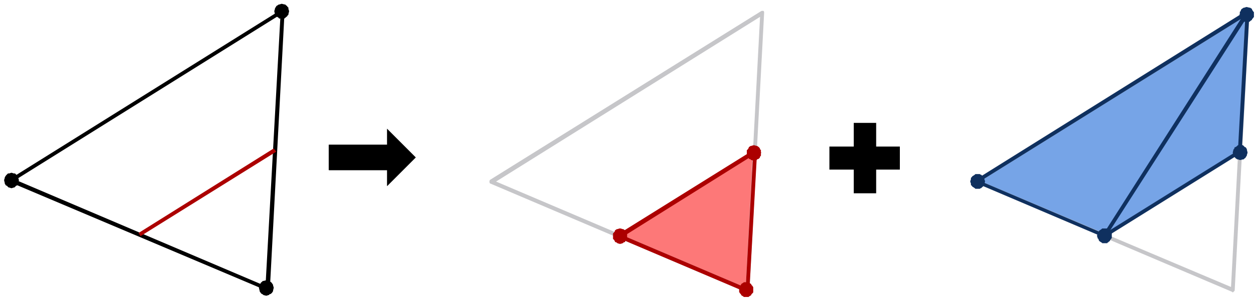
Conformal Decomposition FEM

- CDFEM[1] dynamically generates an interface-conforming mesh
  - Related to XFEM-type approaches: single phase/state per subelement



- Naturally captures C^0 discontinuities
- Complete de Rahm complex available in 3-D (for tetrahedra)
  - Requires cutting and tracking full topology of a tetrahedron to define discretizations for H(grad), H(curl) and H(div) spaces
  - Recently extended to electromagnetics[2]

[1] Noble, Newren and Lechman, IJNMF, 2009
[2] Kramer, Bochev, Siefert and Voth, JCP, 2013

Remap for enriched methods

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- Consider ALE Lagrange + remap formulation for MHD
- How do we remap a [divergence-free] field on an enriched triangular or tetrahedral mesh?

$$\nabla \times \mu^{-1}\mathbf{B} = \sigma\mathbf{E}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

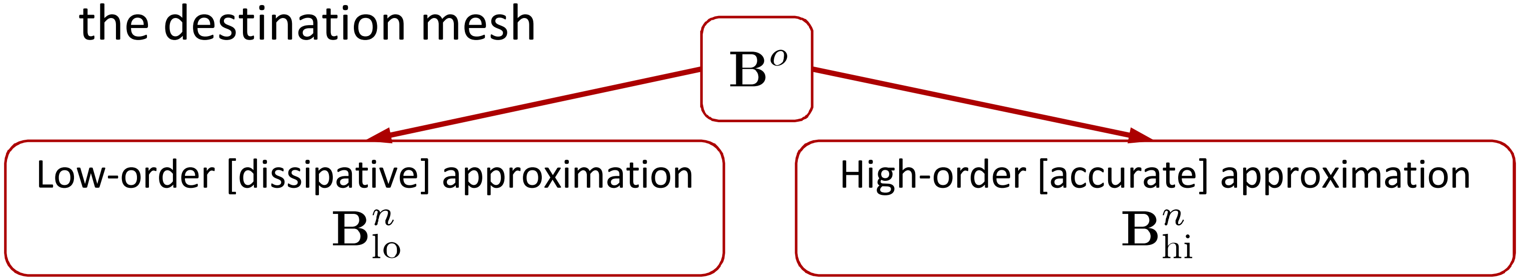
$$\nabla \cdot \sigma\mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

- *Think about remap as a method to transfer a representation of a field from one mesh to another*
  - Applicable to general meshes, general discretizations
  - Consistent with compatible discretizations (edge, face bases)
  - No need to define fluxes from one mesh to another
  - Must preserve physical discontinuities across interfaces

Constrained Interpolation Remap

- Bochev & Shashkov (2005):
  1. Construct ‘low’ and ‘high’ order interpolations of the source field on the destination mesh



  2. Form the desired norm on the source mesh

$$\mathbf{B}^n = \lambda_{\mathrm{opt}} \mathbf{B}^n_{\mathrm{lo}} + (1 - \lambda_{\mathrm{opt}}) \mathbf{B}^n_{\mathrm{hi}}$$

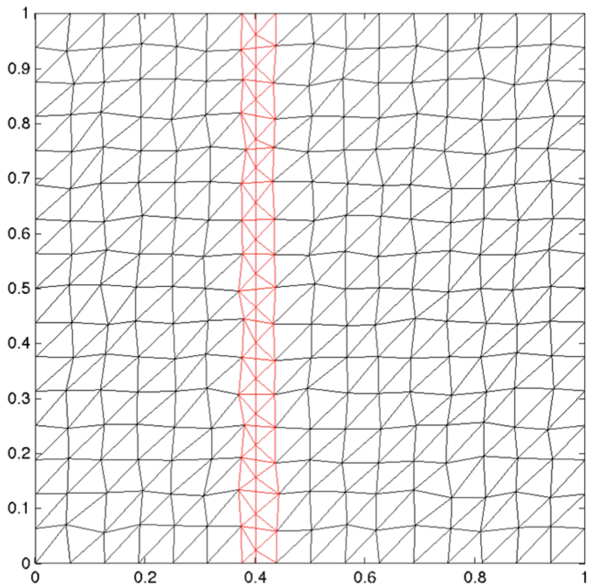
  3. (Locally) optimize the combination of low- and high-order interpolations such that this norm is preserved

$$\lambda^e_{\mathrm{opt}} = \mathrm{argmin} \Big| \|\mathbf{B}^o\|^2_e - \|\mathbf{B}^n(\lambda^e)\|^2_e \Big|^2 \quad \forall e \in T^n.$$

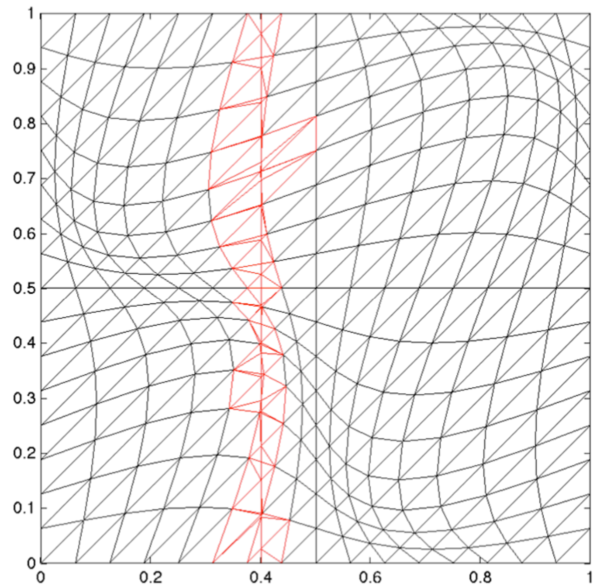
- The 2-D problems are divergence-free by construction

Test Problems

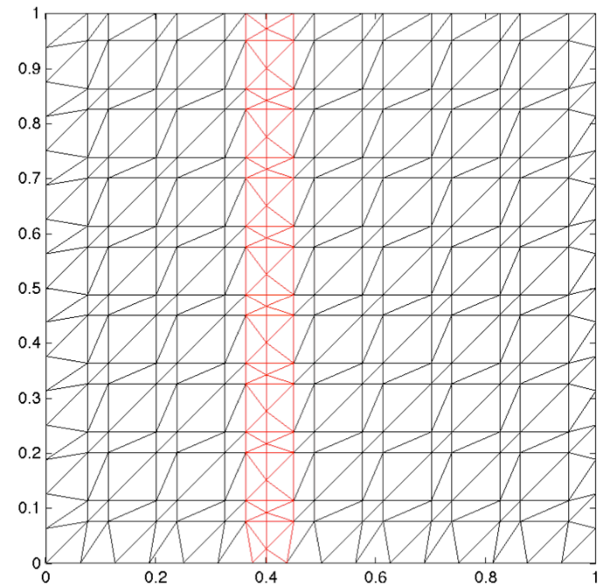
- Linear interface at constant *x*, enriched elements highlighted
- Perform 20 remap cycles with C^∞ and C^0 test fields



RandomFixed
Random motion of interior nodes



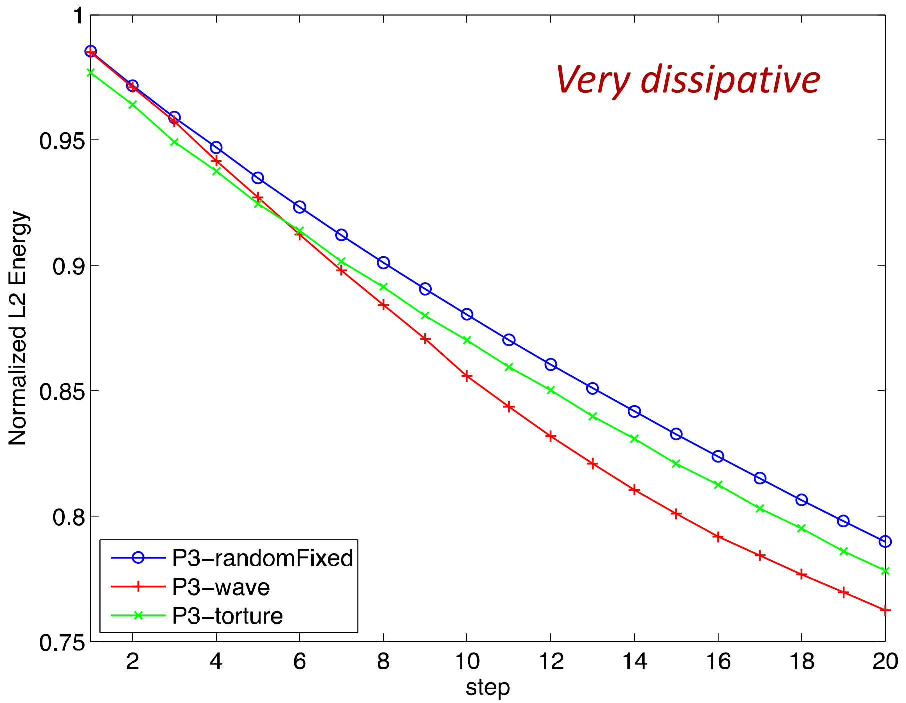
Wave
Sinusoidal motion of interior nodes



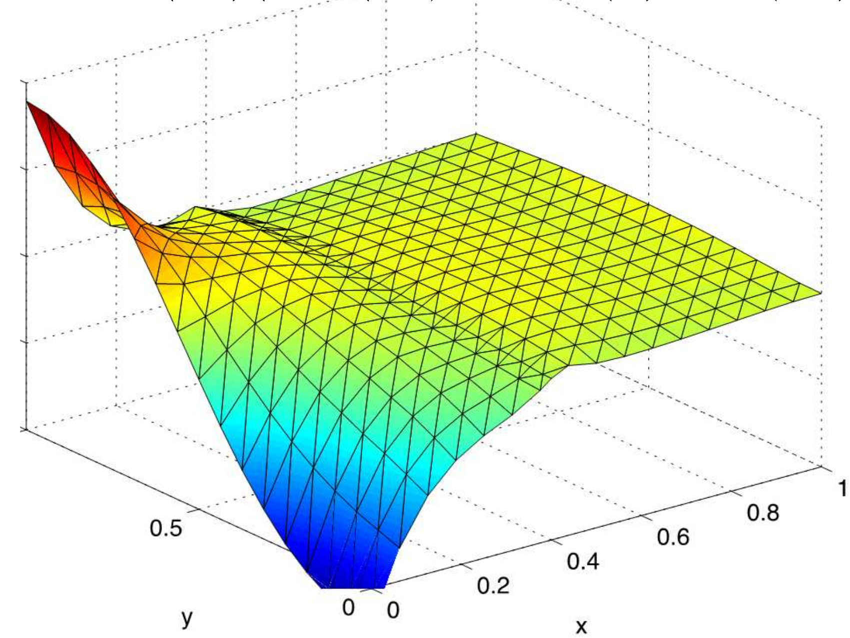
Torture
Mesh compression and expansion

CI Remap: Visually

- Remap using zeroth-order interpolation



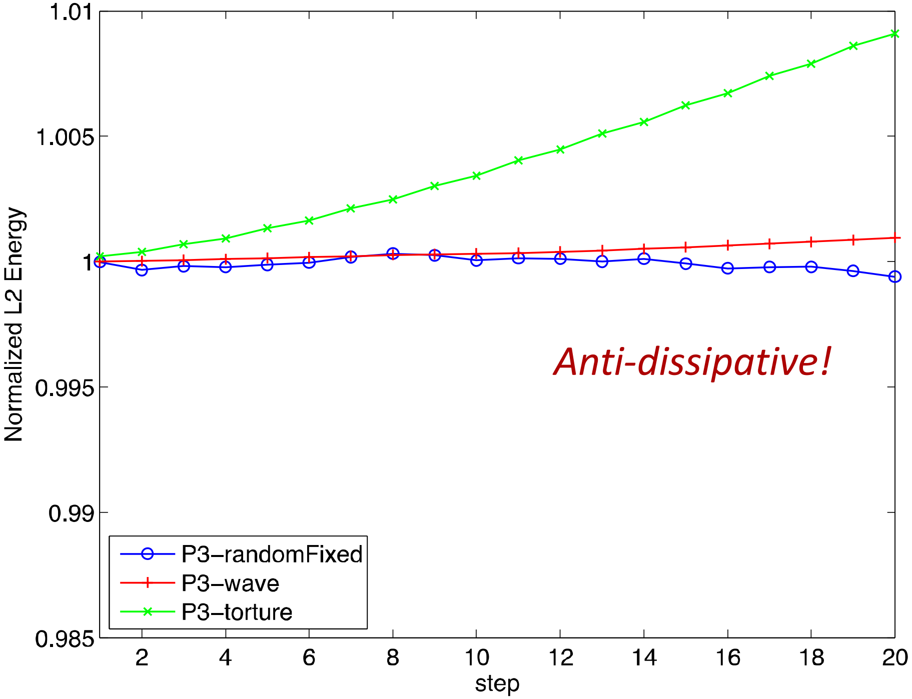
$$f = \cos(\pi y)(\sinh(\pi x) - \coth(\pi)\cosh(\pi x))$$



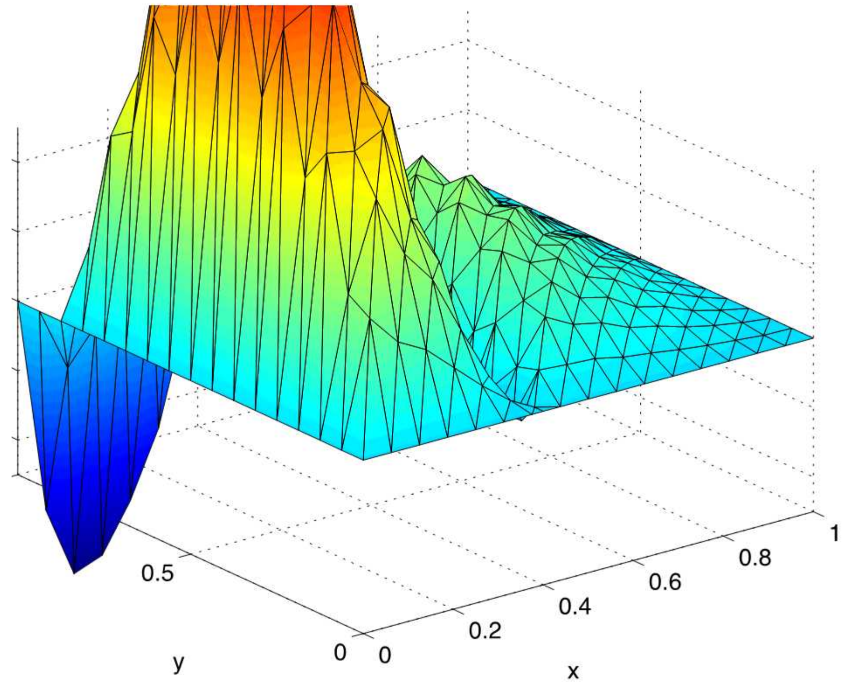
L^2 norm of the nodal field over 20 remap cycles, relative to the initial norm; Difference between initial and final solutions [torture grid motion]

CI Remap: Visually

- Remap using first-order interpolation



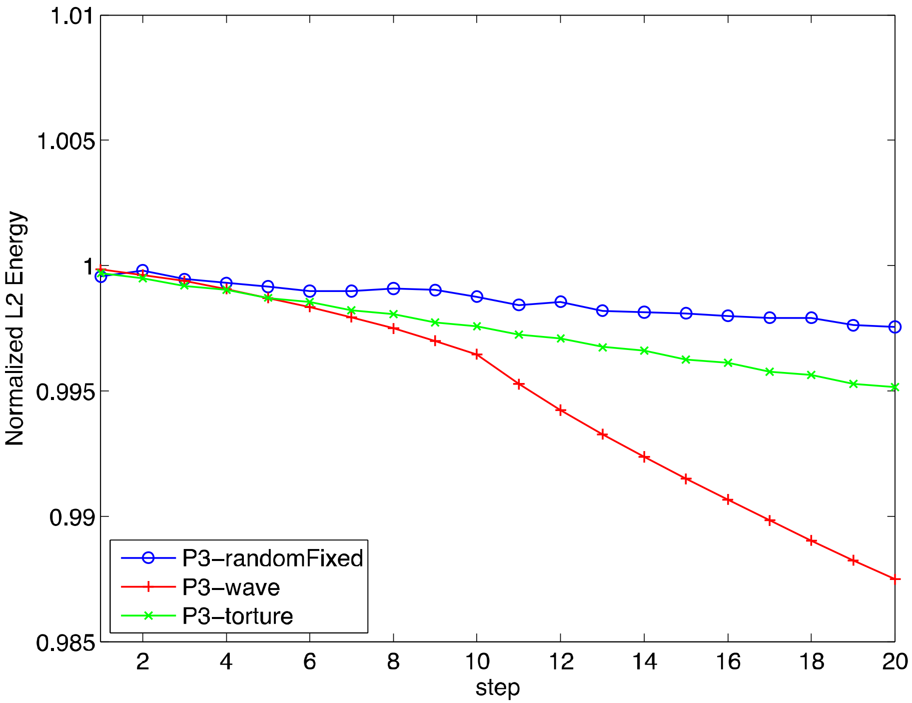
$$f = \cos(\pi y)(\sinh(\pi x) - \coth(\pi)\cosh(\pi x))$$



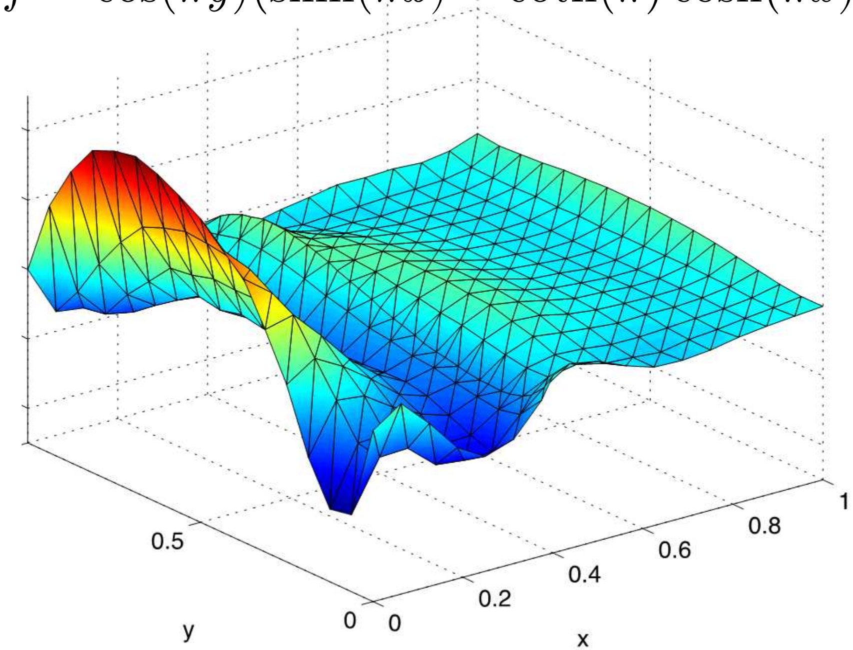
L^2 norm of the nodal field over 20 remap cycles, relative to the initial norm; Difference between initial and final solutions [torture grid motion]

CI Remap: Visually

- Remap using second-order interpolation (patch recovery)



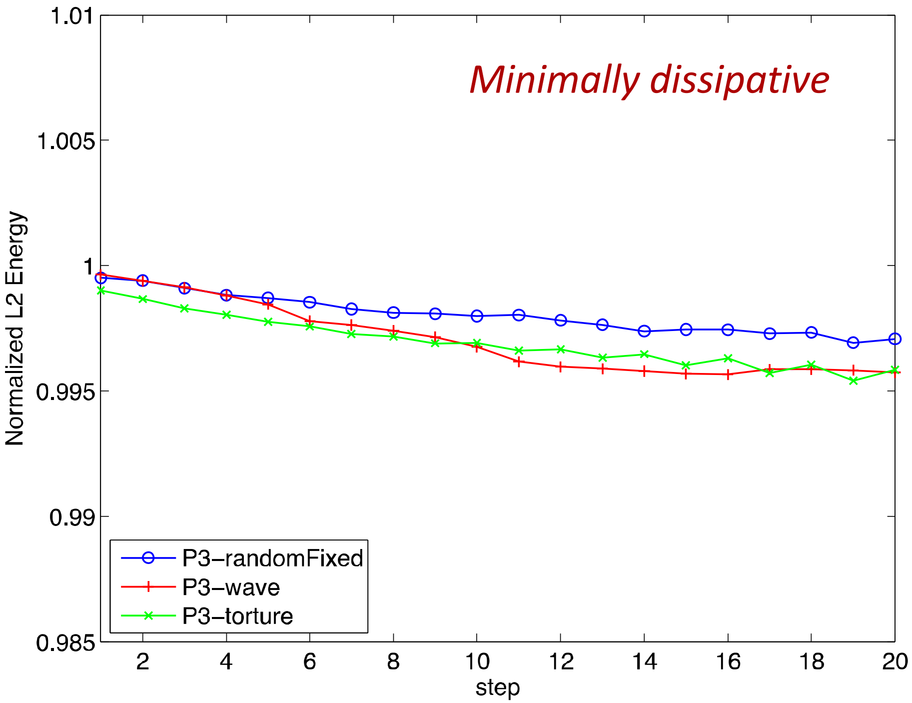
$$f = \cos(\pi y)(\sinh(\pi x) - \coth(\pi)\cosh(\pi x))$$

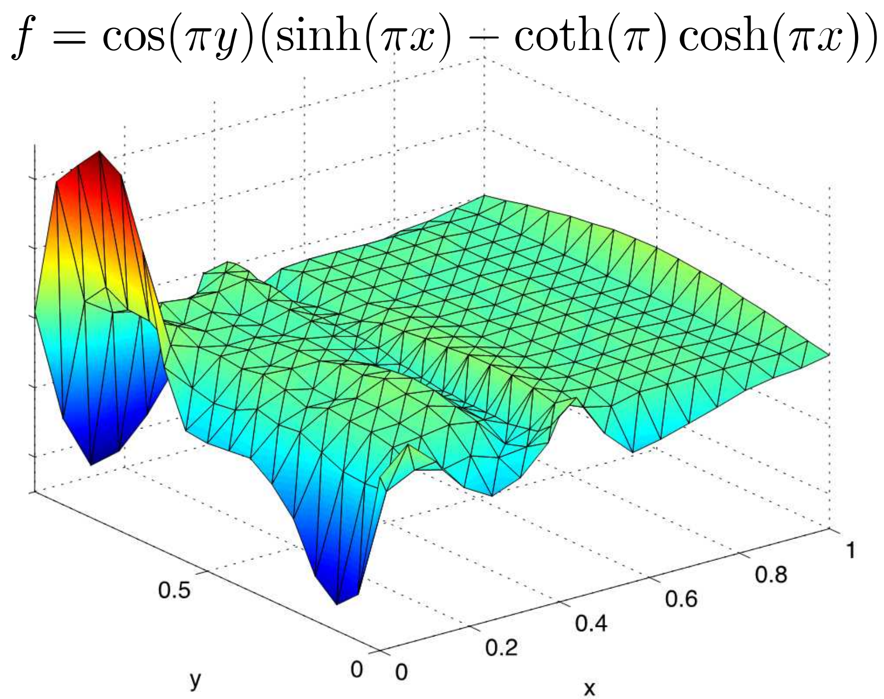


L^2 norm of the nodal field over 20 remap cycles, relative to the initial norm; Difference between initial and final solutions [torture grid motion]

CI Remap: Visually

- CI Remap using 0th + 2nd order (with analytic reference norm)

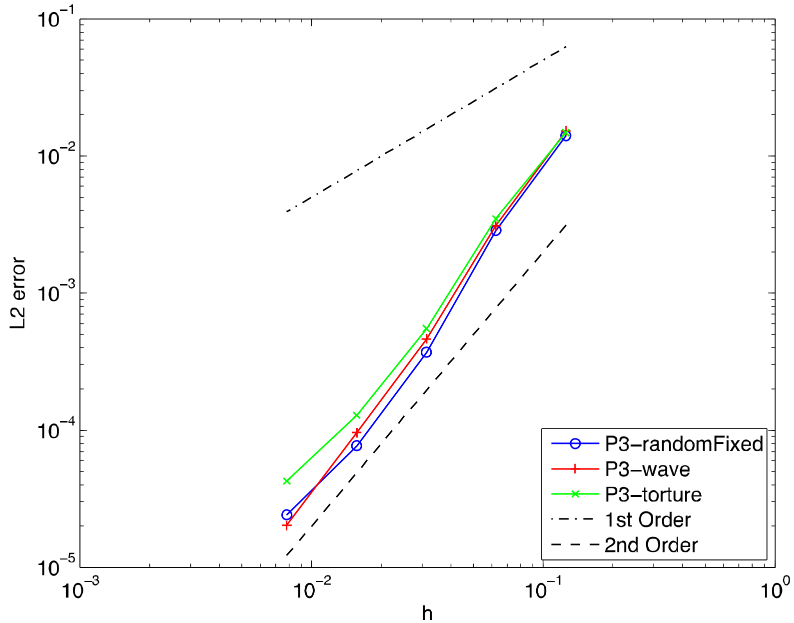




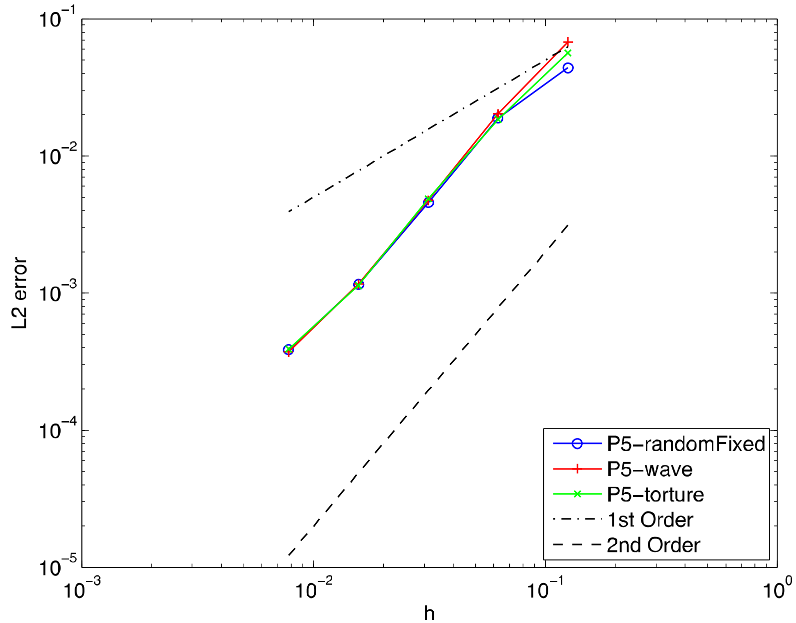
L^2 norm of the nodal field over 20 remap cycles, relative to the initial norm; Difference between initial and final solutions [torture grid motion]

CI Remap for Electromagnetics

- Nodal formulations of 2-D Maxwell equations:
  - B-form: Transverse electric field, magnetic field $\boldsymbol{B} = [0, 0, B_z(x,y)]$
  - Remap preserves the L^2 norm of the solution, $|\boldsymbol{B}^2|$



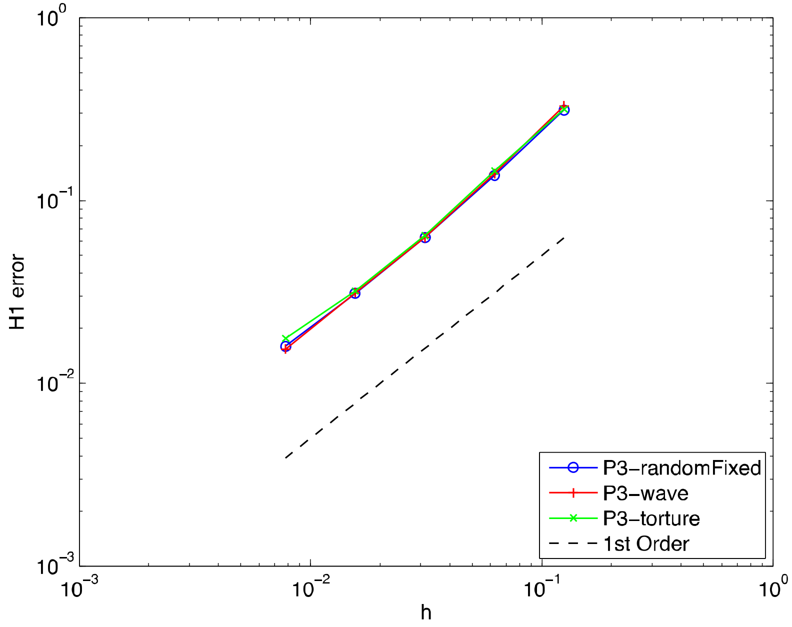
$$B_z = \cos(\pi y)(\sinh(\pi x) - \coth(\pi)\cosh(\pi x))$$

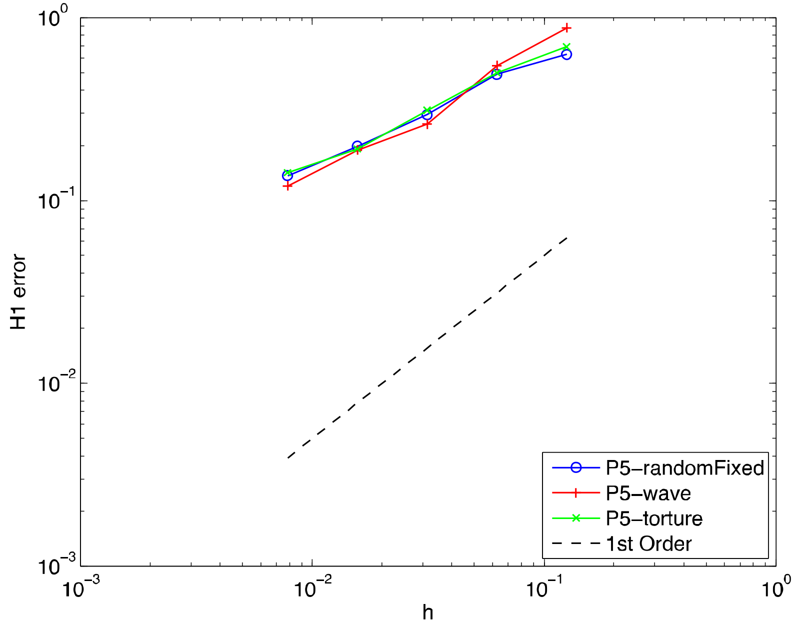


$$B_z = \begin{cases} 2 - r^2/4, & r < 1/3, \\ 1 - r^2, & r \geq 1/3 \end{cases}$$

CI Remap for Electromagnetics

- Nodal formulations of 2-D Maxwell equations:
  - A-form: Transverse magnetic, nodal vector potential **A** = [0, 0, $A_z(x,y)$]
  - Remap preserves the H^1 seminorm, $|\text{grad}(A_z)^2| = |\mathbf{B}^2|$



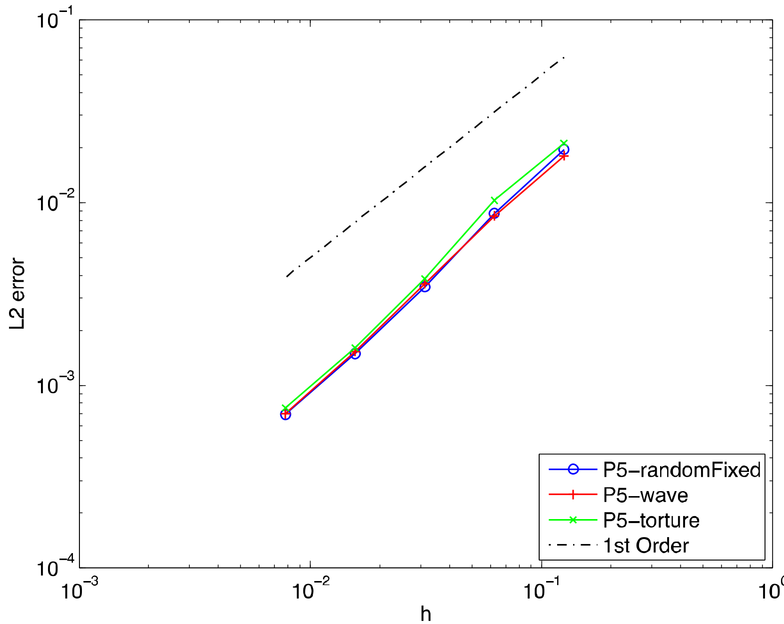


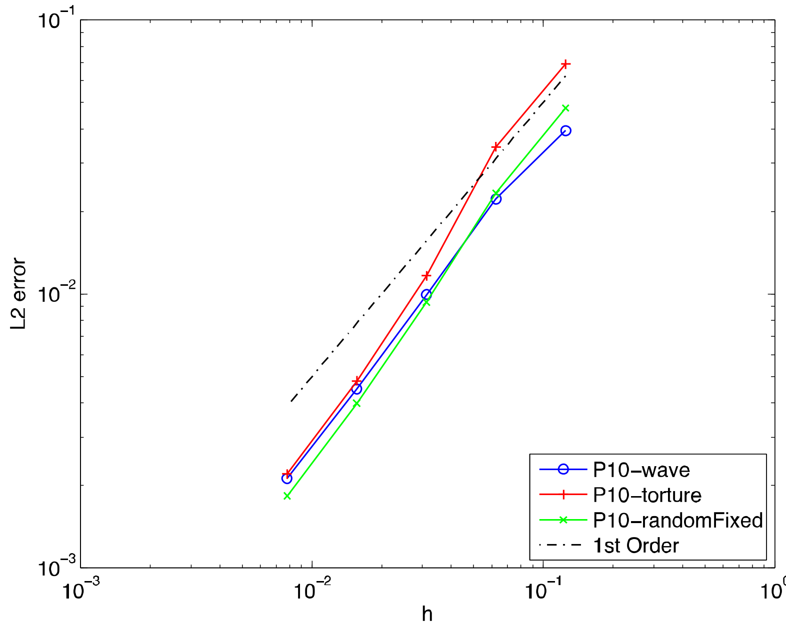
$$A_z = \cos(\pi y)(\sinh(\pi x) - \coth(\pi)\cosh(\pi x))$$

$$A_z = \begin{cases} 2 - r^2/4, & r < 1/3, \\ 1 - r^2, & r \geq 1/3 \end{cases}$$

Compatible CI Remap

- Alternative compatible discretization for transverse electric:
  - Edge-based vector potential **A** = $[A_x, A_y, 0]$
  - Remap preserves the H(curl) seminorm, $|\mathrm{curl}(\mathbf{A})^2| = |\mathbf{B}^2|$
- Direct extension to a compatible 3-D formulation





$$\mathbf{A} = \left[\left(1 - 8x + 2y + 8xy\right)/24, \left(-1 + 2x + 4y - 8x^2\right)/24\right]$$

$$\mathbf{A} = \begin{cases} \left[y\left(\frac{169}{266} - \frac{28}{19}x\right), 0\right], & x < 4/7 \\ \left[y\left(\frac{1}{266} - \frac{98}{266}x\right), 0\right], & x \geq 4/7 \end{cases}$$

Conclusions

- CI remap has been successfully extended to problems with enriched CDFEM discretizations.
- Remap has been demonstrated for both nodal and edge-based fields with different reference norms:
  - Convergence at the expected rates is observed in remapped solutions.
  - Results show small amounts of dissipation that converges to exact conservation in the reference energy norm.
- Further work is needed to improve patch recovery for the edge element formulation.
- Methods for remapping the reference (target) norm need further investigation.