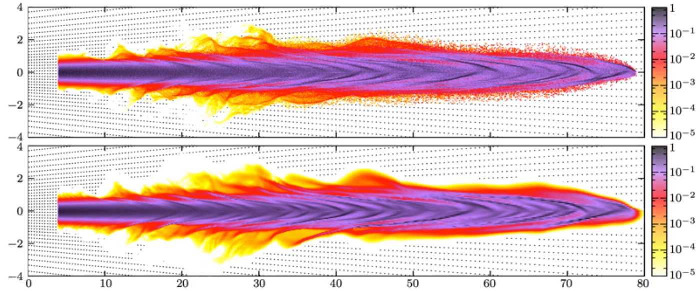
SAND2015-5183C

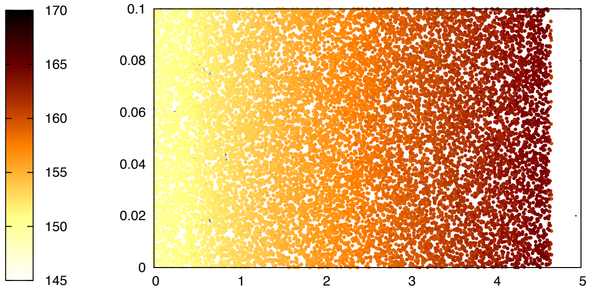
An Eulerian strategy for disperse phase flows

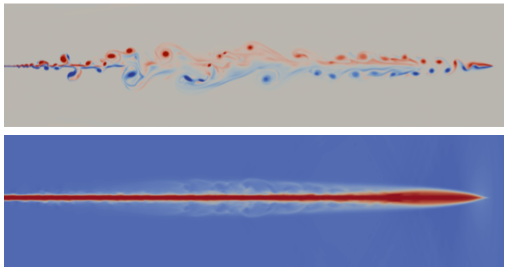
F. Doisneau, M. Arienti, J. Oefelein

Combustion Research Facility

Sandia National Laboratories, Livermore, CA 94551







Support by Sandia National Laboratories' LDRD program

LDRD – Laboratory Directed Research and Development

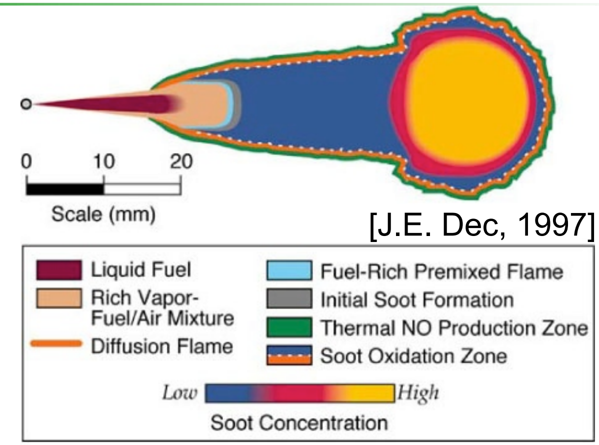
Is gratefully acknowledged.

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

MOTIVATION 1 – Liquid fuel injection

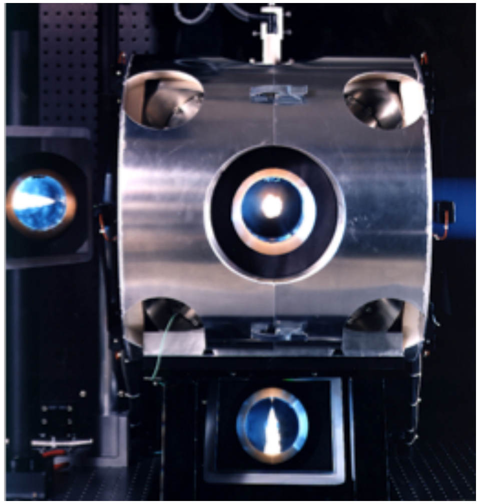
In Diesel and Gasoline engines

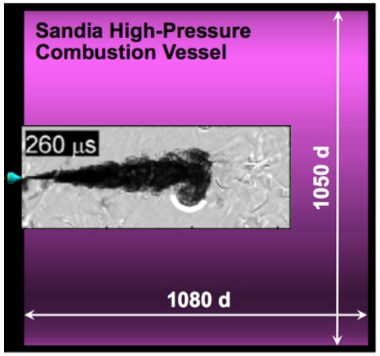
- **Inlet is turbulent** (+ cavitation)
  - **$Re \sim 10^5$, $d = 90\mu m$**
- **High pressure chamber and sonic flow**
  - **$p = 60bar$, $u_l = 600m/s$**
- **Atomization process not understood**
  - **$We \sim 10^4$, $1\mu m < r_l < 100\mu m$**



[J.E. Dec, 1997]

➔ **MULTI-SCALE&MULTI-PHYSICS** drive **MIXING&COMBUSTION**





Experimental background
High pressure vessels
[Pickett 2010, Skeen 2014]

Need for a
High Fidelity Simulation
that is **affordable**

MOTIVATION 2 – Kinetic problems

Versatility of coupled Boltzmann equation

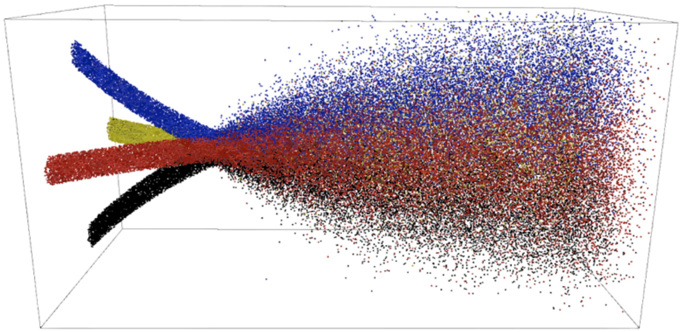
- **Kinetic equations**
  - –Boltzmann
  - –Vlasov
  - –Williams

- **With many applications**
  - –gases
  - –plasmas
  - –sprays
  - –astrophysics

- **and possibly coupled**
  - –multicomponent Boltzmann
  - –Maxwell or subset
  - –Navier-Stokes

Kinetic theory to translate **non-equilibrium**/unknown equilibrium

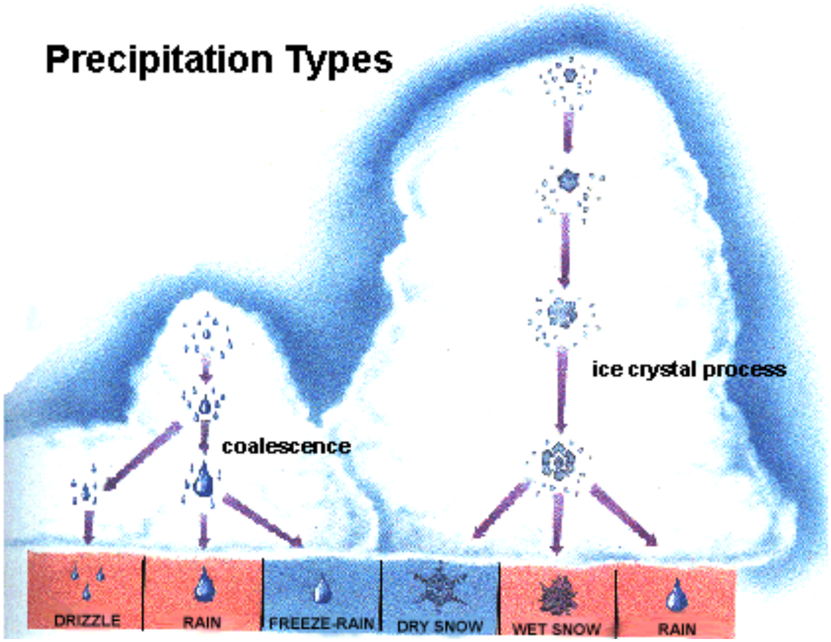
- **Complex behavior**
- **Large dimensionality**
- **Strong couplings**
- **Requires a dedicated approach**



MOTIVATION 3 – Rain formation

Atmospheric sciences

- **Raindrop formation/fall depends on:**
  - Thermodynamics
  - Turbulence
  - Coalescence dynamics
- **Empirical models not accurate enough**



- **Need for detailed+large scale computations**

State of the art – Particle solvers

Transport for out-of-equilibrium systems

- **Direct Simulation Monte-Carlo (aka stochastic Lagrangian)**
  - –straightforward equations
  - –can handle any correlations
  - –converges as n1/2
  - –noise
  - –unexpected errors related to non-linearity
  - –load-balancing problem
  - –coupling to a Eulerian fluid difficult

- **Eulerian resolution (aka Moment Methods)**
  - –deterministic
  - –coupling to Eulerian fluids straightforward
  - –predictable load
  - –solves for some moments
  - –requires assumptions/presumed pdfs
  - –unexpected errors related to the assumptions
  - –requires a dedicated numerical scheme (realizability, robustness)
    - –**Kinetic** [Bouchut 2003, de Chaisemartin 2009]
    - –**MUSCL** [Le Touze 2012, Vie 2015]
    - –**Semi-Lagrangian** [Cheng 1976, Besse 2003]
    - –**Discontinuous Galerkin** [Sabat 2014]

MODEL – Kinetic theory for sprays

A high-fidelity description

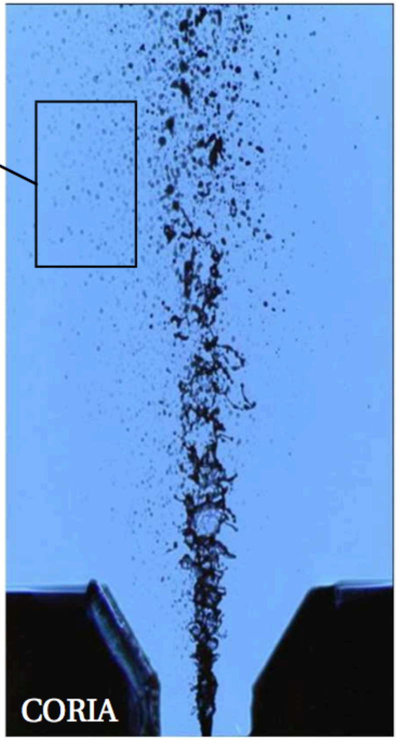
Point particles with velocity $\boldsymbol{c}$, temperature θ, surface S

Number density function **NDF** : describes the disperse phase

$$\mathrm{d}N = f(t,\boldsymbol{x},\boldsymbol{c},S,\theta)\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}\boldsymbol{c}\,\mathrm{d}S\,\mathrm{d}\theta$$

NDF satisfies a Boltzmann-like PDE [Williams, 1958]

$$\partial_t f + \partial_{\boldsymbol{x}}\cdot(\boldsymbol{c}f) + \partial_{\boldsymbol{c}}\cdot(\boldsymbol{F}f) + \partial_\theta(\mathrm{H}f) = \mathfrak{B} + \mathfrak{C}$$



Closures : all the physical modeling job

- Drag force $\boldsymbol{F}$
- Heat transfer H
- Break-up $\mathfrak{B}$
- Collisions $\mathfrak{C}$

MODEL – Kinetic theory for sprays

A high-fidelity description

Inertial droplet coalescence :

$$\mathfrak{C}^{+} = \frac{1}{2}\int_{\boldsymbol{c}^{\star}}\int_{v^{\star}\in[0,v]} f(t,\boldsymbol{x},\boldsymbol{c}^{\diamond},v^{\diamond})f(t,\boldsymbol{x},\boldsymbol{c}^{\star},v^{\star})\mathfrak{K}_{\mathrm{coal}}J\,\mathrm{d}v^{\star}\,\mathrm{d}\boldsymbol{c}^{\star}$$

$$\mathfrak{C}^{-} = \int_{\boldsymbol{c}^{\star}}\int_{v^{\star}} f(t,\boldsymbol{x},\boldsymbol{c},v)f(t,\boldsymbol{x},\boldsymbol{c}^{\star},v^{\star})\mathfrak{K}_{\mathrm{coal}}\,\mathrm{d}v^{\star}\,\mathrm{d}\boldsymbol{c}^{\star}$$

Coalescence source term

- integral
- quadratic
- multivariate kernel
- non-linear kernel

Balistic coalescence kernel modeling

$$\mathfrak{K}_{\mathrm{coal}} = \pi(r^{\diamond}+r^{\star})^{2}\,||\boldsymbol{u}-\boldsymbol{u}^{\star}||\,\mathfrak{E}(......)$$

Efficiency $\mathfrak{E}$ models

- collision efficiency [Langmuir, 1948, Beard and Grover, 1974]
- coalescence efficiency [Brazier-Smith et al., 1972, Ashgriz and Poo, 1990]

$\Rightarrow$ **closed**

MODEL – Kinetic theory for sprays

A high-fidelity description

- Reduction of kinetic theory is
  - Crucial
  - Problem dependent

- Two approaches considered
  - Lagrangian (Direct simulation Monte-Carlo)
  - Eulerian Multi-Fluid (Sectional)

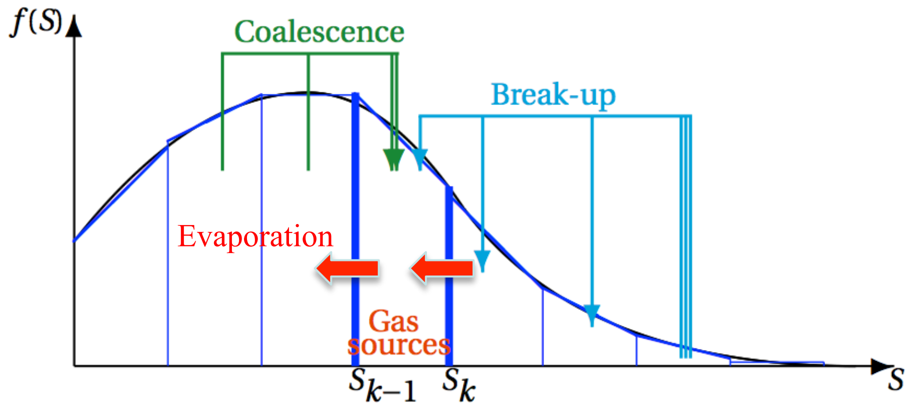
- Objectives
  - Numerical strategy for strongly coupled massively parallel simulations
  - Hybrid approach for wide spectrum of droplet sizes

MODEL – Sectional method (EULERIAN)

A cost-efficient way to capture polydispersity

- **Various drop sizes are treated as a continuum:**

  Multi-Fluid [Laurent 2001, Doisneau 2013]



N_{sec} systems

$$\begin{cases} \partial_t n_k + \partial_{\boldsymbol{x}} \cdot (n_k \boldsymbol{u}_k) = {}^2C_k^n + {}^2B_k^n + {}^2E_k^n \\ \partial_t m_k + \partial_{\boldsymbol{x}} \cdot (m_k \boldsymbol{u}_k) = {}^2C_k^m + {}^2B_k^m + {}^2E_k^m \\ \partial_t (m_k \boldsymbol{u}_k) + \partial_{\boldsymbol{x}} \cdot (m_k \boldsymbol{u}_k \otimes \boldsymbol{u}_k) = m_k \boldsymbol{F}_k + {}^2\boldsymbol{C}_k^u + {}^2\boldsymbol{B}_k^u + {}^2E_k^u \\ \partial_t (m_k h_k) + \partial_{\boldsymbol{x}} \cdot (m_k h_k \boldsymbol{u}_k) = m_k H_k + {}^2C_k^h + {}^2B_k^h + {}^2E_k^h \end{cases}$$

$\rightleftharpoons$ Navier-Stokes with **sources**

…many integral source terms to compute

MODEL – Sectional method (EULERIAN)

A cost-efficient way to capture polydispersity

Coalescence terms : quadratic sums of 2D elementary integrals

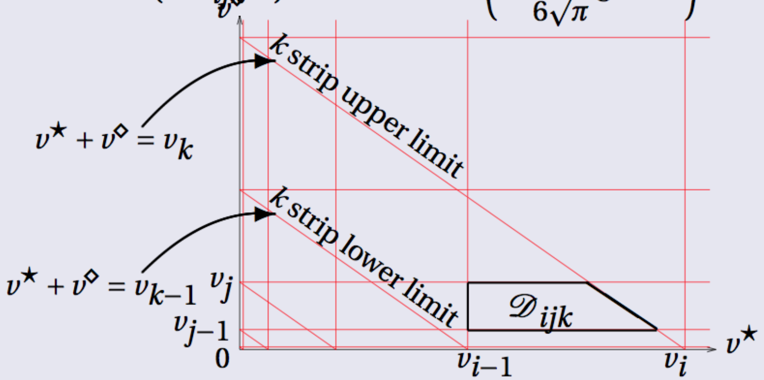
$$^2\mathrm{C}^{n+}_k = \sum_{i=1}^{k}\sum_{j=1}^{i-1} \mathrm{Q}^{n}_{ijk} \qquad ^2\mathrm{C}^{n-}_k = \sum_{i=1}^{N}\sum_{j=1}^{N} \mathrm{Q}^{n}_{kji}$$

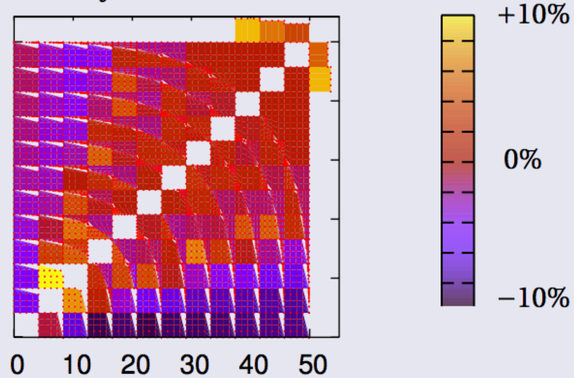
$$^2\mathrm{C}^{m+}_k = \sum_{i=1}^{k}\sum_{j=1}^{i-1} (\mathrm{Q}^{\star}_{ijk} + \mathrm{Q}^{\diamond}_{ijk}) \qquad ^2\mathrm{C}^{m-}_k = \sum_{i=1}^{N}\sum_{j=1}^{N} (\mathrm{Q}^{\star}_{kji} + \mathrm{Q}^{\diamond}_{kji})$$

$$^2\mathbf{C}^{u+}_k = \sum_{i=1}^{k}\sum_{j=1}^{i-1} (\boldsymbol{u}_i \mathrm{Q}^{\star}_{ijk} + \boldsymbol{u}_j \mathrm{Q}^{\diamond}_{ijk}) \qquad ^2\mathbf{C}^{u-}_k = \boldsymbol{u}_k \,.\, ^2\mathrm{C}^{m-}_k$$

Elementary integrals

$$\begin{pmatrix} \mathrm{Q}^{n}_{ijk} \\ \mathrm{Q}^{\star}_{ijk} \\ \mathrm{Q}^{\diamond}_{ijk} \end{pmatrix}(t,\boldsymbol{x}) = \iint_{\mathcal{D}_{ijk}} \begin{pmatrix} 1 \\ \frac{\rho_l}{6\sqrt{\pi}} S^{\star\frac{3}{2}} \\ \frac{\rho_l}{6\sqrt{\pi}} S^{\diamond\frac{3}{2}} \end{pmatrix} {}^2\kappa_i(t,\boldsymbol{x},S^\star)\, {}^2\kappa_j(t,\boldsymbol{x},S^\diamond)\, |\,\boldsymbol{u}_i - \boldsymbol{u}_j\,|\, \mathfrak{E}(\dots)\pi(r^\star + r^\diamond)^2\, \mathrm{d}S^\star\, \mathrm{d}S^\diamond$$





Relative error of integrals for Exp-TSM with (NC5)
13 sections, sizes in radius (μm) ; + quadrature nodes.

If one size moment ⇒ size dependency factorization and **precalculation**

General case ⇒ **integration at all times** !

MODEL – Sectional method (EULERIAN)

A cost-efficient way to capture polydispersity

- **The coupled NS-PGD system**

$$
\left\{
\begin{array}{lll}
\partial_t \rho_g Y_f + \partial_{\boldsymbol{x}} \rho_g Y_f \boldsymbol{u}_g & = \omega_f + \sum_k \mathrm{E}_k^{m-g} & \\
\partial_t \rho_g Y_i + \partial_{\boldsymbol{x}} \rho_g Y_i \boldsymbol{u}_g & = \omega_i \quad , & i \in [\![1; N_{\text{species}}]\!],\ i \neq f \\
\partial_t \rho_g \boldsymbol{u}_g + \partial_{\boldsymbol{x}} \rho_g \boldsymbol{u}_g \otimes \boldsymbol{u}_g & = -\partial_{\boldsymbol{x}} p + \sum_k \left(-\boldsymbol{F}_k + \boldsymbol{u}_k \mathrm{E}_k^{m-g} \right) & \\
\partial_t \rho_g e_g + \partial_{\boldsymbol{x}} \rho_g e_g \boldsymbol{u}_g & = -p \partial_{\boldsymbol{x}} \boldsymbol{u}_g + \sum_k \left(-\mathrm{H}_k + \boldsymbol{F}_k (\boldsymbol{u}_g - \boldsymbol{u}_k) + h_k \mathrm{E}_k^{m-g} \right) & \\
\partial_t m_k + \partial_{\boldsymbol{x}} m_k \boldsymbol{u}_k & = \mathrm{E}_{k+1}^m + \mathrm{B}_k^{m+} + \mathrm{C}_k^{m+} - \left(\mathrm{E}_k^m + \mathrm{E}_k^{m-g} + \mathrm{B}_k^{m-} + \mathrm{C}_k^{m-} \right) & \\
\partial_t m_k \boldsymbol{u}_k + \partial_{\boldsymbol{x}} m_k \boldsymbol{u}_k \otimes \boldsymbol{u}_k & = \boldsymbol{F}_k + \boldsymbol{u}_{k+1} \mathrm{E}_{k+1}^m + \boldsymbol{B}_k^{u+} + \boldsymbol{C}_k^{u+} - \boldsymbol{u}_k \left(\mathrm{E}_k^m + \mathrm{E}_k^{m-g} + \mathrm{B}_k^{m-} + \mathrm{C}_k^{m-} \right) & k \in [\![1; N_{\text{sec}}]\!] \\
\partial_t m_k h_k + \partial_{\boldsymbol{x}} m_k h_k \boldsymbol{u}_k & = \mathrm{H}_k + h_{k+1} \mathrm{E}_{k+1}^m + \mathrm{B}_k^{h+} + \mathrm{C}_k^{h+} - h_k \left(\mathrm{E}_k^m + \mathrm{E}_k^{m-g} + \mathrm{B}_k^{m-} + \mathrm{C}_k^{m-} \right) &
\end{array}
\right.
$$

pressureless

sections

Needs to be closed

- **Key parameter: number of sections**

MODEL – DS Monte-Carlo (LAGRANGIAN)

A reference for particle resolution

- **The coupled NS-parcel system**

$$
\begin{cases}
\partial_t \rho_g Y_f + \partial_{\boldsymbol{x}} \rho_g Y_f \boldsymbol{u}_g & = \omega_f + \sum_k \mathrm{E}_k^{m-g} \\
\partial_t \rho_g Y_i + \partial_{\boldsymbol{x}} \rho_g Y_i \boldsymbol{u}_g & = \omega_i \quad , \qquad\qquad i \in [\![1; N_{\text{species}}]\!],\ i \neq f \\
\partial_t \rho_g \boldsymbol{u}_g + \partial_{\boldsymbol{x}} \rho_g \boldsymbol{u}_g \otimes \boldsymbol{u}_g & = -\partial_{\boldsymbol{x}} p + \sum_k \left(-\boldsymbol{F}_k + \boldsymbol{u}_k \mathrm{E}_k^{m-g} \right) \\
\partial_t \rho_g e_g + \partial_{\boldsymbol{x}} \rho_g e_g \boldsymbol{u}_g & = -p \partial_{\boldsymbol{x}} \boldsymbol{u}_g + \sum_k \left(-\mathrm{H}_k + \boldsymbol{F}_k (\boldsymbol{u}_g - \boldsymbol{u}_k) + h_k \mathrm{E}_k^{m-g} \right) \\
\\
\mathrm{d}_t \boldsymbol{X}_k = \boldsymbol{u}_k ; \quad \mathrm{d}_t \boldsymbol{u}_k = \boldsymbol{F}_k ; \quad c_{p,l} \mathrm{d}_t \theta_s = H_k ; \quad \mathrm{d}_t r_k = R_k .
\end{cases}
$$

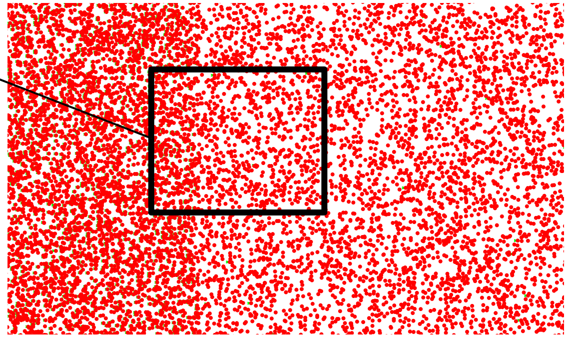
- **Collision step is split**

- **Key parameter: number of parcels**

MODEL – DS Monte-Carlo (LAGRANGIAN)

A reference for particle resolution

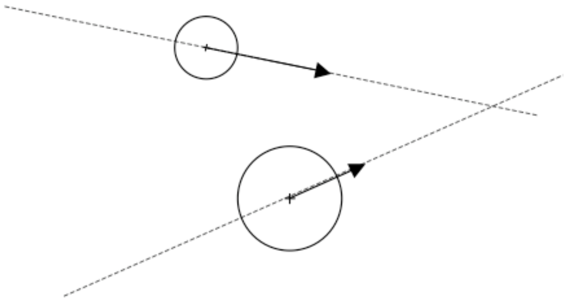
- **Stochastic Lagrangian coalescence step**
  - **Pick N/2 random pairs in a control volume**
  - **Collision probability**



$$P(\nu_p) = \frac{\lambda_{12}^{\nu_p}}{\nu_p!} \exp(-\lambda_{12})$$

with $\lambda_{12} = \dfrac{n_{p1}(N_J - 1)\Delta t}{\mathcal{V}(C_J)} \mathfrak{K}_{\mathrm{coal}}(|\boldsymbol{u}_{p1} - \boldsymbol{u}_{p2}|, r_{p1}, r_{p2})$

  - **Variable update**



$$n'_{p1} = n_{p1} - n_{p2}$$

$$v'_{p2} = v_{p2} + \nu_p v_{p1}$$

$$\boldsymbol{u}'_{p2} = \frac{v_{p2}\boldsymbol{u}_{p2} + \nu_p v_{p1} \boldsymbol{u}_{p1}}{v_{p2} + \nu_p v_{p1}}$$

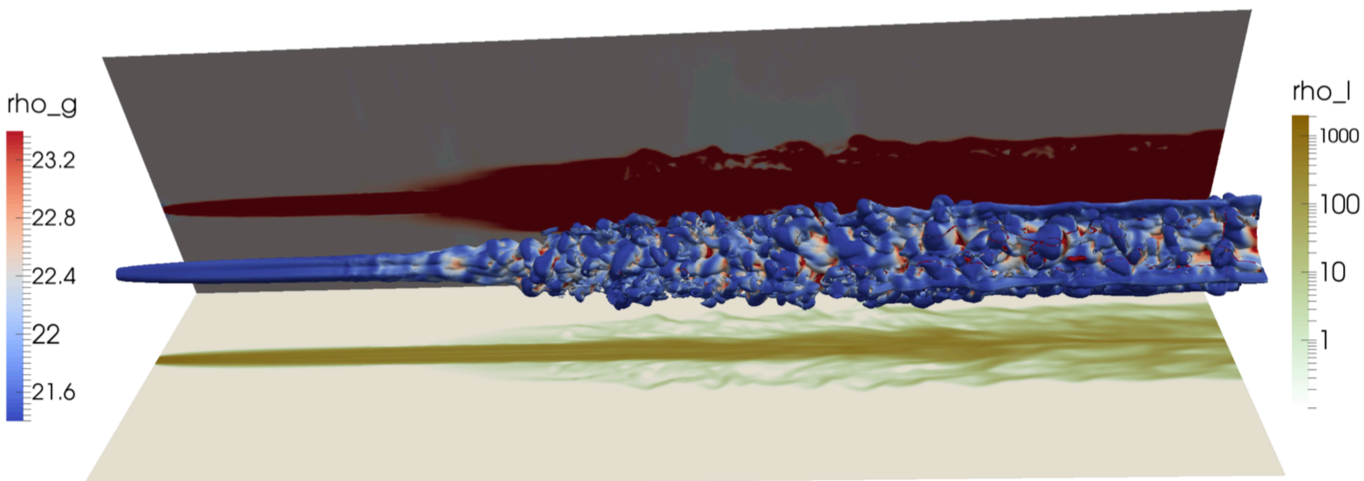
NUMERICS – Euler-Euler coupling needs

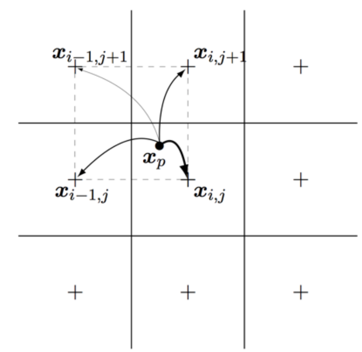
Effort on numerical methods for multi-scale coupled flows

- **1) Time integration tailored splitting**

Gas transport $\mathscr{T}_g$	=	Coupling $\mathscr{R}$ $\mathscr{F}+\mathscr{H}+\mathscr{E}$		
Section transport $\mathscr{T}_k$	=		+	Spray sources $\mathscr{B}+\mathscr{C}$

- **2) Space transport novel semi-Lagrangian scheme**

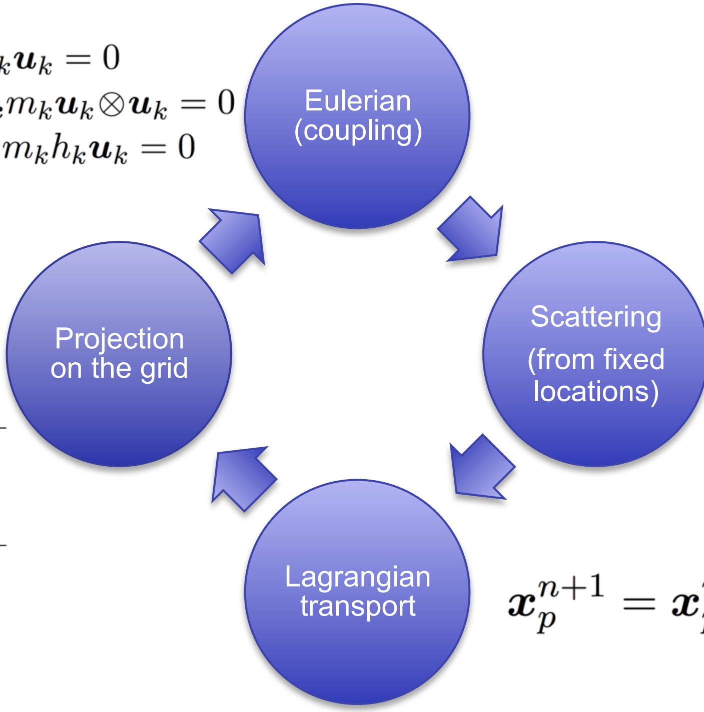




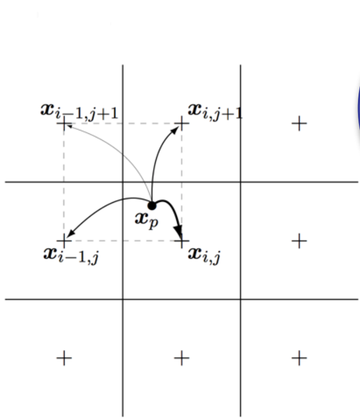
NUMERICS – PGD transport

A robust and accurate answer to PGD peculiarities

$$\begin{cases} \partial_t m_k + \partial_{\boldsymbol{x}} m_k \boldsymbol{u}_k = 0 \\ \partial_t m_k \boldsymbol{u}_k + \partial_{\boldsymbol{x}} m_k \boldsymbol{u}_k \otimes \boldsymbol{u}_k = 0 \\ \partial_t m_k h_k + \partial_{\boldsymbol{x}} m_k h_k \boldsymbol{u}_k = 0 \end{cases}$$



$$\begin{pmatrix} m_k \\ m_k \boldsymbol{u}_k \\ m_k T_k \end{pmatrix}(t, \boldsymbol{x}) \to \begin{pmatrix} \boldsymbol{x}_p \\ m_p \\ \boldsymbol{u}_p \\ T_p \end{pmatrix}(t)$$



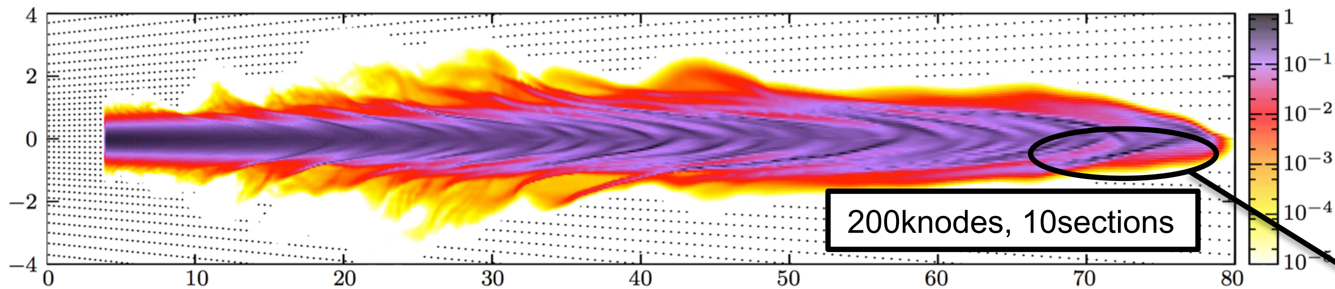
$$\boldsymbol{x}_p^{n+1} = \boldsymbol{x}_p^n + \boldsymbol{u}_p^n \Delta t$$

- **Novel semi-Lagrangian PGD transport scheme**
  - **Deterministic: no noise**
  - **Localizes spray info at mesh nodes: good for coupling**
  - **Easier load balancing**
  - **No fluxes to be computed: reduce cost and numerical diffusion**

NUMERICS – PGD transport

2D test with prescribed flow field

- **Obtained cost-efficient and accurate results**

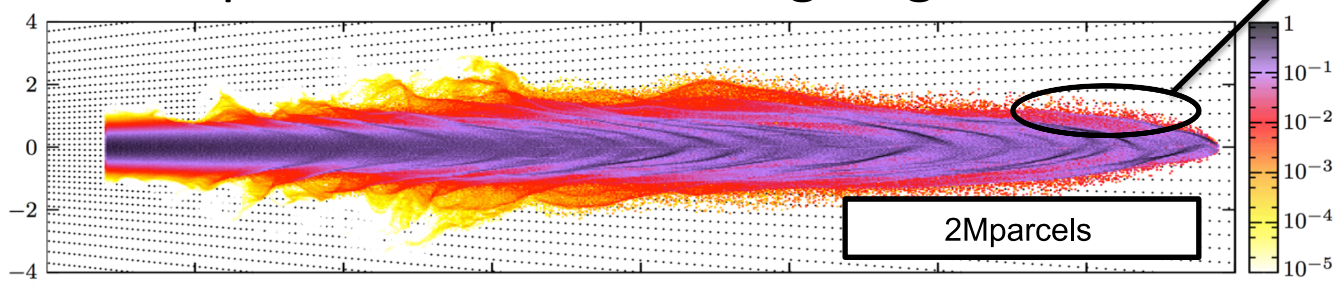


Mass concentration (Eulerian)

δ-shock?

... compared to stochastic Lagrangian

noise

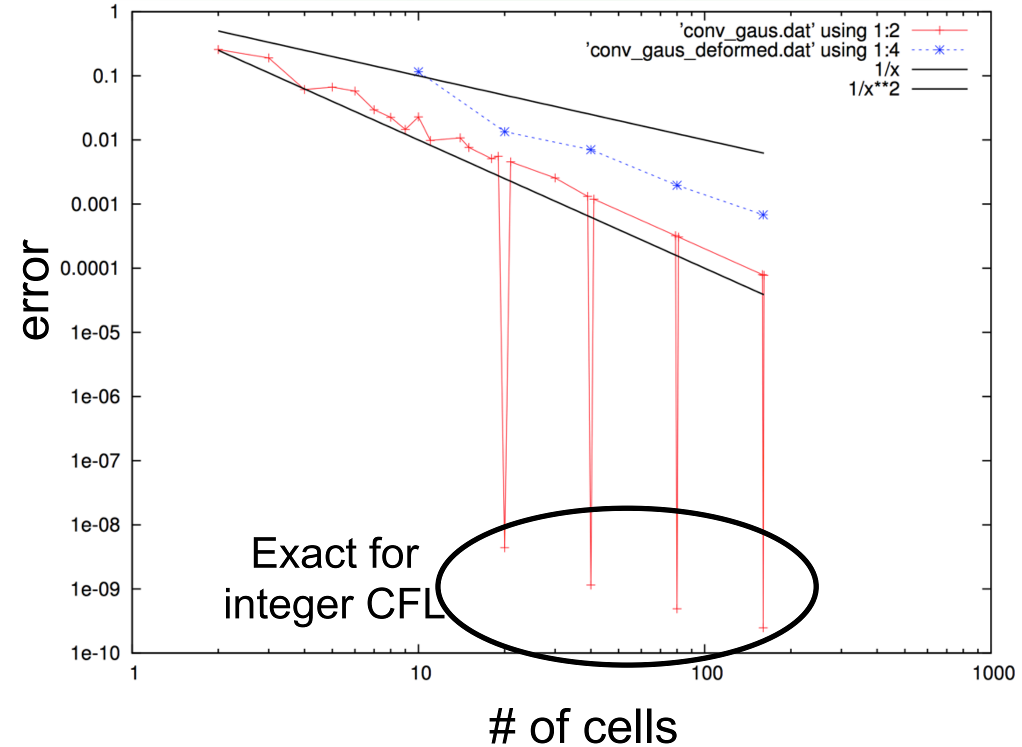


Mass concentration (Lagrangian)

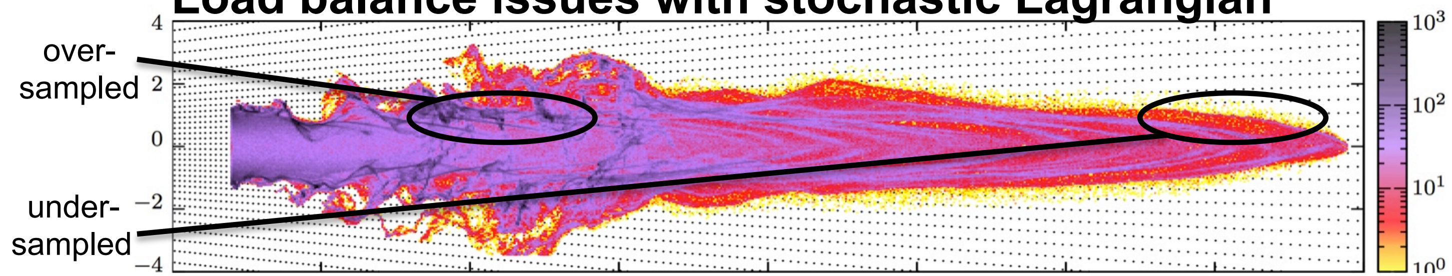
NUMERICS – PGD transport

Transport is 2nd order in space

- **No CFL constraint** (unconditionally stable)
- **Handles vacuum**
- **Handles δ-shocks**
- **Predictable load**



Load balance issues with stochastic Lagrangian



APPLICATIONS

1) **Test cases for fuel injection**
2) Coalescence studies

TEST 1 – Momentum Coupling

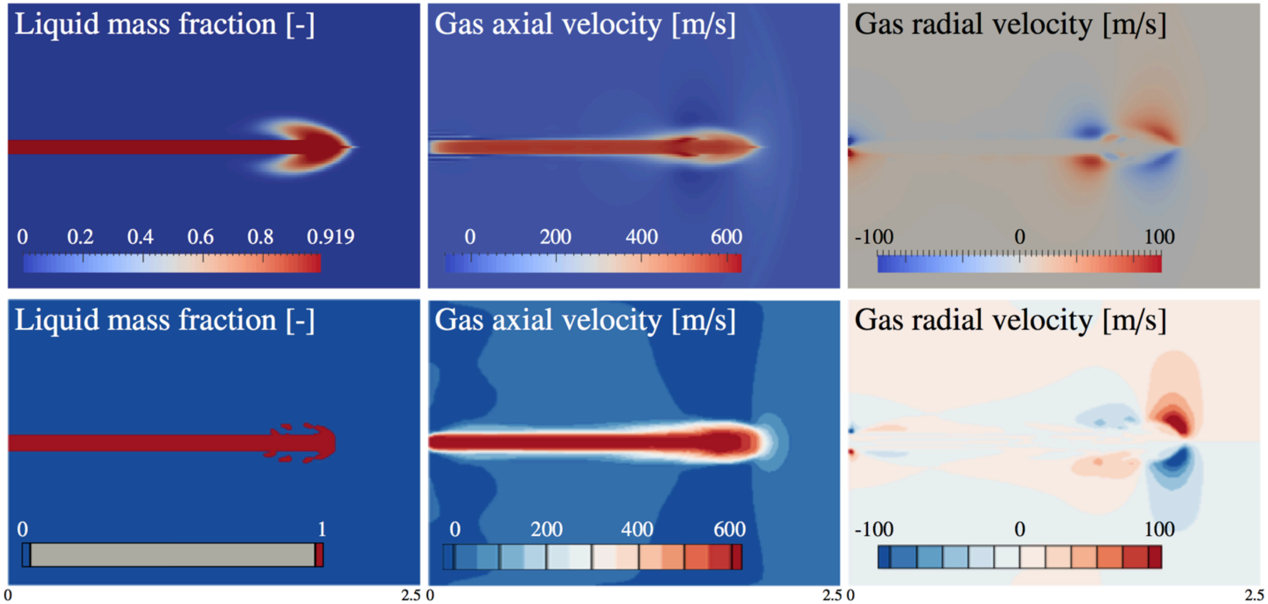
Comparison between E-ES and CLSVOF

✓ **Supersonic dense injection** (toy problem)

- **Large liquid/gas loading ratio (35/1)**
- **velocity plug-flow boundary**
- **no thermal transfer**
- **T_{end} = 4μs**

Raptor E-ES

Δx = 12.5 μm, Δt = 8 ns



CLSVOF

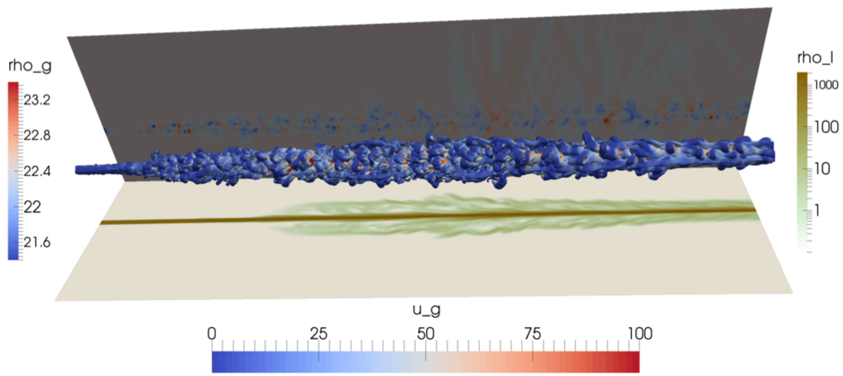
Δx = 13.3 μm, Δt ~ 6 ns

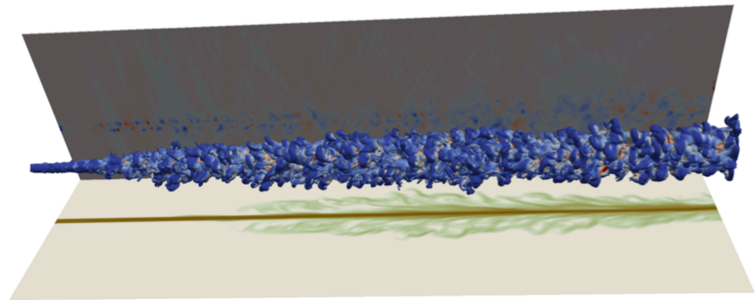
- ✓ Agreement on **gas entrainment**
- ✸ Liquid density discrepancy from **pressureless** assumption
- ✸ Jet tip is different because of **lack of surface tension**

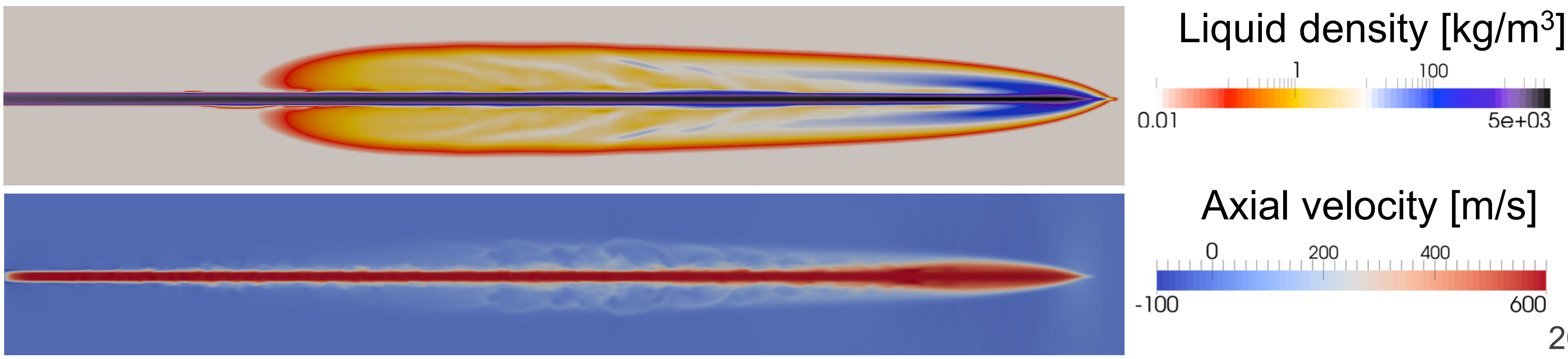
TEST 2 – Induced turbulence

Entrainment and induced turbulence by jet injection

- Executed with **RAPTOR E-ES**
  - **Box 3x3x10mm**
  - **d_{inj}=90µm, T_{end}=40µs**
  - **quiescent gas at 60bar, 900K**
  - **n-dodecane at 702kg/m³, 600m/s**
  - **50Mcells** (cartesian mesh)
  - **Δx=12.5µm, Δt=8ns, T_{end}=40µs**
  - **1 section** (prescribed initial size)
  - **PGD transport** (δ-shocks)



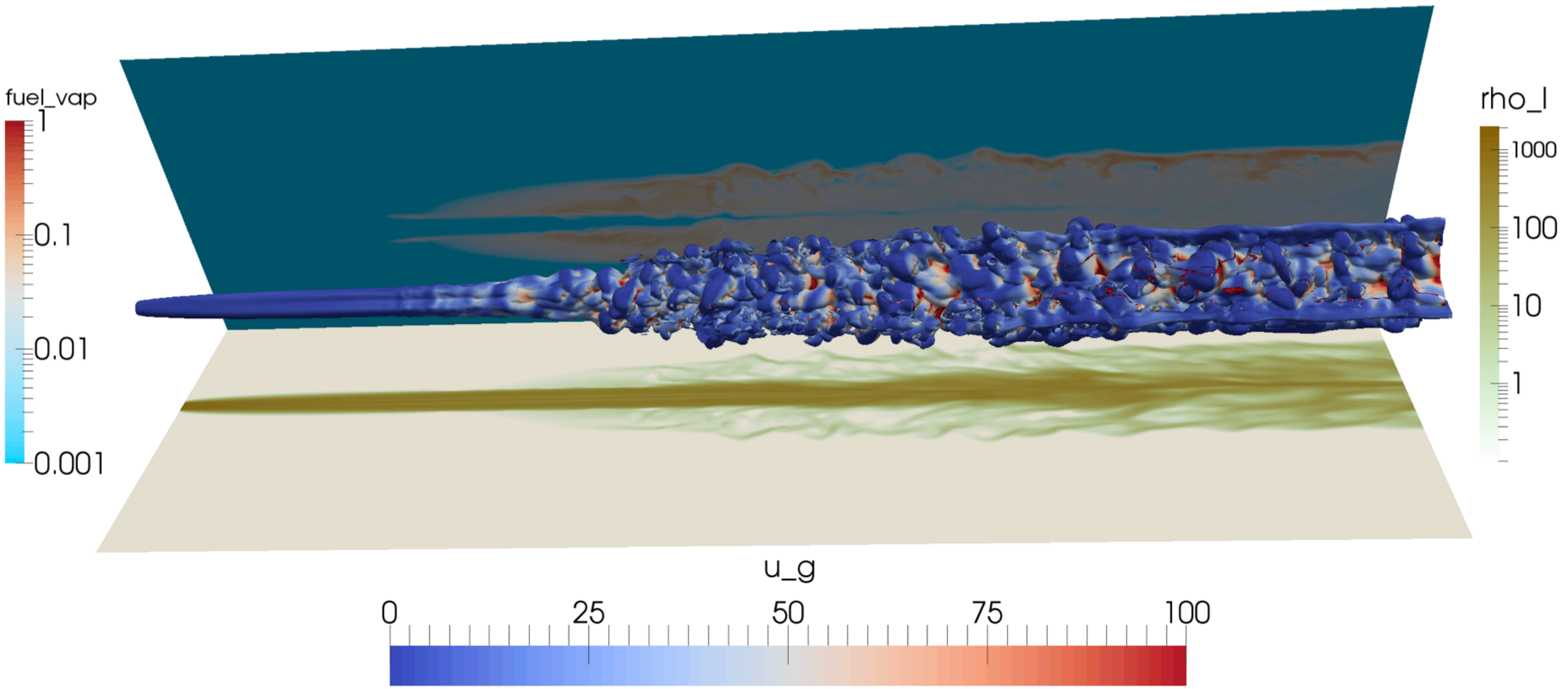




TEST 3 – Fuel vaporization

Fuel vapor footprint

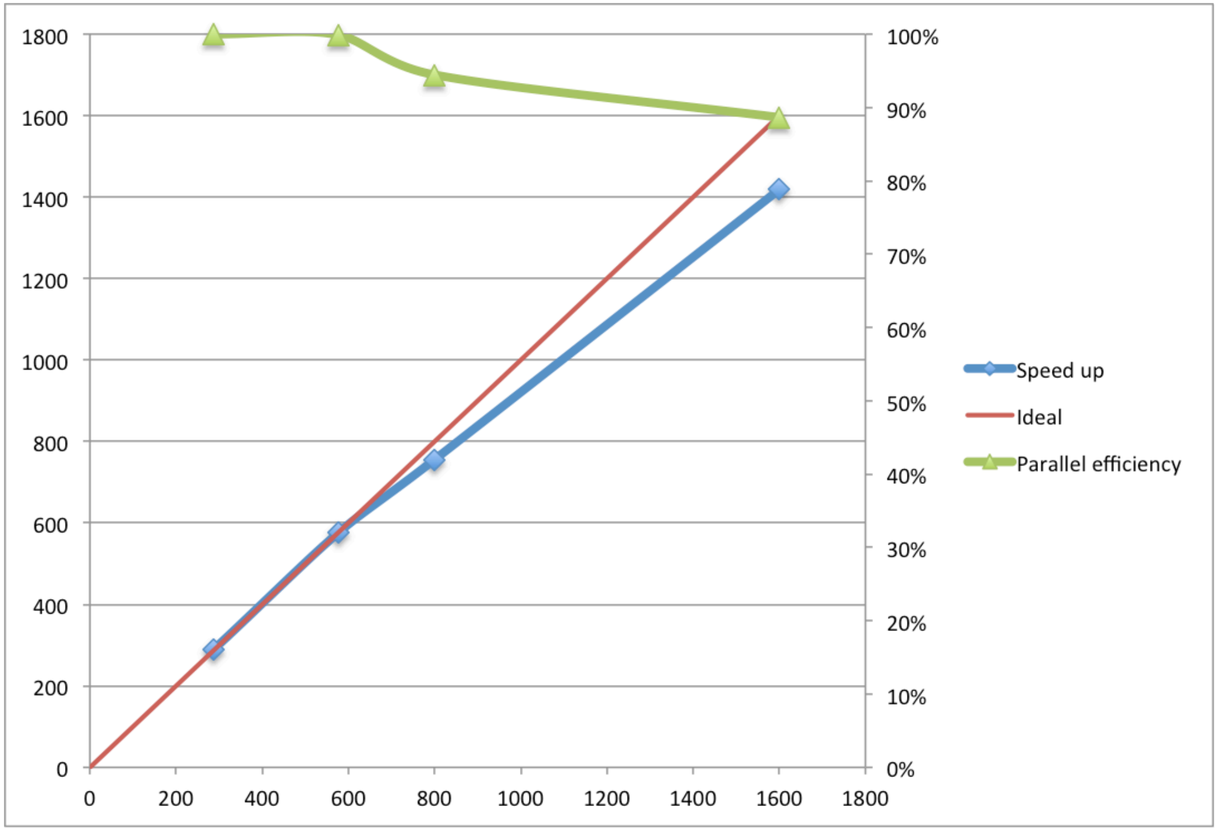
- Executed with **RAPTOR E-ES**
  - **Box 3x3x10mm**
  - **d_{inj}=90µm, T_{end}=40µs**
  - **quiescent gas at 60bar, 900K**
  - **n-dodecane at 702kg/m³, 600m/s**
  - **50Mcells** (cartesian mesh)
  - **Δx=12.5µm, Δt=8ns, T_{end}=40µs**
  - **1 section** (prescribed initial size)
  - **PGD transport** (δ-shocks)
  - **d^2-law**



TEST 4 – Weak scaling

Towards massively parallel computations

- Executed with **RAPTOR E-ES**
  - **Injection case, 50Mcells**
  - **resolution of the full length of the jet (maximum number of "particles")**
  - **including I/O**



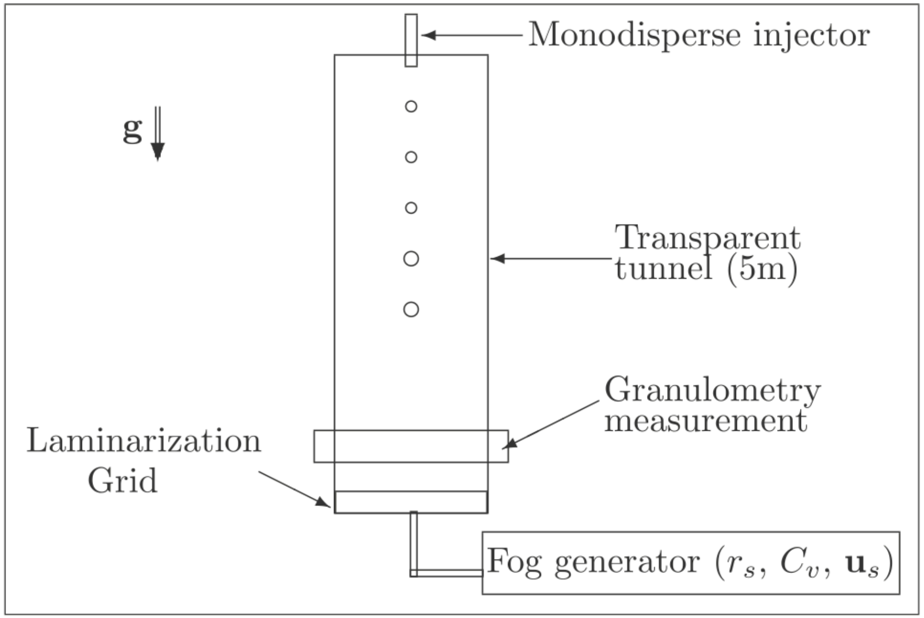
APPLICATIONS

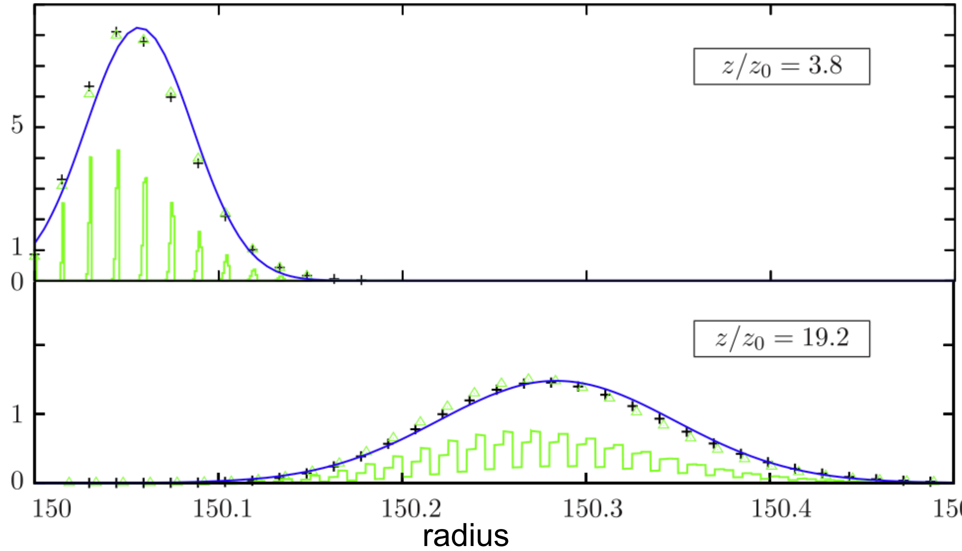
1) Test cases for fuel injection
2) **Coalescence studies**

The D'Herbigny coalescence problem

A benchmark for coalescence

- **Linearized coalescence problem**
  - **Big drops (150µm) falling**
  - **Mist of small drops (3µm)**
  - **No auto-coalescence**
  - **Analytical solutions for various regimes** [Doisneau 2013]
    - Early (exact poisson)
    - Linear increase of radius and dispersion
    - Variation of cross-section



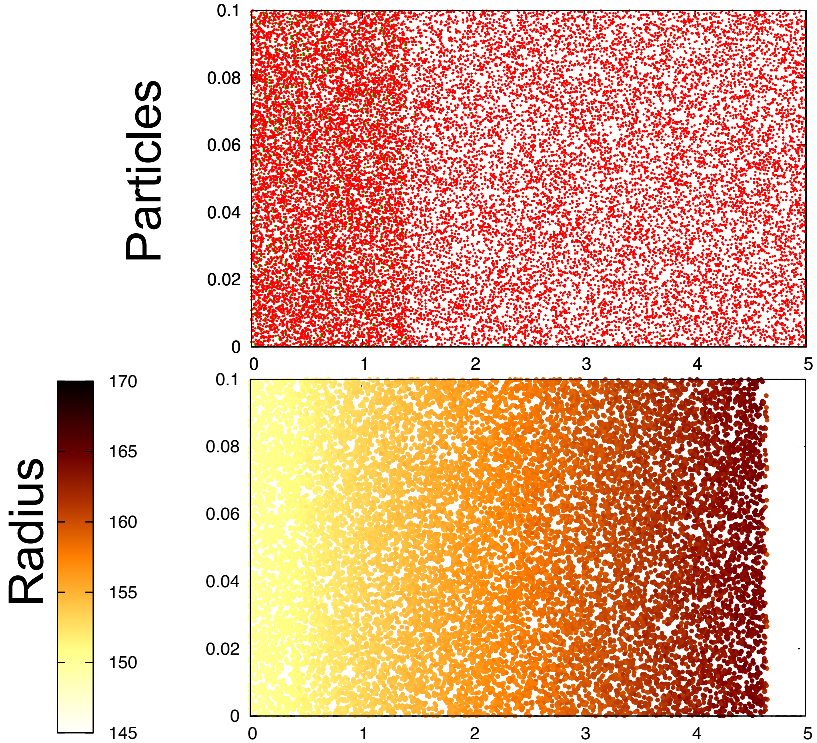


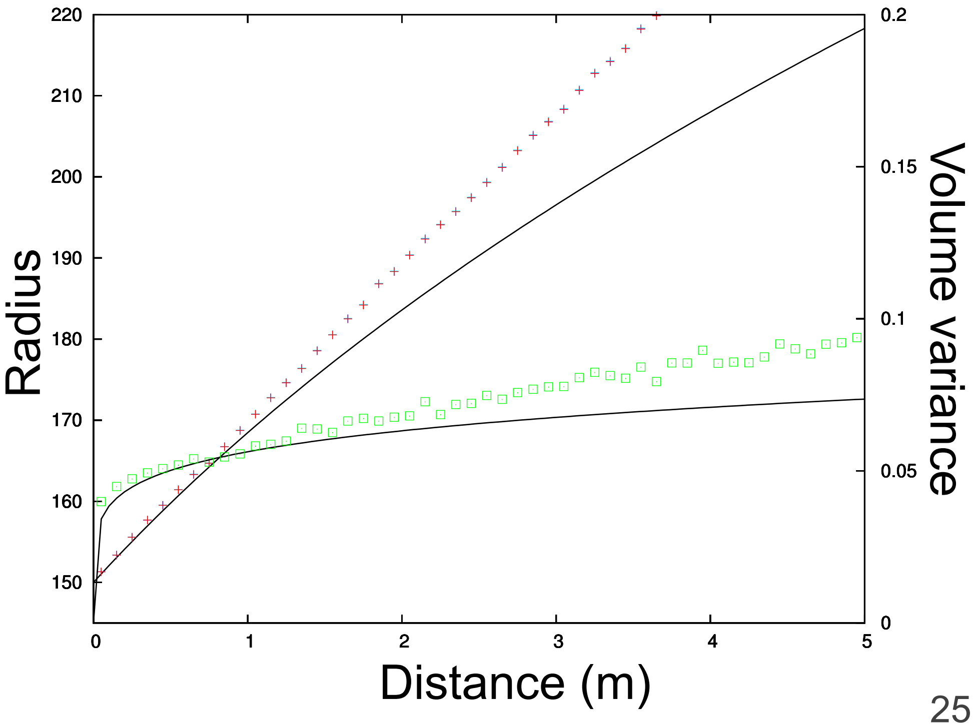
Transition from Poisson to Gaussian and refined sectional approximation (200 sections)

D'Herbigny – Stochastic Lagrangian

A costly approach

- Executed with in-house code
  - Cv = 60 ppm, L = 5 m
  - Random pair algorithm ~O(n)
  - 2x20,000 parcels
  - ~400 parcels/collisional cell
  - Algorithm verified

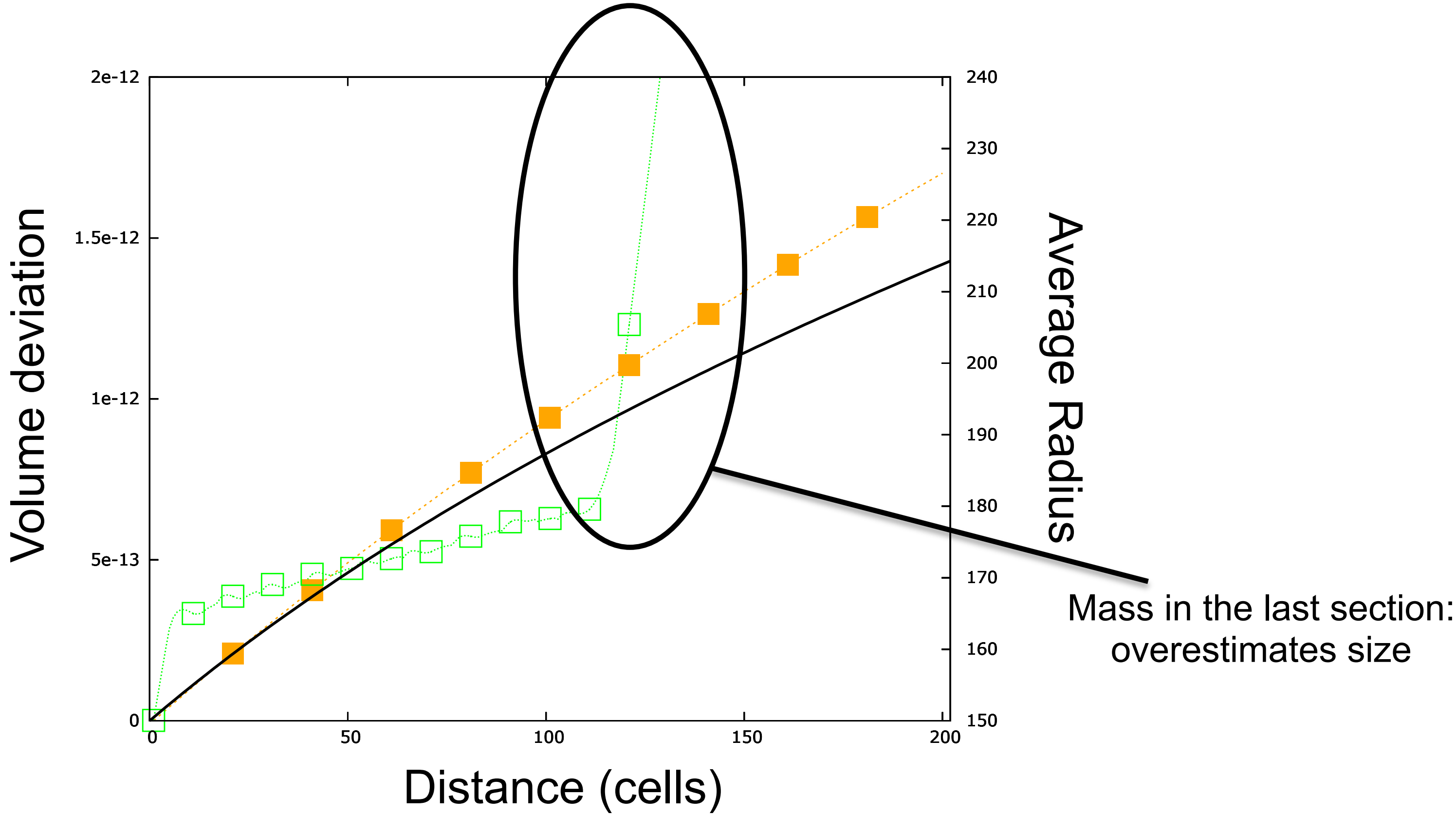




D'Herbigny – Deterministic sectional

Efficient approach for low inertia droplets

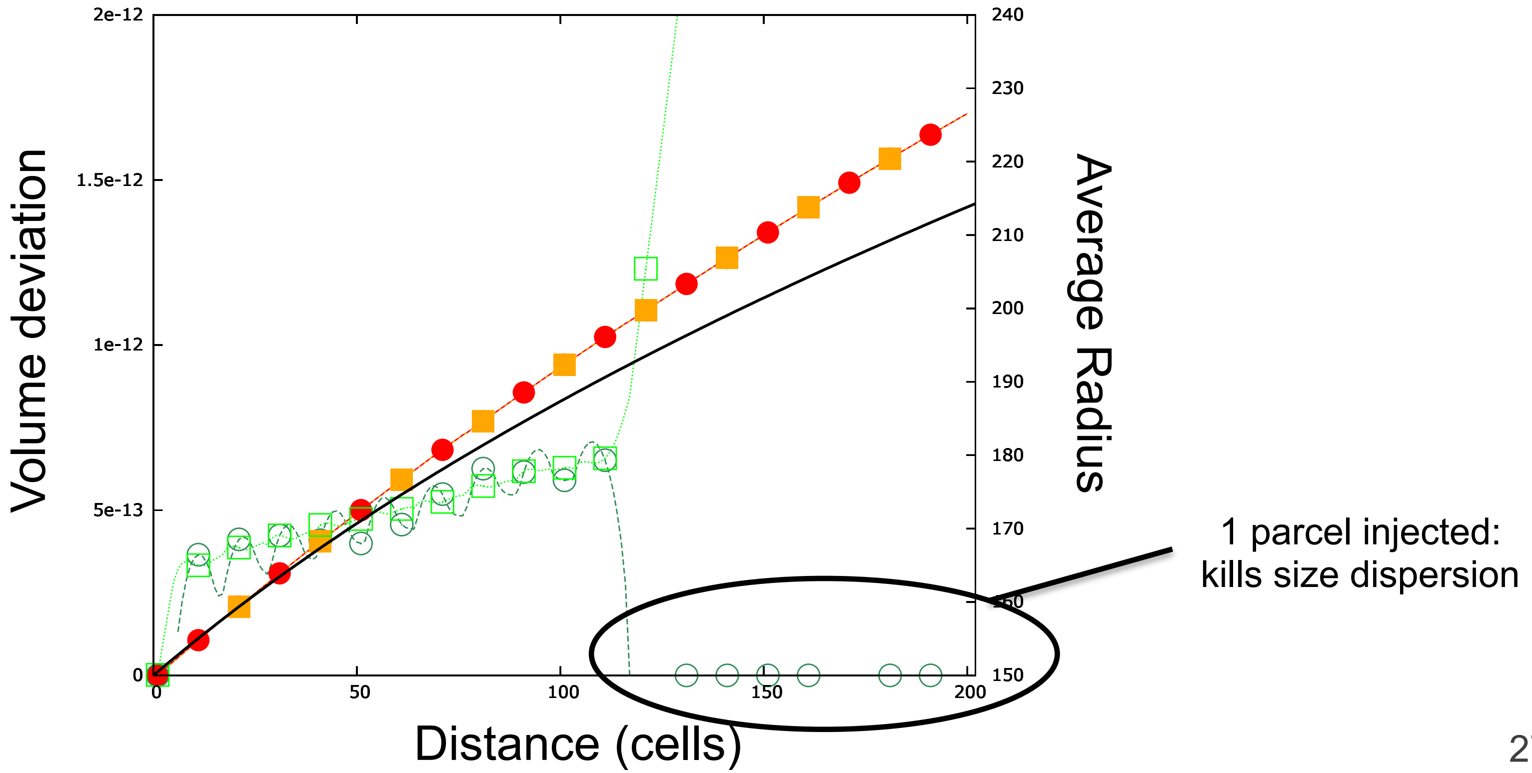
- Executed with **RAPTOR E-ES**
  - **Cv = 60 ppm, L = 5 m**
  - **Cartesian mesh (200 cells)**
  - **11 sections**
  - **Standard sectional approach**
  - **Code verified**



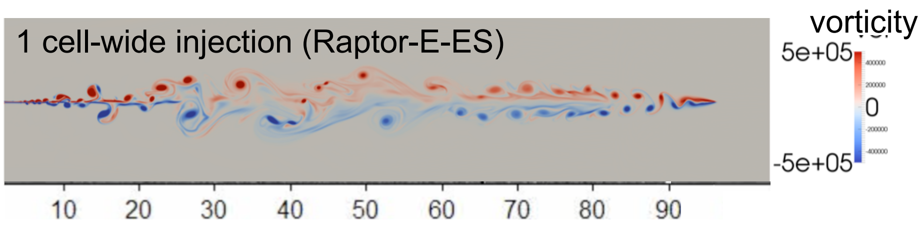
D'Herbigny – Hybrid Sectional-Lagrangian

A versatile method

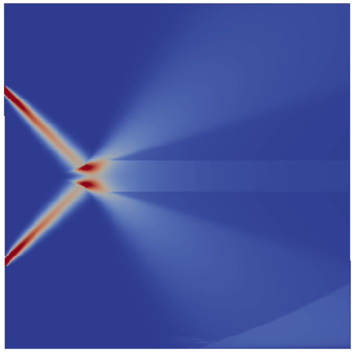
- Executed with **RAPTOR E-ES**
  - **When mass outflows the last section, it is injected in the cell as a cloud of stochastic parcels**
  - **Good agreement on average radius**
  - **Needs more parcels for dispersion**



CONCLUSION



- The semi-Lagrangian formalism is efficient for two-way coupled LES
  - Provides all qualities of Eulerian solvers
  - Robust at high liquid/gas density ratio
  - Provide fluctuating data to avoid relying on RANS models

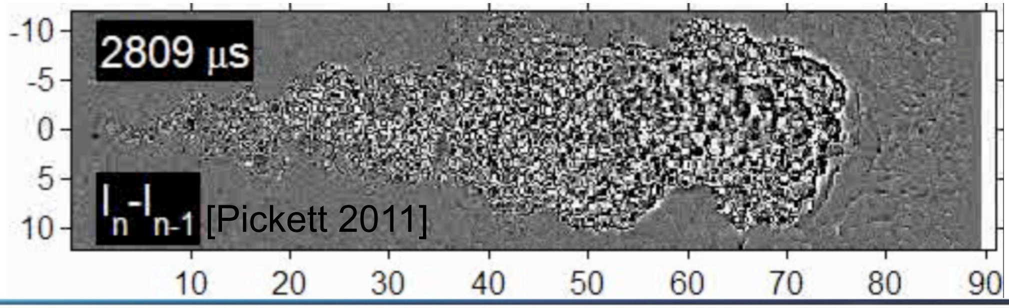


AG-QMOM with RAPTOR

- Flexibility
  - Can host spray models (here coalescence)
  - Dynamically interfaced to Lagrangian approach for inertial droplets
  - Other interfacings possible (high order velocity moments)

Perspectives

- **Verification**
  - vs CLSVOF
  - vs stochastic Lagrangian
  - vs Real-Gas solver
- **Validation** vs ECN results (spray A)



REFERENCES

[Chesnel 2011] Chesnel, J., Reveillon, J., Menard, T., and Demoulin, F.-X. (2011) **Large eddy simulation of liquid jet atomization** Atomization and Sprays, 21(9).

[J.E. Dec, 1997] Dec, J. E. **A conceptual model of DI diesel combustion based on laser-sheet imaging** SAE transactions, 106(3):1319-1348.

[Desjardins 20XX] Desjardins, O. and Pitsch, H. (2010) **Detailed numerical investigation of turbulent atomization of liquid jets** Atomization and Sprays, 20(4).

[Doisneau 2013] Doisneau, F., Laurent, F., Murrone, A., Dupays, J., and Massot, M. (2013). **Eulerian Multi-Fluid models for the simulation of dynamics and coalescence of particles in solid propellant combustion** Journal of Computational Physics, 234 :230–262.

[Laurent 2001] Laurent, F. and Massot, M. (2001) **Multi-fluid modeling of laminar poly-dispersed spray flames: origin, assumptions and comparison of the sectional and sampling methods** Combustion Theory and Modelling, 5(4):537-572

[Menard 2007] Menard, T., Tanguy, S., and Berlemont, A. (2007). **Coupling level set/VOF/ghost fluid methods : Validation and application to 3D simulation of the primary break-up of a liquid jet I**nternational Journal of Multiphase Flow, 33(5) :510–524.

[Pickett 2010] Pickett L.M., Genzale C.L., Bruneaux G., Malbec L.-M., Hermant L., Christiansen C., Schramm J. (2010) **Comparison of diesel spray combustion in different high-temperature, high-pressure facilities** SAE Int. J. Engines 3(2): 156-181.

[Pickett 2011] Pickett L.M., Manin J., Genzale C.L., Siebers D.L., Musculus M.P.B., Idicheria C.A. (2011) **Relationship between diesel fuel spray vapor penetration/dispersion and local fuel mixture fraction** SAE Int. J. Engines 4(1):764-799.

[Skeen 2014] Skeen, S.A. and Manin, J. and Pickett, L.M. **Simultaneous formaldehyde PLIF and high-speed schlieren imaging for ignition visualization in high-pressure spray flames** Proceedings of the Combustion Institute,35(3):3167-3174.

[Som 2013] Xue, Q. and Som, S. and Senecal, P. K. and Pomraning, E. (2013). **Large eddy simulation of fuel-spray under non-reacting IC engine conditions** Atomization and Sprays, 23(10).

[Tillou 2014] Tillou, J., Michel, J.-B., Angelberger, C., and Veynante, D. (2014). **Assessing LES models based on tabulated chemistry for the simulation of diesel spray combustion** Combustion and Flame, 161(2) :525–540.