SAND2015-3848C

Inverse Problems in Transient Structural Acoustics

Timothy Walsh

Computational Solid Mechanics and Structural Dynamics

Sandia National Laboratories, Albuquerque, NM

Collaborators: Sierra-SD team (Sandia), ROL team(Sandia), Wilkins Aquino (Duke)

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000

Outline

- Inverse problems - motivation
- Inverse problems – mathematical formulation in the time domain
- Examples of inverse problems in large-scale finite element applications

Inverse Problems- Motivation

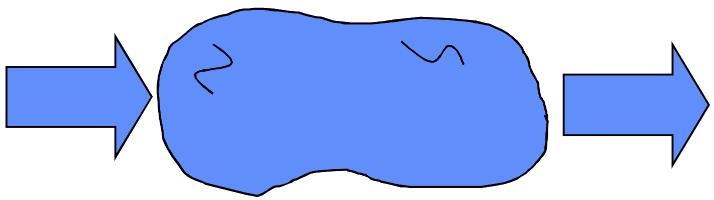
- Characterizing energy sources from experimental measurements is a common need in structural acoustics
  - Earthquake modeling, nonproliferation, acoustic testing, damage or defect identification from acoustic emission
- Determining unknown material properties from measurements is a common need in model calibration
  - Subsurface modeling, medical ultrasonics
- For applications that involve complex geometries and/or sources, finite element modeling is needed for an accurate solution of the forward problem.
- Goal: leverage existing massively parallel finite element technology developed for forward problems to solve the inverse problem.

Inverse Problems: The physical View

The direct or forward problem

External inputs (known)

e.g. forces, fluxes, etc.



System response (unknown)

e.g. displacements, temperature, concentrations, etc.

The System (known)

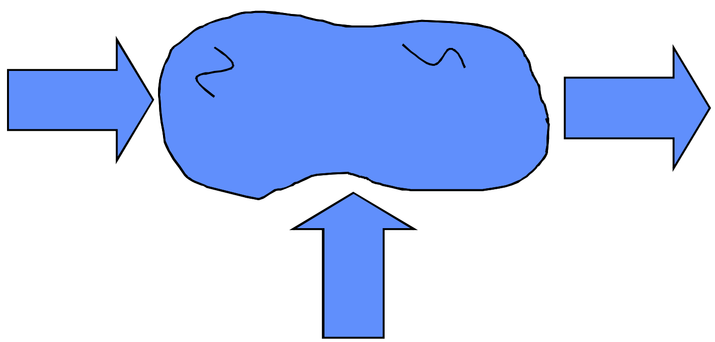
e.g. geometry, material properties, etc.

Inverse Problems: The physical View (2)

One type of inverse problem

External inputs (unknown)

e.g. forces, fluxes, etc.



System response (partially known)

e.g. displacements, temperature, concentrations, etc.

The System (unknown)

e.g. geometry, material properties, etc.

Inverse Problems (3) Challenges

- Usually, inverse problems are ill-posed.
  - Solution may not exist.
  - Solution may not be unique.
  - Solution may be unstable. That is, it may be sensitive to small changes in the input data.
- Can be very computationally demanding.

Abstract Optimization Formulation

Abstract optimization formulation (Inverse, Design, Control)

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$

Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$

PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}^T\boldsymbol{g}$$

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \boldsymbol{g}_u^T \boldsymbol{w} \\ J_p + \boldsymbol{g}_p^T \boldsymbol{w} \\ \boldsymbol{g} \end{Bmatrix} = \{0\}$$

First order optimality conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \boldsymbol{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \boldsymbol{g}_p^T \\ \boldsymbol{g}_u & \boldsymbol{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta\boldsymbol{u} \\ \delta\boldsymbol{p} \\ \boldsymbol{w}^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \boldsymbol{g} \end{Bmatrix}$$

Newton iteration

$$\boldsymbol{W}\Delta\boldsymbol{p} = -\hat{J}',$$

$$\boldsymbol{W} = \boldsymbol{g}_p^T \boldsymbol{g}_u^{-T}(\mathcal{L}_{uu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu}\boldsymbol{g}_u^{-1}\boldsymbol{g}_p + \mathcal{L}_{pp}$$

Hessian calculation

Structural Acoustic Equations of Motion

acoustics

$$\nabla^2 \phi = \frac{1}{c^2}\ddot{\phi}, \text{ in } \Omega_f \times (0,T)$$

$$\nabla \phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \text{ on } \partial\Omega_f^N \times [0,T]$$

$$\phi = 0, \text{ on } \partial\Omega_f^D \times [0,T]$$

$$\phi(0,T) = 0, \text{ in } \Omega_f$$

$$\dot{\phi}(0,T) = 0, \text{ in } \Omega_f$$

solid mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho \ddot{\boldsymbol{u}}, \text{ in } \Omega \times (0,T)$$

$$\boldsymbol{\sigma n} = \boldsymbol{h}, \text{ on } \partial\Omega^N \times [0,T]$$

$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\nabla u}, \text{ in } \Omega \times [0,T]$$

$$\boldsymbol{u} = \boldsymbol{0}, \text{ on } \partial\Omega^D \times [0,T]$$

$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \text{ in } \Omega$$

$$\ddot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \text{ in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$

$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

Sandia National Laboratories

Structural Acoustic Equations of Motion

Fully coupled formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ -L & C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ f_a \end{bmatrix}$$

Condensed notation

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

We will use the condensed notation in following slides

Statement of Inverse Problem

Minimize objective function

$$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\Big(\{\boldsymbol{u}\} - \{\boldsymbol{u}_m\}\Big)^{\mathrm{T}}[Q]\Big(\{\boldsymbol{u}\} - \{\boldsymbol{u}_m\}\Big) + \mathscr{R}(\{\boldsymbol{p}\}),$$

- $\{\boldsymbol{u}\}$ State variables (displacement, pressure)
- $\{\boldsymbol{u}_m\}$ Measured data (displacement, pressure)
- $\{\boldsymbol{p}\}$ Unknown parameters (loads, material parameters)
- $[Q]$ Weight matrix

Subject to equations of motion

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Statement of Inverse Problem (2)

The Lagrangian

$$\mathscr{L}(\{\boldsymbol{d}\},\{\hat{\boldsymbol{d}}\},\{\boldsymbol{p}\}) = \tilde{J}(\{\boldsymbol{p}\}) + \hat{\boldsymbol{u}}_0^{\mathrm{T}}\Big([M]\boldsymbol{a}_0 + [C]\boldsymbol{v}_0 + [K]\boldsymbol{u}_0 - \boldsymbol{f}_0(\{\boldsymbol{p}\})\Big)$$
$$+ \sum_{k=1}^{N}\Big\{\hat{\boldsymbol{u}}_k^{\mathrm{T}}\Big([M]\boldsymbol{a}_k + [C]\boldsymbol{v}_k + [K]\boldsymbol{u}_k - \boldsymbol{f}_k(\{\boldsymbol{p}\})\Big)$$
$$+ \hat{\boldsymbol{v}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{v}_k - \boldsymbol{v}_{k-1} - \Delta t\left[(1-\gamma)\boldsymbol{a}_{k-1} + \gamma\boldsymbol{a}_k\right]\Big)$$
$$+ \hat{\boldsymbol{a}}_k^{\mathrm{T}}[M]\Big(\boldsymbol{u}_k - \boldsymbol{u}_{k-1} - \Delta t\,\boldsymbol{v}_{k-1} - \frac{\Delta t^2}{2}\left[(1-2\beta)\boldsymbol{a}_{k-1} + 2\beta\boldsymbol{a}_k\right]\Big)\Big\}$$

where

$$\{\boldsymbol{d}(\{\boldsymbol{p}\})\} = \Big\{\{\boldsymbol{u}\},\{\boldsymbol{v}\},\{\boldsymbol{a}\}\Big\}$$

Sandia National Laboratories

Optimality conditions

- Optimality is obtained by setting derivatives of Lagrangian to zero
- We will adopt a reduced space approach where we derive reduced gradients from full space approach
- Reduced space approach can be derived from full space

Statement of Inverse Problem (3)

Gateaux derivatives of the Lagrangian with respect to adjoint variables

$$
\begin{aligned}
\nabla_{\boldsymbol{a}_0}\mathscr{L}\cdot\delta\boldsymbol{a}_0 &= \delta\boldsymbol{a}_0^{\mathrm{T}}\left([M]\hat{\boldsymbol{u}}_0 - \frac{\Delta t^2}{2}(1-2\beta)[M]\hat{\boldsymbol{a}}_1 - \Delta t(1-\gamma)[M]\hat{\boldsymbol{v}}_1\right),\\
\nabla_{\boldsymbol{u}_k}\mathscr{L}\cdot\delta\boldsymbol{u}_k &= \delta\boldsymbol{u}_k^{\mathrm{T}}\left([M](\hat{\boldsymbol{a}}_k - \hat{\boldsymbol{a}}_{k+1}) + [K]\hat{\boldsymbol{u}}_k + \kappa[Q](\boldsymbol{u}_k - \boldsymbol{u}_{m_k}),\right),\\
\nabla_{\boldsymbol{v}_k}\mathscr{L}\cdot\delta\boldsymbol{v}_k &= \delta\boldsymbol{v}_k^{\mathrm{T}}\left([C]\hat{\boldsymbol{u}}_k - \Delta t[M]\hat{\boldsymbol{a}}_{k+1} + [M]\hat{\boldsymbol{v}}_k - [M]\hat{\boldsymbol{v}}_{k+1}\right),\\
\nabla_{\boldsymbol{a}_k}\mathscr{L}\cdot\delta\boldsymbol{a}_k &= \delta\boldsymbol{a}_k^{\mathrm{T}}\left([M]\hat{\boldsymbol{u}}_k - \beta\Delta t^2[M]\hat{\boldsymbol{a}}_k - \frac{\Delta t^2}{2}[M](1-2\beta)\hat{\boldsymbol{a}}_{k+1},\right.\\
&\quad \left. -\Delta t[M](\gamma\hat{\boldsymbol{v}}_k + (1-\gamma)\hat{\boldsymbol{v}}_{k+1})\right),\\
\nabla_{\boldsymbol{u}_N}\mathscr{L}\cdot\delta\boldsymbol{u}_N &= \delta\boldsymbol{u}_N^{\mathrm{T}}\left([M]\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N + \kappa[Q](\boldsymbol{u}_N - \boldsymbol{u}_{m_N})\right),\\
\nabla_{\boldsymbol{v}_N}\mathscr{L}\cdot\delta\boldsymbol{v}_N &= \delta\boldsymbol{v}_N^{\mathrm{T}}\left([C]\hat{\boldsymbol{u}}_N + [M]\hat{\boldsymbol{v}}_N\right),\\
\nabla_{\boldsymbol{a}_N}\mathscr{L}\cdot\delta\boldsymbol{a}_N &= \delta\boldsymbol{a}_N^{\mathrm{T}}\left([M]\hat{\boldsymbol{u}}_N - \Delta t^2\beta[M]\hat{\boldsymbol{a}}_N - \Delta t\gamma[M]\hat{\boldsymbol{v}}_N\right).
\end{aligned}
$$

Sandia National Laboratories

Statement of Inverse Problem (4)

(i) Final conditions

$$
\begin{aligned}
[C]\,\hat{\boldsymbol{u}}_N + [M]\hat{\boldsymbol{v}}_N &= \mathbf{0} \\
\hat{\boldsymbol{u}}_N &= \Delta t^2 \beta\, \hat{\boldsymbol{a}}_N + \Delta t\,\gamma \hat{\boldsymbol{v}}_N \\
[M]\,\hat{\boldsymbol{a}}_N + [K]\hat{\boldsymbol{u}}_N &= \kappa[Q](\boldsymbol{u}_{m_N} - \boldsymbol{u}_N)
\end{aligned}
$$

(ii) Backward transition equations

$$
\begin{aligned}
\hat{\boldsymbol{u}}_k - \beta \Delta t^2 \hat{\boldsymbol{a}}_k - \Delta t\,\gamma \hat{\boldsymbol{v}}_k &= \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_{k+1} + \Delta t(1-\gamma)\hat{\boldsymbol{v}}_{k+1} \\
[C]\,\hat{\boldsymbol{u}}_k + [M](\hat{\boldsymbol{v}}_k - \Delta t\,\hat{\boldsymbol{a}}_{k+1} - \hat{\boldsymbol{v}}_{k+1}) &= \mathbf{0} \\
[M]\,\hat{\boldsymbol{a}}_k + [K]\hat{\boldsymbol{u}}_k &= [M]\hat{\boldsymbol{a}}_{k+1} + \kappa[Q](\boldsymbol{u}_{m_k} - \boldsymbol{u}_k)
\end{aligned}
$$

(iii) Last transition equation

$$
\hat{\boldsymbol{u}}_0 = \frac{\Delta t^2}{2}(1-2\beta)\hat{\boldsymbol{a}}_1 + \Delta t(1-\gamma)\hat{\boldsymbol{v}}_1
$$

Solution of Inverse Problem

Do until tolerance < eps

- Solve forward problem
- ↓
- Solve adjoint problem
- ↓
- Compute gradients, Hessians
- ↓
- Optimization step
- ↓
- Receive design variable updates from optimization solver

end

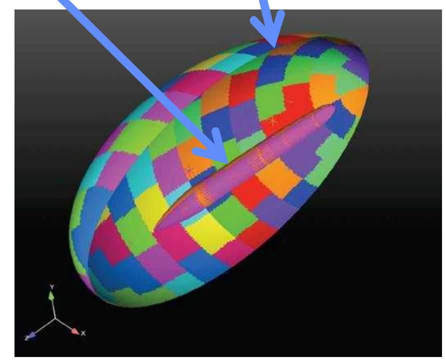
Structural Acoustics in Sierra-SD

Acoustic source inversion

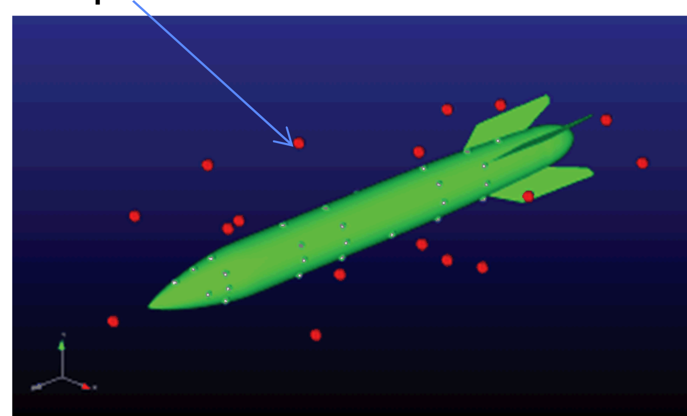
Goal:
Solve inverse problem to obtain acoustic patch inputs that produce the given microphone measurements.

2 approaches:
1. Frequency domain
   - broadband frequency sweep
2. Time domain
   - implicit time integration that covers frequency range of interest

Surface with 172 acoustic patches

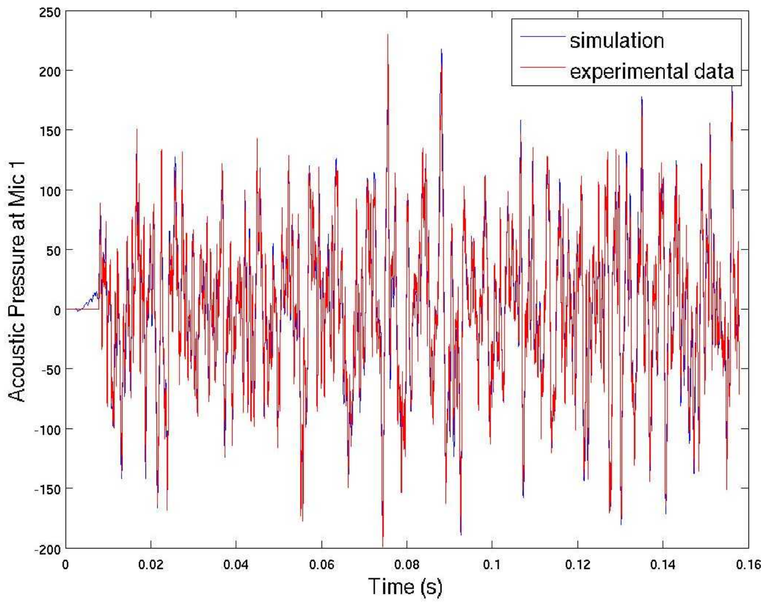


Microphone locations

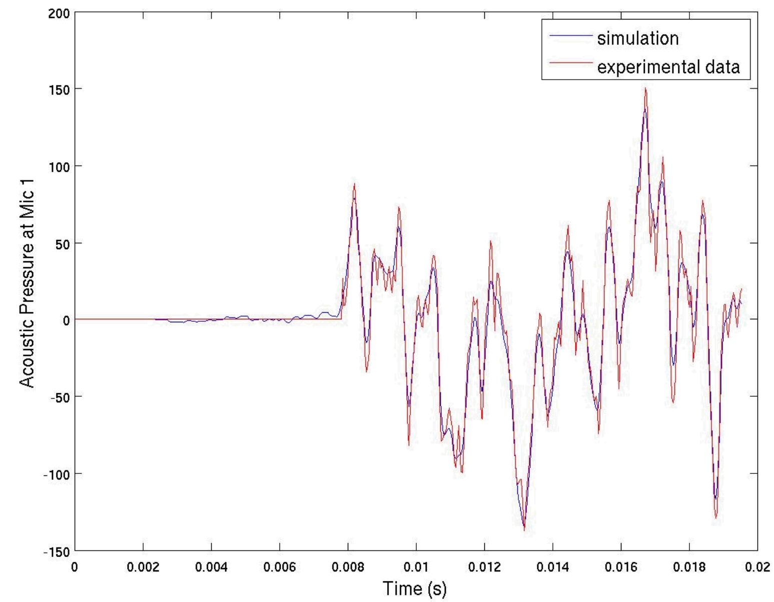


Time Domain Source Inversion

Results for Microphone 1 (other mics were similar)



Full time history

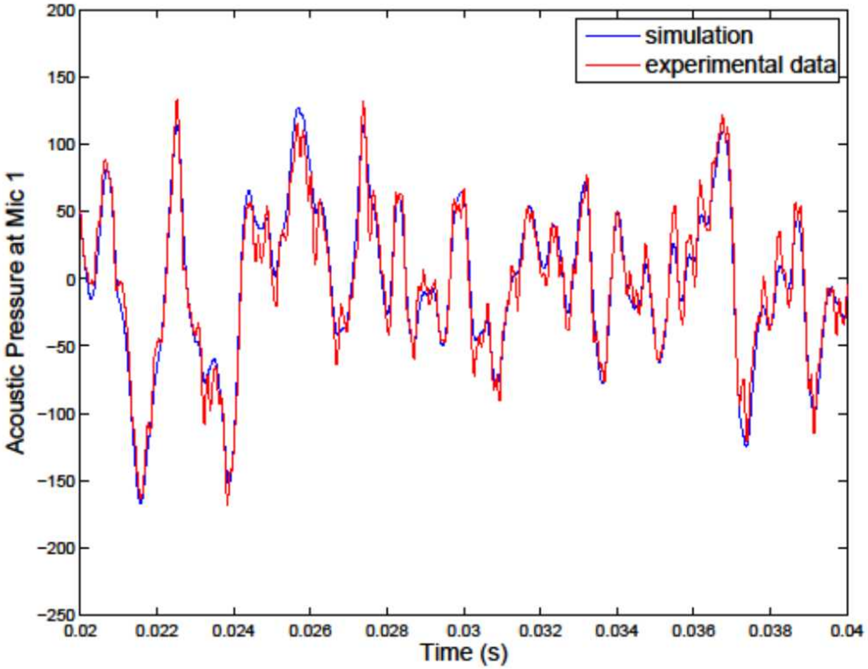


Blow-up near origin of wave

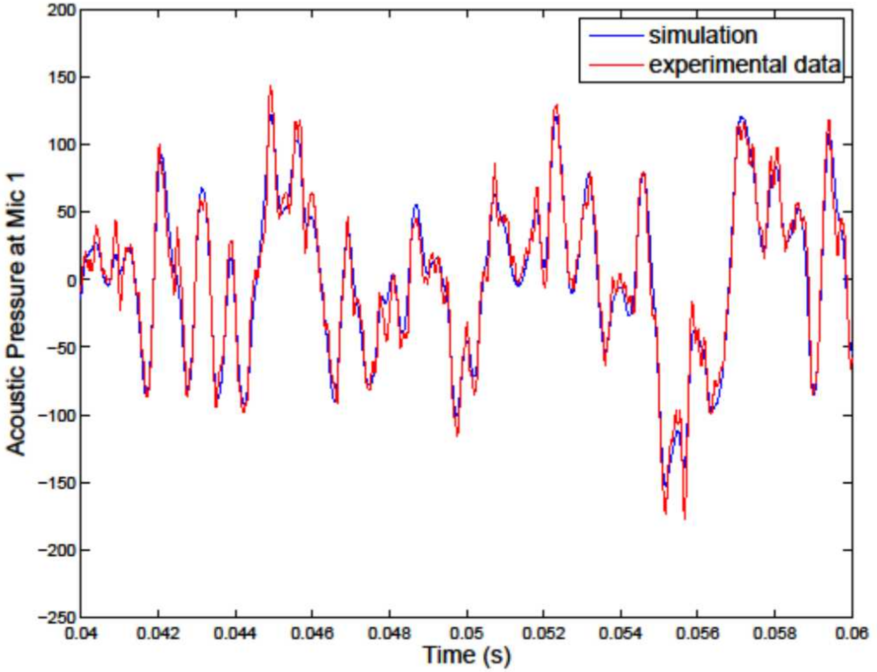
Sandia National Laboratories

Time Domain Source Inversion

Results for Microphone 1



0.02(s) < t < 0.04(s)



0.04(s) < t < 0.06(s)

Sandia National Laboratories

Dissipative Material Inversion

Generic formulation based on complex modulus

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$ Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{0}$$ PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}_R^T \boldsymbol{g}_R + \boldsymbol{w}_I^T \boldsymbol{g}_I = J + \Re(\boldsymbol{w}^h \boldsymbol{g})$$ Lagrangian

$$\boldsymbol{\sigma}(\omega) = \boldsymbol{D}(\omega)\boldsymbol{\epsilon} = (b(\omega)\boldsymbol{D}_b + G(\omega)\boldsymbol{D}_G)\boldsymbol{\epsilon}(\omega)$$ Constitutive Law

Viscoelasticity

$$b(\omega) = b_R(\omega) + i b_I(\omega)$$
$$G(\omega) = G_R(\omega) + i G_I(\omega)$$

Block Proportional Damping

$$b(\omega) = b + i\omega\beta b$$
$$G(\omega) = G + i\omega\beta G$$

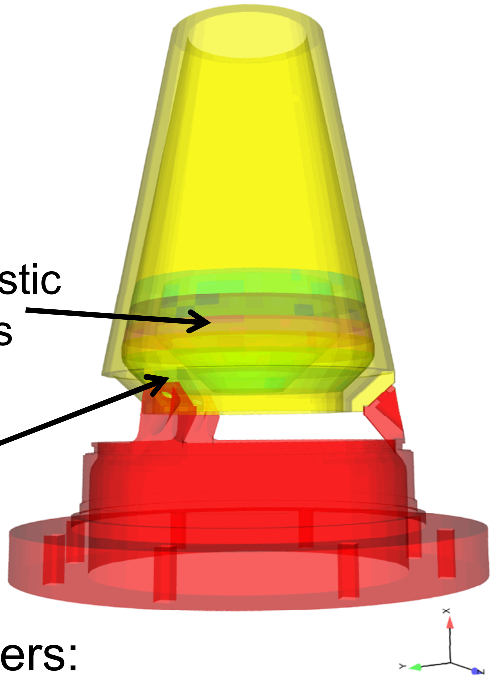
Dashpots

$$E_R = 0$$
$$E_I = \omega c$$

Sandia National Laboratories

Eigenvalue-Based Material Inversion

- Spring/foam calibration on mass-mock
- Goal: Match measured modes to computed modes
- Synthetic modal data used as input
- Initial guess of parameters a factor of 10 away from true values



Mode number	Initial Guess	Results from inversion	Exact modes
1	2.759e6	1.240e6	1.240e6
2	2.957e6	1.358e6	1.3607e6
3	1.157e7	7.071e6	7.052e6
4	1.186e7	1.041e7	1.043e7
5	1.266e7	1.102e7	1.103e7
6	1.695e7	1.262e7	1.256e7
7	2.041e8	2.025e8	2.025e8

Initial guess of parameters:

Table 1. Joint2G parameters for mass-mock model.

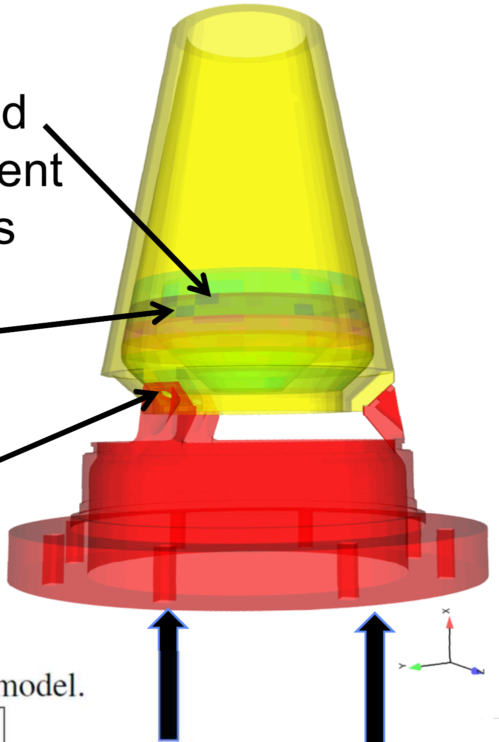
	kx	ky	kz	krz	kry	krz
exact	2.5e5	2.0e8	2.0e8	N/A	N/A	N/A
initial guess	2.5e4	2.0e8	2.0e8	N/A	N/A	N/A

Table 2. Elastic foam parameters for mass-mock model.

	shear modulus (G)	bulk modulus (K)
exact	1.585e4	4.134e4
initial guess	1.585e3	4.134e3

Frequency-Domain Material Inversion

- **Dashpot/foam calibration on mass-mock**
- **Full Newton with adjoint-based Hessians**
- **Measured displacements on foam block**
- **Stiffness parameters from previous slide**
- **Initial guess: zero damping**



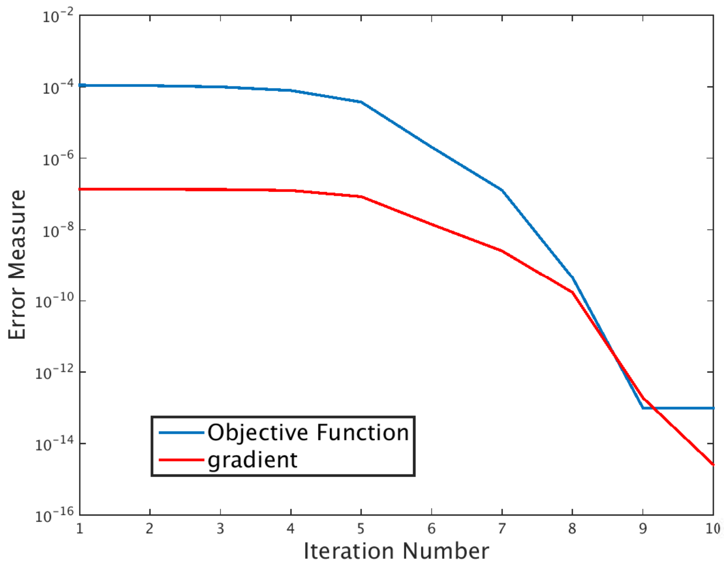


Table 3. Dashpot parameters for mass-mock model.

	cx	cy	cz
exact	1000.0	N/A	N/A
computed	1000.0	N/A	N/A
initial guess	0	N/A	N/A

Table 4. Viscoelastic foam parameters for mass-mock model.

	Imaginary part of G	Imaginary part of K
exact	362.4	785.2
computed	362.3	785.0
initial guess	0	0

Sandia National Laboratories

Some Additional Inverse Problems of Interest

- Impedance boundary conditions for acoustics
  - For modeling dissipation at walls in DFAT tests
- Inversion under uncertainty
- Interfaces in structures
  - Impedance conditions, interface elements, etc

Conclusions

- Massively parallel finite element structural acoustics capability Sierra-SD has been developed for large-scale analysis
- Applicable to large-scale models with many degrees of freedom.
- Sierra-SD and optimization codes have been loosely coupled for the solution of source and material inversion problems.
- Capability has been applied to a variety of problems in structural acoustics