

# Unified Creep Plasticity Damage (UCPD) Model for Rigid Polyurethane Foams

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Sandia National Laboratories



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**SEM 2015 Annual Conference and Exposition  
on Experimental and Applied Mechanics**

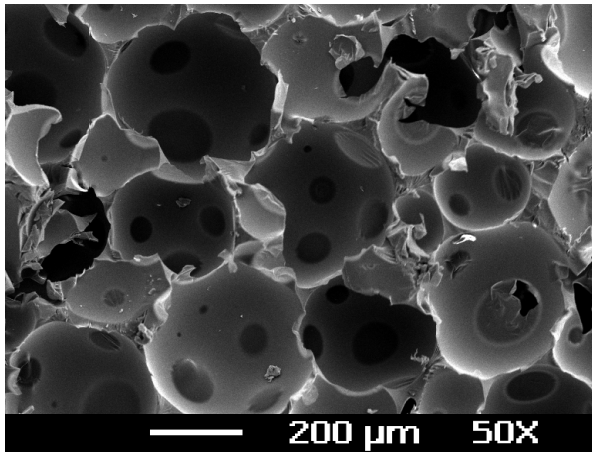


# Outline

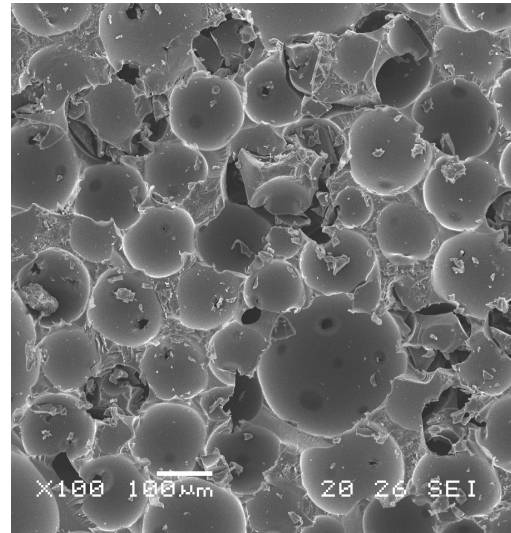
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- ❑ Experimental Characterization
- ❑ Foam Micromechanics (cell level)
- ❑ Unified Creep Plasticity Damage (UCPD) Model
- ❑ Simulations
- ❑ Summary and Future Work

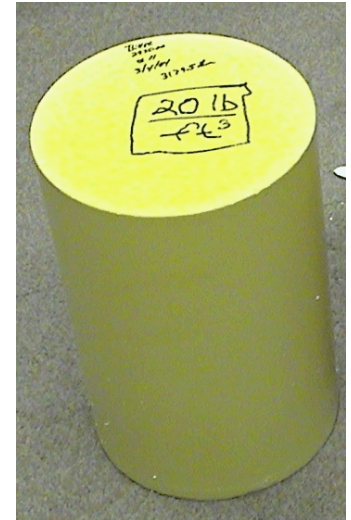
# Rigid Polyurethane Foam



176 kg/m<sup>3</sup> (11 pcf) rigid  
polyurethane foam

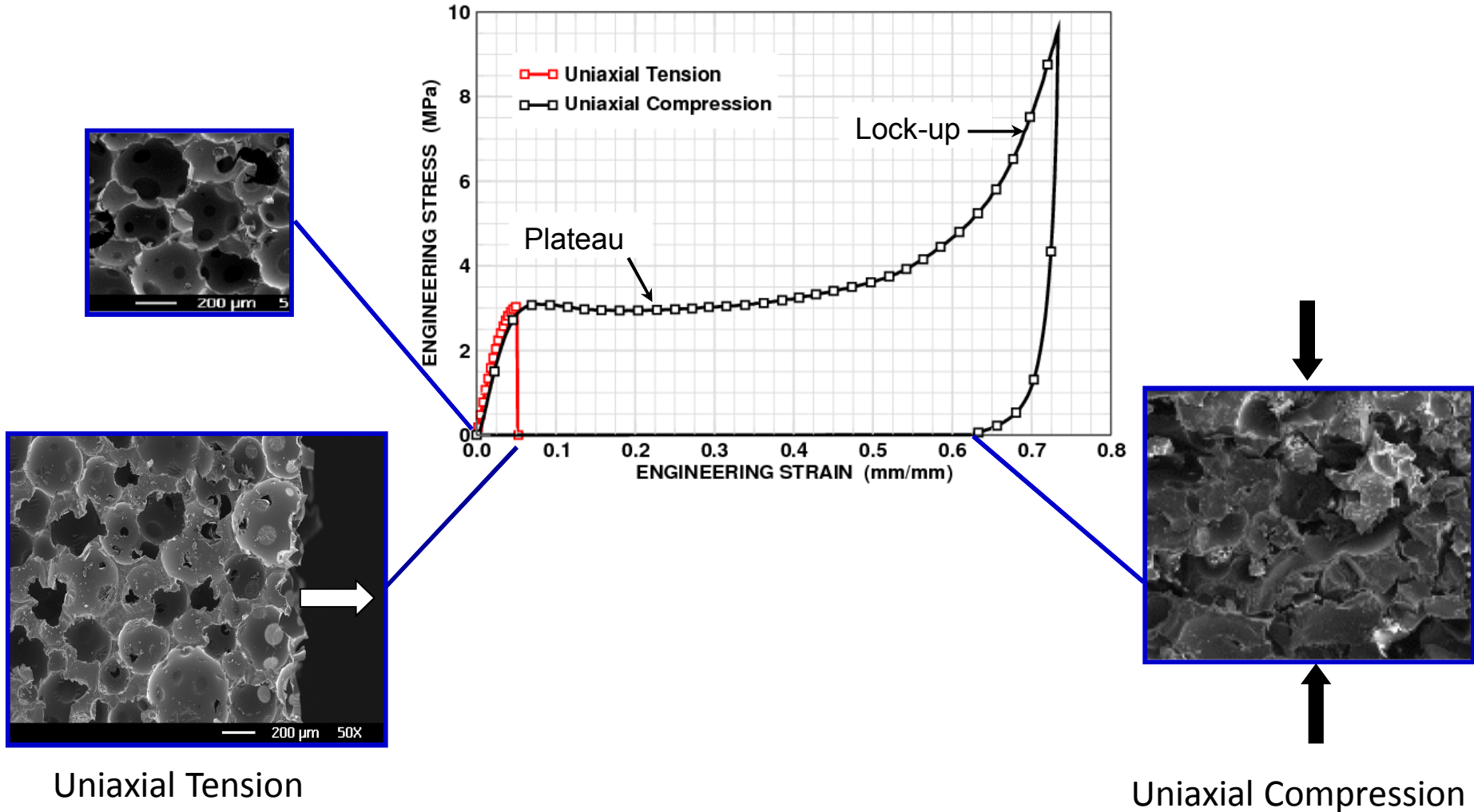


320 kg/m<sup>3</sup> (20 pcf) rigid  
polyurethane foam

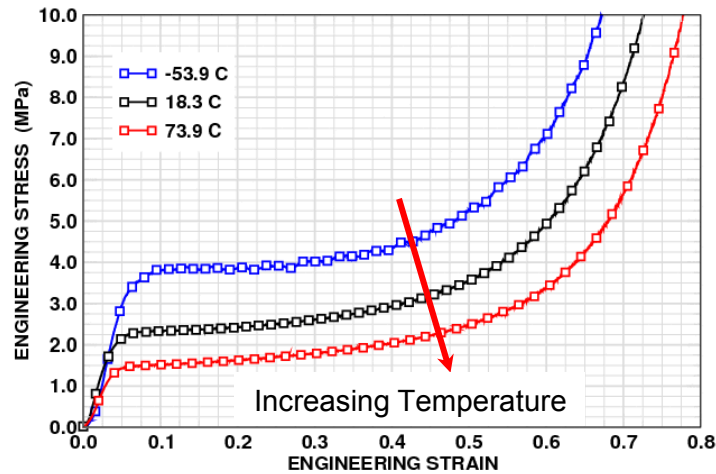


30.5 cm (12 in.) tall billet

# Experiments on 176 kg/m<sup>3</sup> (11 pcf) Foam

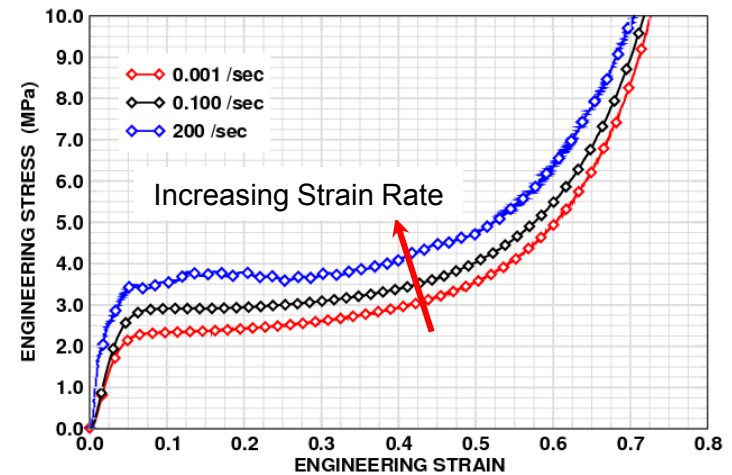


# Experiments on 192 kg/m<sup>3</sup> (12 pcf) Foam



## Temperature Effects

Constant Strain Rate = 0.001 / second



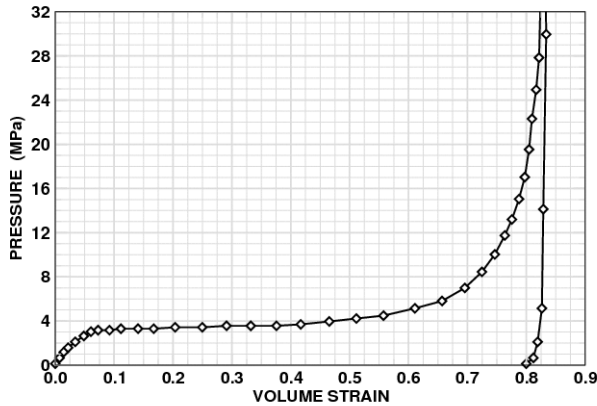
## Strain-Rate Effects

Constant Temperature = 18.3 °C

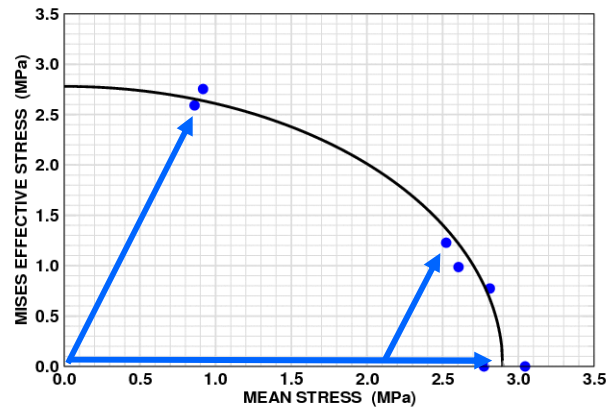
Uniaxial Compression of FR3712

# Experiments on 192 kg/m<sup>3</sup> (12 pcf) Foam

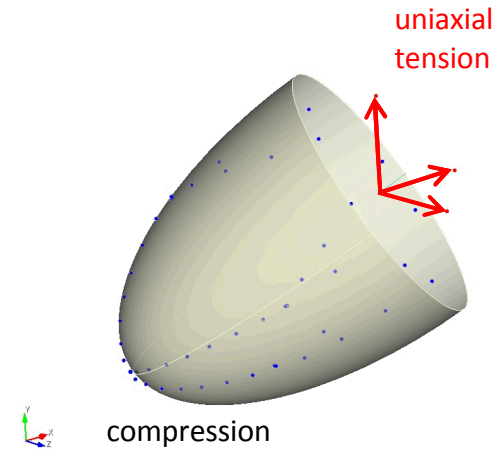
Unlike metals, foams exhibit large permanent volume changes – volumetric plasticity



Hydrostatic Compression  
Pressurization Rate 0.1 MPa/sec



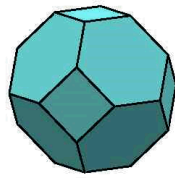
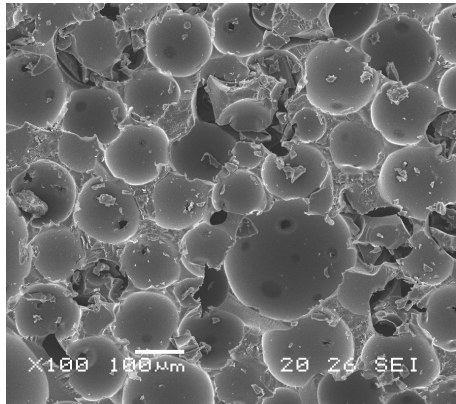
Triaxial Compression  
W.A. Olsson, 2006



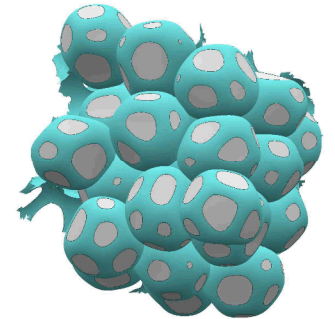
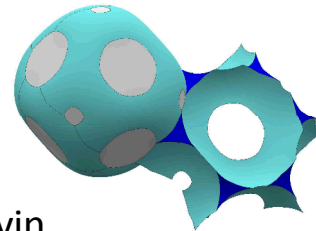
Yield Surface in  
Principal Stress Space

M.K. Neilsen, W-Y. Lu, W.A. Olsson, A.M. Kraynik, W.M. Scherzinger, 'Foam Constitutive Models from Complementary Experiments and Cell-Level Simulations,' 15<sup>th</sup> UCTAM, Univ. of Colorado at Boulder, 2006.

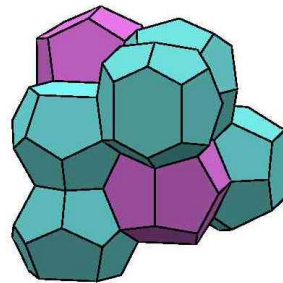
Need spatially-periodic representative volume



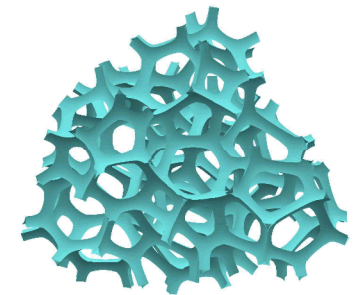
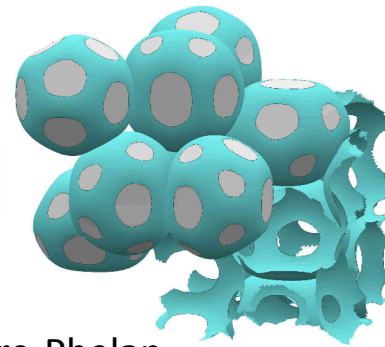
Kelvin



Random 27 bubbles



Weaire-Phelan



Plateau borders

A.M. Kraynik, "Foam structure: From soap froth to solid foams." *MRS Bulletin* 28.04 (2003): 275-278.

W. Thomson, Lord Kelvin, "On the division of space with minimum partitional area", *Phil. Mag.* vol. 24 (1887), 503.

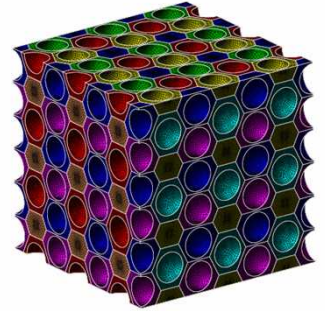
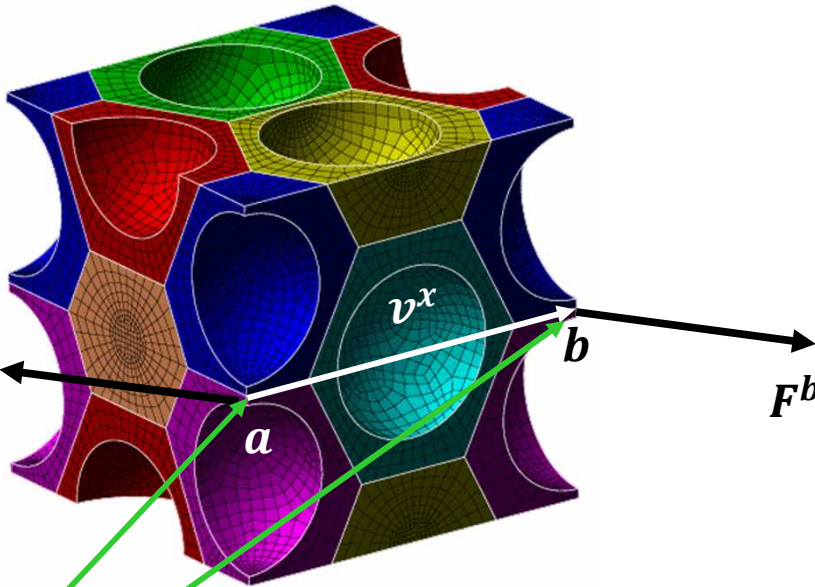
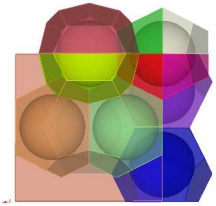
D. Weaire and R. Phelan, "A counterexample to Kelvin's conjecture on minimal surfaces", *Phil. Mag. Lett.* **69** (1994), 107.

Prof. K. Brakke, Susquehanna University, **Surface Evolver**, [www.susqu.edu/brakke/evolver](http://www.susqu.edu/brakke/evolver)

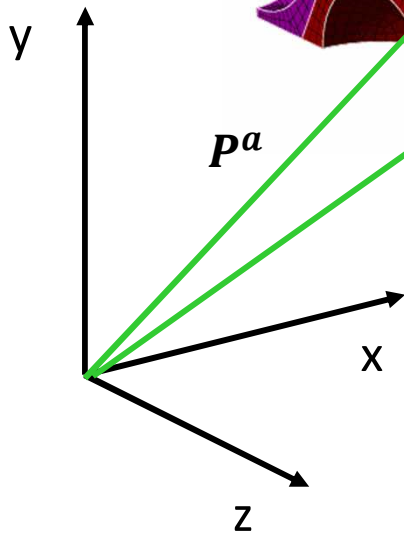
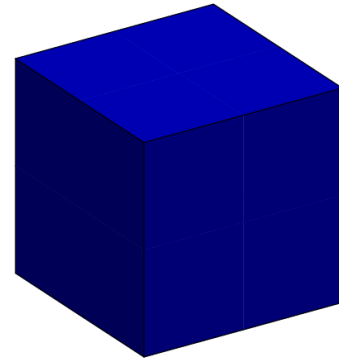
# Cell-Level to Macro Connection

Weaire-Phelan Unit Cell

Periodic b.c. :  $\mathbf{p}^b = \mathbf{p}^a + \mathbf{v}^x$



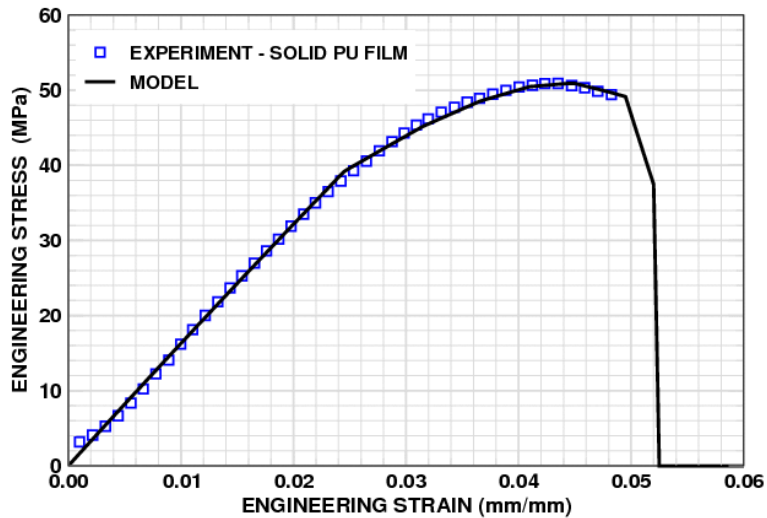
“Equivalent”  
Continuum



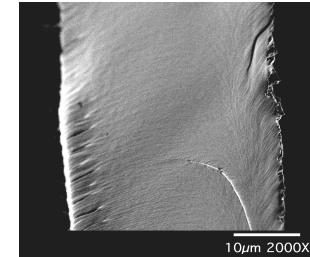
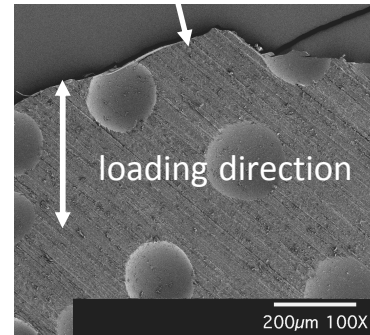
$$\bar{\sigma}_{ij} = \frac{1}{\text{volume}} \int \sigma_{ij} dv = \frac{1}{\text{volume}} \sum_{n=\text{periodic nodes}} \mathbf{P}_i^n \mathbf{F}_j^n$$

Macroscopic Cauchy Stress for “Equivalent” Continuum

# Tension of Solid Polyurethane (PU)



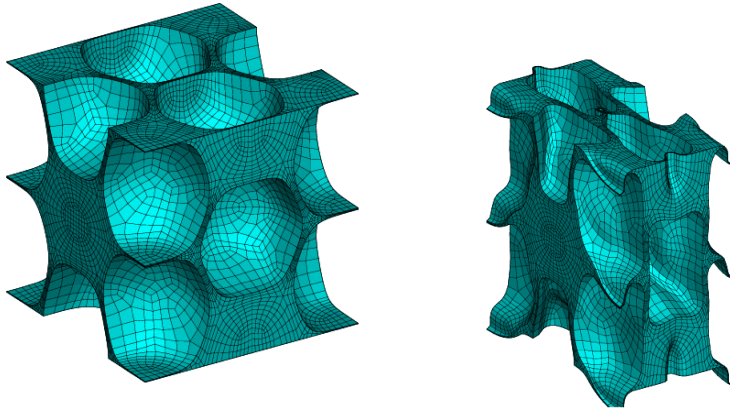
fracture edge



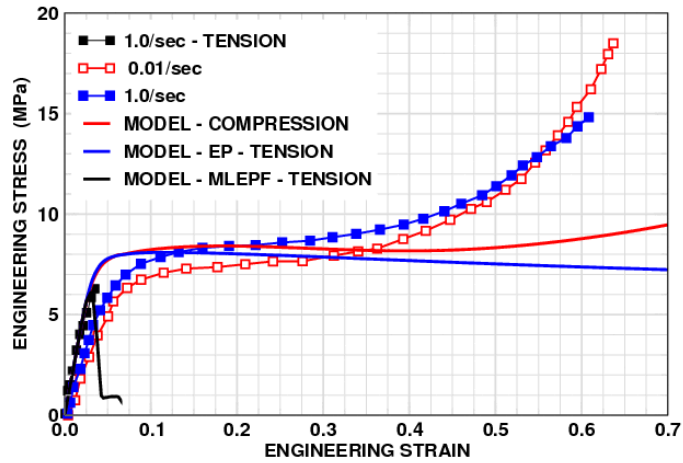
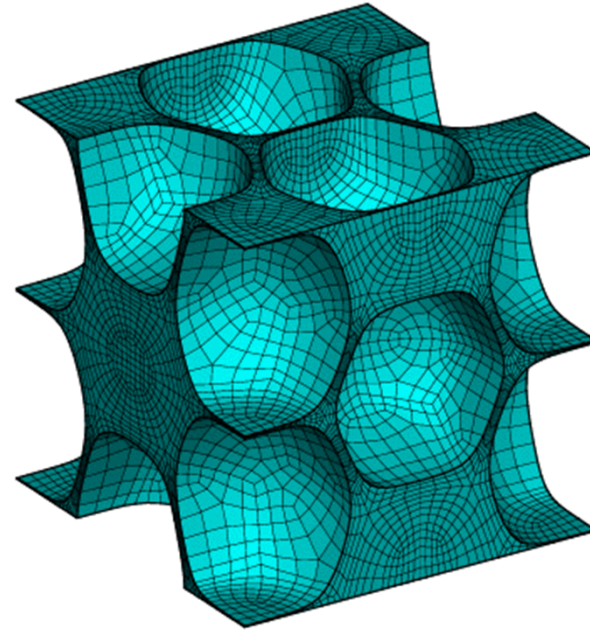
FR3712 Solid Film

G.W. Wellman, 'A Simple Approach to Modeling Ductile Failure,' SAND2012-1343, Sandia National Labs., June 2012.

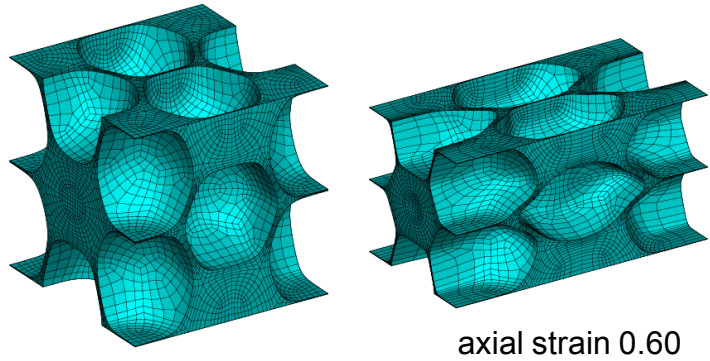
# WP Cell Model Compression



Compression - 100

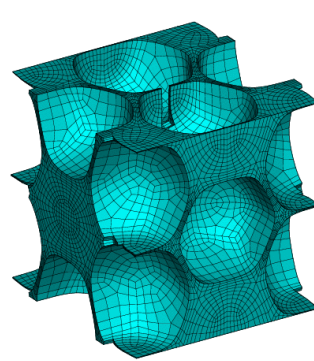


# WP Cell Model - Tension



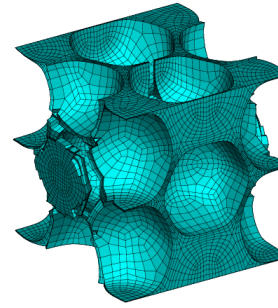
axial strain 0.60

Tension – Elastic Plastic

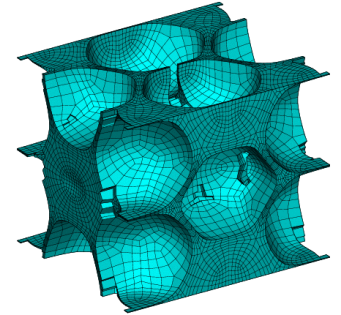


axial strain 0.03

Tension – MLEPF

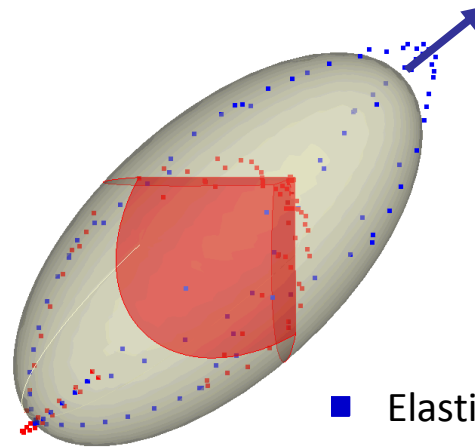
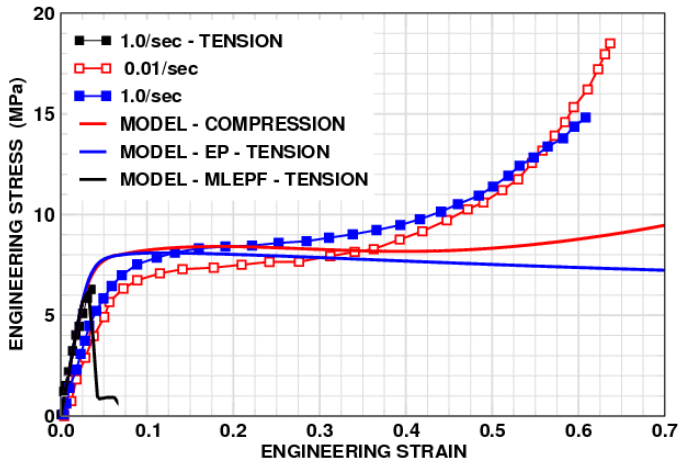


axial strain 0.06



axial strain 0.10

Tension – MLEPF with lateral compression



Hydrostat

■ Elastic-Plastic Polymer

■ MLEPF – Tearing Polymer

# Yield and Damage Surfaces for Foams

## Yield /Damage Surfaces Proposed for Rigid Foam

Triantafillou et al. (1989) Stretched ellipsoid w/ buckling cap.

Neilsen et al. (1995). Principal stress criterion.

Puso and Govindjee (1995) Stretched ellipsoid w/ ellipsoidal cap.

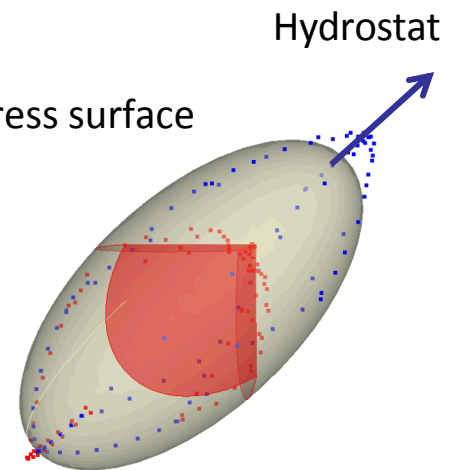
Zhang et al. (1998). Ellipsoid about hydrostat, pressure offset.

Deshpande and Fleck (2000) Ellipsoid about hydrostat – metal foams

Deshpande and Fleck (2001) Ellipsoid capped by max. compressive principal stress surface

Experiments and Cell Level Models indicate we should use

Ellipsoid capped by maximum tensile stress **damage surfaces**



- Gibson, L.J., and Ashby, M.F., **Cellular Solids – Structural and Properties**, Pergamon Press, New York, 1988
- Neilsen, M. K., Krieg, R. D., and Schreyer, H.L., *Polymer Engineering and Science*, **35**, No. 5, pp. 387-94, 1995
- Puso, M.A., and Govindjee, S., ASME MD-Vol. 68/AMD-Vol. 215, **Mechanics of Plastics & Composites**, 1995
- Zhang, J., Kikuchi, N., Li, V., Yee, A., and Nusholtz, G., *Intl. J. Impact Engr.*, **21**, No. 5, pp. 369-386, 1998.
- Deshpande, V.S., and Fleck, N.A., *J. Mech. Phys. Solids*, **48**, pp. 1253-83, 2000.
- Deshpande, V.S., and Fleck, N.A., *Acta. Mater.*, **49**, pp. 1859-1866, 2001.

# Unified Creep Plasticity Damage (UCPD) Model

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}}^e = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in})$$

$$\varphi = \frac{\bar{\sigma}^2}{a^2} + \frac{p^2}{b^2} - 1.0$$

$$p = \frac{1}{3} \boldsymbol{\sigma} : \mathbf{i}$$

$$\bar{\sigma} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}}$$

$$\mathbf{s} = \boldsymbol{\sigma} - p \mathbf{i}$$

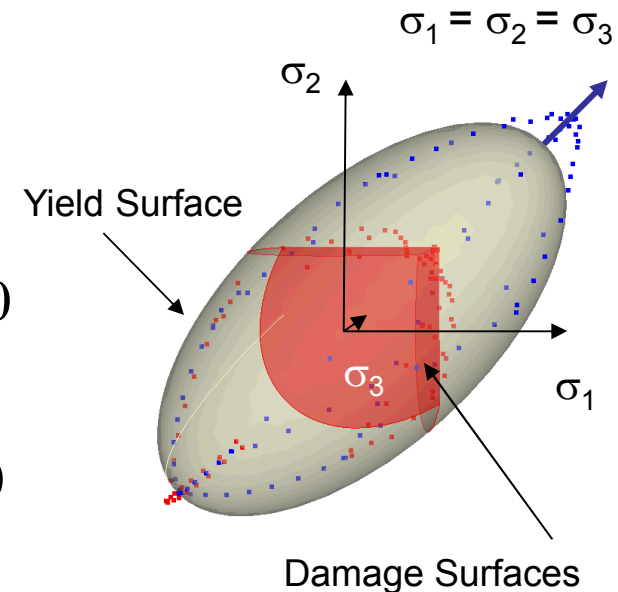
$$\varphi = \sigma^* - a$$

$$\sigma^* = \sqrt{\bar{\sigma}^2 + \frac{a^2}{b^2} p^2}$$

$$\dot{\boldsymbol{\varepsilon}}^{in} = \begin{cases} \lambda \mathbf{g} = e^h \left\langle \frac{\sigma^*}{a} - 1 \right\rangle^n \mathbf{g} \\ \mathbf{0} \end{cases}$$

$$\text{when } \frac{\sigma^*}{a} - 1 > 0$$

$$\text{when } \frac{\sigma^*}{a} - 1 \leq 0$$



# Unified Creep Plasticity Damage (UCPD) Model

$$\mathbf{g}_{associated} = \frac{\frac{\partial \phi}{\partial \boldsymbol{\sigma}}}{\left| \frac{\partial \phi}{\partial \boldsymbol{\sigma}} \right|} = \frac{\frac{3}{a^2} \mathbf{s} + \frac{2}{3b^2} p \mathbf{i}}{\left| \frac{3}{a^2} \mathbf{s} + \frac{2}{3b^2} p \mathbf{i} \right|}$$

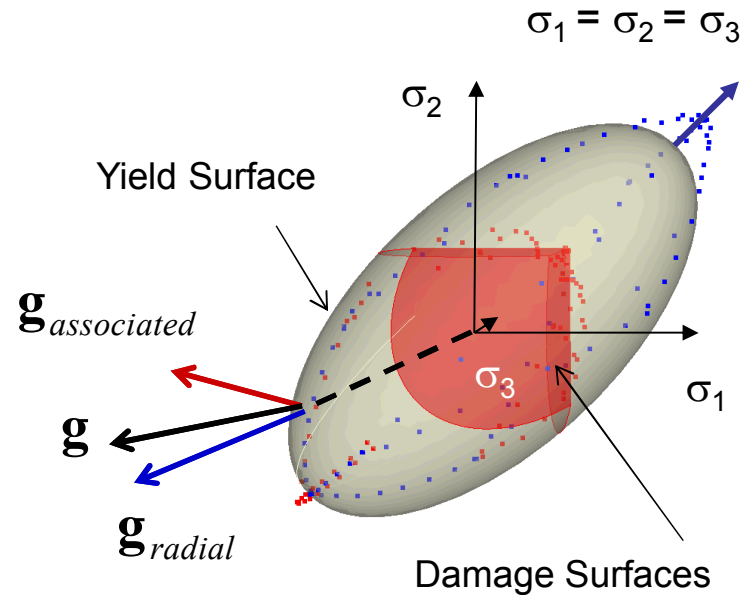
$$\mathbf{g}_{radial} = \frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}|} = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{\sigma} : \boldsymbol{\sigma}}}$$

$$\mathbf{g} = \frac{(1 - \beta) \mathbf{g}_{associated} + \beta \mathbf{g}_{radial}}{|(1 - \beta) \mathbf{g}_{associated} + \beta \mathbf{g}_{radial}|}$$

$$\phi = \frac{\phi_0 V_0}{V}$$

$$\varphi_{Damage}^i = \sigma^{**i} - c(1 - w) = 0, \quad i = 1, 3$$

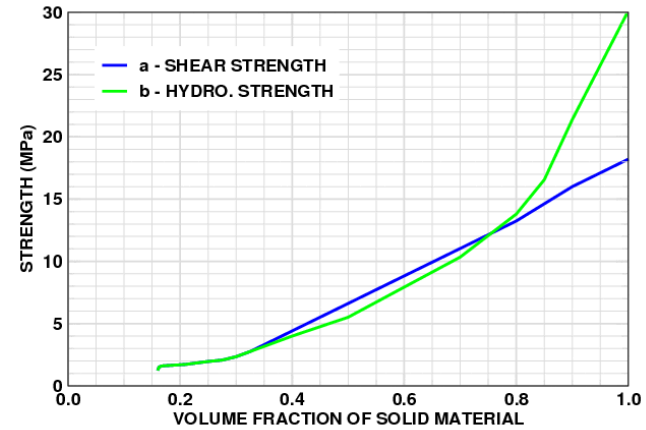
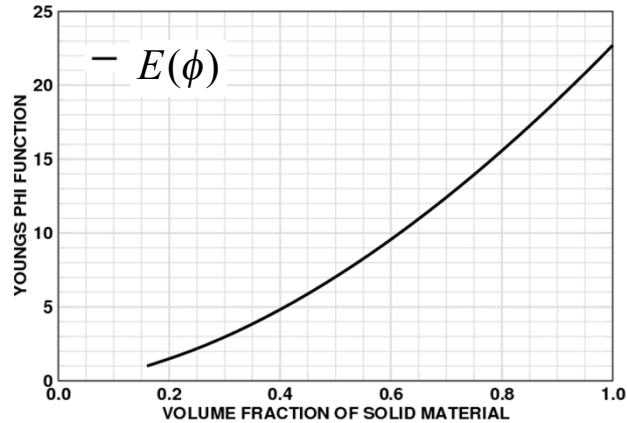
$$w = w(\boldsymbol{\varepsilon}_{dam}) = w\left(a_{dam} \boldsymbol{\varepsilon}_{max} + b_{dam} \boldsymbol{\varepsilon}_{vol}^p\right)$$



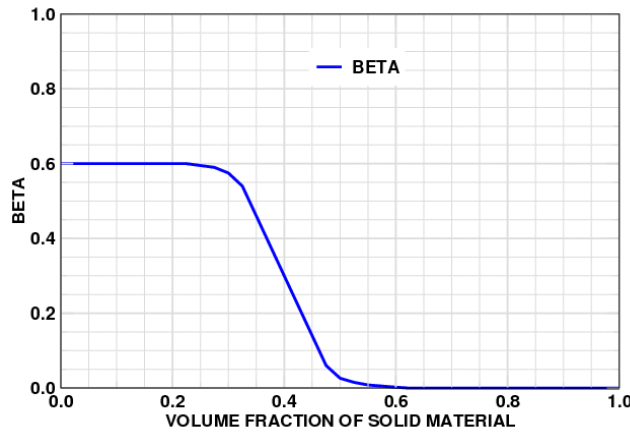
# FR3712 Material Parameters

Parameter		Units	Value	Value	Value
Temperature		C	-53.9	18.3	73.9
Young's Modulus	$E_r E(\theta)$	psi	11,550	11,550	9,240
Poisson's Ratio	$\nu$	-	0.250		
Initial Volume Fraction	$\phi_0$	-	0.160		
Flow Rate	$h_r h(\theta)$	-	-10.0	2.60	7.50
Power Exponent	$n_r n(\theta)$	-	18.0	14.0	6.00
Tensile Strength	$c$	psi	280.0		
Adam	$a_{dam}$	-	1.00		
Bdam	$b_{dam}$	-	0.50		
Thermal Expansion Coefficient		1/C	$60.0 \times 10^{-6}$		

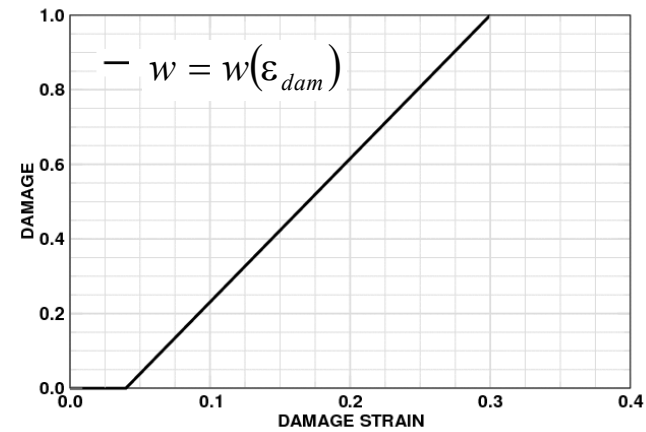
# FR3712 Material Parameter Functions



Radial Flow

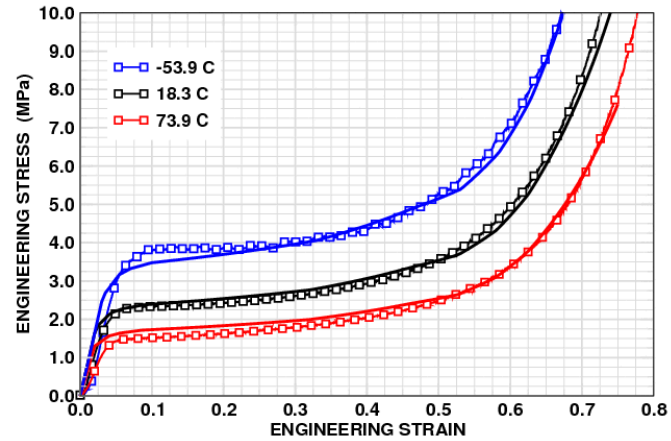
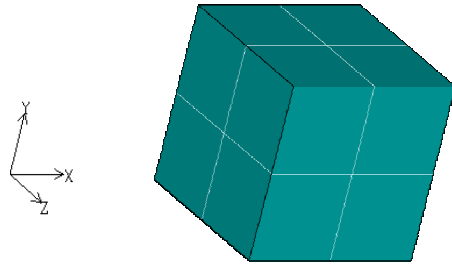


Associated Flow

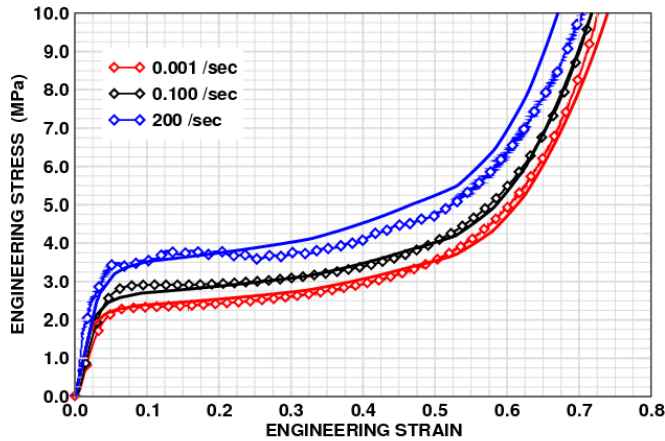


Material parameter functions for FR3712.

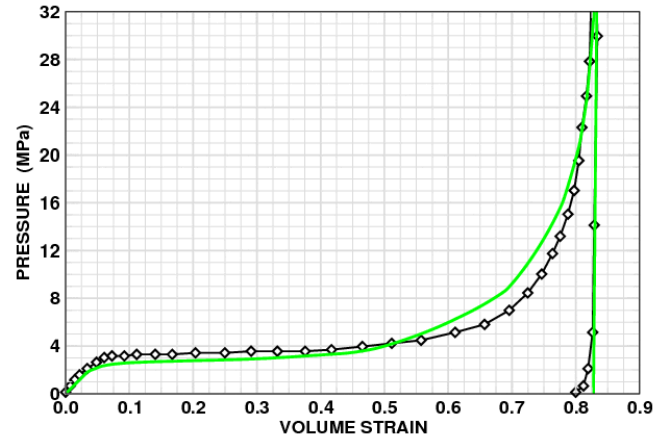
# FR3712 Comparison of Model Predictions with Experiments



Uniaxial Compression at 0.001/second

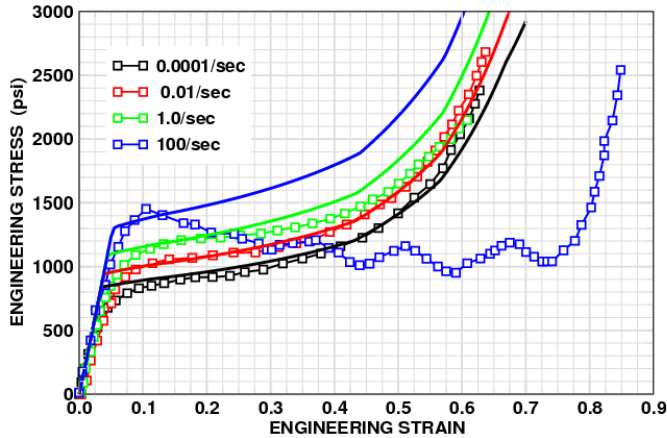


Uniaxial Compression at 18.3 C

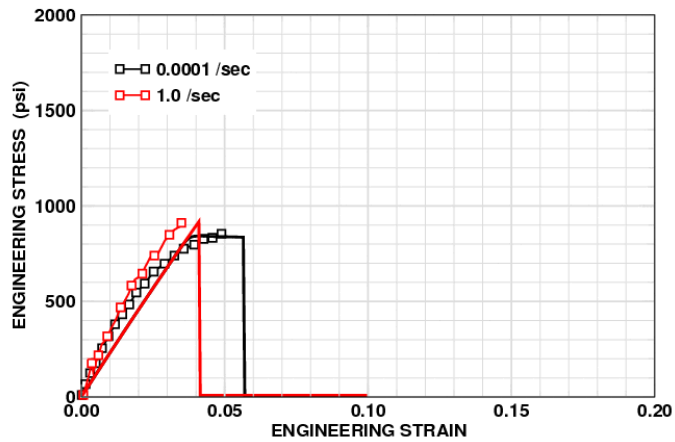


Hydrostatic Compression at 20.0 C

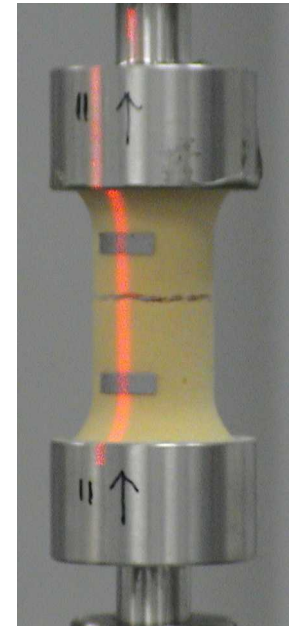
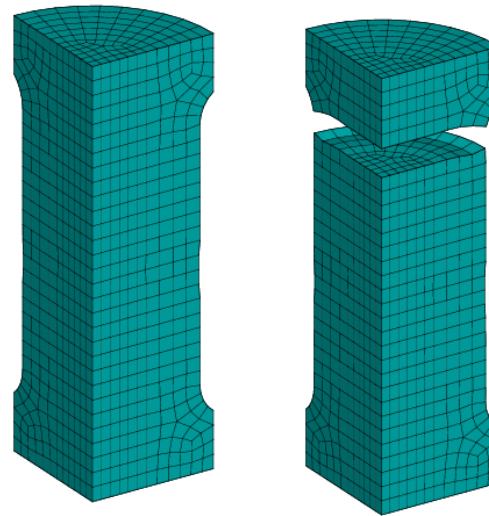
# PMDI20 Comparison of Model Predictions with Experiments



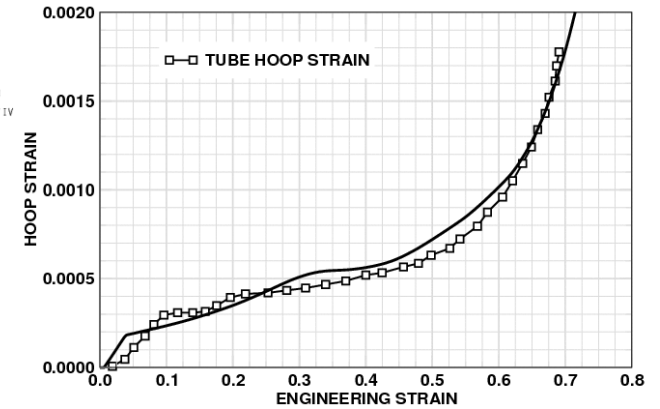
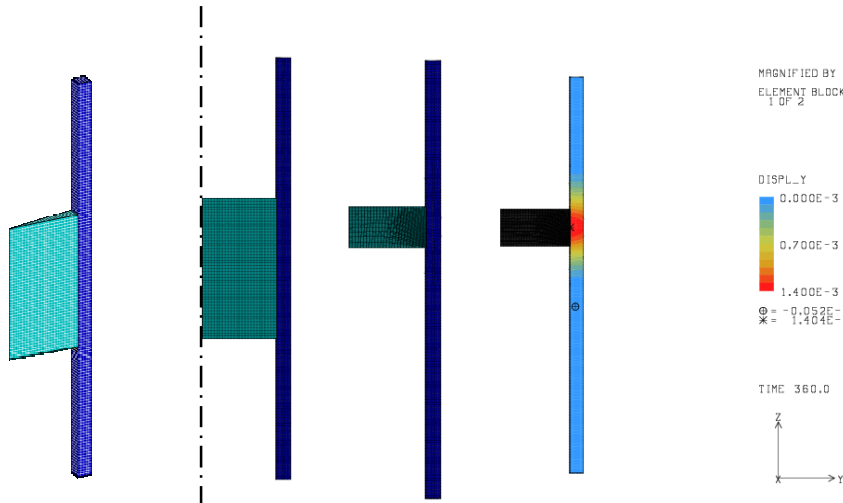
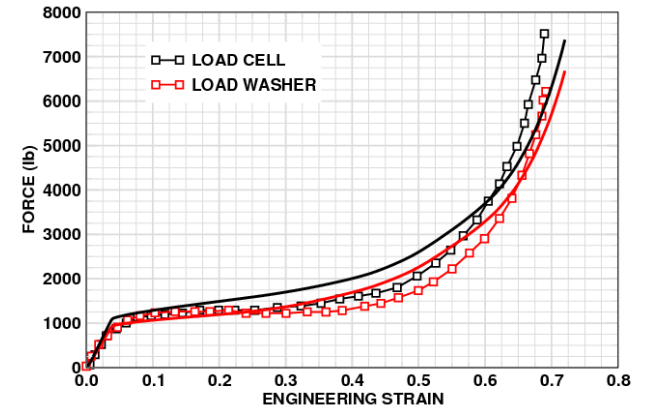
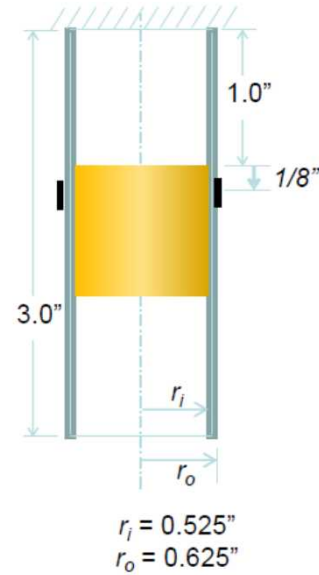
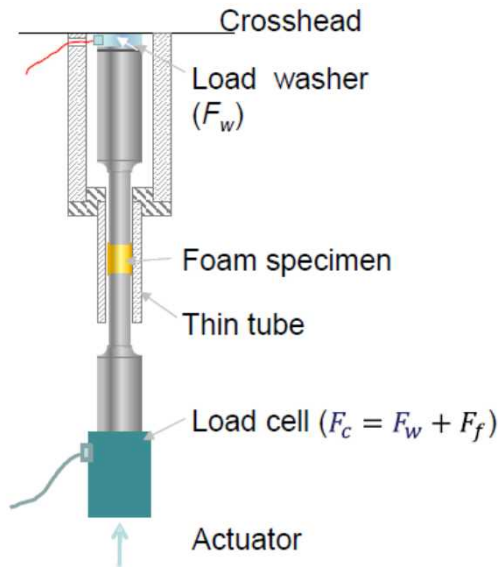
Uniaxial Compression at 21.1 C



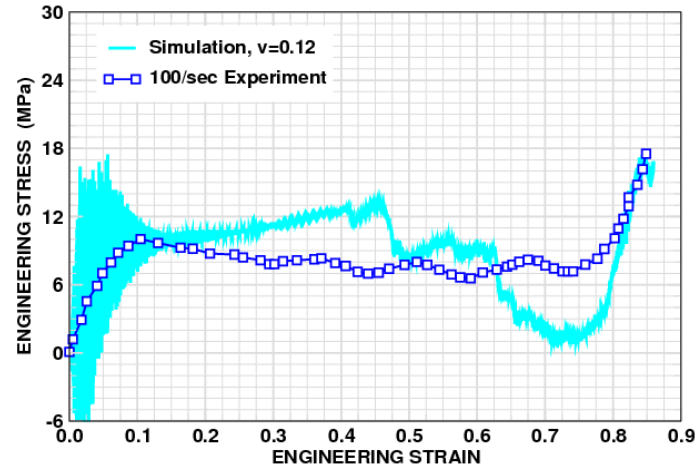
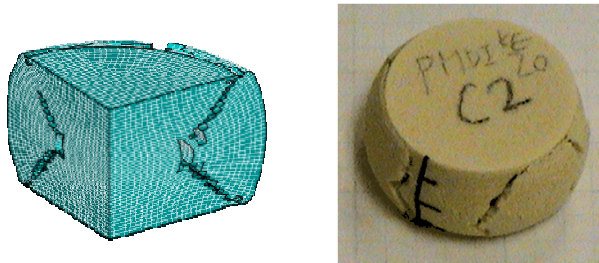
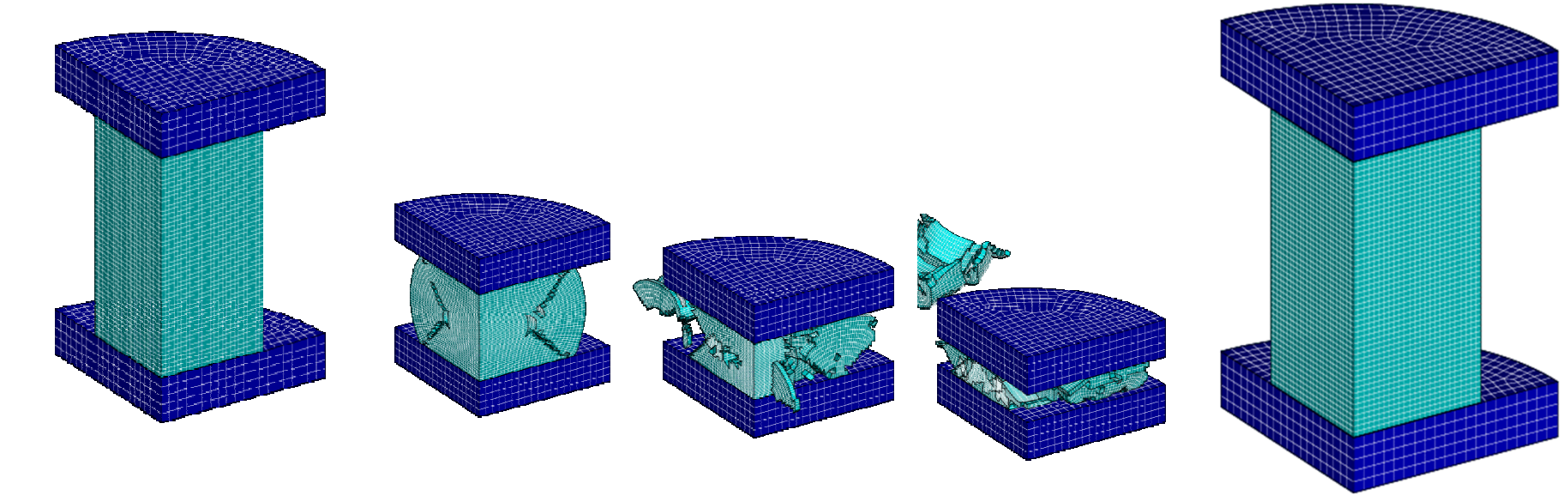
Uniaxial Tension at 21.1 C



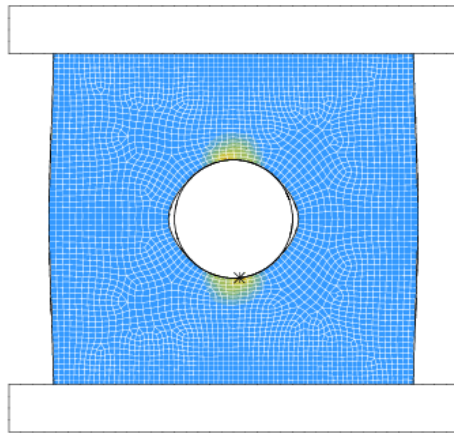
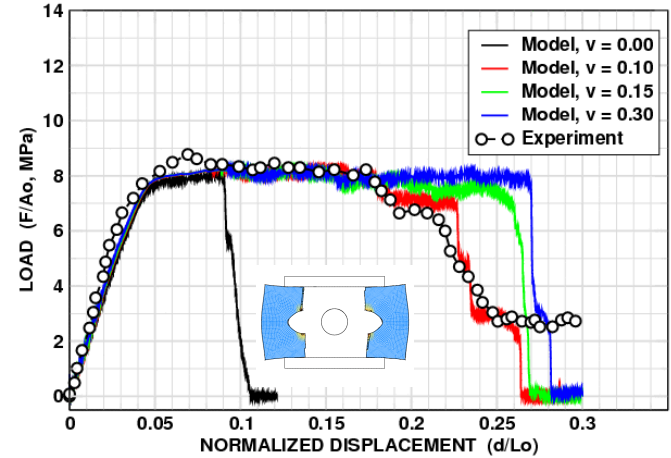
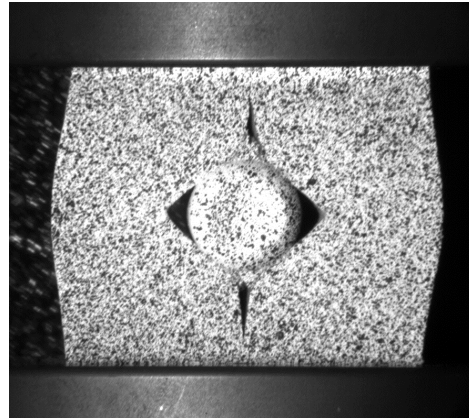
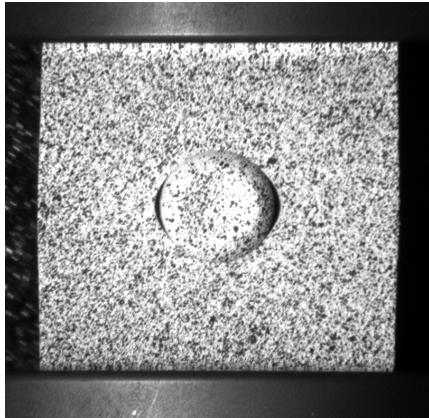
# PMDI20 Comparison of Model Predictions with Experiments



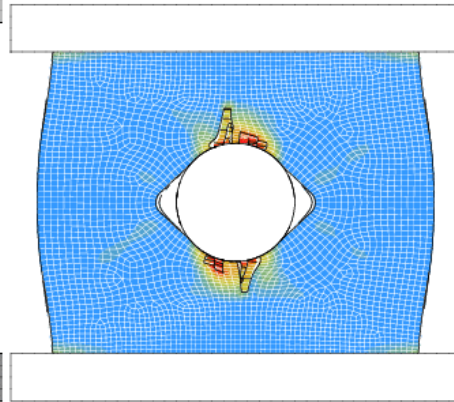
# PMDI20 Comparison of Model Predictions with Experiments



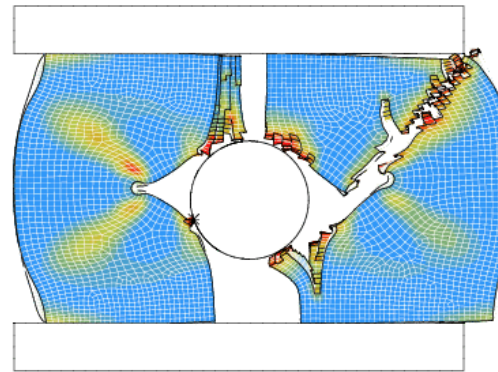
# PMDI20 Validation Experiment – Steel Rod in Foam Block



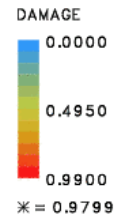
7% compression  
Peak load



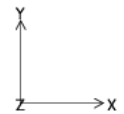
15% compression



24% compression

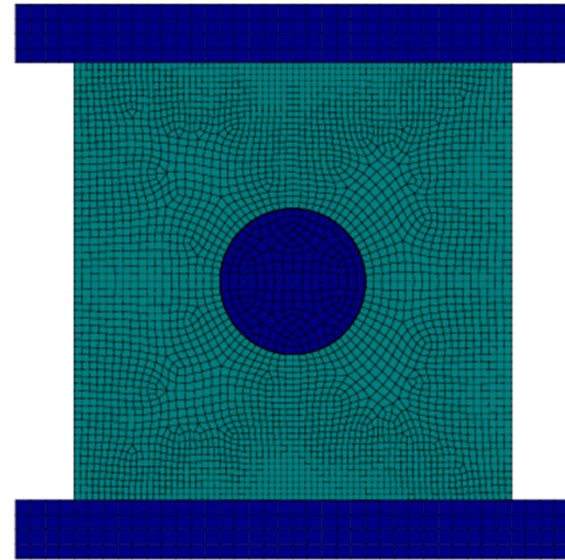
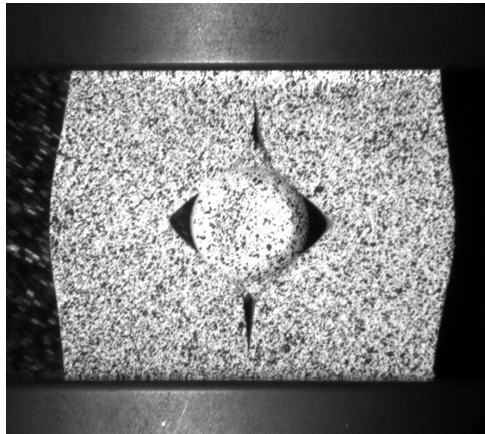
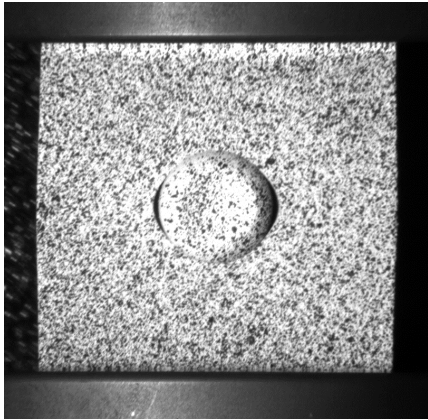


TIME 36.00E-3



## Uniaxial Compression of Foam Block with Steel Rod

# PMDI20 Validation Experiment – Steel Rod in Foam Block



Uniaxial Compression of Foam Block with Steel Rod

# Summary and Future Work

- Polyurethane foam response depends on temperature and strain-rate
- UCPD model captures inelastic deformation and fracture of foam
- Failure is based on a damage strain =  $f(\text{max. tensile strain, plastic volume strain})$
- Foam exhibits a small amount of strain hardening in tension that is not captured
- Foam strain to failure depends on loading rate and temperature which is also not captured.