

Nanoparticle Diffusion in a Polymer Matrix

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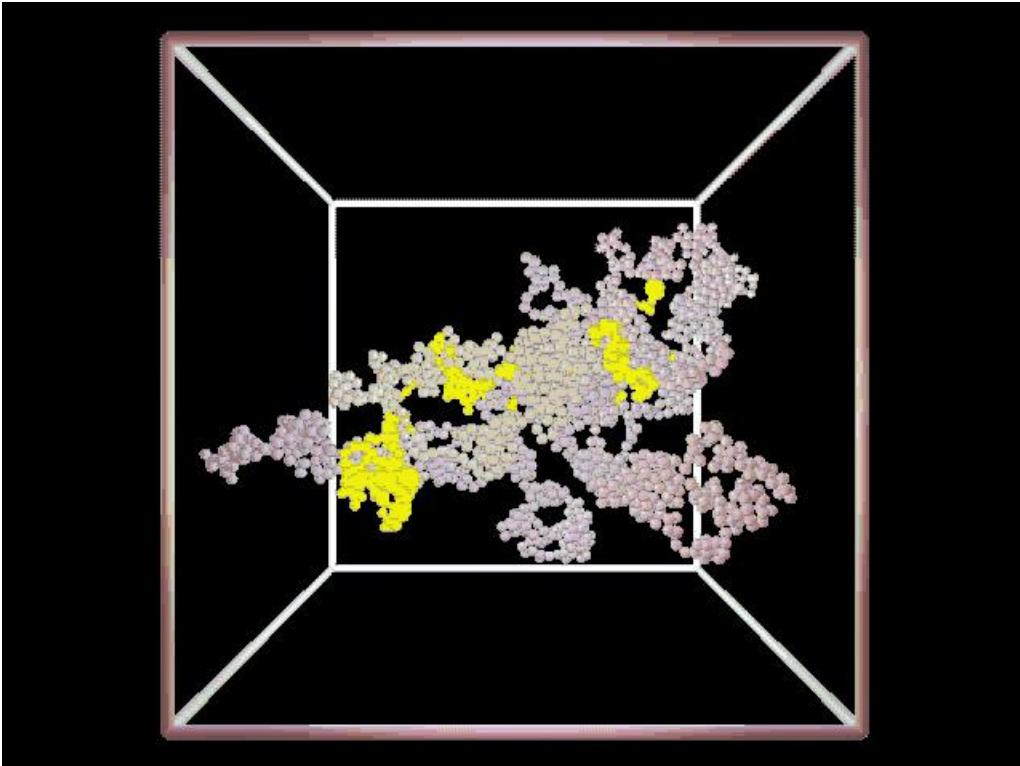


Outline

- Unique Properties of Entangled Polymers
- Computational Challenges
- Dynamics of Linear Polymers
- Dynamics of Ring Polymers
- Polymer Nanocomposites
- Future Directions

Why are Polymers Interesting?

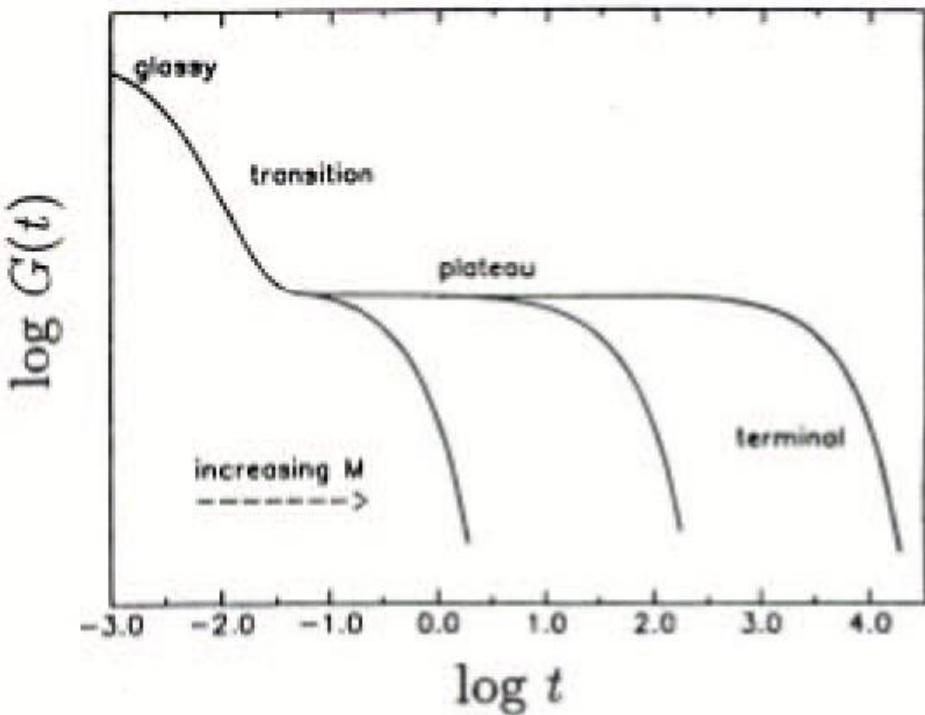
- Polymers can simultaneously be hard and soft
 - Unique Viscoelastic Behavior



- Motion of a polymer chain is subject to complicated topological constraints

Entangled Polymer Liquids

Viscoelastic Response



Stress Relaxation after strain

- **Macroscopic**

- Intermediate frequency, time polymer melt acts as a solid
- Long time, low frequency polymer acts as a liquid

- **Microscopic**

- Gaussian coils, $R \sim N^{1/2}$
- Stress is due to entropy loss of stretched chains
- Polymers as "entropic springs"
- Stress relaxation due to Brownian motion of topologically constrained chains

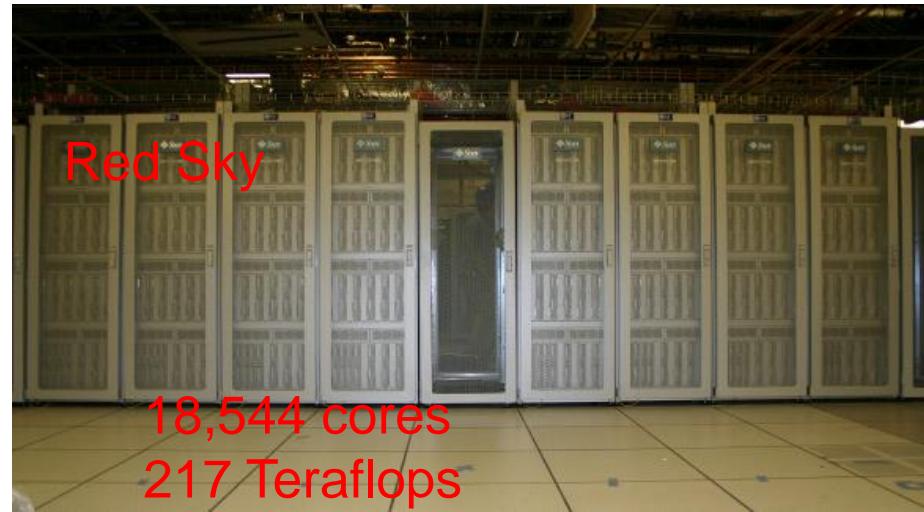
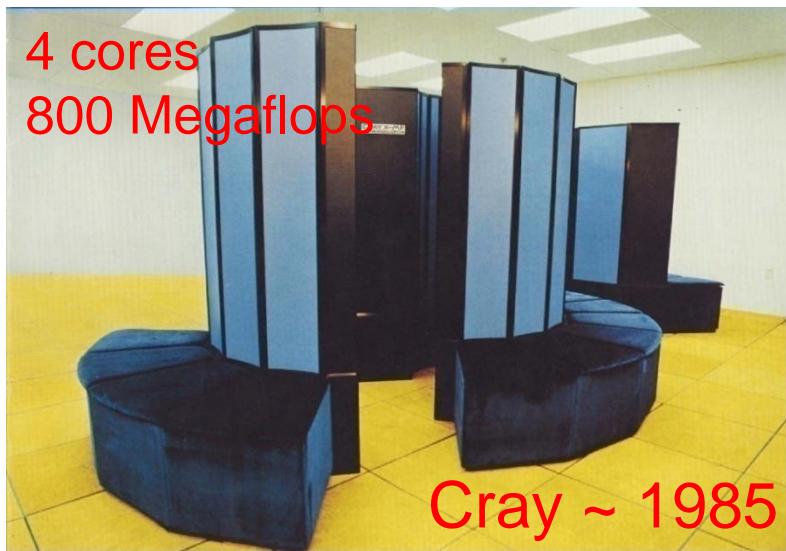
Computational Challenges

- Longest relaxation time $\tau \sim N^3$
- Chains are Gaussian coils – $R \sim N^{1/2}$
 - Number of chains must increase as $R^3 \sim N^{3/2}$ so polymer chains do not see themselves through periodic boundary conditions
- Double chain length – cpu required increases by at least a factor of $2^{4.5} \sim 23$
 - 1-2 month simulation becomes 2-4 years
- Number of processors limited: $\sim 500-1000$ particles/processor

Computational Challenges

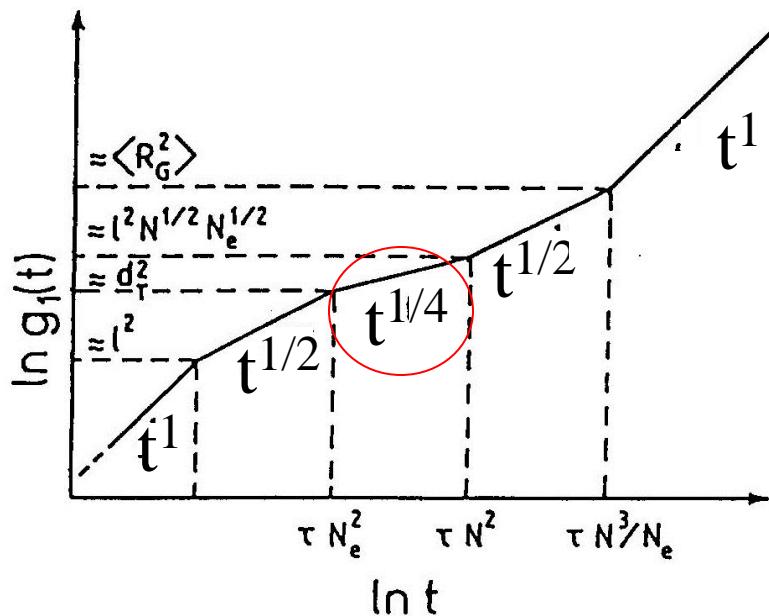
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- Double chain length – cpu required increases by at least a factor of $2^{4.5} \sim 23$
 - 1-2 month simulation becomes 2-4 years
- Number of processors limited: ~ 1000 particles/processor
- Software/hardware advances have been significant

Toys for the Simulator



Polymer Diffusion

- Simple Liquids
 - $D \sim M^{-1}$, $\eta \sim M$
- Short Polymer Chains ($M < M_e$)
 - Longest relaxation time $\tau_R \sim M^2$
 - Intermediate $t^{1/2}$ time regime in mean square displacement
 - $D \sim M^{-1}$, $\eta \sim M$
- Long Polymer Chains ($M > M_e$) - Reptation



$$\begin{aligned}D &\sim M^{-2} \\ \eta &\sim M^3 \\ \tau_d &\sim M^3\end{aligned}$$

Characteristic signature of reptation – intermediate $t^{1/4}$ regime

Bead-Spring Model

- Short range - excluded volume interaction

$$U_{\text{LJ}}(r) = \begin{cases} 4\epsilon \left\{ \left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6 + \frac{1}{4} \right\} & r \leq r_c \\ 0 & r \geq r_c \end{cases}$$

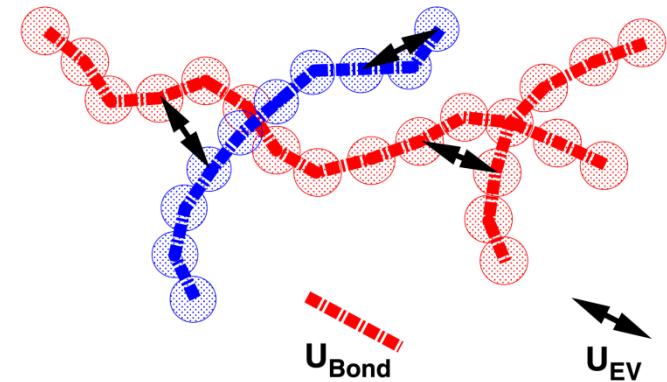
- Bonded interaction - FENE spring

$$U_{\text{FENE}}(r) = \begin{cases} -0.5kR_0^2 \ln(1 - (r/R_0)^2) & r \leq R_0 \\ \infty & r > R_0 \end{cases} \quad k=30\epsilon/\sigma^2, R_0=1.5\sigma$$

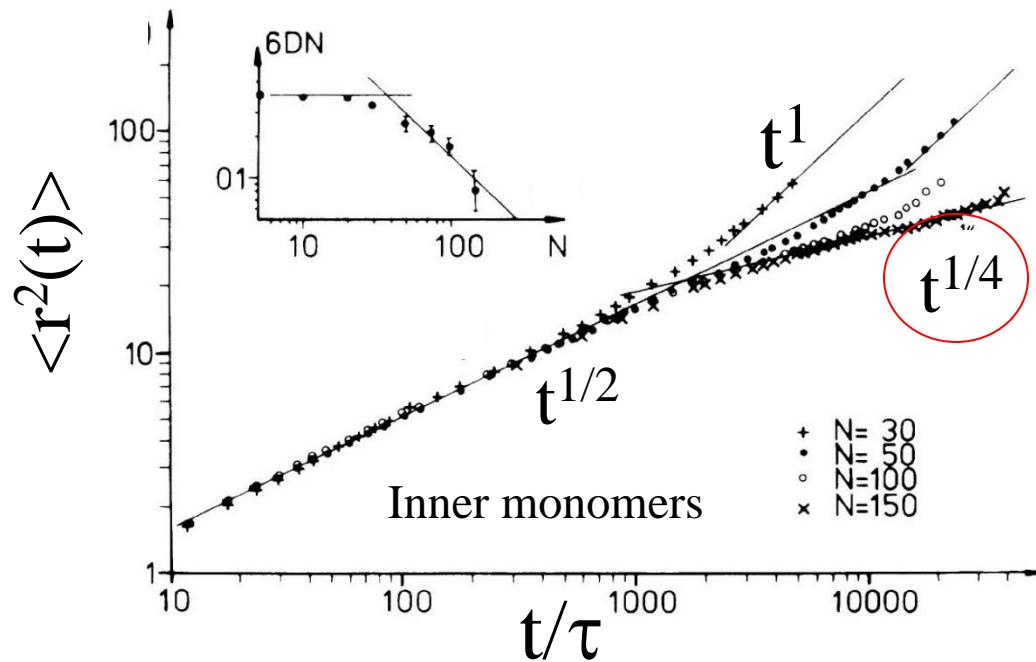
- Energy barrier prohibits chains from cutting through each other
 - topology conserved

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = -\vec{\nabla} \cdot U_i - m_i \Gamma \frac{d\vec{r}_i}{dt} + \vec{W}_i(t)$$

Time step $\Delta t \sim 0.01\tau$, $\tau=\sigma(m/\epsilon)^{1/2}$



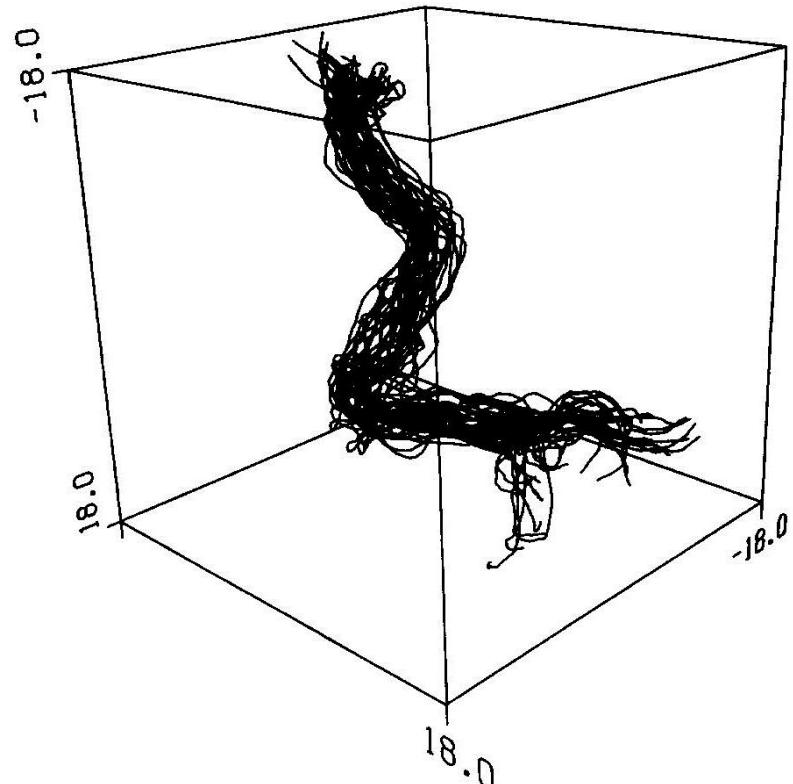
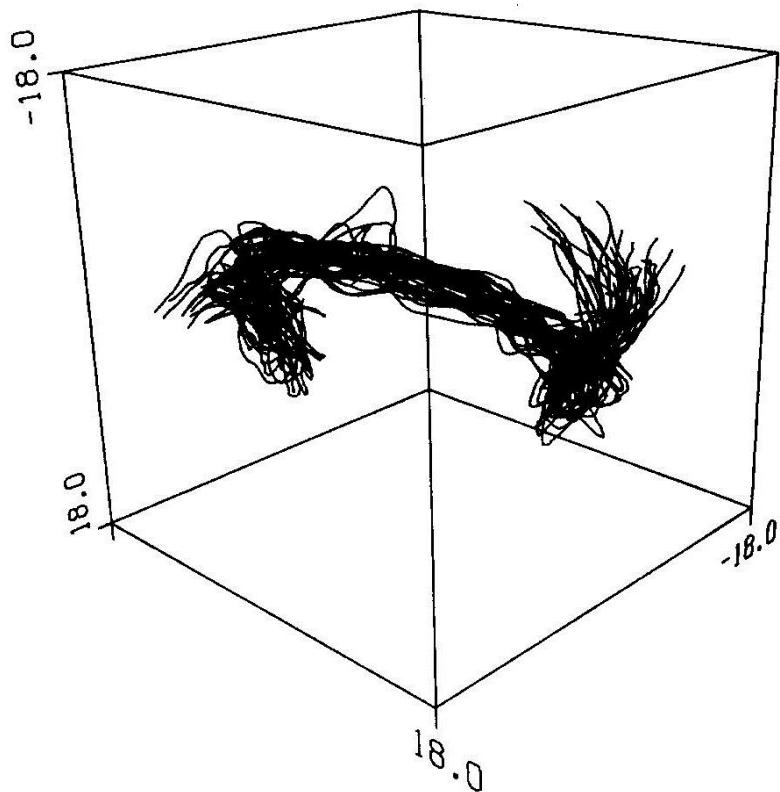
Polymers do Reptate!



- $t^{1/4}$ reptation regime for $N > 100$
- First direct evidence from simulation or experiment

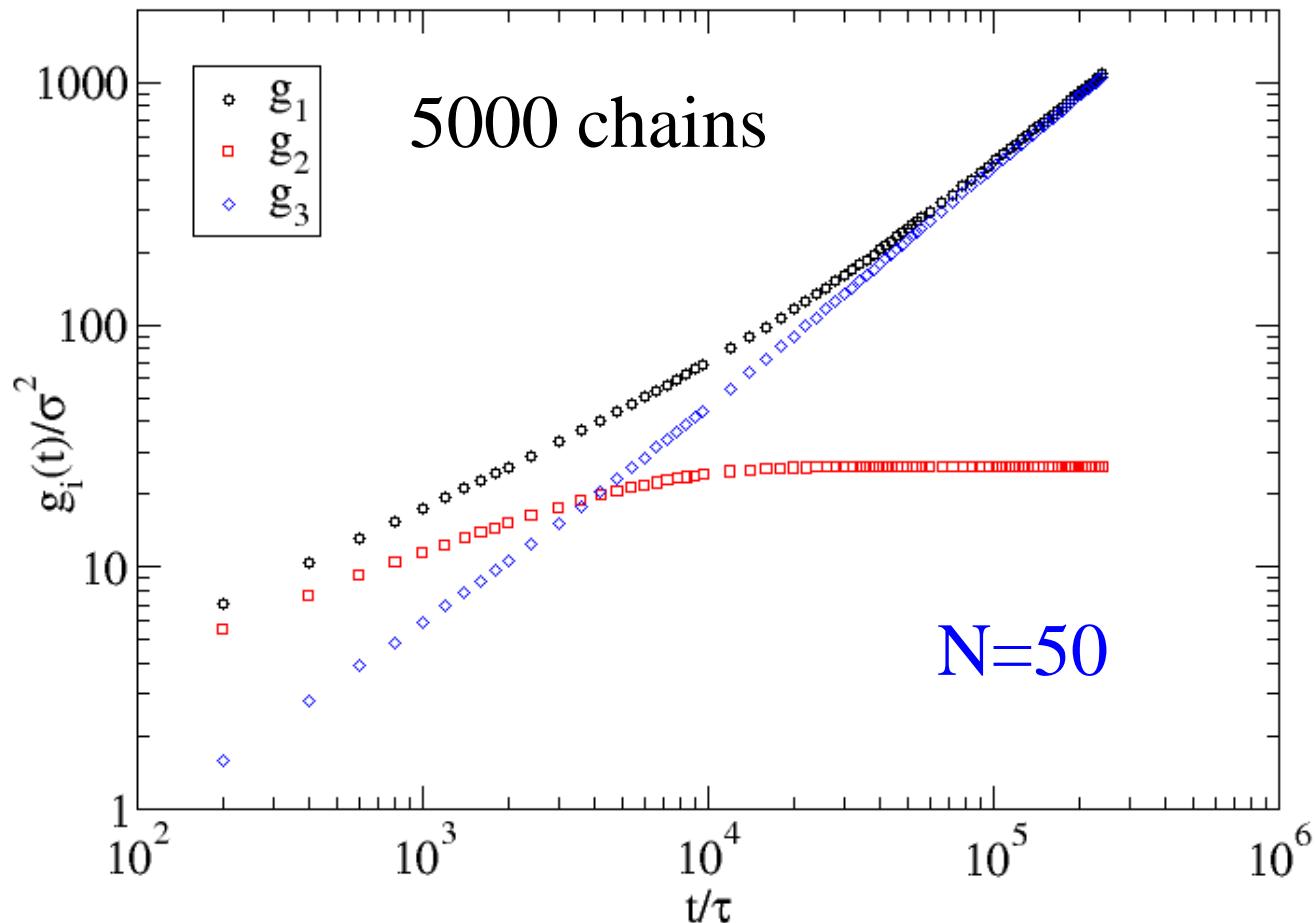
Polymer Chain Confined to Tube

- Coarse grained chain



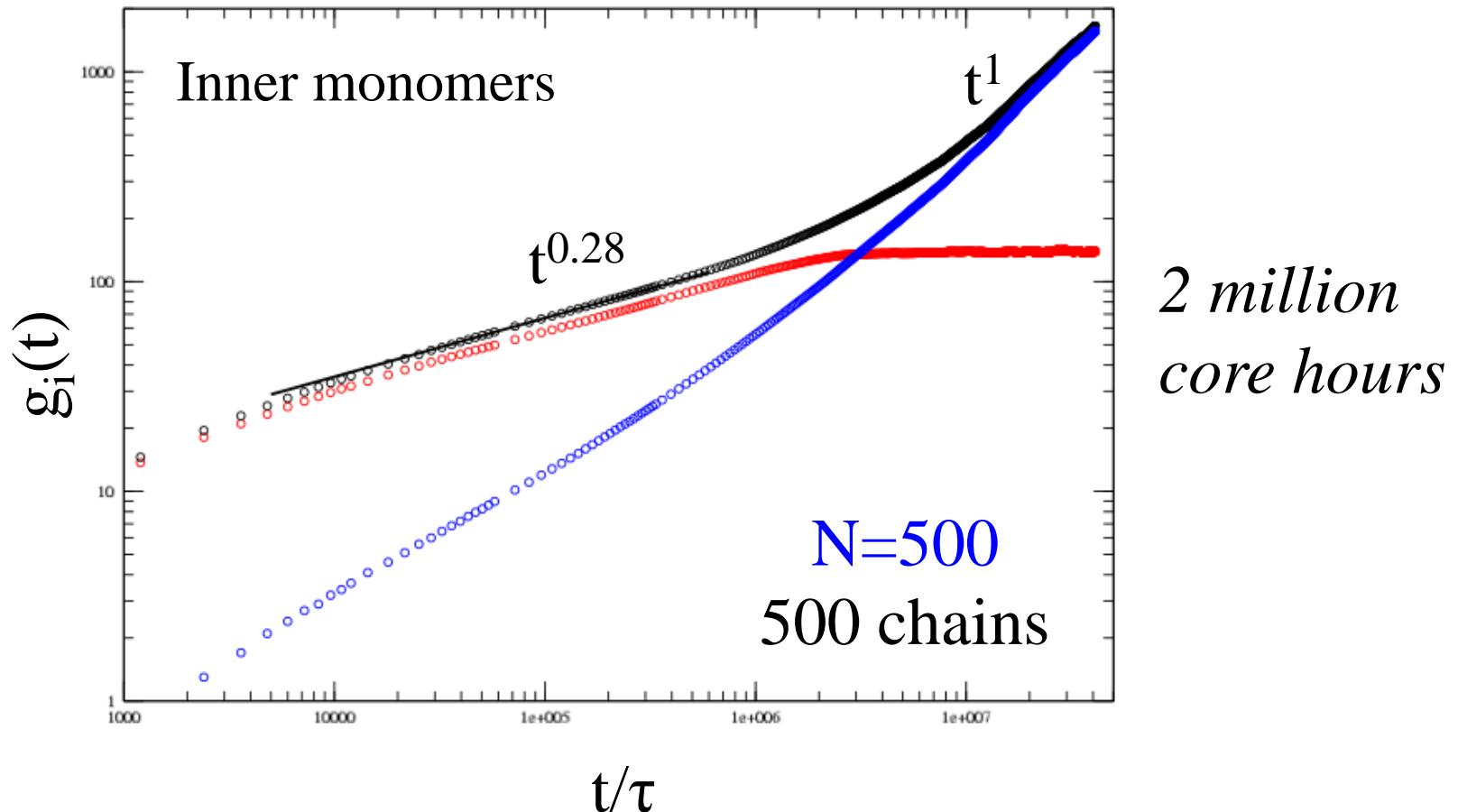
$N=400$ – 20 plots, 600 τ apart

Motion of Unentangled Polymer



- Once polymer move their own size, unentangled polymers move like normal liquids

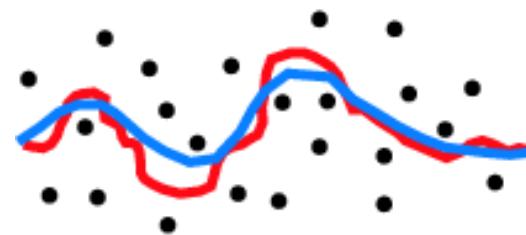
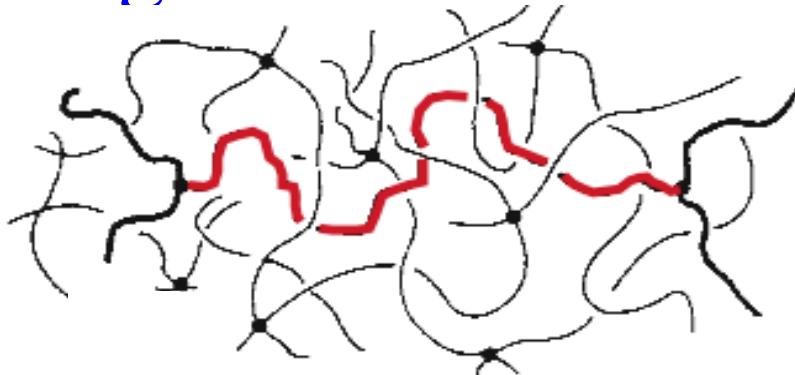
Motion of Entangled Polymer



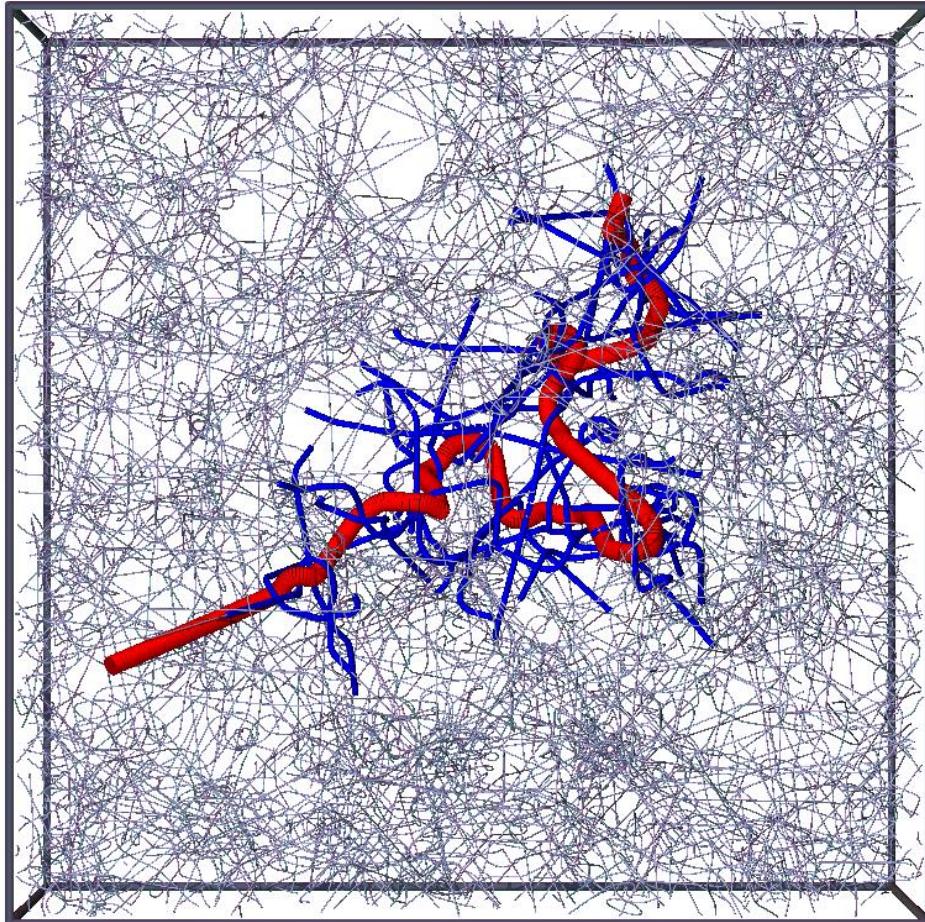
- $t^{1/4}$ motion is clearly seen for inner monomers
- Second $t^{1/2}$ region still unresolved

Topological Approach to Identify Entanglements

- Microscopic conformation
- Shortest path into which a chain can contract with fixed endpoints and without crossing obstacles
- Tube axis = primitive path
- Need a topological analysis which can follow motion of chain



Primitive Path Analysis



- Primitive paths of a cluster of entangled chains

- Shorter Contour Length

$$L_{pp} = N b_{pp} < L$$

- Larger Kuhn Length

$$a_{pp} > l_k$$

- Same spatial extent

$$a_{pp} L_{pp} = R^2 = l_k L$$

- Entanglement Length

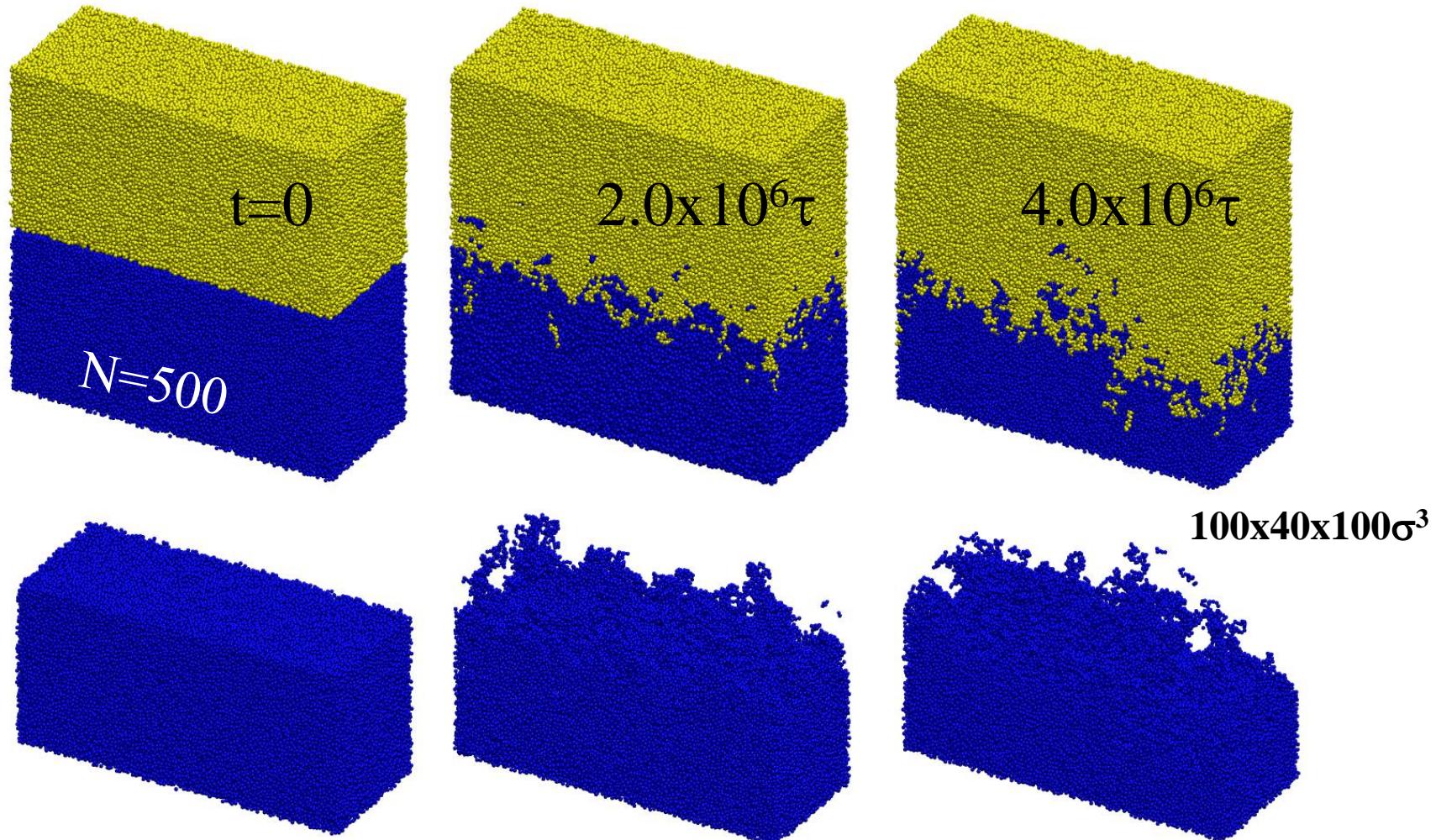
$$N_e = a_{pp} / b_{pp}$$

- Packing Length

$$p = 1 / \rho_{chain} R^2$$

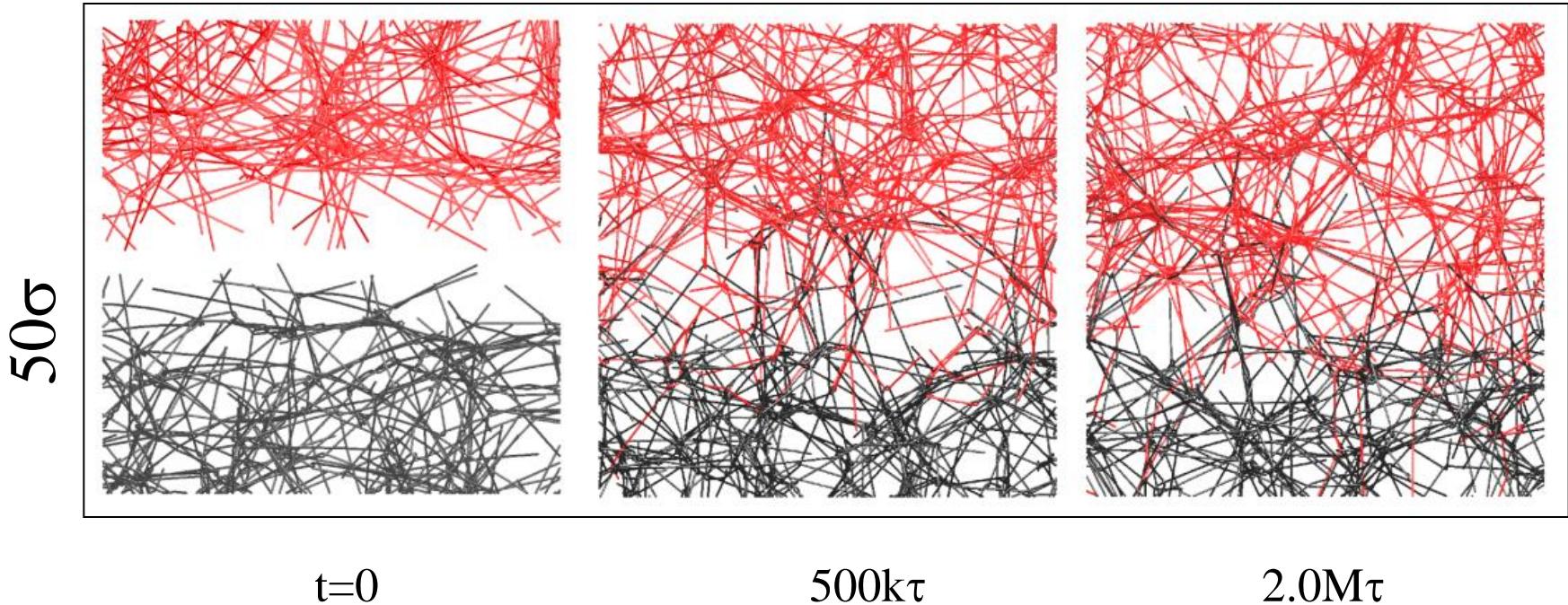
Self-Healing of Polymer Films

- Development of Entanglements Across an Interface



Entanglements at Interface

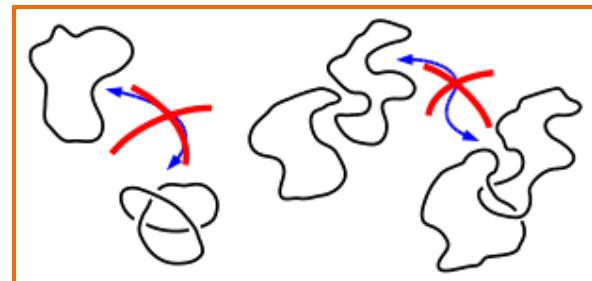
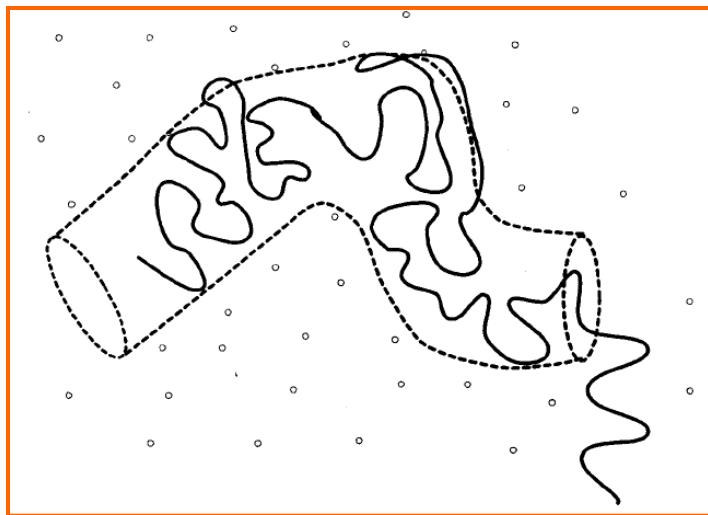
- Primitive Path Analysis



- Interfacial Entanglements form between chains from opposite sides
- Bulk response is fully recovered when the density of entanglements at the interface reaches the bulk value

Dynamics of Ring Polymers

- As chain size increases, linear polymers entangle and are forced to move ('reptate') along their contours
- Branched polymers relax via a hierarchy of modes from dangling ends moving inward
- Remaining mystery: How do ring polymers relax without beginning or end?



Configurations of Ring Polymers

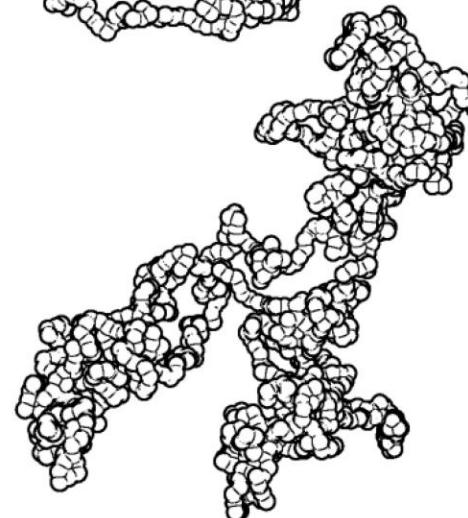
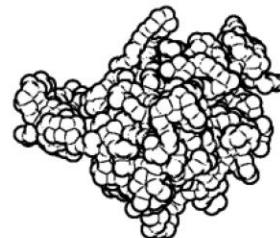
(a) $N = 100$



(b) $N = 400$

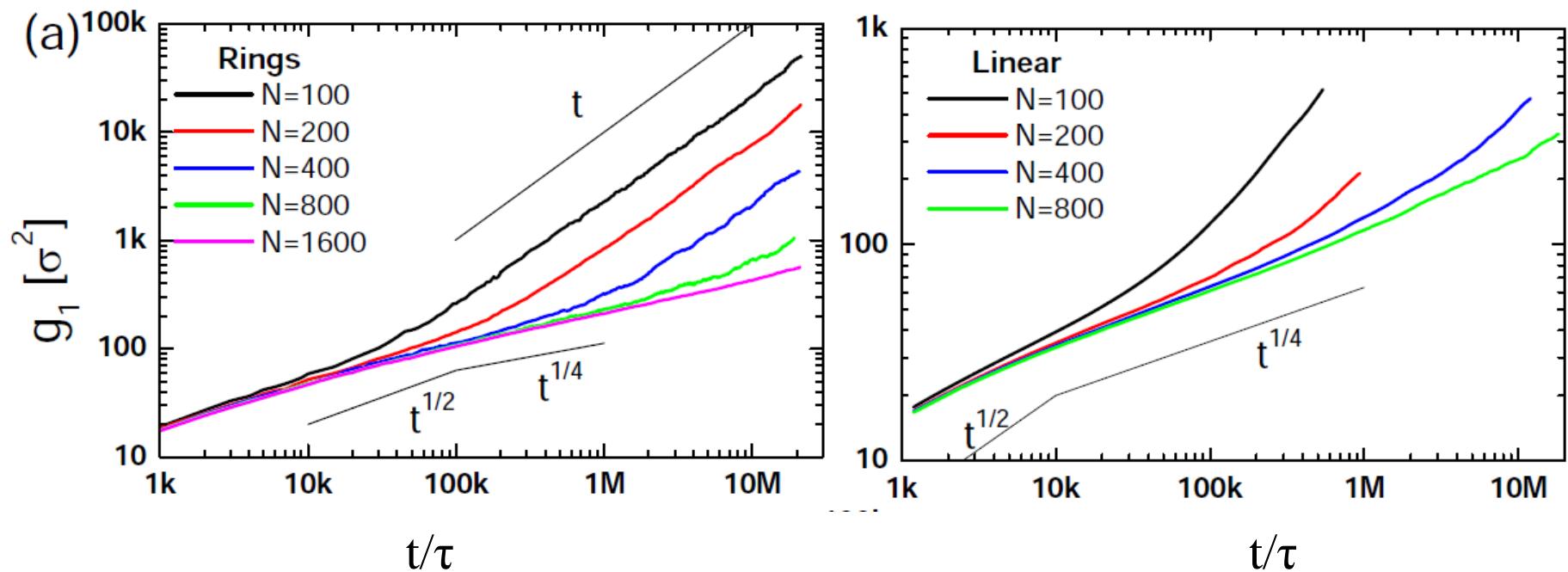


(c) $N = 1600$



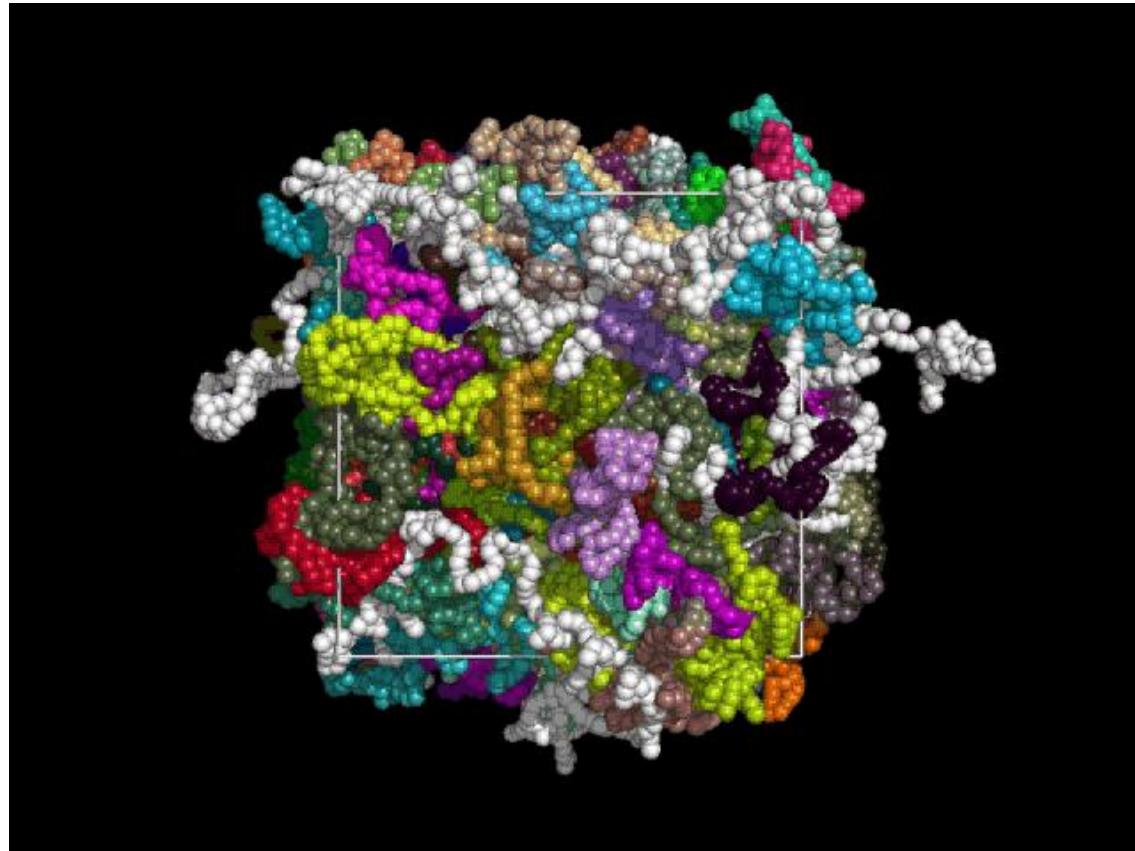
- Rings more compact, less entangled than linear chains
- Radius of Gyration $R_g^2 \sim N^{2/3}$ for rings
 $\sim N$ for linear

Dynamics of Ring Polymers



- Ring polymers move much faster than linear chains
- Longest relaxation time $\tau \sim N^2$ for rings, N^3 for linear chains

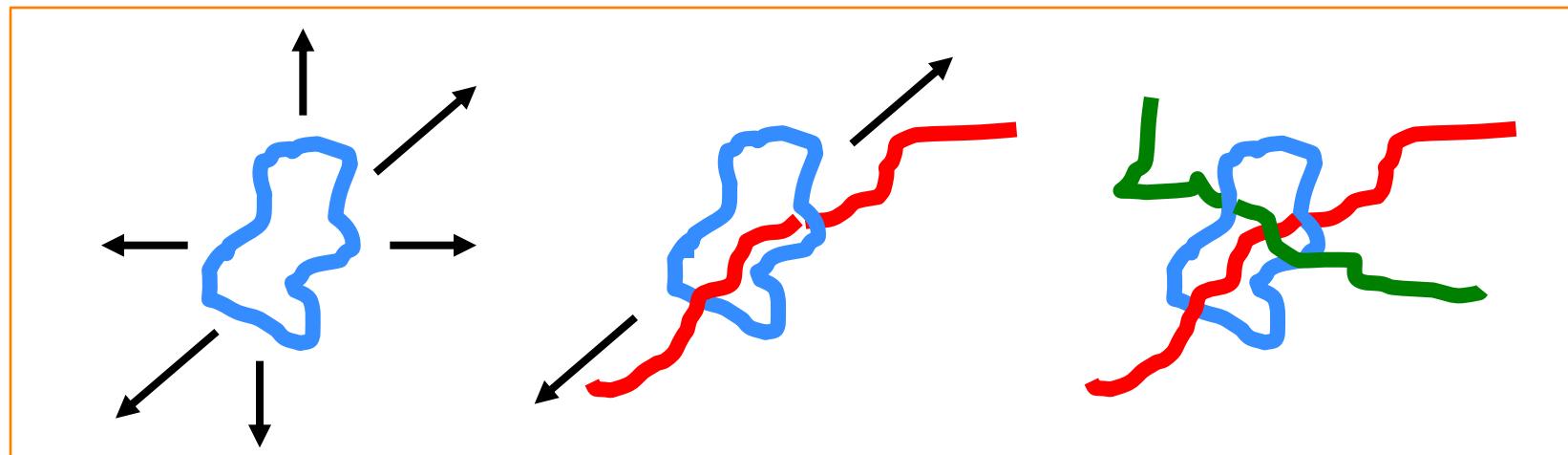
Mixture of Ring and Linear Polymers



$N = 200, M_{\text{rings}} = 200, M_{\text{linear}} = 26$ $t = 0 \tau$

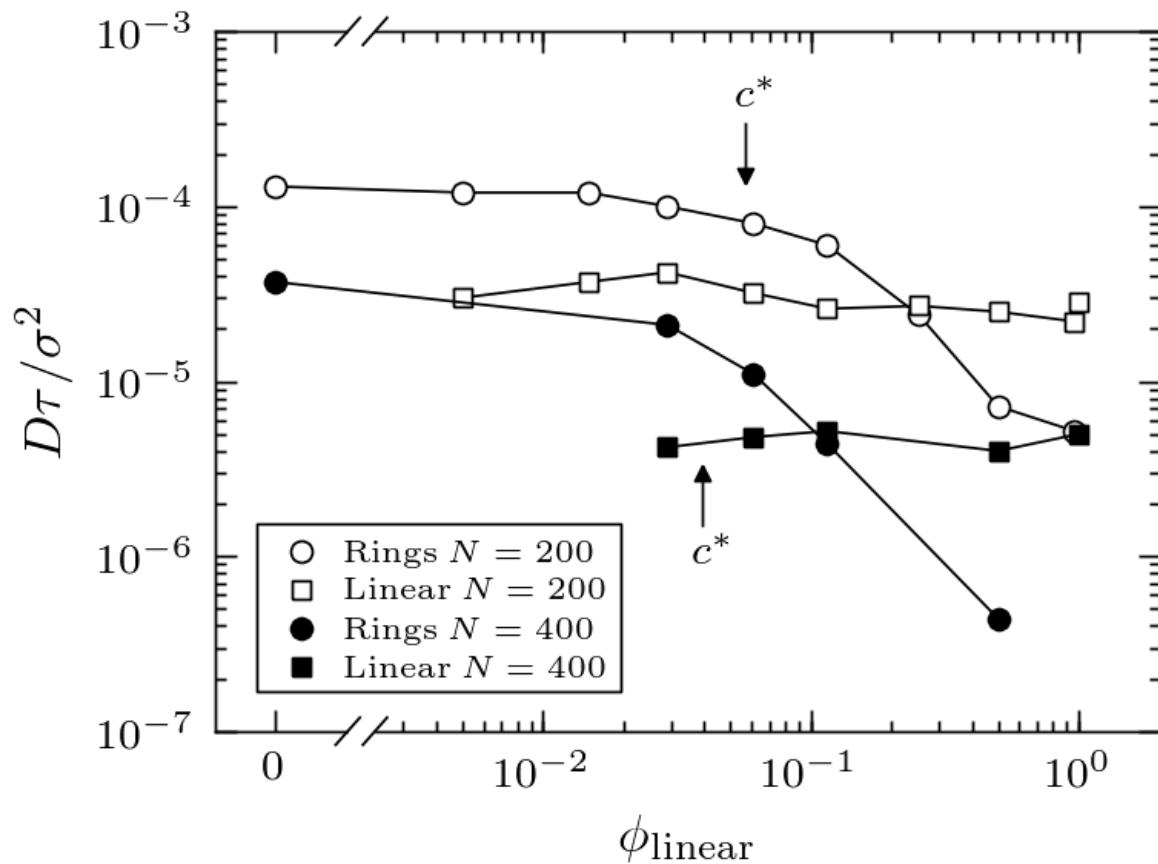
Constrained Motion of Rings in Linear Melt

- Threading causes rings to diffuse more slowly



- Threaded rings can only diffuse along the contour of the linear chain
- Time for constraint release scales as $N^{3.4}$

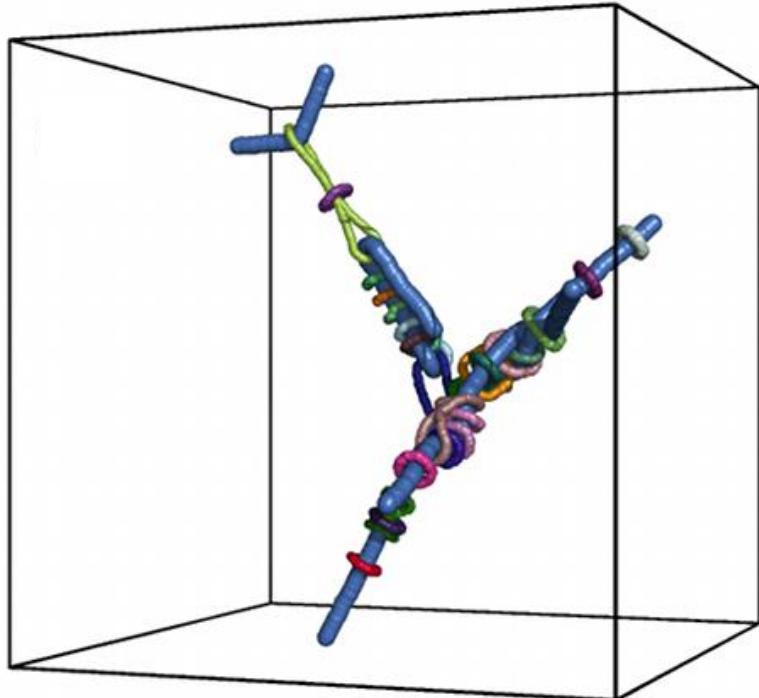
Dynamics of Ring/Linear Blends



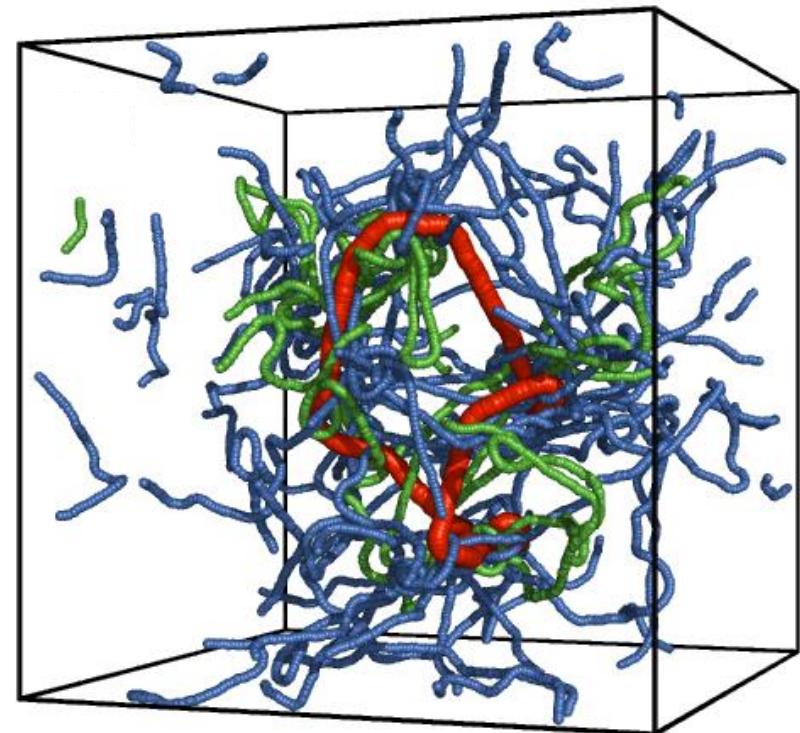
- D_{ring} decreases dramatically when threaded by multiple linear chains ($\phi_{\text{linear}} > 0.1$)
- D_{linear} is approximately independent of ϕ_{linear}

Primitive Path Analysis

$N=400$

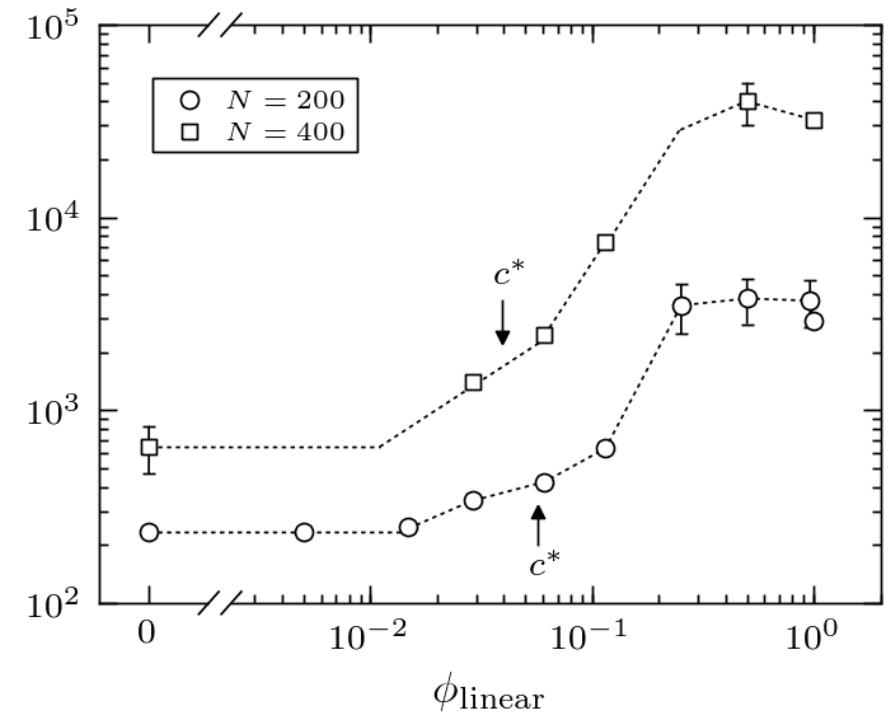
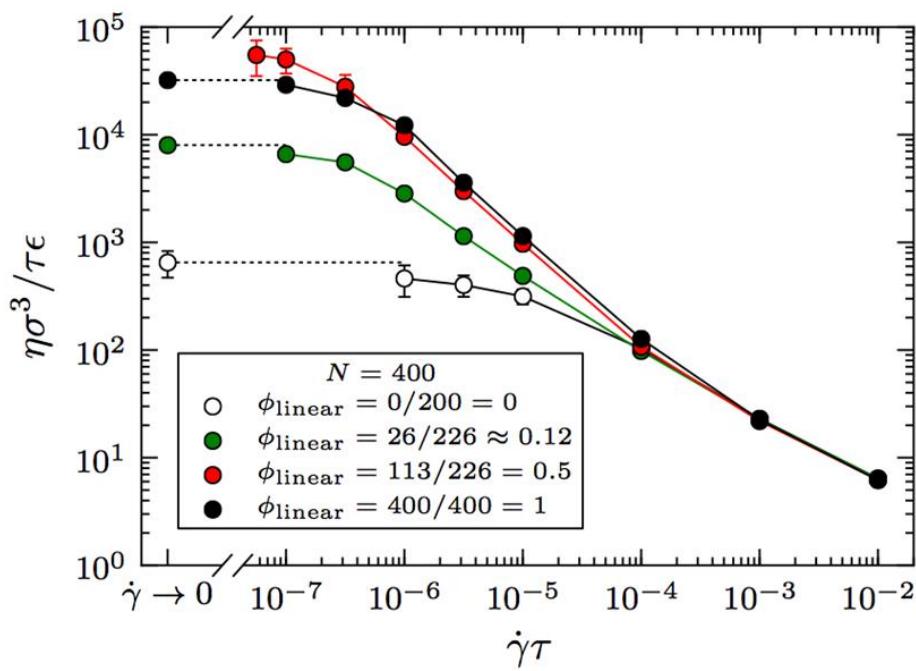


$$\phi_{\text{linear}} = 3/203 \sim 0.015$$



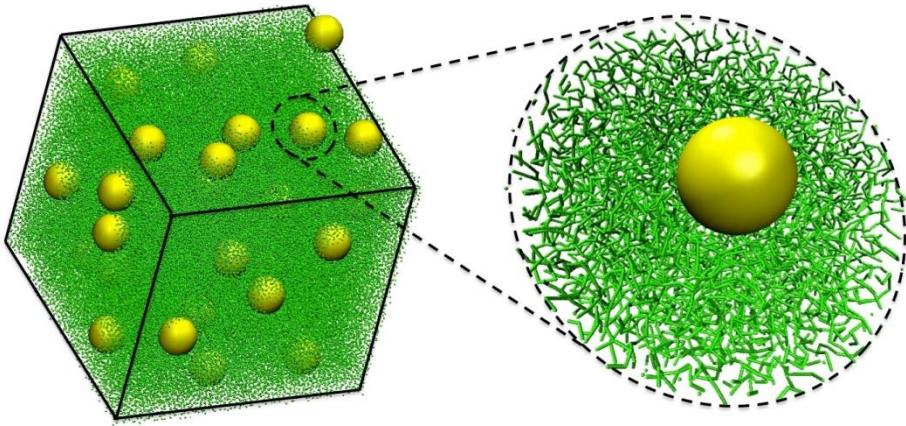
$$\phi_{\text{linear}} = 113/226 = 0.5$$

Viscosity of Ring/linear Blends

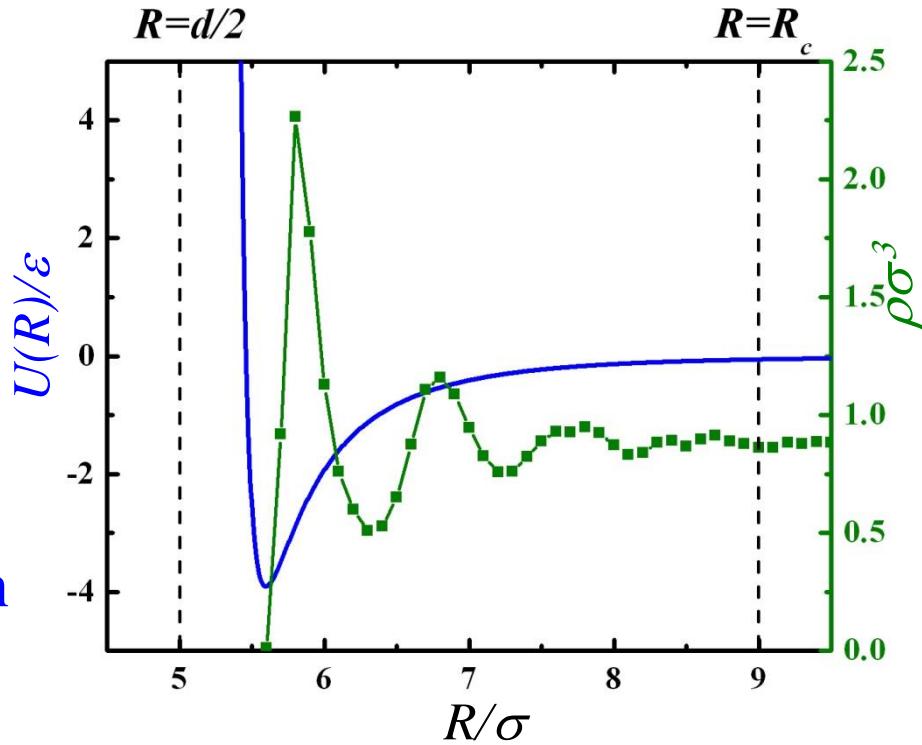


- Onset concentration of $\phi_{\text{linear}} \approx 0.01$
- Peak in viscosity for $\phi_{\text{linear}} \approx 0.5$

Diffusion of Nanoparticles in Polymers

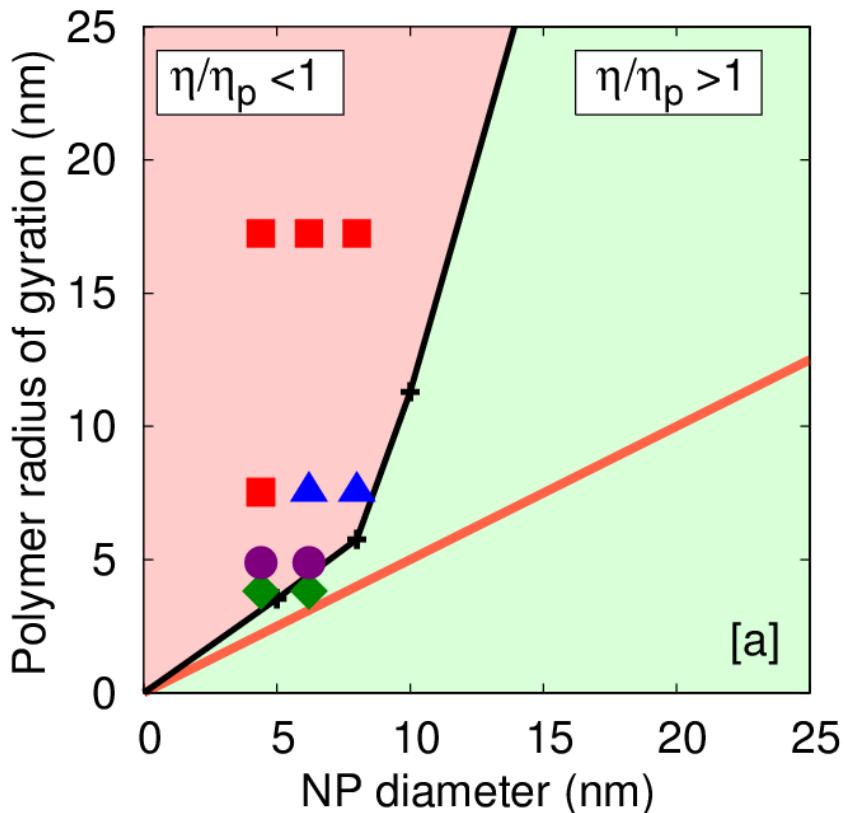


Athermal NP-NP Interaction
Attractive NP-Polymer Interaction
for Miscibility



- Weakly interacting mixtures of nanoparticles (NPs) and ring/linear polymers
- NPs of diameter d are well dispersed at $\phi_{NP} \sim 0.1$

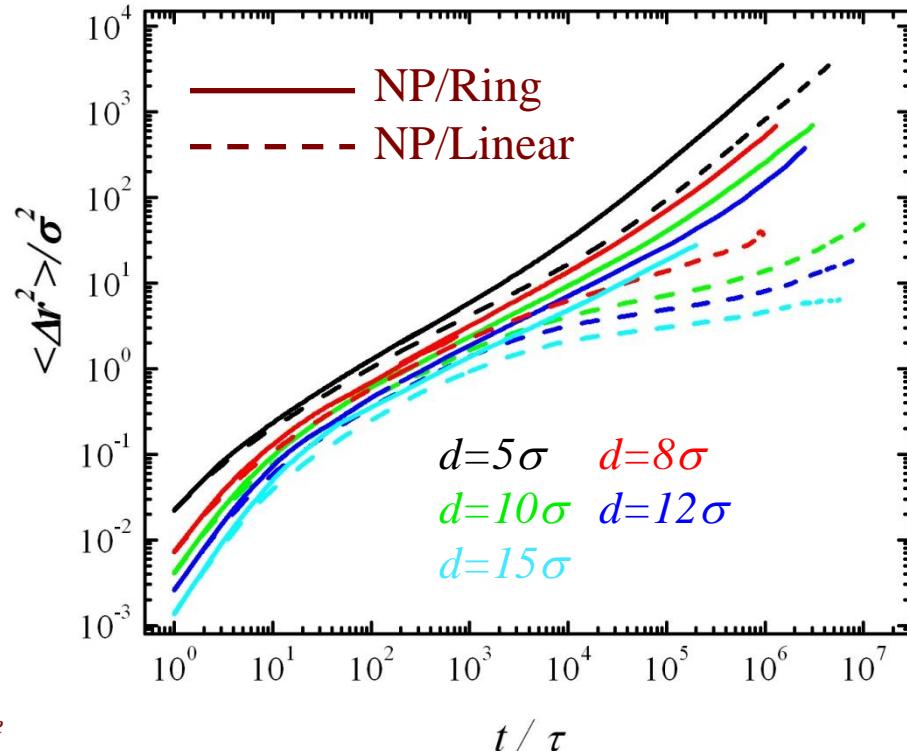
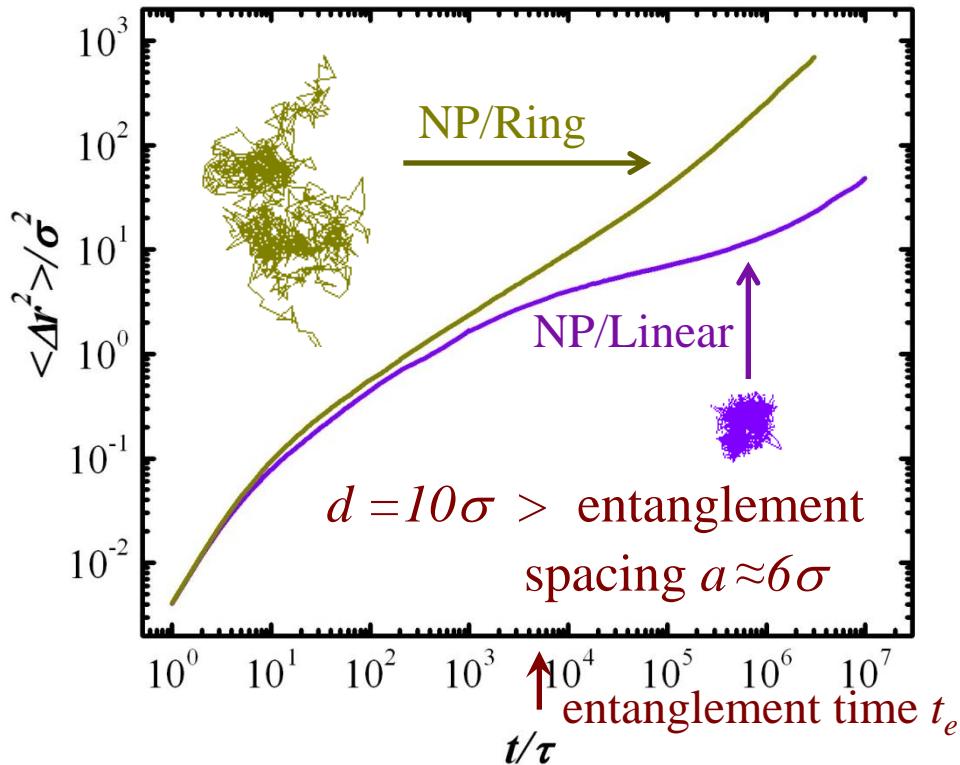
Viscosity of Polymer Nanocomposites



- Small, neutral NPs act akin to plasticizers
 - reduce the viscosity of polymer melt
- Effect persists for particles whose sizes are as large as chain size or entanglement mesh size
 - Overcome by making the chain-NP interactions significantly attractive

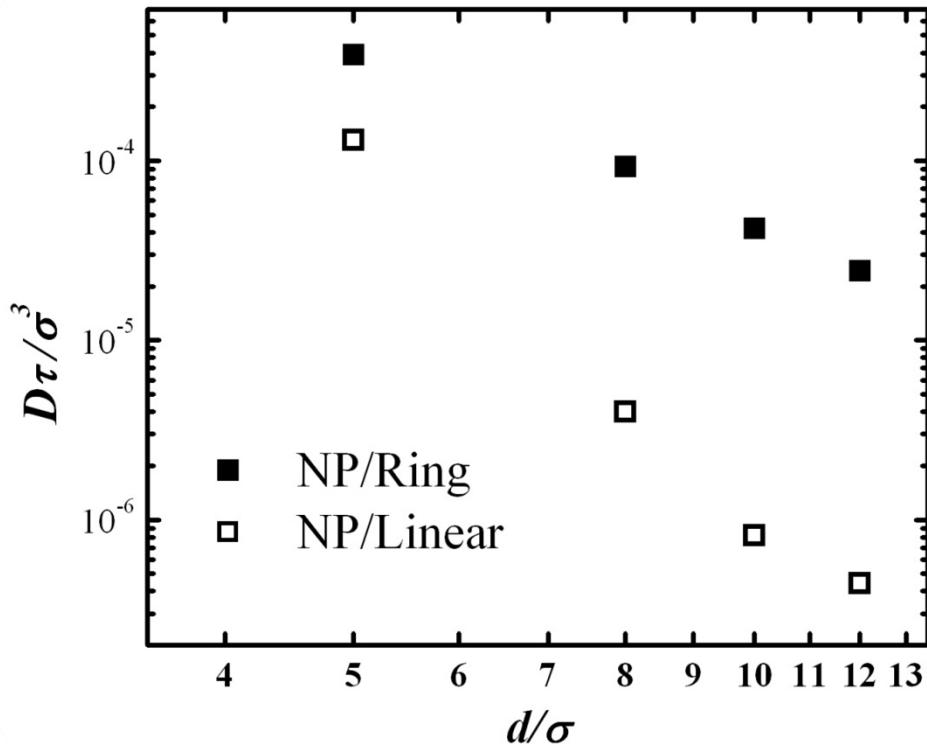
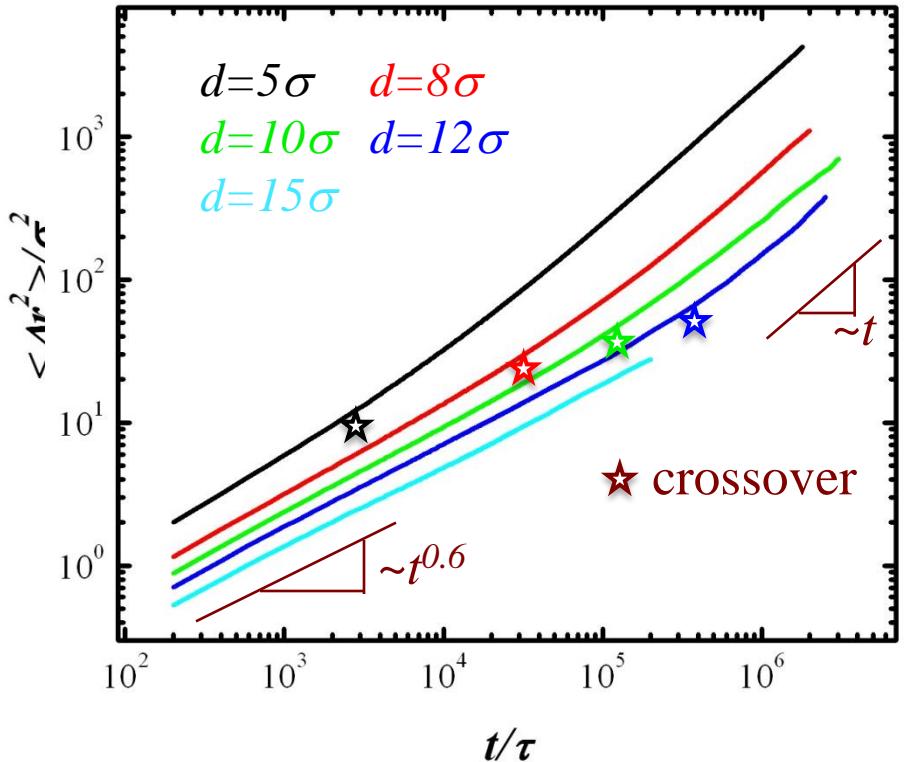
A. Tuteja et al., Macromolecules 38, 8000 (2005)
J. Kalathi, PRL 109, 198301 (2012)

Mean-Squared Displacement of NPs



- $t_0 < t < t_e$, sub-diffusive motion due to coupling with dynamics of the subsections of polymer chains
- $t > t_e$, before Fickian diffusion, motion of NPs with $d > a$
 - Trapped by the entanglement mesh in linear polymers
 - Remains subdiffusive in rings, no entanglement mesh

Fickian Diffusion of Nanoparticles



- Crossover occurs as NP motion couples with coherent motion of chain subsections of size $R_g \sim d$
- Rings: $D \sim d^{-3.2}$, theory predicts $D \sim d^{-4.5}$
- Linear: $D \sim d^{-4.5}$, theory predicts d^{-3}
- Crossover to Stokes-Einstein $d > 20\sigma$

Highlights

- Simulations identified for the first time the reptation motion of polymers
- Followed entanglements of polymer using Primitive Path Analysis to predict macroscopic properties
- Ring polymers move much faster than linear chains
 - Relaxation time $\sim N^2$ for rings
- Threading causes rings to diffuse very slowly in ring/linear mixtures
- Small nanoparticles act as diluent, decrease viscosity

Future Directions

- Outlook for computer modeling is exciting
 - Faster, cheaper computers
 - Efficient parallel MD codes
- Larger Systems, Longer Chains, Longer Times
- Smaller strain, shear rates
 - Viscosity
 - Relaxation after shear
- Constraint Release - Polydispersity
- Semidilute polymers – explicit solvent
- Primitive Path Dynamics – Melts/Networks
- Branched Polymers, Stars,

Acknowledgements

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- K. Schweizer and U. Yamamoto (University of Illinois)

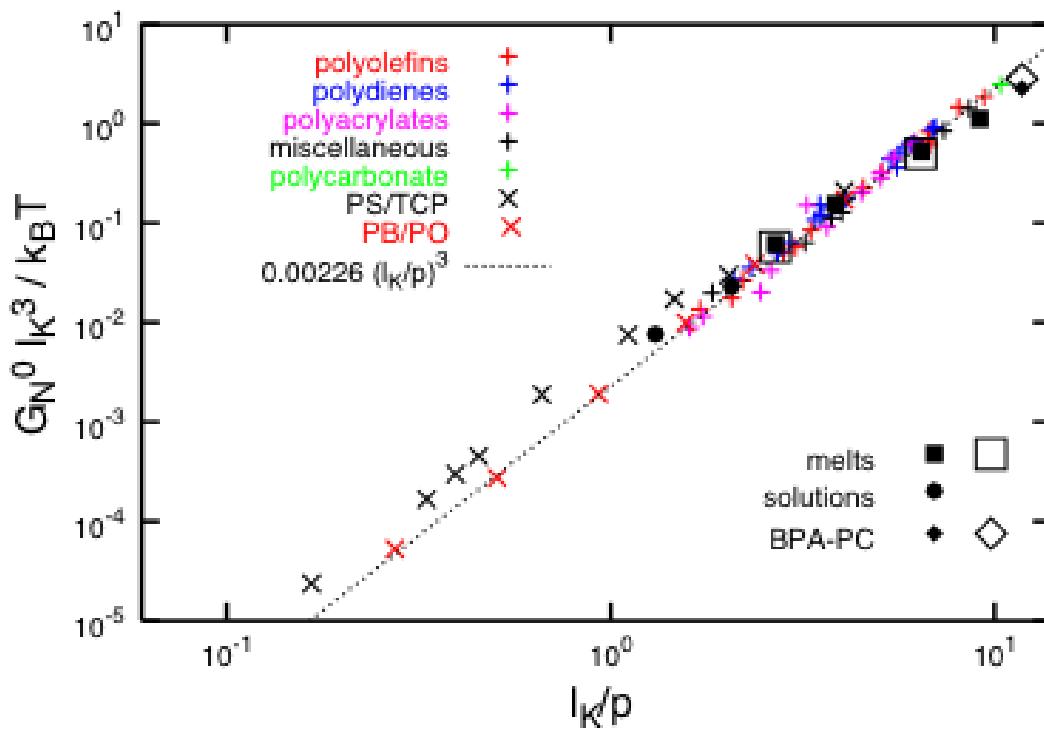
Funding:

- Center for Integrated Nanotechnologies (CINT)
- DOE (BES)

Computer Resources:

- Advanced Scientific Computing Research (ASCR) Leadership Computing Challenge (ALCC) at the National Energy Research Scientific Computing Center (NERSC)
- Sandia National Laboratories

Predicting the Plateau Modulus from PPA



$$G_N^0 = 0.00226 k_B T / p^3$$

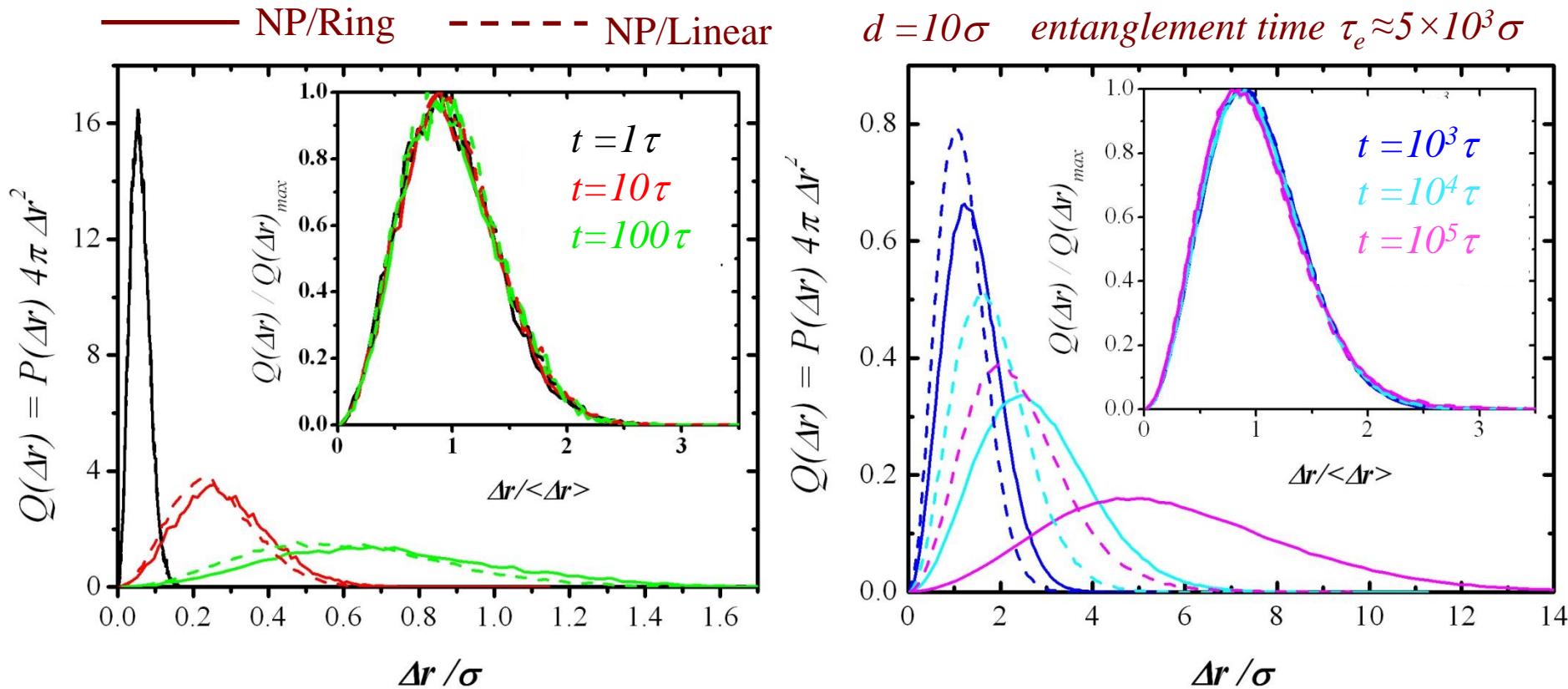
L. Fetters *et al.*, J. Polym. Sci B: Polym. Phys. 37, 1023 (1999)

$$G_N^0 = \frac{4}{5} \frac{k_B T}{a_{pp}^2 p}$$

- Parameter free prediction for plateau modulus

Extended to solutions of semi-flexible polymers –
N. Uchida *et al.*, JCP 128, 044902 (2008)

Probability Distribution of NP Displacement



- Nearly Gaussian distribution of NP displacement
- Entanglement mesh reduces the mean displacement for linear chains but does not change the distribution
- No evidence of NP hopping