

# Nanoparticle Diffusion in a Polymer Matrix

**Gary S. Grest**

**Sandia National Laboratories**

**Center for Integrated Nanotechnologies**

**Albuquerque, NM**



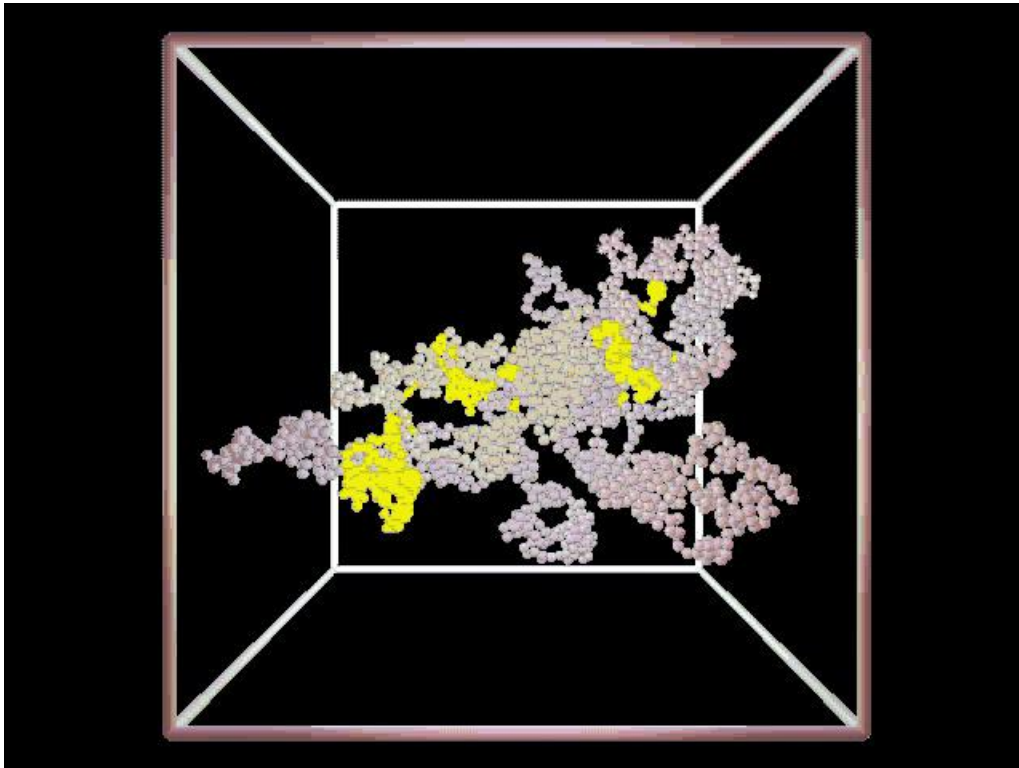
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# Outline

- Unique Properties of Entangled Polymers
- Computational Challenges
- Dynamics of Linear Polymers
- Dynamics of Ring Polymers
- Polymer Nanocomposites
- Future Directions

# Why are Polymers Interesting?

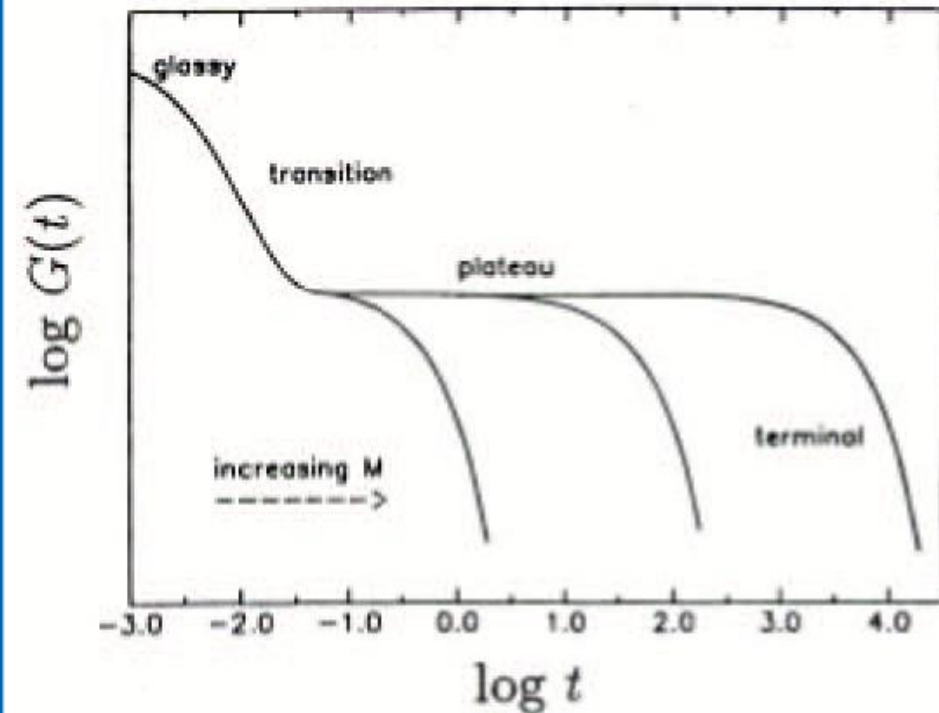
- Polymers can simultaneously be hard and soft
  - Unique Viscoelastic Behavior



- Motion of a polymer chain is subject to complicated topological constraints

# Entangled Polymer Liquids

## Viscoelastic Response



Stress Relaxation  
after strain

## • Macroscopic

- Intermediate frequency, time polymer melt acts as a solid
- Long time, low frequency polymer acts as a liquid

## Microscopic

- Gaussian coils,  $R \sim N^{1/2}$
- Stress is due to entropy loss of stretched chains
- Polymers as "entropic springs"
- Stress relaxation due to Brownian motion of topologically constrained chains

# Computational Challenges

- Longest relaxation time  $\tau \sim N^3$
- Chains are Gaussian coils –  $R \sim N^{1/2}$ 
  - Number of chains must increase as  $R^3 \sim N^{3/2}$  so polymer chains do not to see themselves through periodic boundary conditions
- Double chain length – cpu required increases by at least a factor of  $2^{4.5} \sim 23$ 
  - 1-2 month simulation becomes 2-4 years
- Number of processors limited: ~500-1000 particles/processor

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  - 1-2 month simulation becomes 2-4 years
- Number of processors limited:  $\sim 1000$  particles/processor
- Software/hardware advances have been significant

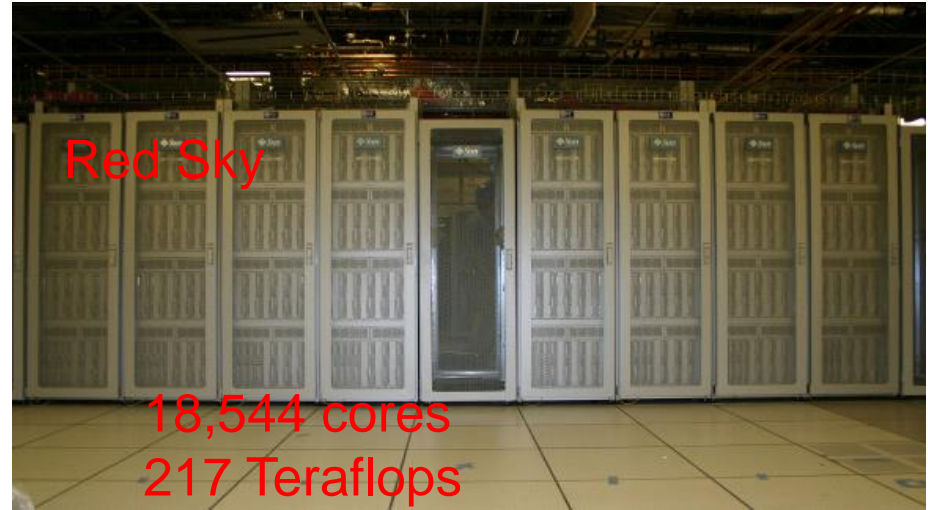
# Toys for the Simulator

4 cores  
800 Megaflops



Cray ~ 1985

Red Sky



18,544 cores  
217 Teraflops

Edison- NERSC  
Cray XC30



124,608 cores  
2.4 Petaflops

Titan - ORNL



299,00 Cores plus  
18,688 Nvidia Tesla  
K20X GPUs  
17.6 Petaflops

# Polymer Diffusion

- Simple Liquids

- $D \sim M^{-1}$  ,  $\eta \sim M$

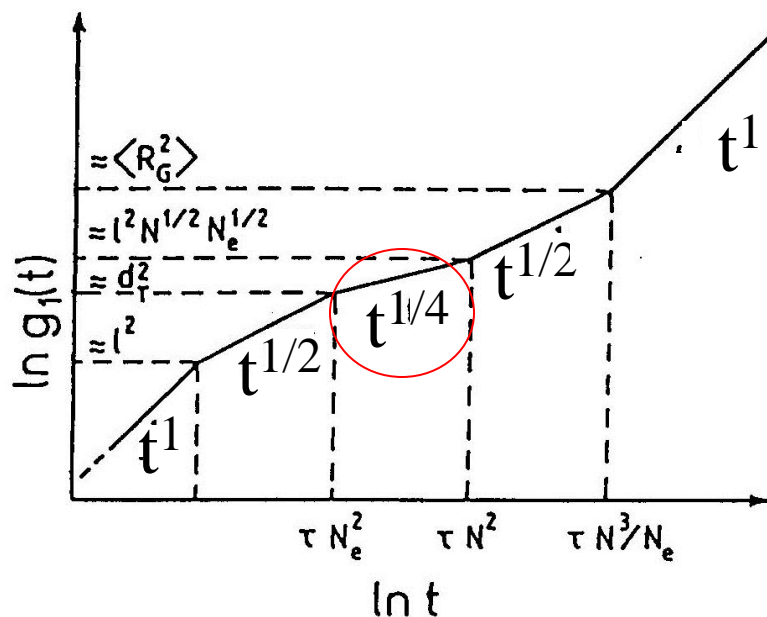
- Short Polymer Chains ( $M < M_e$ )

- Longest relaxation time  $\tau_R \sim M^2$

- Intermediate  $t^{1/2}$  time regime in mean square displacement

- $D \sim M^{-1}$  ,  $\eta \sim M$

- Long Polymer Chains ( $M > M_e$ ) - Reptation



$$D \sim M^{-2}$$

$$\eta \sim M^3$$

$$\tau_d \sim M^3$$

Characteristic signature of reptation – intermediate  $t^{1/4}$  regime



# Bead-Spring Model

- Short range - excluded volume interaction

$$U_{\text{LJ}}(r) = \begin{cases} 4\epsilon \left\{ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 + \frac{1}{4} \right\} & r \leq r_c \\ 0 & r \geq r_c \end{cases}$$

- Bonded interaction - FENE spring

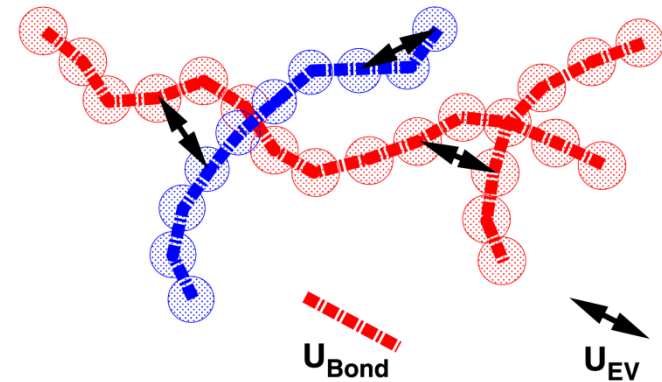
$$U_{\text{FENE}}(r) = \begin{cases} -0.5kR_0^2 \ln(1 - (r/R_0)^2) & r \leq R_0 \\ \infty & r > R_0 \end{cases}$$

$$k=30\epsilon/\sigma^2, R_0=1.5\sigma$$

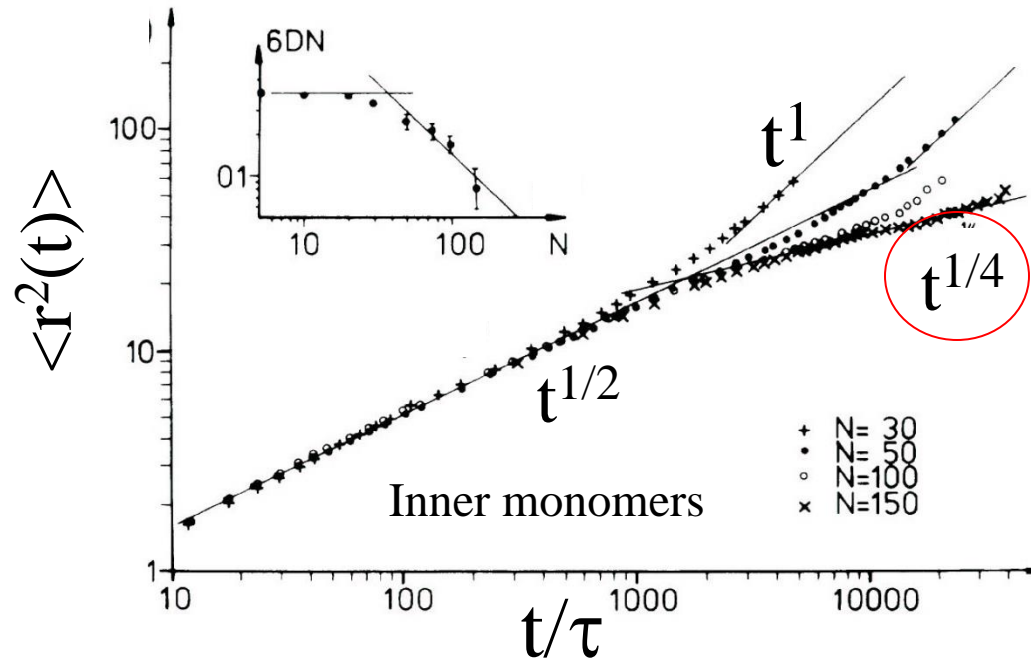
- Energy barrier prohibits chains from cutting through each other – topology conserved

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = -\vec{\nabla} \cdot U_i - m_i \Gamma \frac{d\vec{r}_i}{dt} + \vec{W}_i(t)$$

$$\text{Time step } \Delta t \sim 0.01\tau, \tau = \sigma(m/\epsilon)^{1/2}$$



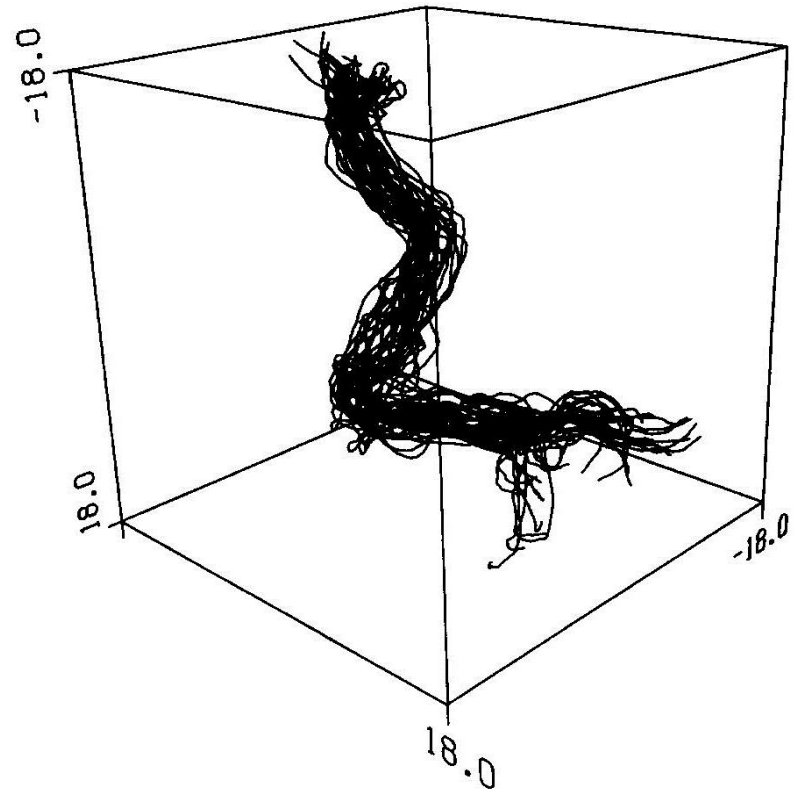
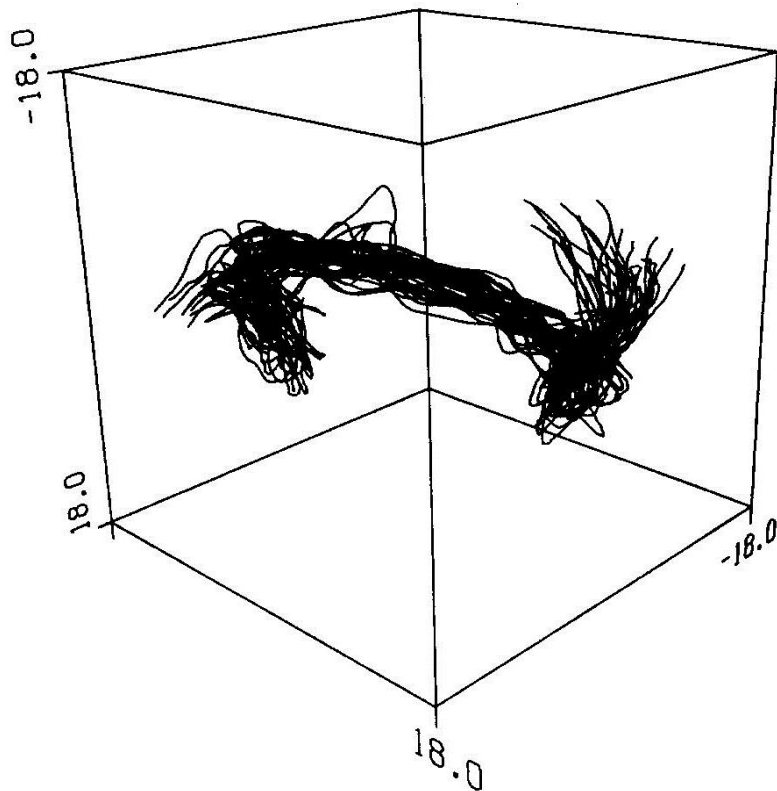
# Polymers do Reptate!



- $t^{1/4}$  reptation regime for  $N > 100$
- First direct evidence from simulation or experiment

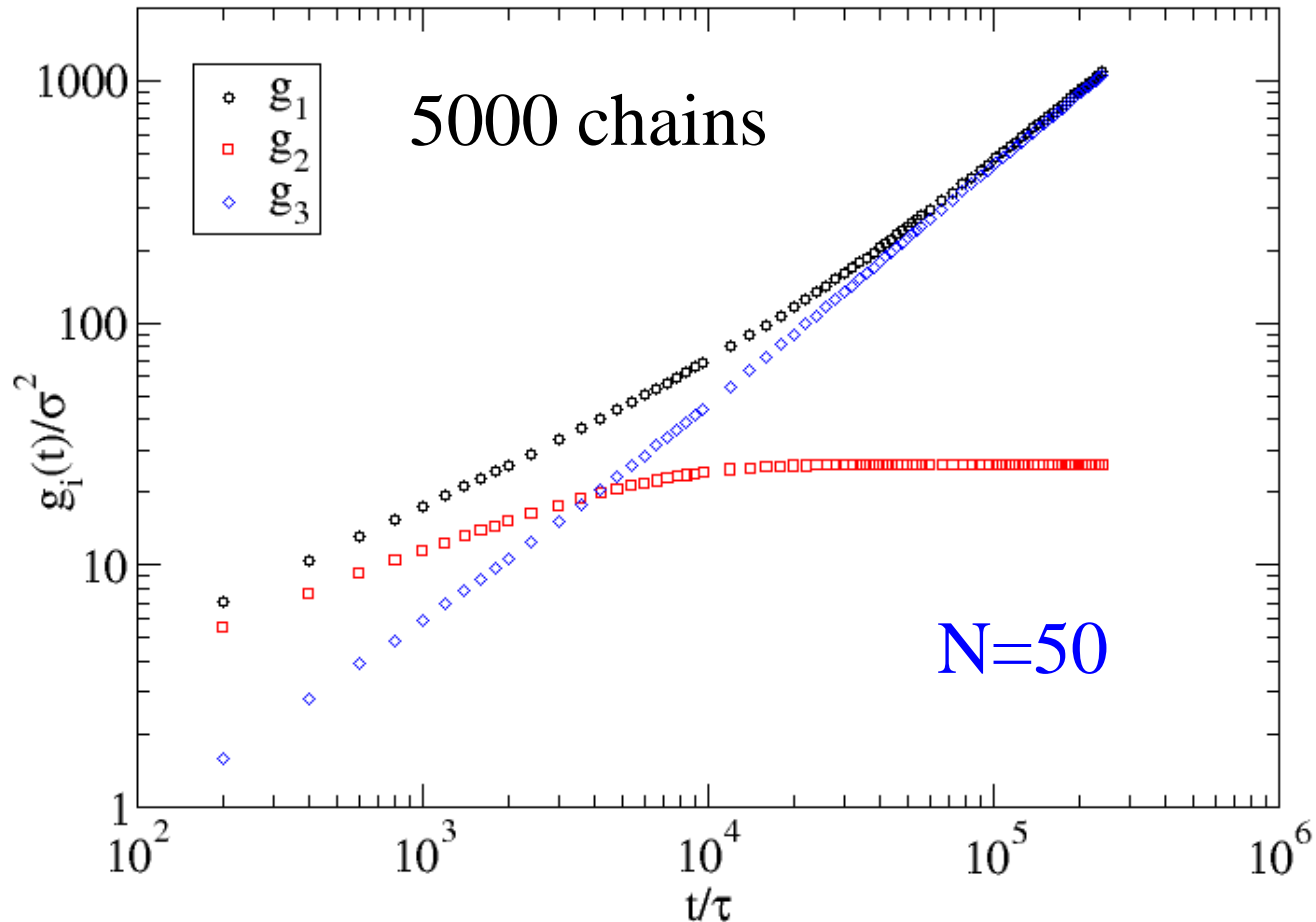
# Polymer Chain Confined to Tube

- Coarse grained chain



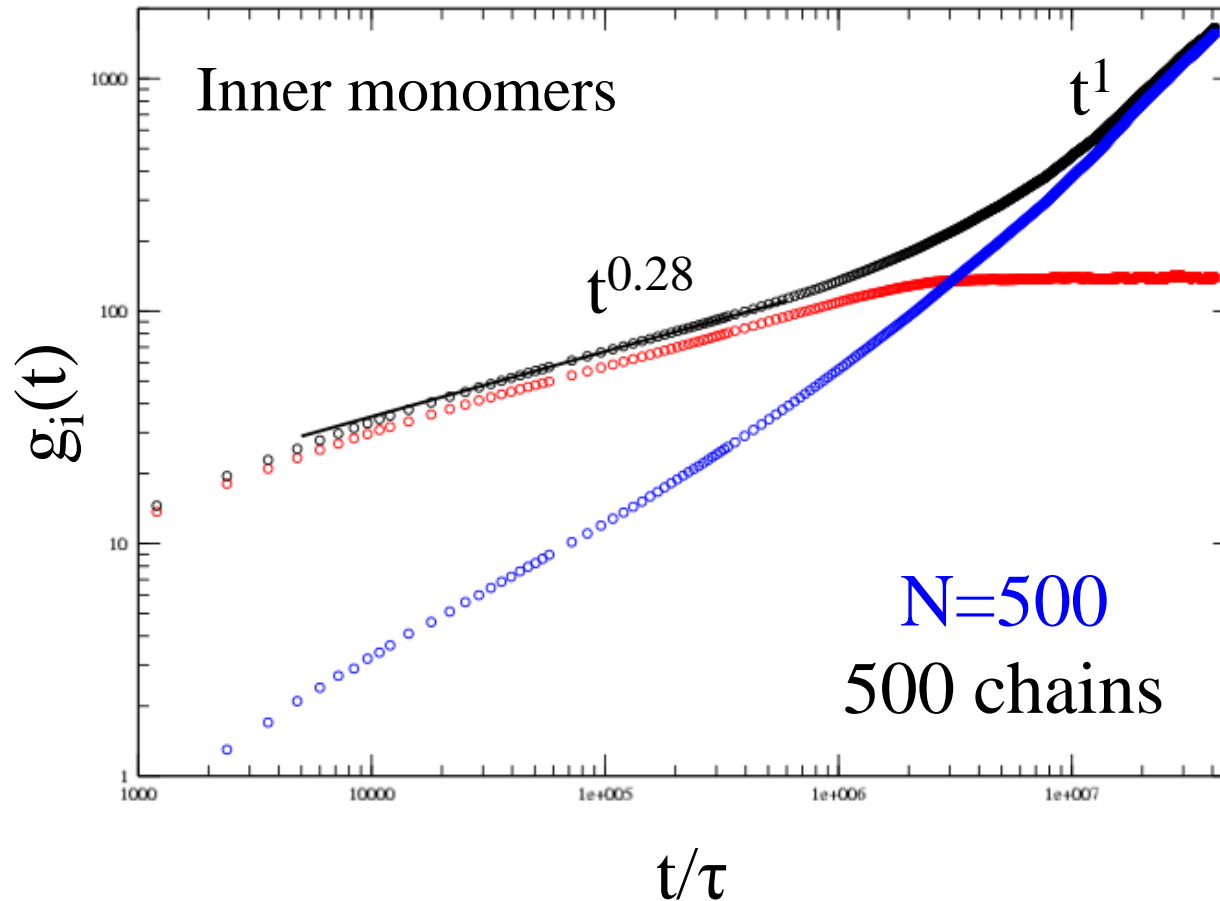
$N=400$  – 20 plots,  $600 \tau$  apart

# Motion of Unentangled Polymer



- Once polymer move their own size, unentangled polymers move like normal liquids

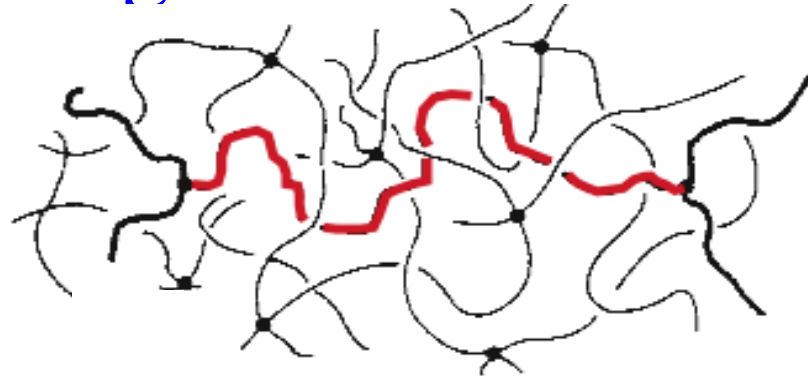
# Motion of Entangled Polymer



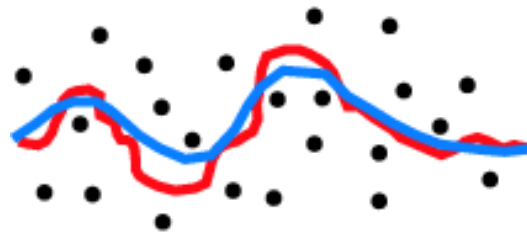
- $t^{1/4}$  motion is clearly seen for inner monomers
- Second  $t^{1/2}$  region still unresolved

# Topological Approach to Identify Entanglements

- Microscopic conformation



- Shortest path into which a chain can contract with fixed endpoints and without crossing obstacles



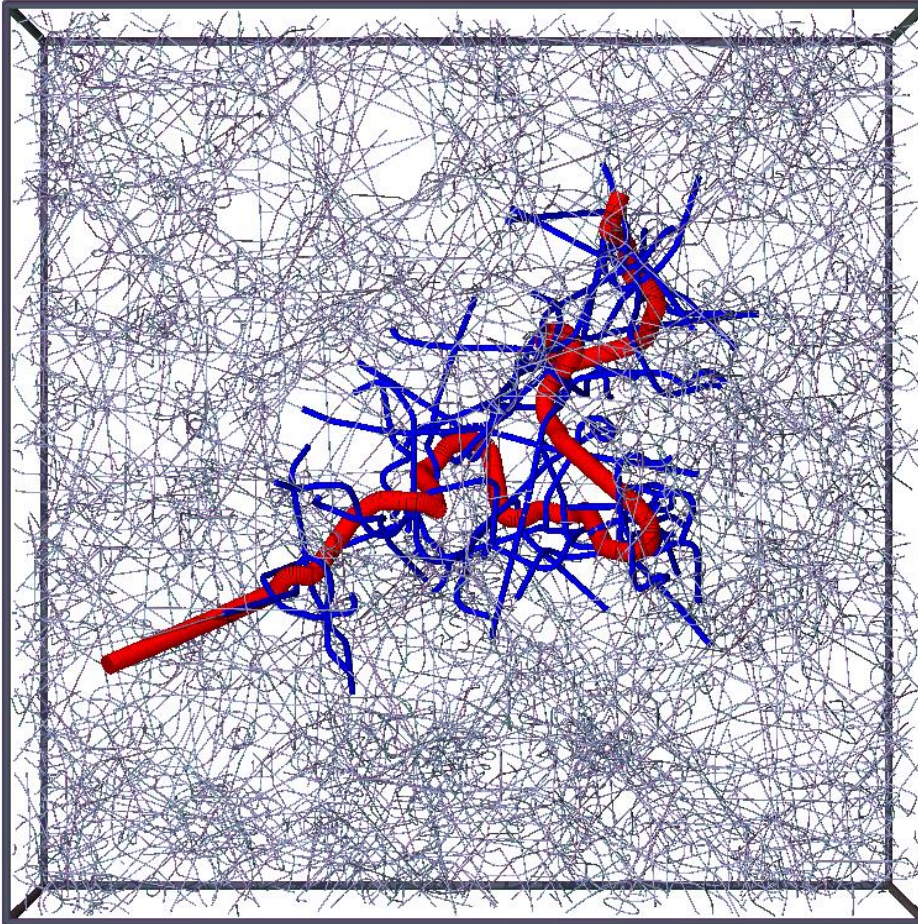
- Tube axis = primitive path



- Need a topological analysis which can follow motion of chain



# Primitive Path Analysis



$N=700$

- Shorter Contour Length

$$L_{pp} = N b_{pp} < L$$

- Larger Kuhn Length

$$a_{pp} > l_k$$

- Same spatial extent

$$a_{pp} L_{pp} = R^2 = l_k L$$

- Entanglement Length

$$N_e = a_{pp} / b_{pp}$$

- Packing Length

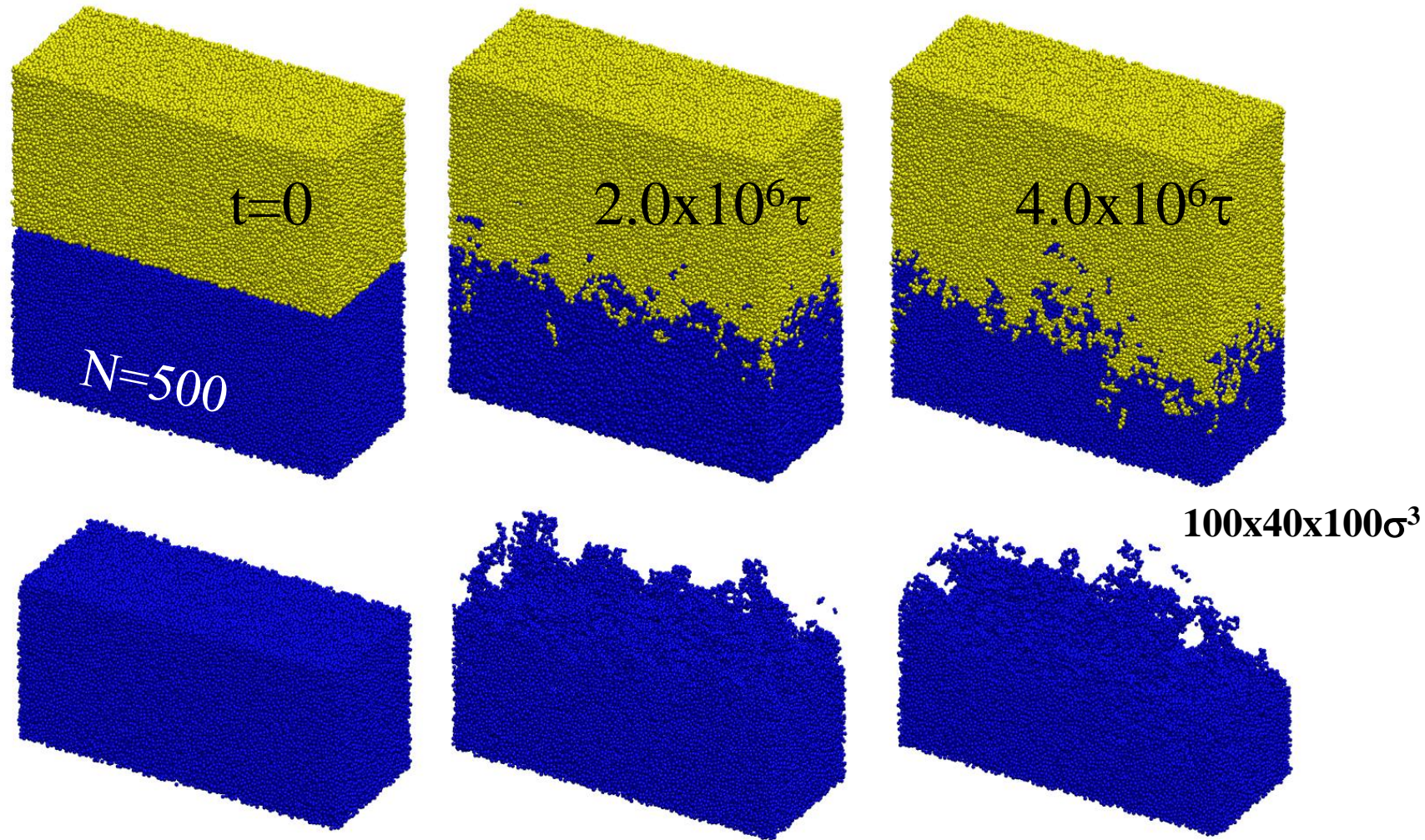
$$p = 1 / \rho_{\text{chain}} R^2$$

- Primitive paths of a cluster of entangled chains



# Self-Healing of Polymer Films

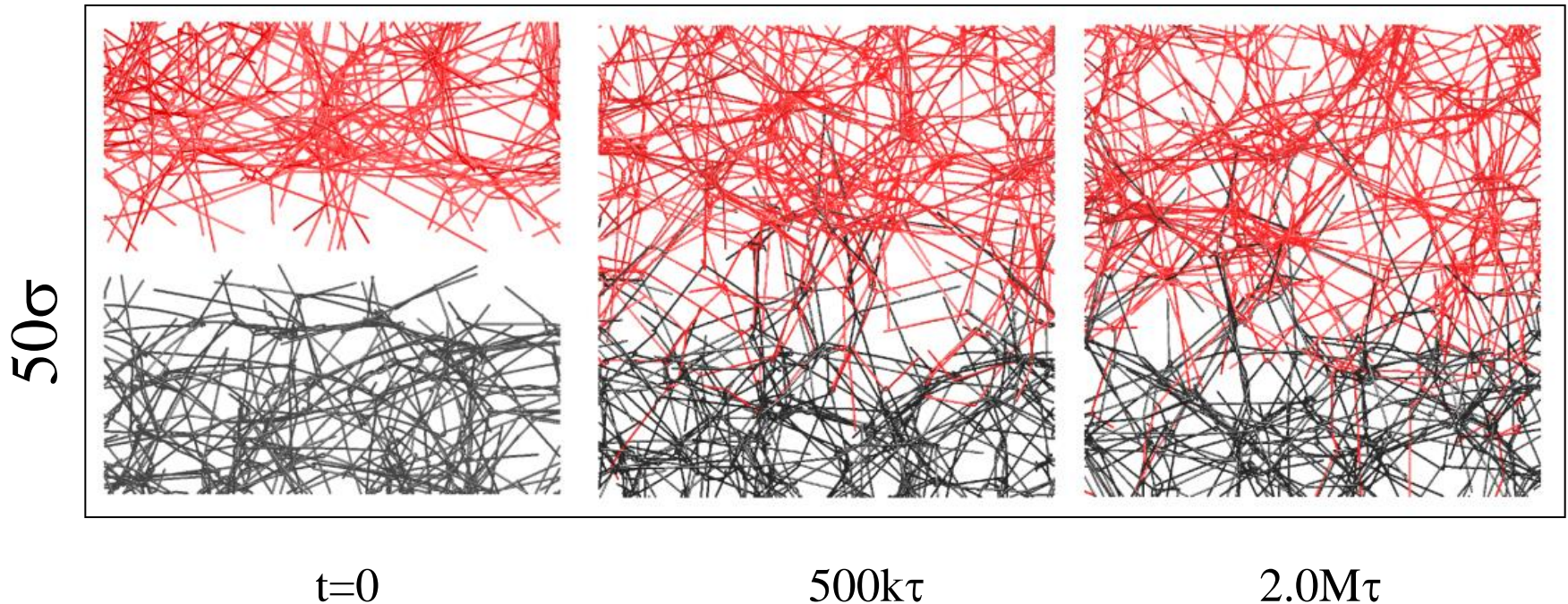
- Development of Entanglements Across an Interface





# Entanglements at Interface

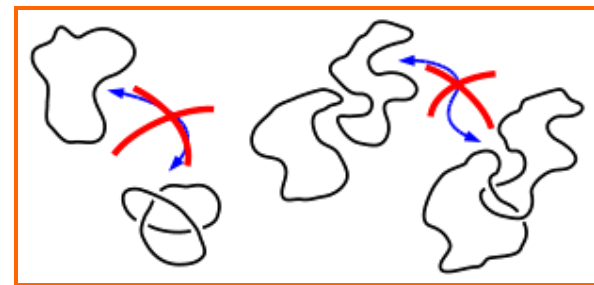
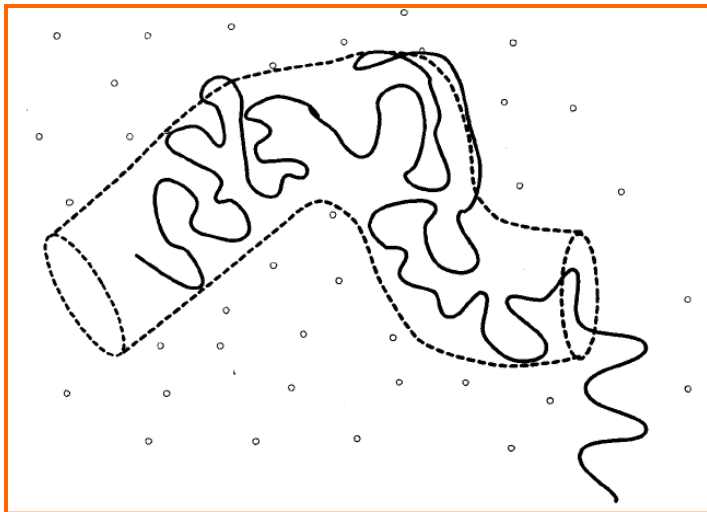
- Primitive Path Analysis



- Interfacial Entanglements form between chains from opposite sides
- Bulk response is fully recovered when the density of entanglements at the interface reaches the bulk value

# Dynamics of Ring Polymers

- As chain size increases, linear polymers entangle and are forced to move ('reptate') along their contours
- Branched polymers relax via a hierarchy of modes from dangling ends moving inward
- Remaining mystery: How do ring polymers relax without beginning or end?



# Configurations of Ring Polymers

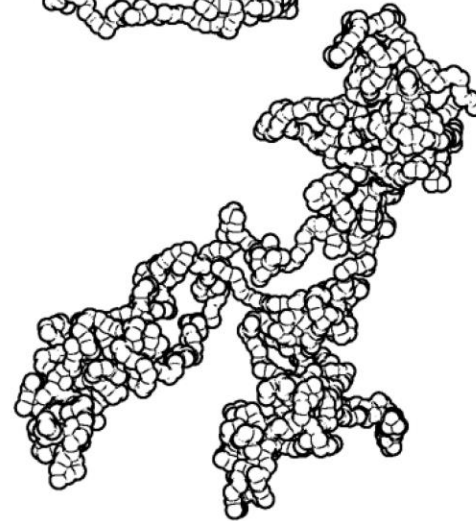
(a)  $N = 100$



(b)  $N = 400$

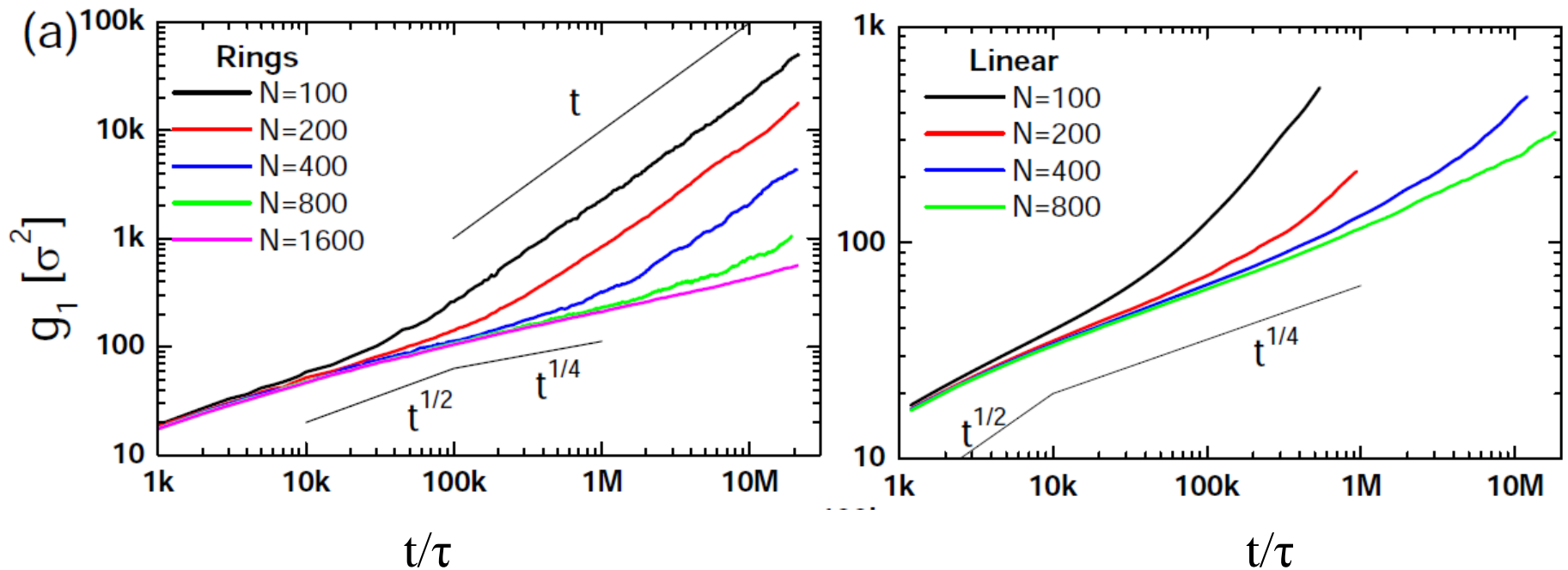


(c)  $N = 1600$



- Rings more compact, less entangled than linear chains
- Radius of Gyration  $R_g^2 \sim N^{2/3}$  for rings  
 $\sim N$  for linear

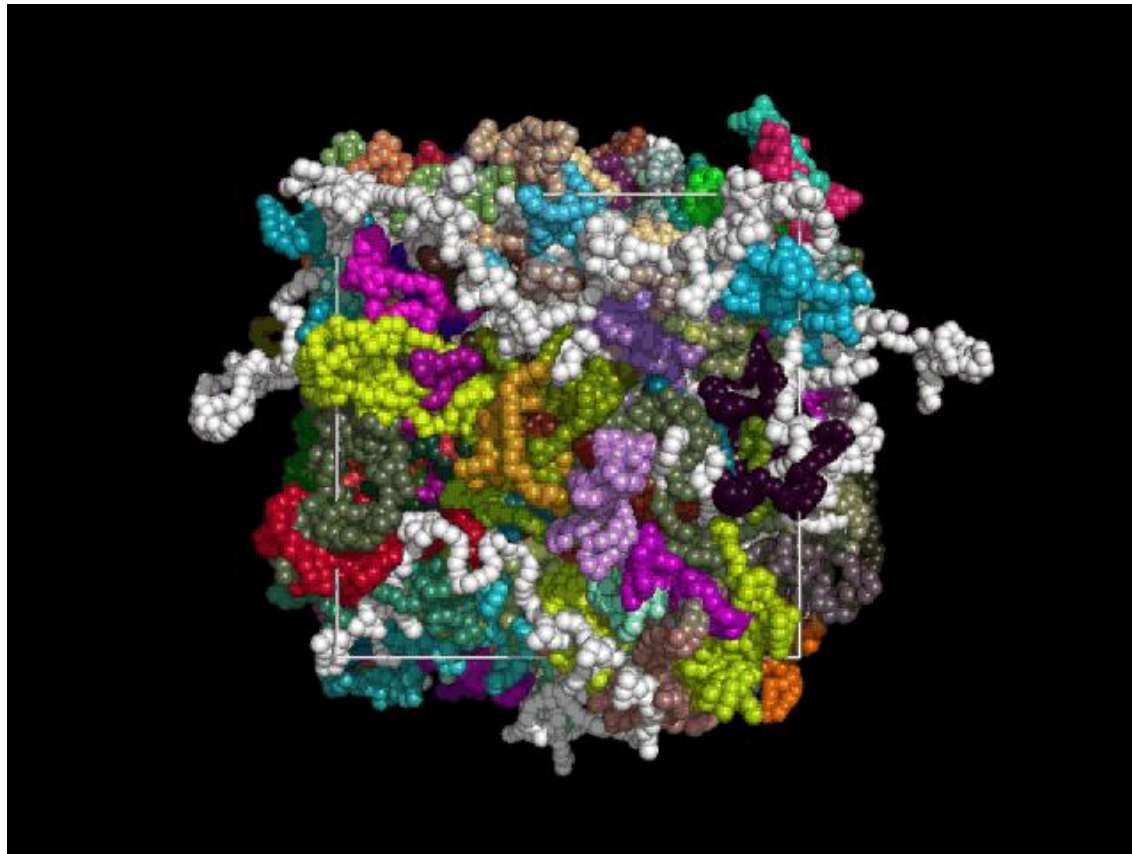
# Dynamics of Ring Polymers



- Ring polymers move much faster than linear chains
- Longest relaxation time  $\tau \sim N^2$  for rings,  $N^3$  for linear chains



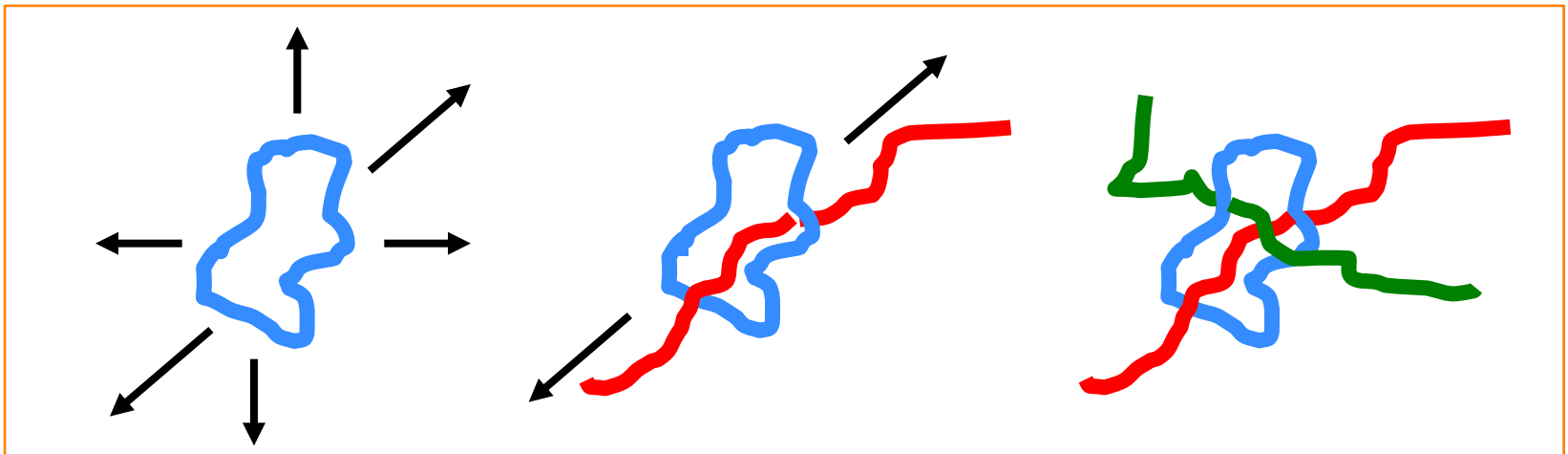
# Mixture of Ring and Linear Polymers



$N = 200$ ,  $M_{\text{rings}} = 200$ ,  $M_{\text{linear}} = 26$   $t = 0 \tau$

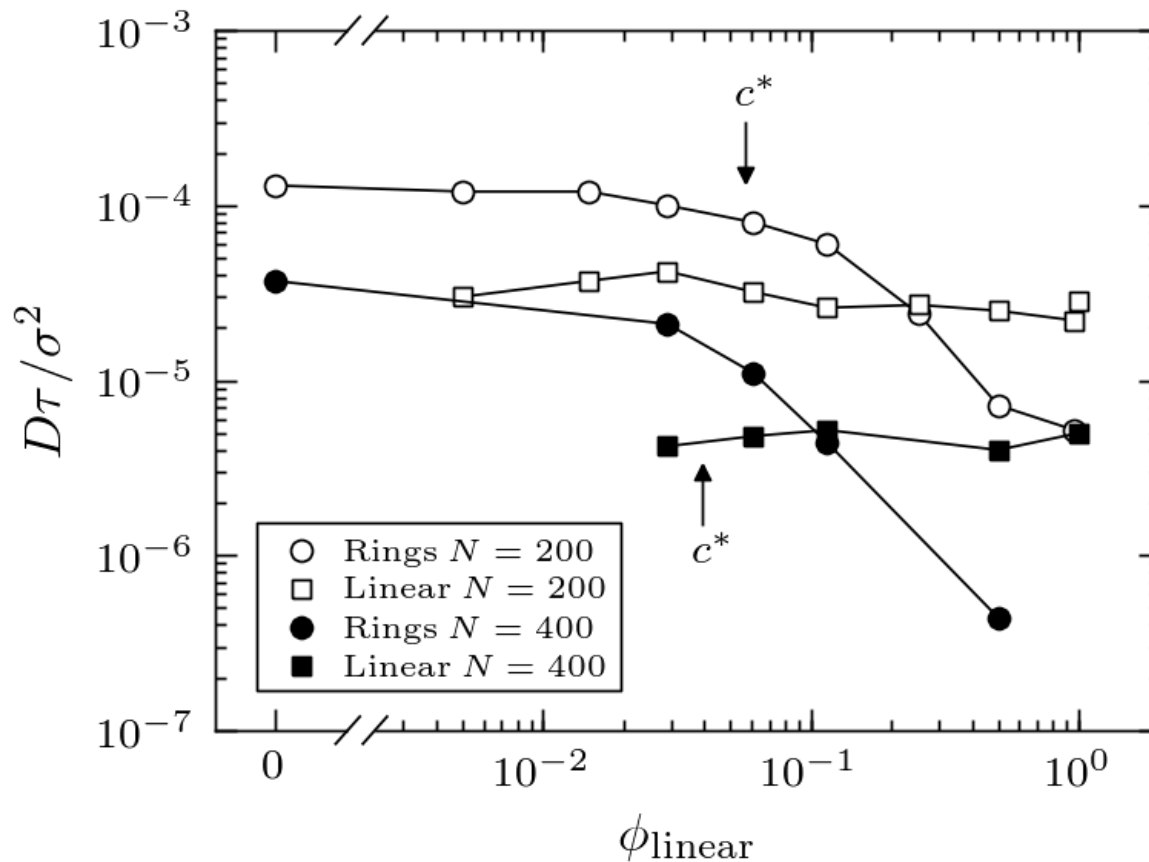
# Constrained Motion of Rings in Linear Melt

- Threading causes rings to diffuse more slowly



- Threaded rings can only diffuse along the contour of the linear chain
- Time for constraint release scales as  $N^{3.4}$

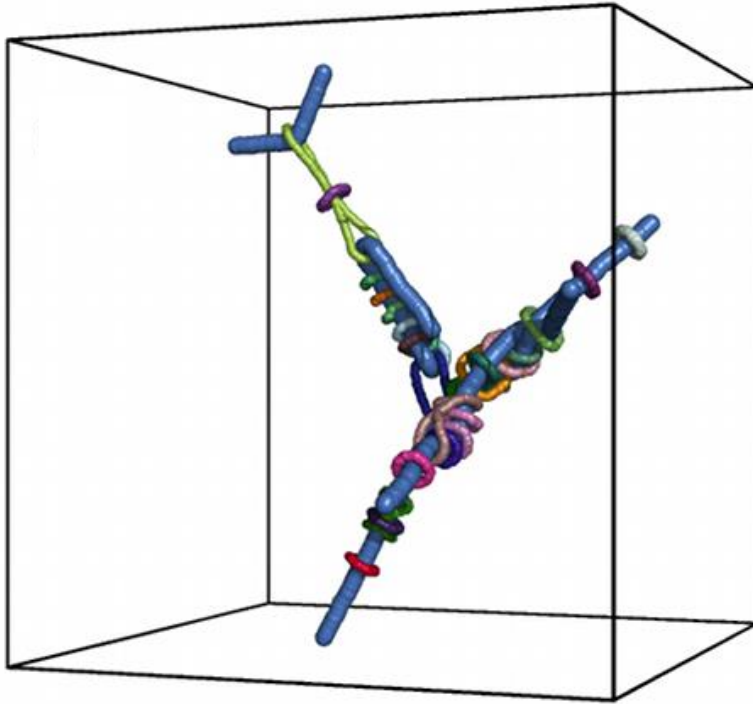
# Dynamics of Ring/Linear Blends



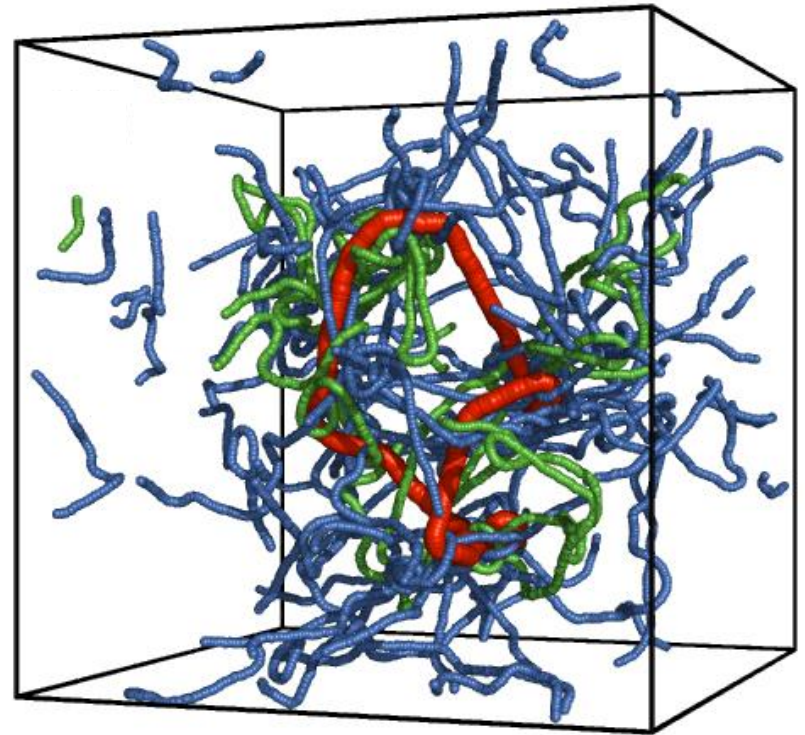
- $D_{\text{ring}}$  decreases dramatically when threaded by multiple linear chains ( $\phi_{\text{linear}} > 0.1$ )
- $D_{\text{linear}}$  is approximately independent of  $\phi_{\text{linear}}$

# Primitive Path Analysis

N=400



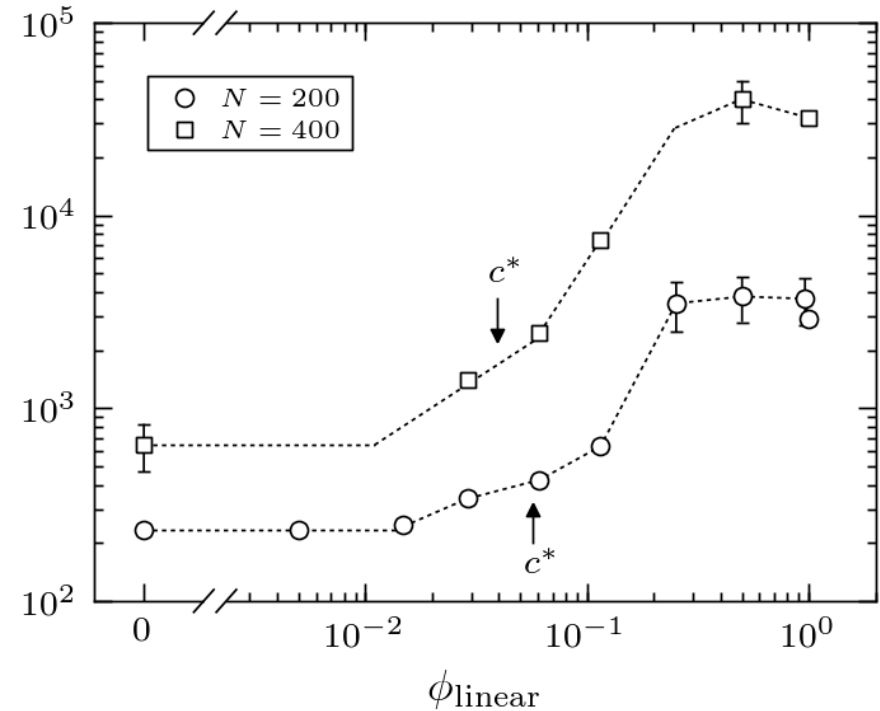
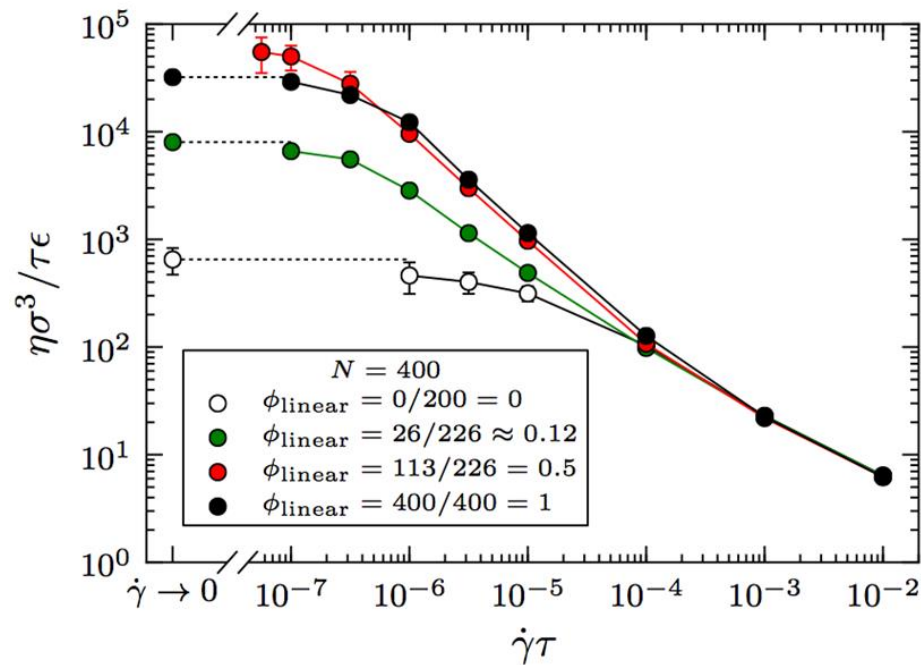
$$\phi_{\text{linear}} = 3/203 \sim 0.015$$



$$\phi_{\text{linear}} = 113/226 = 0.5$$

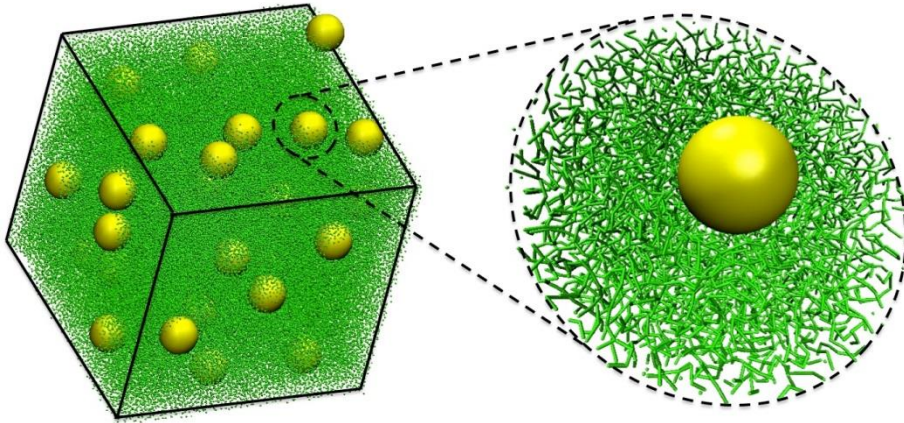


# Viscosity of Ring/linear Blends

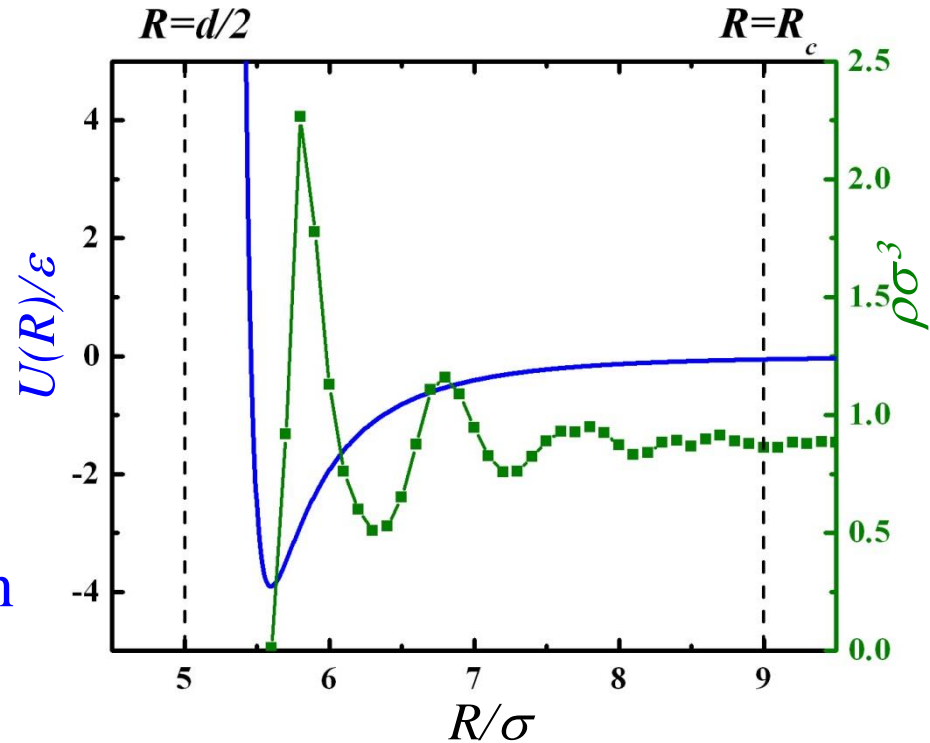


- Onset concentration of  $\phi_{\text{linear}} \approx 0.01$
- Peak in viscosity for  $\phi_{\text{linear}} \approx 0.5$

# Diffusion of Nanoparticles in Polymers

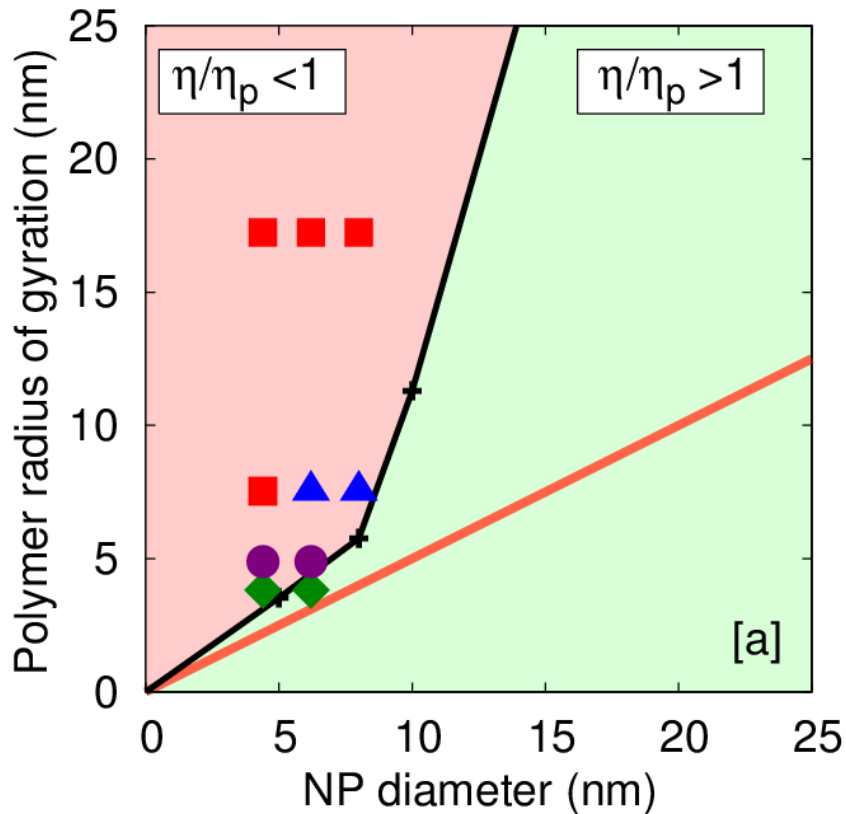


Athermal NP-NP Interaction  
Attractive NP-Polymer Interaction  
for Miscibility



- Weakly interacting mixtures of nanoparticles (NPs) and ring/linear polymers
- NPs of diameter  $d$  are well dispersed at  $\phi_{\text{NP}} \sim 0.1$

# Viscosity of Polymer Nanocomposites

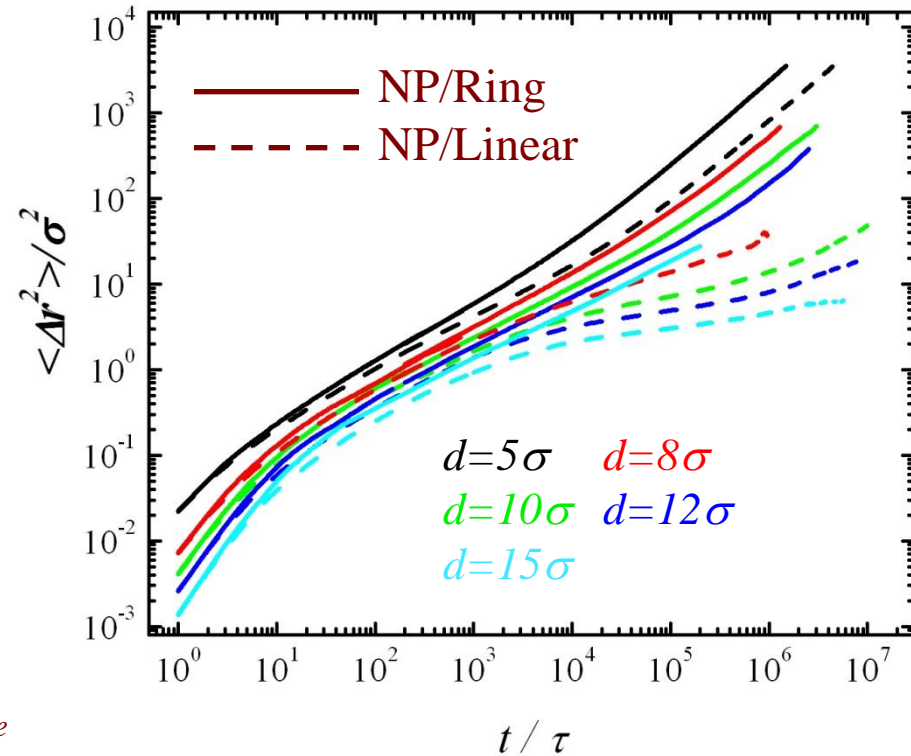
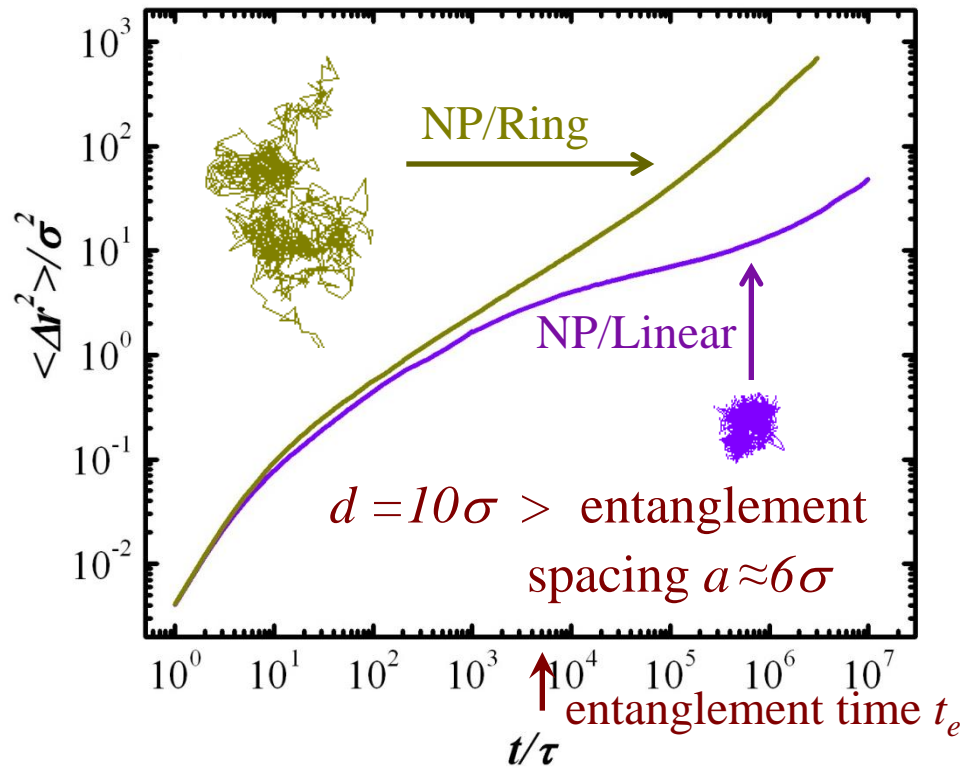


- Small, neutral NPs act akin to plasticizers
  - reduce the viscosity of polymer melt
- Effect persists for particles whose sizes are as large as chain size or entanglement mesh size
- Overcome by making the chain-NP interactions significantly attractive

A. Tuteja et al., Macromolecules 38, 8000 (2005)

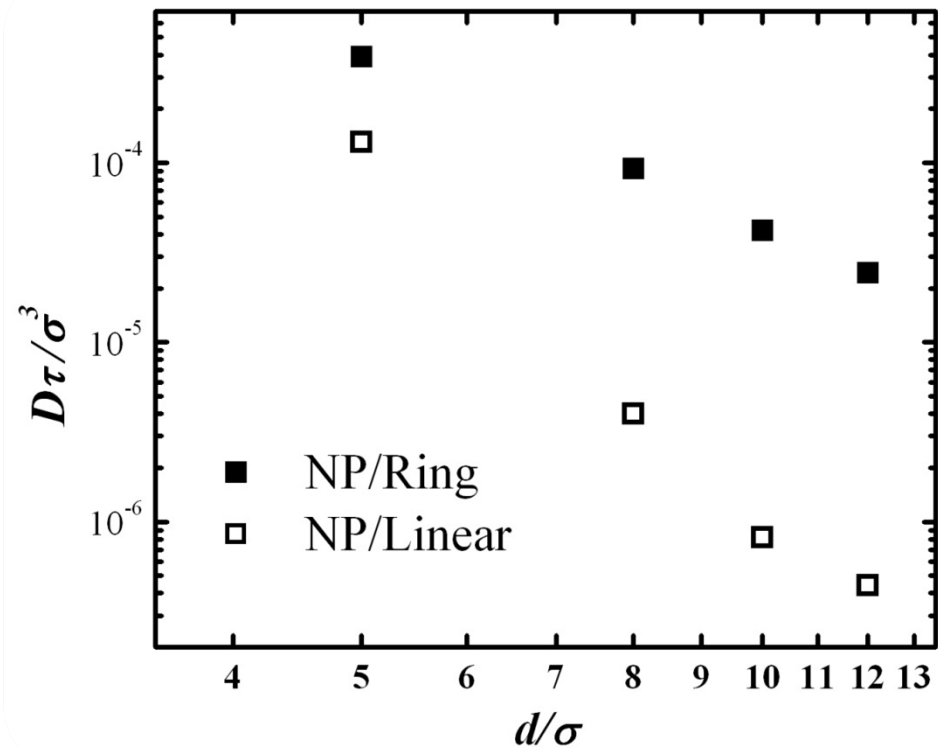
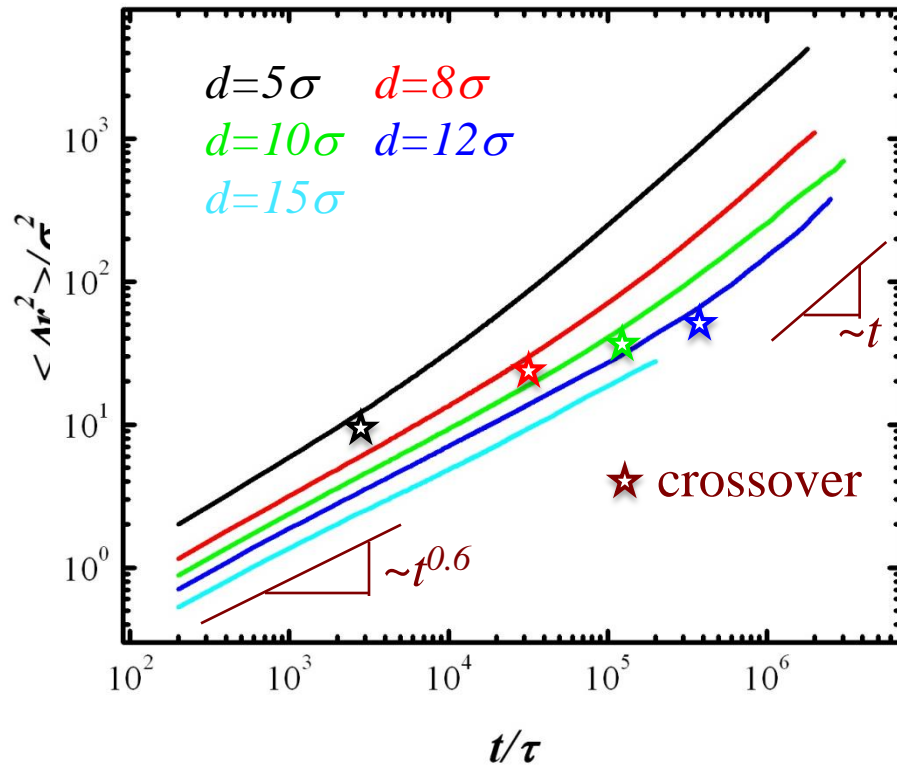
J. Kalathi, PRL 109, 198301 (2012)

# Mean-Squared Displacement of NPs



- $t_0 < t < t_e$ , sub-diffusive motion due to coupling with dynamics of the subsections of polymer chains
- $t > t_e$ , before Fickian diffusion, motion of NPs with  $d > a$ 
  - Trapped by the entanglement mesh in linear polymers
  - Remains subdiffusive in rings, no entanglement mesh

# Fickian Diffusion of Nanoparticles



- Crossover occurs as NP motion couples with coherent motion of chain subsections of size  $R_g \sim d$
- Rings:  $D \sim d^{-3.2}$ , theory predicts  $D \sim d^{-4.5}$
- Linear:  $D \sim d^{-4.5}$ , theory predicts  $d^{-3}$
- Crossover to Stokes-Einstein  $d > 20\sigma$

# Highlights

- Simulations identified for the first time the reptation motion of polymers
- Followed entanglements of polymer using Primitive Path Analysis to predict macroscopic properties
- Ring polymers move much faster than linear chains
  - Relaxation time  $\sim N^2$  for rings
- Threading causes rings to diffuse very slowly in ring/linear mixtures
- Small nanoparticles act as diluent, decrease viscosity

# Future Directions

- Outlook for computer modeling is exciting
  - Faster, cheaper computers
  - Efficient parallel MD codes
- Larger Systems, Longer Chains, Longer Times
- Smaller strain, shear rates
  - Viscosity
  - Relaxation after shear
- Constraint Release - Polydispersity
- Semidilute polymers – explicit solvent
- Primitive Path Dynamics – Melts/Networks
- Branched Polymers, Stars, ....

# Acknowledgements

## Collaborations:

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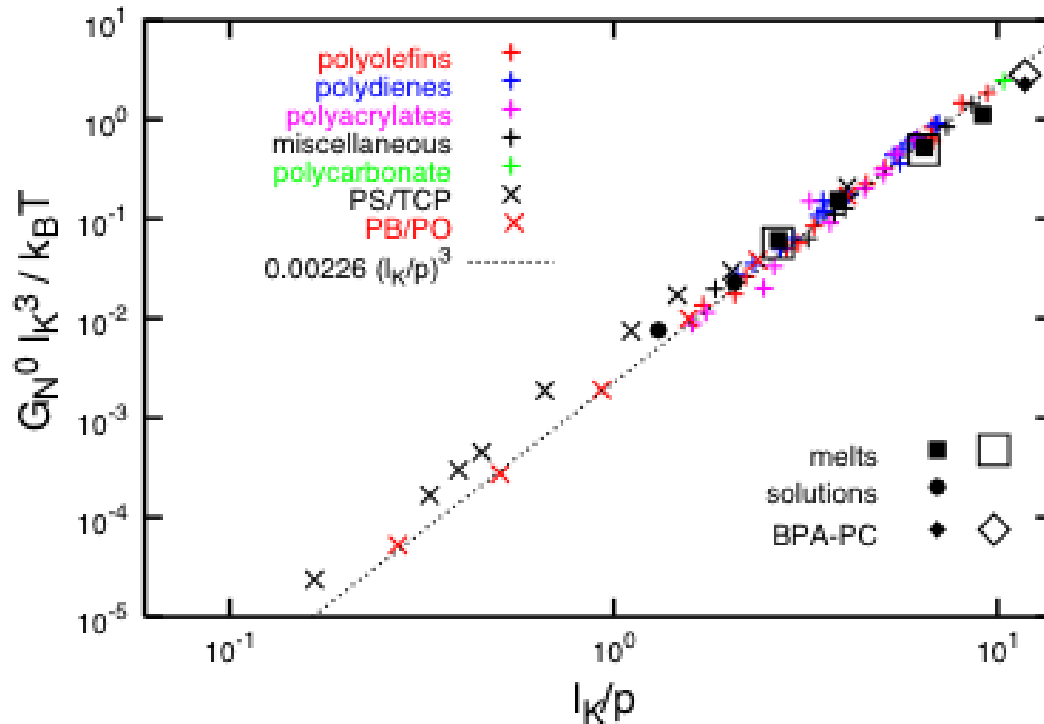
- Center for Integrated Nanotechnologies (CINT)
- DOE (BES)

## Computer Resources:

- Advanced Scientific Computing Research (ASCR) Leadership Computing Challenge (ALCC) at the National Energy Research Scientific Computing Center (NERSC)
- Sandia National Laboratories



# Predicting the Plateau Modulus from PPA



$$G_N^0 = 0.00226 k_B T / p^3$$

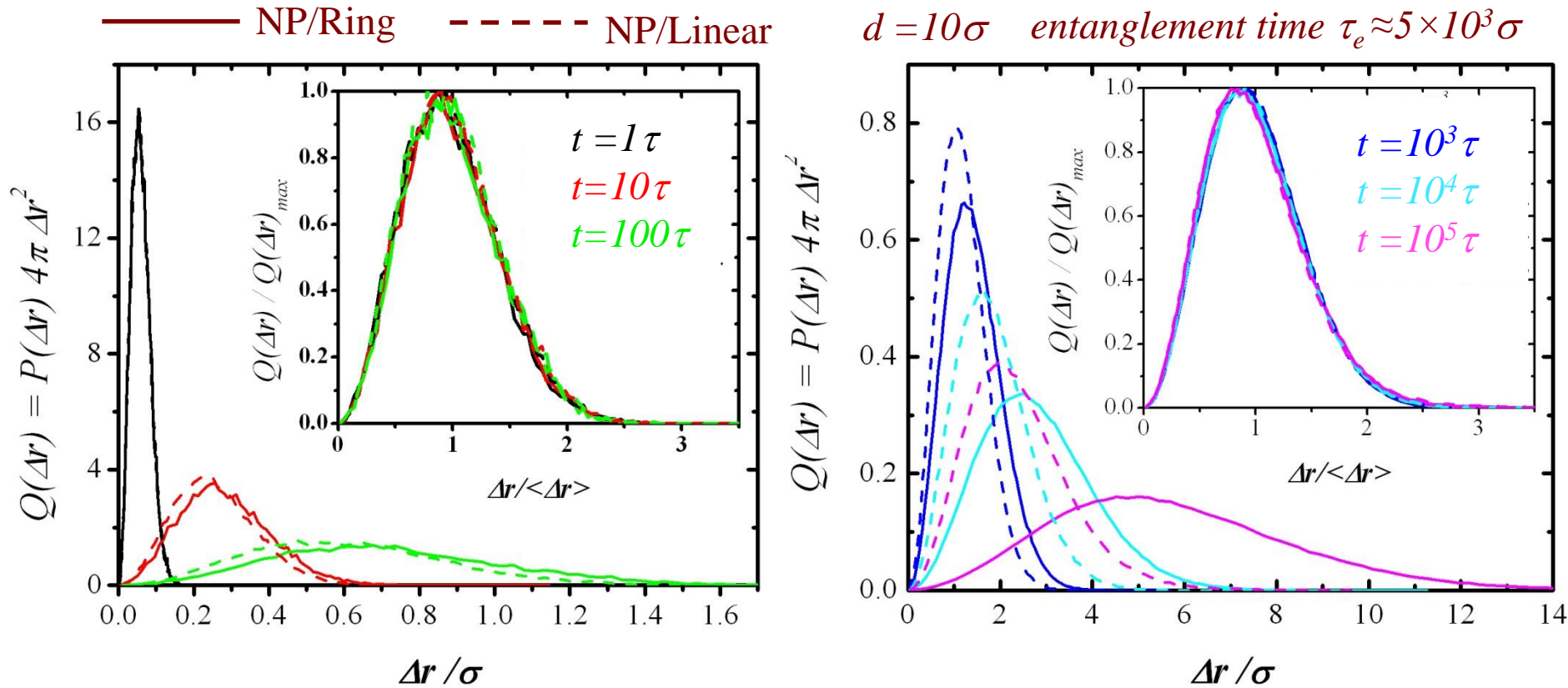
L. Fetters *et al.*, J. Polym. Sci  
B: Polym. Phys. 37, 1023  
(1999)

$$G_N^0 = \frac{4}{5} \frac{k_B T}{a_{pp}^2 p}$$

- Parameter free prediction for plateau modulus

Extended to solutions of semi-flexible polymers –  
N. Uchida *et al.*, JCP 128, 044902 (2008)

# Probability Distribution of NP Displacement



- Nearly Gaussian distribution of NP displacement
- Entanglement mesh reduces the mean displacement for linear chains but does not change the distribution
- No evidence of NP hopping