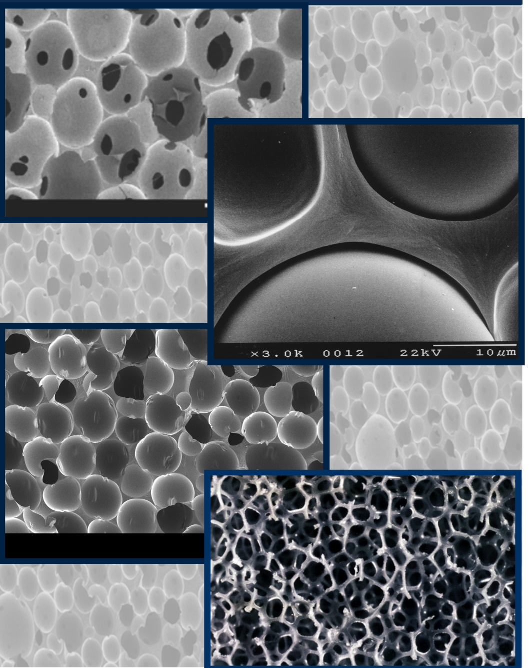
Component Science and Mechanics



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Instabilities Predicted by Plasticity Models for Metals

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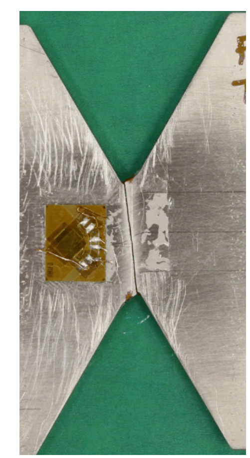
Outline

- Bifurcation Criteria
- vonMises Plasticity
- Numerical Simulations
- Summary and Future Work

Motivation

Accurate predictions of ductile tearing in metals require accurate predictions of bifurcations.



Reference:

B. Boyce and S. Kramer, "The 2nd Sandia Fracture Challenge", *Imechanica web site* accessed May 30th, 2014. http://imechanica.org/node/16708

Bifurcation Criteria

Criterion	Equation	Mode
General	$\mathbf{e}:\mathbf{T}^{s}:\mathbf{e} = 0$	Diffuse or Localized
Limit Point	$\mathbf{T}:\mathbf{e} = \mathbf{0}$	Diffuse or Localized
Loss of Strong Ellipticity	$\mathbf{m}\cdot\mathbf{A}^{s}\cdot\mathbf{m} = 0$	Localized
Classical Discontinuous	$\mathbf{A}\cdot\mathbf{m} = \mathbf{0}$	Localized

*Acoustic Tensor $\mathbf{A} = \mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n}$

References:

Drucker, D.C. (1950) *Q. Appl. Math.* 7, 411-418.

Hill, R. (1958) *J. Mech. Phys. Solids* **6**, 236-249.

Rice, J.R. (1976) Theoretical & Applied Mechanics, 14thIUTAM, Ed. W.T. Koiter, North-Holland, Amsterdam

Valanis, K.C. (1989) *Acta Mech.* **79**, 113-141.

Bigoni, D. and Hueckel, T. (1991) *Int. J. Solids Struct.* 28, 197-213

Neilsen, M.K. and Schreyer, H.L. (1993) Int. J. Solids Struct. 30, 521-544.

Bifurcation Criteria as an Eigenproblem

General Bifurcation Criterion $\mathbf{e}:\mathbf{T}^{s}:\mathbf{e} = 0$

Eigenvalue problem $\mathbf{T}^{s}:\mathbf{x} = \lambda\mathbf{x}$

$$\mathbf{x}_{\boldsymbol{i}}:\mathbf{x}_{\boldsymbol{j}} = \delta_{ij}$$

$$\mathbf{e} = \sum_{i=1}^{6} \alpha_i \, \mathbf{x}_i$$

General Bifurcation Criterion $\mathbf{e}:\mathbf{T}^{s}:\mathbf{e} = \sum_{i=1}^{6} \alpha_i^2 \, \lambda_i = 0$

Criterion is first satisfied when $\lambda_1 = 0$

Bifurcation is characterized by $\mathbf{e} = \mathbf{x}_1$

Linear Isotropic Elasticity

$$\mathbf{T} = \mathbf{E} = 3\mathrm{K}\mathbf{P}^{sp} + 2G\mathbf{P}^{d} \qquad \mathrm{K} = \frac{\mathrm{E}}{3(1-2\upsilon)} \qquad \mathrm{G} = \frac{\mathrm{E}}{2(1+\upsilon)}$$

$$\mathbf{P}^{d} = \mathbf{I} - \mathbf{P}^{sp}$$

$$\mathbf{P}^{sp} = \frac{1}{3}\mathbf{i} \otimes \mathbf{i}$$

$$\mathbf{T}\!:\mathbf{x} = \lambda\mathbf{x}$$

$$\mathbf{x}\text{:} \quad \frac{1}{\sqrt{6}}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\lambda\text{:} \quad 2\mathrm{G}, \quad 2\mathrm{G}, \quad 2\mathrm{G}, \quad 2\mathrm{G}, \quad 2\mathrm{G}, \quad 3\mathrm{K}$$

Reference: Sutcliffe, S. (1992) *J. Appl. Mech.* **59**, 762-773.

Plasticity

$$\psi = \alpha(\boldsymbol{\sigma}) - \kappa$$

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\rho}\mathbf{g}$$

$$\mathbf{f} = \frac{\partial\psi}{\partial\boldsymbol{\sigma}}$$

$$\mathbf{T} = \mathbf{E} - \frac{1}{H + \mathbf{g}:\mathbf{E}:\mathbf{f}}\mathbf{E}:\mathbf{g} \otimes \mathbf{f}:\mathbf{E}$$

$$\mathbf{T}^s:\mathbf{x} = \lambda\mathbf{x}$$

$$H^{gb} = \frac{1}{2}\left(\sqrt{\mathbf{f}:\mathbf{E}:\mathbf{f}}\sqrt{\mathbf{g}:\mathbf{E}:\mathbf{g}} - \mathbf{f}:\mathbf{E}:\mathbf{g}\right)$$

$$\mathbf{x} = \frac{\mathbf{f}}{\sqrt{\mathbf{f}:\mathbf{E}:\mathbf{f}}} + \frac{\mathbf{g}}{\sqrt{\mathbf{g}:\mathbf{E}:\mathbf{g}}}$$

References:

Mroz, Z. (1963) *J. Mechanique* **2**, 21-42.

Hueckel, T. and Maier, G. (1977) *Int. J. Solids Struct.* **13**, 1-15.

Raniecki, B. and Bruhns, O.T. (1981), *J. Mech. Phys. Solids* **29**, 153-172.

Runesson, K. and Mroz, Z. (1989) *Int. J. Plasticity **5**, 639-658.*

Szabo, L. (1998), *Int. J. Plasticity **13**, 809-835.*

Discontinuous Bifurcation Criterion
= General Bifurcation + Additional Constraint

General Bifurcation $$\mathbf{x} : \mathbf{T}^s : \mathbf{x} = 0$$

Additional Constraint $$\mathbf{x} = \frac{1}{2}(\mathbf{m} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{m})$$

2nd order tensor w/ zero intermediate eigenvalue

$$\mathbf{A} = \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n}$$

Loss of Strong Ellipticity $$\mathbf{m} \cdot \mathbf{A}^s \cdot \mathbf{m} = 0$$

Reference:
Neilsen, M.K. and Schreyer, H.L. (1993) *Int. J. Solids Struct.* **30**, 521-544.

vonMises Plasticity with Associated Flow

$$\psi = \sqrt{\mathrm{J}_2} - \kappa$$

$$\mathrm{J}_2 = \frac{1}{2}\mathbf{s}:\mathbf{s}$$

$$\mathbf{f} = \mathbf{g} = \frac{\partial\psi}{\partial\boldsymbol{\sigma}} = \frac{\boldsymbol{s}}{\mathbf{2}\sqrt{\mathrm{J}_2}}$$

$$\mathbf{T} = \mathbf{E} - \frac{2\mathrm{G}^2}{(H + \mathrm{G})}\frac{\mathbf{s}\otimes\mathbf{s}}{(\mathbf{s}:\mathbf{s})}$$

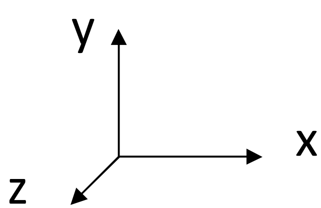
character of first possible bifurcation given by stress deviator

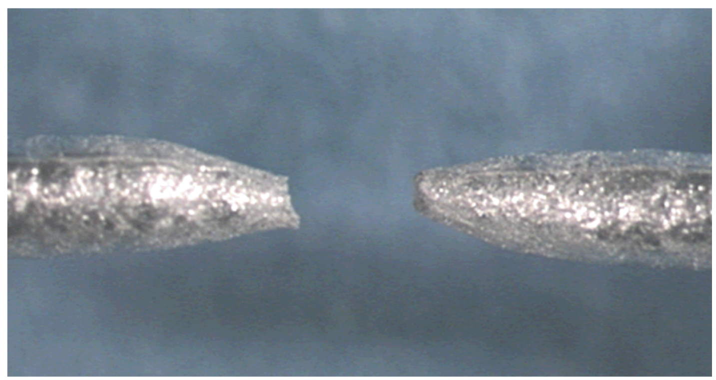
$$H = 0, \qquad \lambda_{\mathbf{1}} = 0, \qquad \mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}}$$

vonMises Plasticity with Associated Flow

uniaxial tension, general bifurcation, necking

$$\mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}} \rightarrow \frac{1}{\sqrt{6}} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$





Sn-Pb solder rod subjected to uniaxial tension.

Constitutive Model: UCP Fail

Stress Rate: $$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}}^{e} = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in})$$

Inelastic Rate: $$\dot{\boldsymbol{\varepsilon}}^{in} = \frac{3}{2}\dot{\gamma}\,\mathbf{n} = \frac{3}{2} e^{f} \sinh^{p}\left(\frac{\tau}{\alpha D\left(1 - c w^{d}\right)}\right)\mathbf{n}$$

Evolutions Eqn: $$D = \hat{D}(\gamma)$$

Flow Direction: $$\mathbf{n} = \frac{\mathbf{s}}{\tau} \qquad \tau = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$$

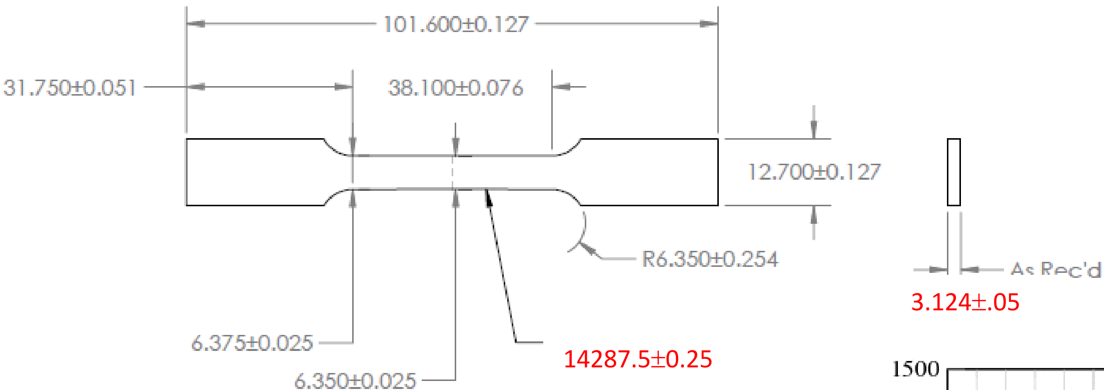
Damage: $$w = \int \left(\frac{1}{1 + \frac{p}{\hat{p}}}\right)^{\hat{a}} (2 - A)^{\hat{\beta}}\, d\varepsilon_p$$

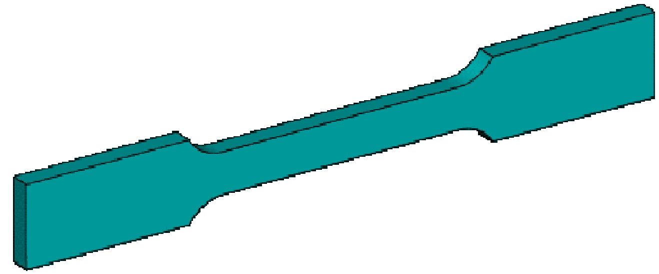
$$s_1 \geq s_2 \geq s_3 \qquad A = Max\left(\frac{s_2}{s_1}, \frac{s_2}{s_3}\right) \qquad p = \frac{-1}{3}\sigma : i$$

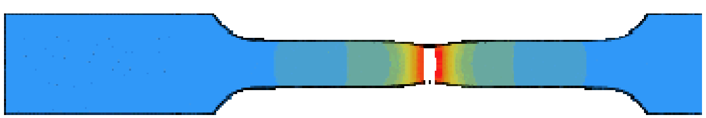
Reference:

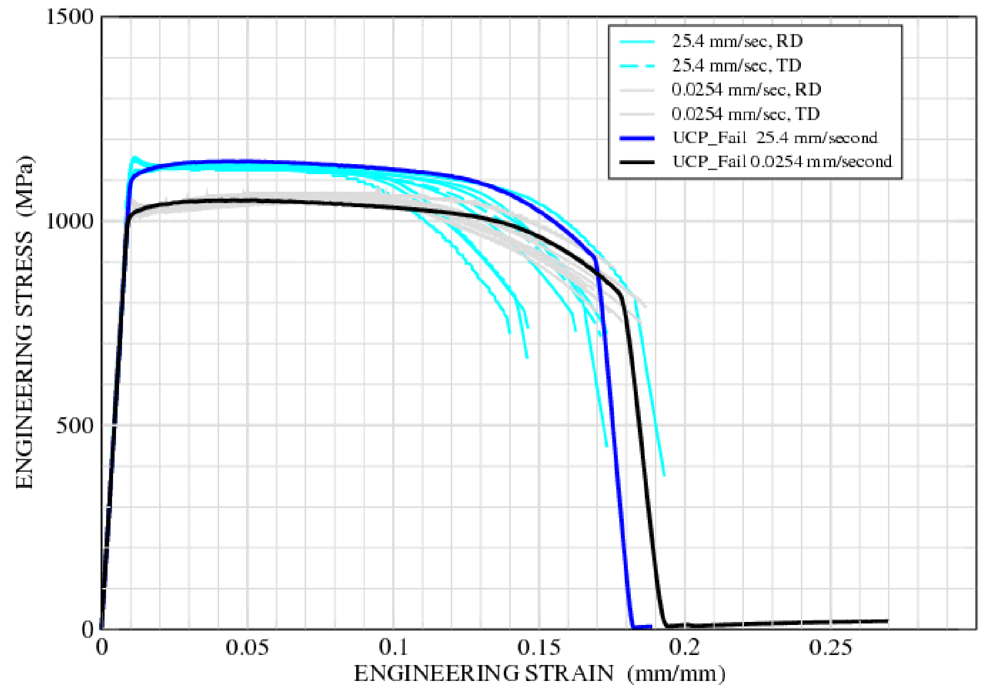
Wilkins, M.L., Streit, R.D., and Reaugh, J.E., 'Cumulative-Strain-Damage Model of Ductile Fracture: Simulation and Prediction of Engineering Fracture Tests,' UCRL-53058, Lawrence Livermore National Laboratory, Oct. 1980

Uniaxial Tension – Ti6Al4V

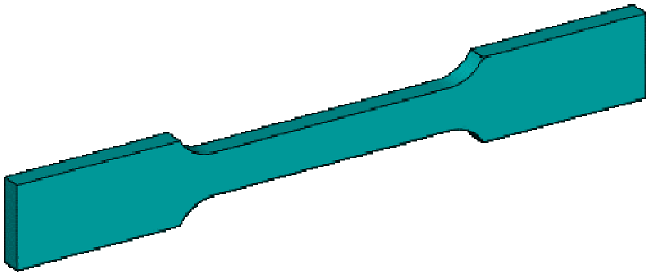


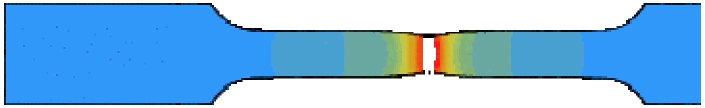


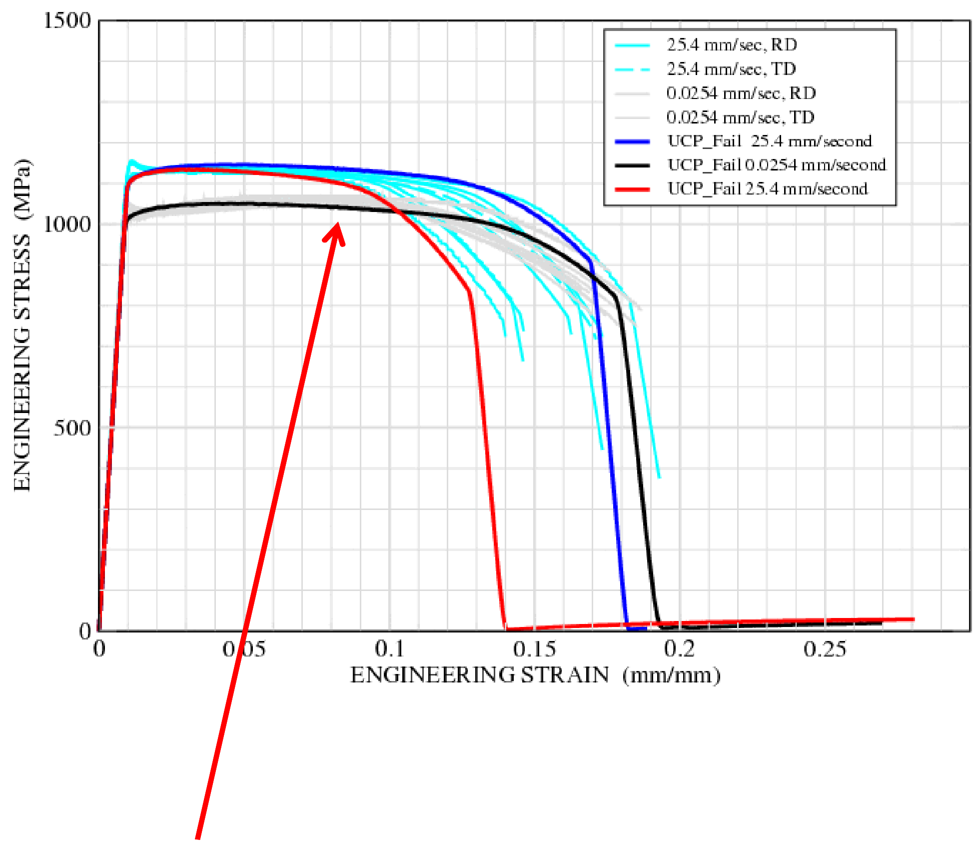




Uniaxial Tension – Ti6Al4V with Heating Due to Plastic Work







Heating due to plastic work reduces effective H and get bifurcation at smaller strain.

Bifurcations for vonMises and Tresca

vonMises Plasticity

$$\psi = \sqrt{J_2} - \kappa$$

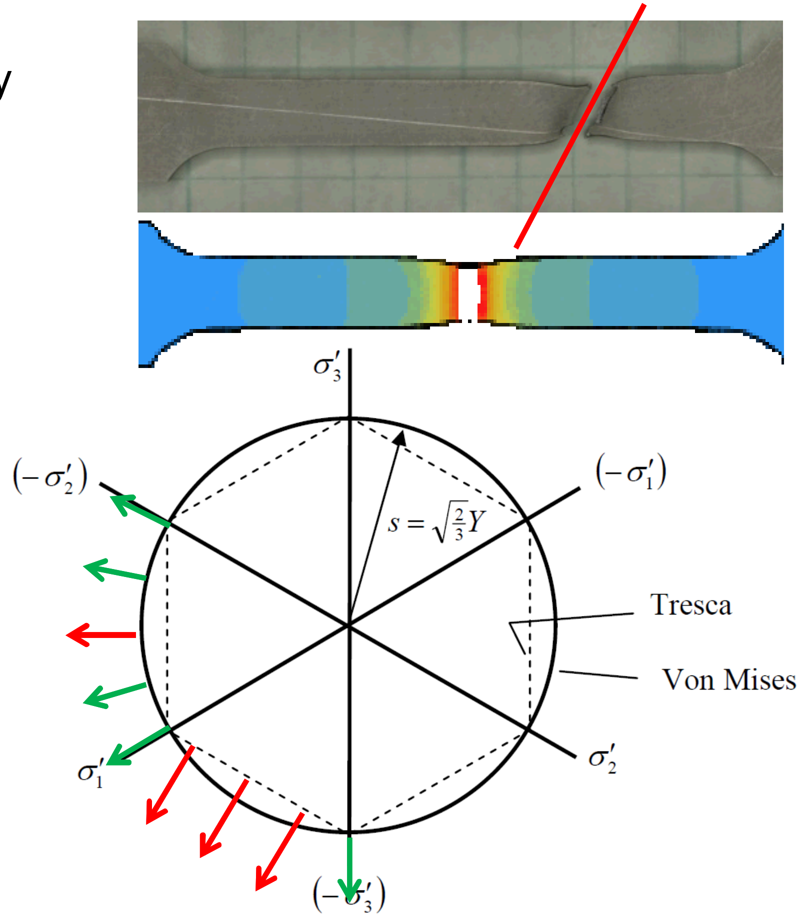
$$\mathbf{f} = \mathbf{g} = \frac{\partial \psi}{\partial \boldsymbol{\sigma}} = \frac{\boldsymbol{s}}{2\sqrt{J_2}}$$

$$\mathbf{T} = \mathbf{E} - \frac{2G^2}{(H+G)} \frac{\mathbf{s} \otimes \mathbf{s}}{(\mathbf{s}:\mathbf{s})}$$

$$H = 0, \qquad \lambda_1 = 0, \qquad \mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}}$$

Tresca Plasticity

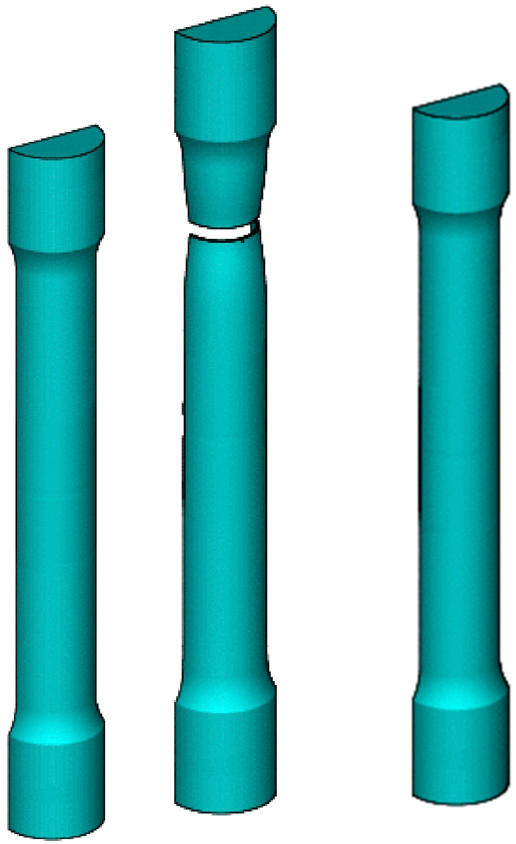
$$\mathbf{x} \Rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$



discontinuous bifurcation (shear band)

diffuse bifurcation (necking)

Uniaxial Tension



$$\boldsymbol{\sigma} \rightarrow \sigma \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \text{hoop} \\ \text{axial} \\ \text{radial} \end{matrix}$$

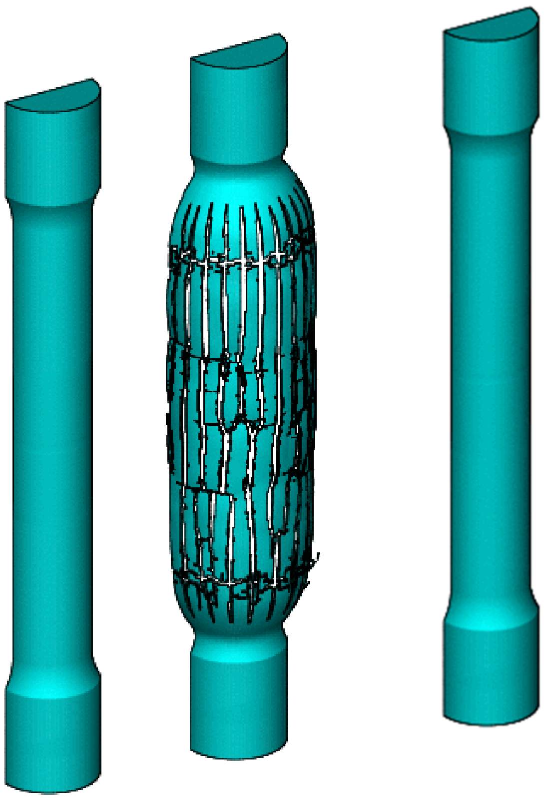
$$\mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}} \rightarrow \frac{1}{\sqrt{6}} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

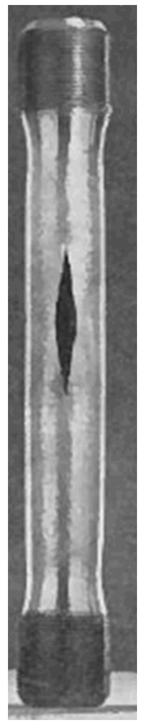
Predict and see general bifurcation, necking followed later by formation of crack.

Reference:

Nadai, A., **Theory of Fracture and Flow of Solids**, Vol. 1, McGraw-Hill, 1950.

Uniform Internal Pressure





$$\boldsymbol{\sigma} \rightarrow \frac{\sigma}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \text{hoop} \\ \text{axial} \\ \text{radial} \end{matrix}$$

$$\mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}} \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

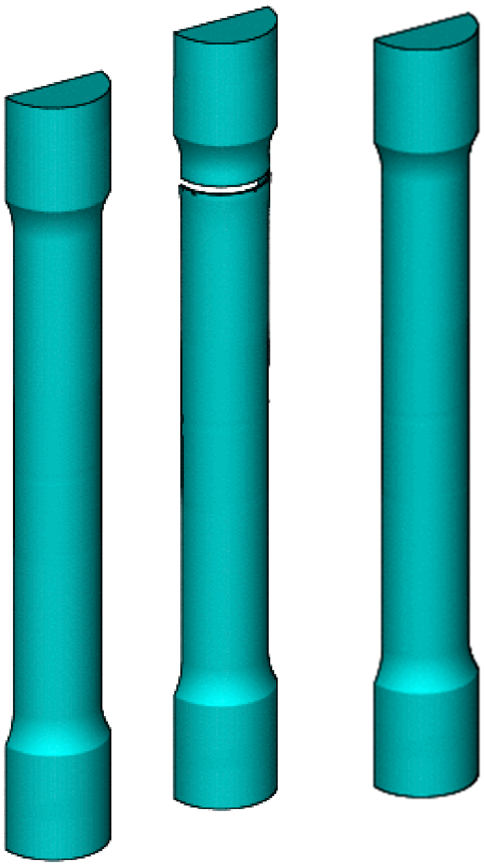
Predict and see discontinuous bifurcation, cracking 45 degrees thru thickness right away.

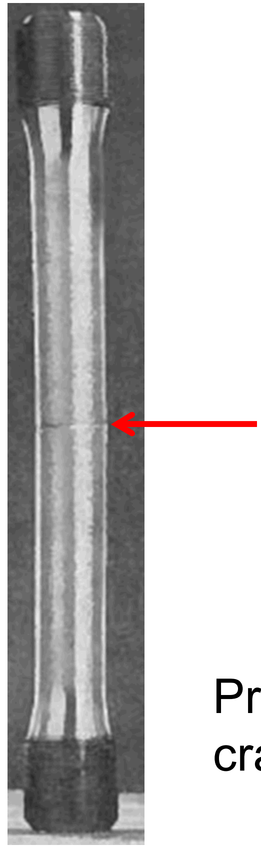
Experiment reduces pressure after crack but simulation leaves pressure on.

Reference:

Nadai, A., **Theory of Fracture and Flow of Solids**, Vol. 1, McGraw-Hill, 1950.

Axial Stress is 2x Hoop Stress





$$\boldsymbol{\sigma} \rightarrow \frac{\sigma}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \text{hoop} \\ \text{axial} \\ \text{radial} \end{matrix}$$

$$\mathbf{x}_1 = \frac{\mathbf{s}}{\sqrt{\mathbf{s}:\mathbf{s}}} \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Predict and see discontinuous bifurcation, cracking 45 degrees from axis right away.

Experiment reduces pressure after crack but simulation leaves pressure on.

Reference:

Nadai, A., **Theory of Fracture and Flow of Solids**, Vol. 1, McGraw-Hill, 1950.

Summary

- vonMises plasticity models predict bifurcations that depend on stress because fundamental eigentensor for tangent modulus tensor is stress deviator

- An eigenanalysis of the tangent modulus tensor provides information about material bifurcations.

- This eigenanalysis along with experimental observations of localization can be used to evaluate proposed material models.

- Accurate predictions of material behavior and bifurcations are essential for generating subsequent material failure predictions.