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Towards a Generalized Framework for DIC

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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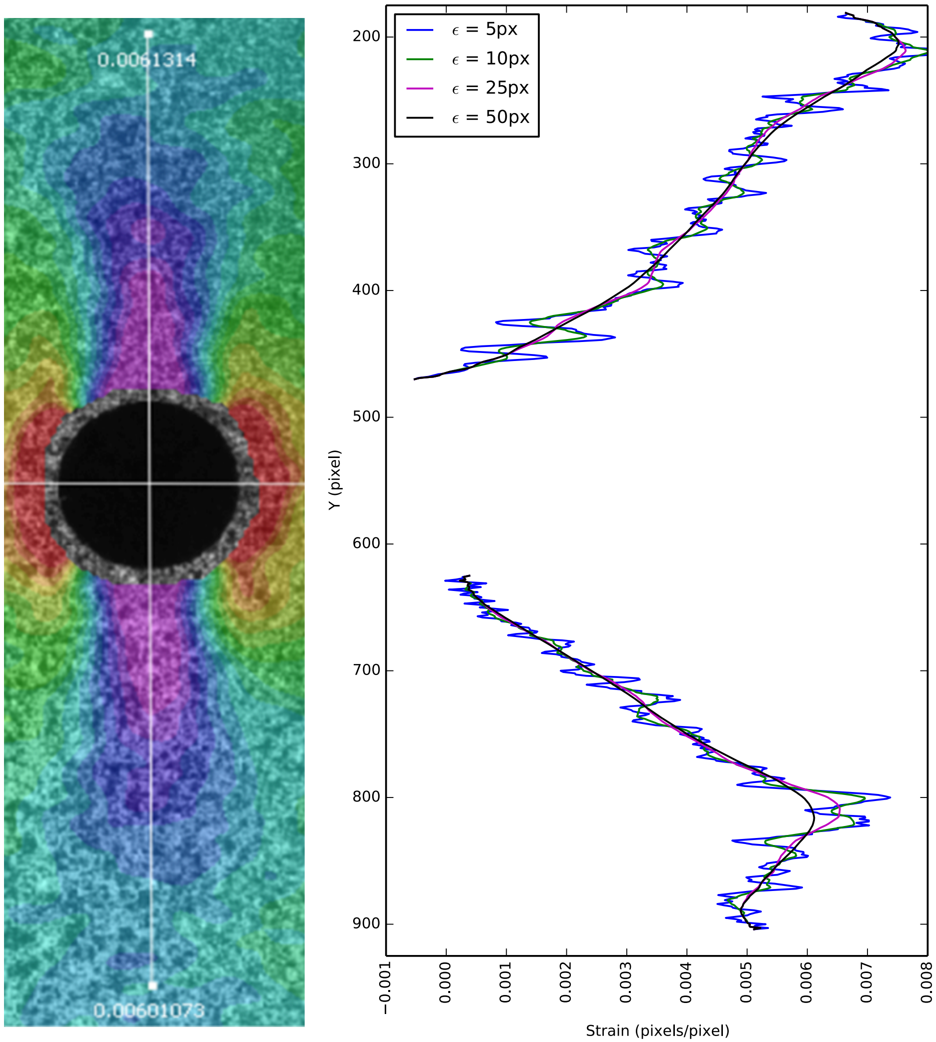
Noise: The Bane of Our Existence



DIC challenge: Plate with a hole,
strain results for various strain window sizes

A New Age of DIC is Beginning

Rigorous theory and framework needed

DIC of the past

Optical flow theory

Assumptions / approximations

Image correlation

Noisy, inaccurate strains and poor model calibration

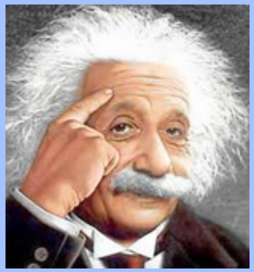
Current DIC trends

Image correlation

Optical flow theory

Material behavior

Regularization



Accurate strains and greatly improved model calibration

Hierarchy of Methods and Motivation

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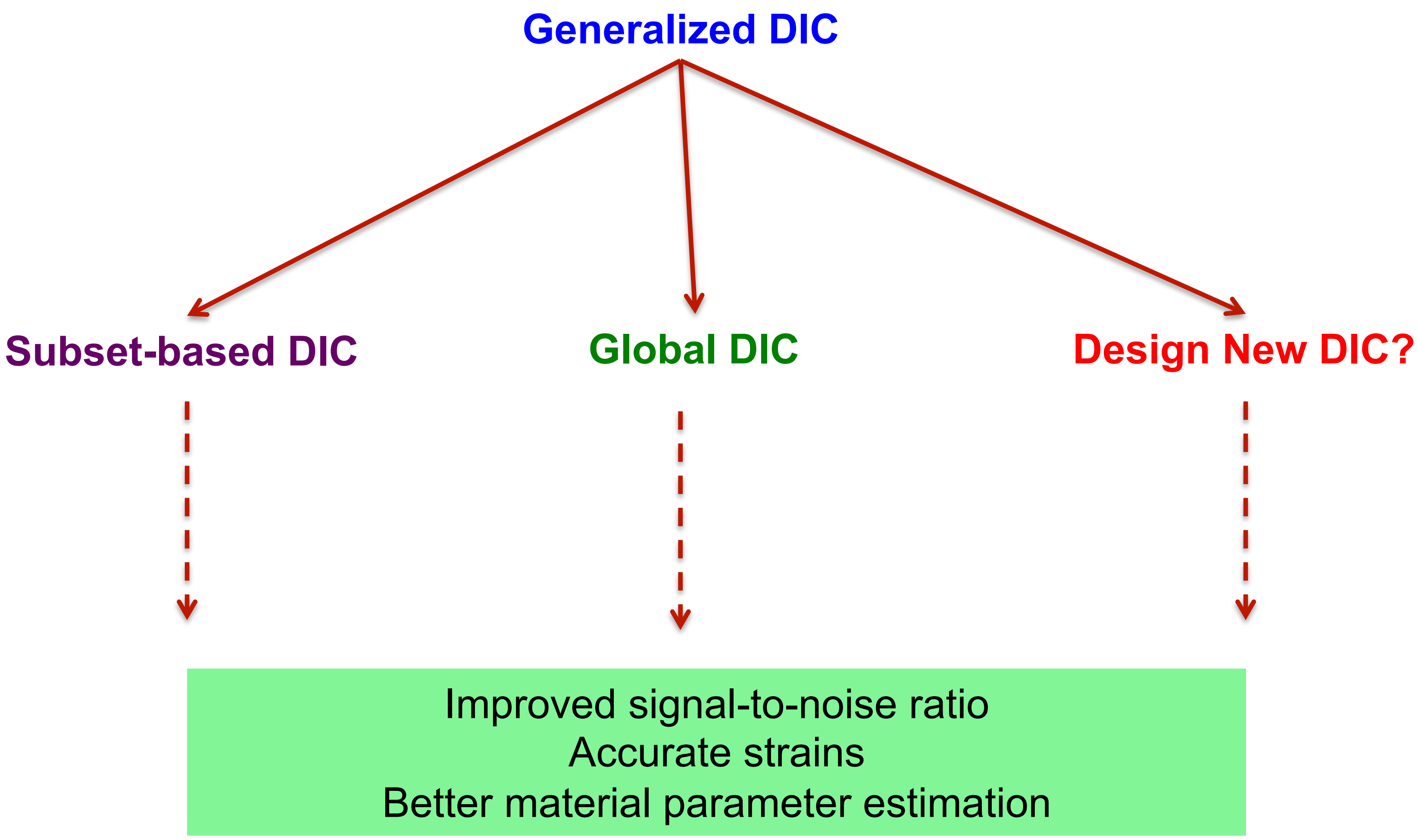
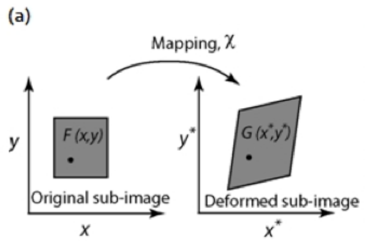


Outline

- Description/comparison of methods
- Generalized DIC
- Research opportunities
- Nonlocal strain

Comparison of Methods

Subset-based DIC



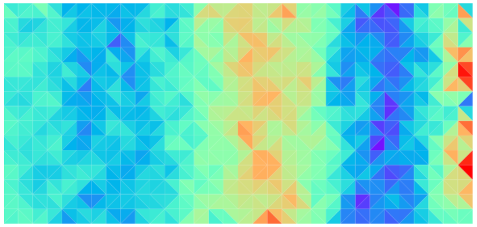
Independent, piecewise-constant deformation for subsets of the image

Least-squares minimization over each subset

Typically no regularization

Strains computed by regression

Global DIC



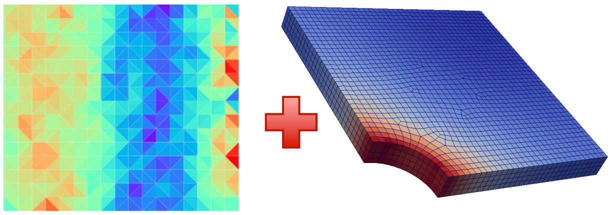
Mesh-based fields with continuity enforced

Global least-squares minimization

Optional regularization, may be physics-based

Strain computed using element shape functions

Generalized DIC (PDE-Constrained)



Discretization flexible (mesh or mesh-free)

Global or subset least-squares minimization

Optional regularization

Solution constrained to satisfy balance laws

Operators flexible (standard or nonlocal)

Noise is "modeled"

Strains computed directly in the minimization

Generalized DIC (in words)

Minimization problem:

Least-squares min of solution field vs. image data

Add quantities of interest (strain) directly to the minimization objective via regularization term

(Strain calculation is not a separate step!)

Constrain the solution to satisfy balance laws

Generalized DIC (in eq's)

Minimization problem:

$$\min_{(\phi,b)\in\mathcal{P}\times\mathcal{V}} \frac{1}{2}\int_0^{\tau}\int_{\Omega_t}\left(\phi(x,t)-\hat{\phi}(x,t)\right)^2 dx\,dt + \frac{\beta}{2}\int_{\Omega_t} W(x,t)\,dx\,,$$

$$+\frac{\alpha}{2}\int_{\Omega_t}\left(\tau^2\nabla b(x)^T\nabla b(x)-I\right):\left(\tau^2\nabla b(x)^T\nabla b(x)-I\right)dx\,,$$

$$\text{subject to}\begin{cases}\dfrac{\partial}{\partial t}\phi + b\cdot\nabla\phi = \sigma\,\Delta\phi & 0<t\le\tau\,,\ \text{over }\Omega_t\,,\\ n\cdot\nabla\phi = 0 & \text{over }\partial\Omega_t\,,\\ \phi(x,0)=\phi_0(x)\,. & \end{cases}$$

subject to $n\cdot b = 0$ over $\partial\Omega$

subject to the balance of linear momentum

Generalized DIC (opportunities)

Minimization problem:

Nonlocal operators (discontinuous fields, fracture)

Noise modeling

$$\min_{(\phi,b)\in\mathcal{P}\times\mathcal{V}} \frac{1}{2}\int_0^{\tau}\int_{\Omega_t}\left(\phi(x,t)-\hat{\phi}(x,t)\right)^2 dx\,dt + \frac{\beta}{2}\int_{\Omega_t} W(x,t)\,dx$$

$$+\frac{\alpha}{2}\int_{\Omega_t}\left(\tau^2\nabla b(x)^T\nabla b(x)-I\right):\left(\tau^2\nabla b(x)^T\nabla b(x)-I\right)dx\,,$$

Novel discretizations

$$\text{subject to}\begin{cases}\frac{\partial}{\partial t}\phi + b\cdot\nabla\phi = \sigma\,\Delta\phi & 0<t\leq\tau\,,\ \text{over } \Omega_t\,,\\ n\cdot\nabla\phi = 0 & \text{over } \partial\Omega_t\,,\\ \phi(x,0)=\phi_0(x)\,. & \end{cases}$$

Other "optimal" quantities of interest

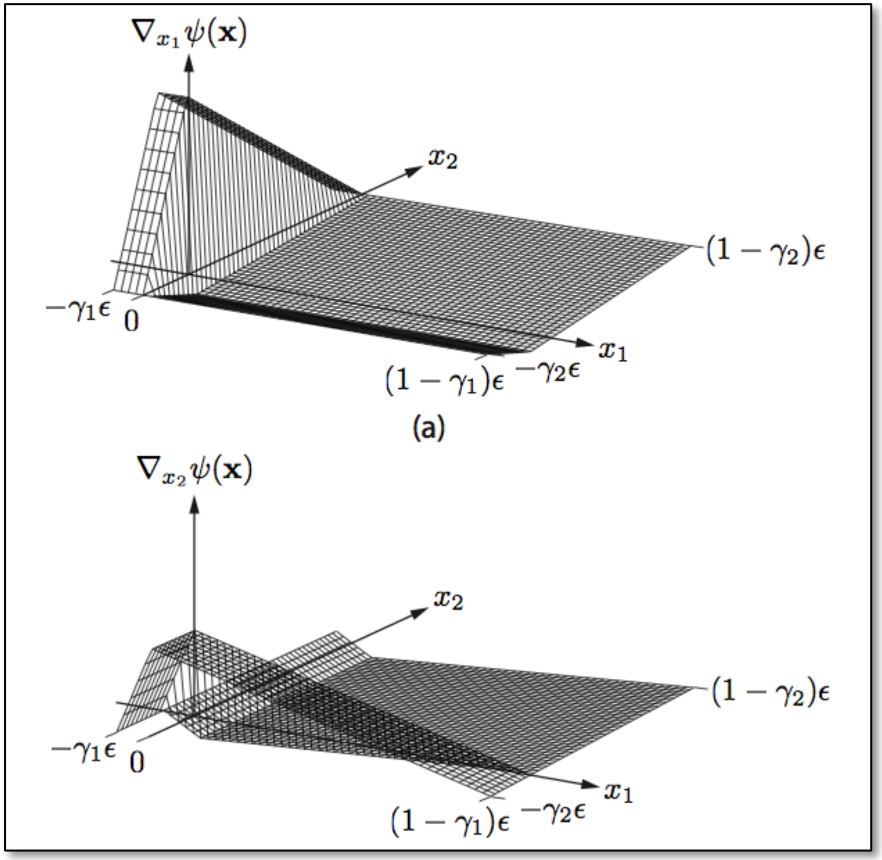
subject to $n\cdot b = 0$ over $\partial\Omega$

subject to the balance of linear momentum

Physics-of-interest-oriented balance laws

"Built-in" material ID schemes

Nonlocal Strain



The kernel is built up from tensor products of functions that integrate to zero over the domain

The kernel can be “designed” to for particular objectives

$$\tilde{\nabla}\,\mathbf{f}(\mathbf{x}) := -\int_{\mathbb{R}^n} \mathbf{f}(\mathbf{y}) \otimes \boldsymbol{\alpha}_\epsilon(\mathbf{y}-\mathbf{x})\,d\mathbf{y}$$

Nonlocal gradient — Data — Kernel

$$\tilde{\mathbf{F}} := \mathbf{I} + \tilde{\nabla}\mathbf{u}$$

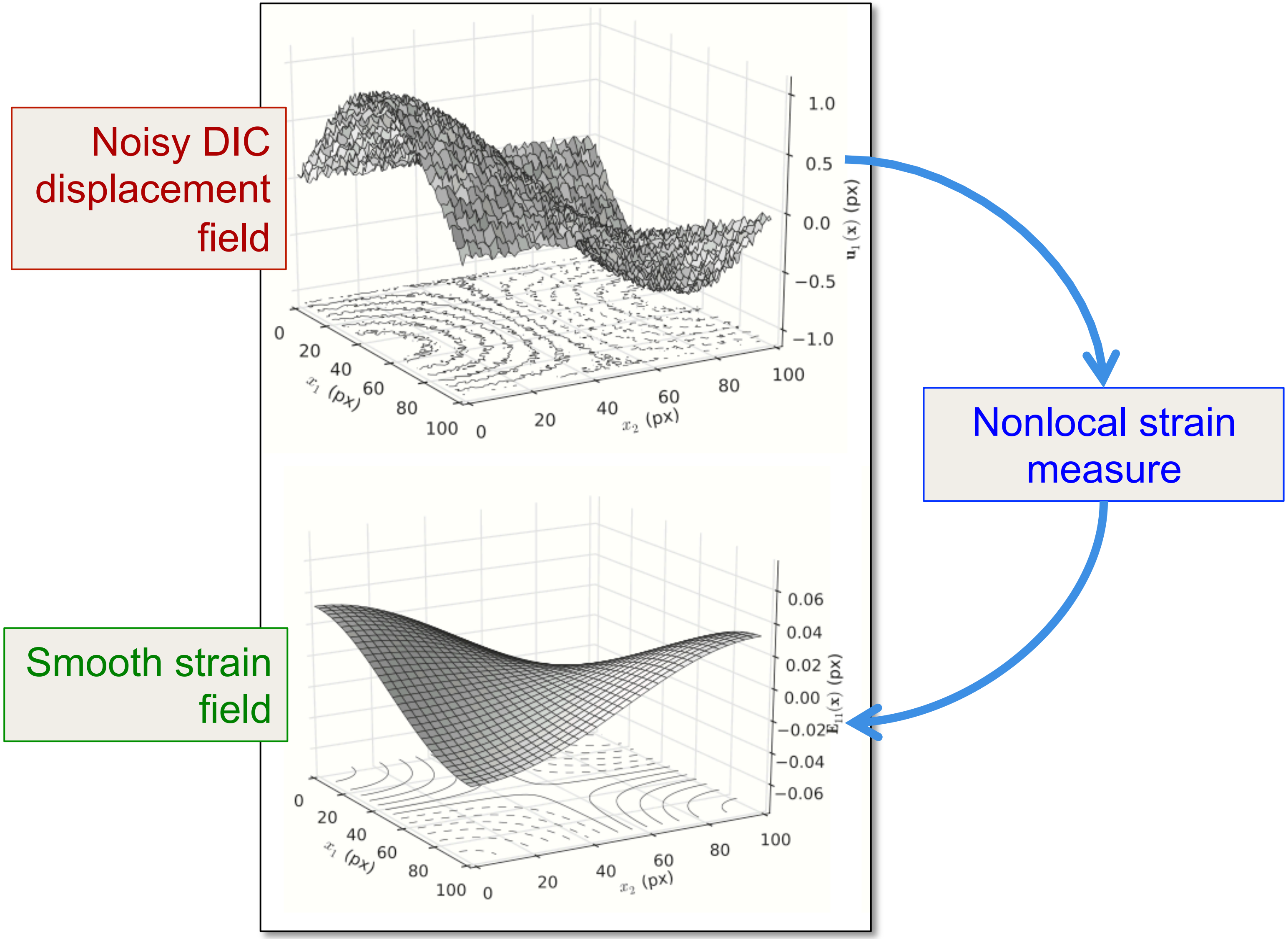
Deformation gradient

$$\tilde{\mathbf{E}} := \frac{1}{2}\left(\tilde{\mathbf{F}}^T\tilde{\mathbf{F}} - \mathbf{I}\right)$$

Strain

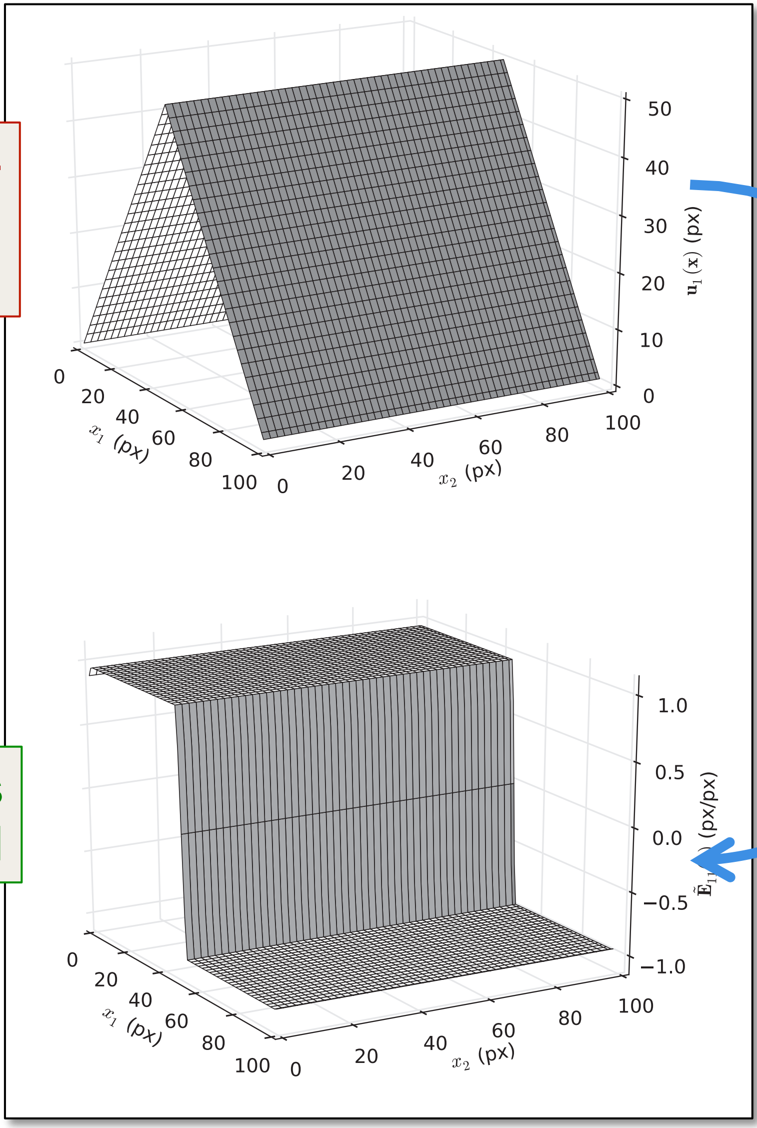
This strain measure is mathematically consistent, even if the displacement field is discontinuous

Computing Smooth Strains from Noisy Data



Appropriate for Discontinuous Fields

Non-globally-differentiable displacement field



Nonlocal strain measure

Discontinuous strain field

DICE

DIGITAL IMAGE CORRELATION engine

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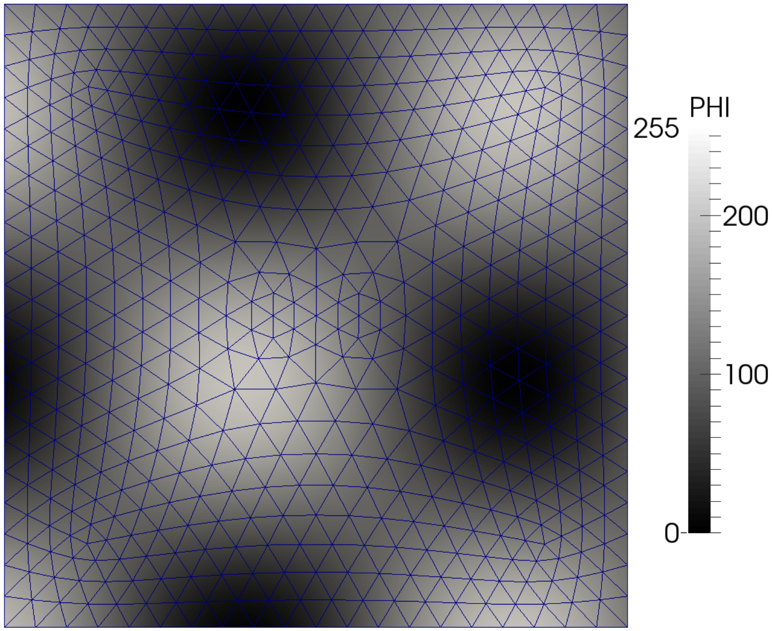
- Funded by WSEAT, ASCR (DOE Office of Science), Stronglink HSV, LDRD
- Focus: high performance computing DIC + novel algorithms
- PDE constrained optimization
- Nonlocal (integral-based) formulation
- Machine portable: OSX, Linux, Windows 7
- MPI parallel + (GPU + OpenMP)



(Generalized DIC) + Global DIC variants + Subset-based DIC available

Collaborators: Paul Crozier, Alvaro Cruz-Cabrera, Rich Lehoucq, Phil Reu, and Scott Walkington
Summer Students 2015: Hasan Jamal and Carlos Garavito Garzon