ARRA Funding

Final Technical Report for Years 1-4
of the Early Career Research Project
"Viscosity and equation of state of hot and dense QCD matter"

Period: 04/15/2010 - 04/14/2014 (first four years of the project)

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1 Main research activities and accomplishments

The Section below summarizes research activities and achievements during the first four years of the PI's Early Career Research Project (ECRP). Two main areas have been advanced: i) radiative $3 \leftrightarrow 2$ radiative transport, via development of a new computer code MPC/Grid that solves the Boltzmann transport equation in full 6+1D (3X+3V+time) on both single-CPU and parallel computers; ii) development of a self-consistent framework to convert viscous fluids to particles, and application of this framework to relativistic heavy-ion collisions, in particular, determination of the shear viscosity. Year 5 of the ECRP is under a separate award number, and therefore it has its own report document 'Final Technical Report for Year 5 of the Early Career Research Project "Viscosity and equation of state of hot and dense QCD matter"' (award DE-SC0008028).

The PI's group was also part of the DOE JET Topical Collaboration, a multi-institution project that overlapped in time significantly with the ECRP. Purdue achievements as part of the JET Topical Collaboration are in a separate report "Final Technical Report summarizing Purdue research activities as part of the DOE JET Topical Collaboration" (award DE-SC0004077).

1.1 Radiative $3 \leftrightarrow 2$ transport

One striking observation in ultra-relativistic heavy ion collisions is that the hot and dense system created appears to be nearly thermal. While the mechanism responsible is still not understood, rapid thermalization lead to the interpretation that the quark-gluon plasma created must be strongly coupled ('sQGP' paradigm[1]). Indeed, elastic $2 \to 2$ scattering rates from perturbative quantum-chromodynamics (pQCD) do not thermalize sufficiently fast (see, e.g., [2]). On the other hand, it has been claimed [3] based on the Boltzmann Approach to Multi-Particle Scattering (BAMPS) code that radiative pQCD $2 \leftrightarrow 3$ rates, specifically $ggg \leftrightarrow gg$, shorten thermalization time-scales in $A+A$ at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies to ~ 1 fm/c or smaller. Other groups later argued, however, (e.g., [4, 5]) that the BAMPS calculation may have overestimated the rates by a factor of $3! = 6$. Investigation of this claim and controversy was one key goal of the ECRP. Radiative transport is also important for studying equilibration in systems that do not have a conserved particle number.

1.1.1 Covariant Boltzmann solver MPC/Grid

We have developed a new C++ computer package MPC/Grid that solves the on-shell, nonlinear Boltzmann transport equation

$$p^\mu \partial_\mu f(x, \vec{p}, t) = C_{2\to 2}[f] + C_{2\leftrightarrow 3}[f] \quad (1)$$

with $2 \to 2$, $2 \to 3$, and $3 \to 2$ rates (MPC stands for Molnar's Parton Cascade). This work was done by graduate student Dustin Hemphill. MPC/Grid employs an algorithm that uses test particles on a 3D spatial grid. In each time step, in each cell, pairs and triplets of test particles are tested for two-body and three-body scatterings, stochastically, with probabilities $P_{2\to 2}$, $P_{2\to 3}$, and $P_{3\to 2}$ dictated by the transport equation. This is quite similar to BAMPS. Formally, the algorithm obtains the correct transport solution in the limit of small time steps, small cell sizes, and large number of test particles. We proved that accuracy in the rates is controlled by the total number of test particles, not the number of test particles per cell. In practice, the computational challenge for $3 \to 2$ is primarily combinatorics ($\mathcal{O}(n^3)$ triplets vs. only $\mathcal{O}(n^2)$ pairs for two-body collisions), and with pQCD matrix elements also that the determination of $P_{3\to 2}$ requires numerical integration in 2D for each triplet. For the integration, we use adaptive, deterministic routines from the GNU Scientific Library (GSL) (BAMPS claims to use the stochastic routine VEGAS, but we found that routine inaccurate in this case). The main challenge for $2 \to 3$ is the correct generation of outgoing momenta according to pQCD matrix elements, which requires the generation of a complicated random distribution in 4D.

MPC/Grid efficiently handles both static box and free expansion simulations with elastic $2 \to 2$ and/or radiative $3 \to 2$ scatterings. It has been verified extensively against a series of analytic tests, and it also reproduces results from the PI's previous Boltzmann solver MPC/Cascade that is based on a geometric closest-approach scattering prescription (cascade algorithm). For $2 \to 2$, both isotropic and Debye screened (forward-peaked) scattering are included. For $2 \leftrightarrow 3$, we have implemented isotropic scattering, and screened leading-order pQCD matrix elements, including several approximations, such as the Bertsch-Gunion form[6] in BAMPS or isotropic outgoing momenta.

Moreover, we have developed an MPC/Grid version for parallel computers as well using the Message Passing Interface (MPI) communication protocol. Parallelization has been done by graduate student Mridula Damodaran. We explored scaling up to 960 CPU cores at the National Energy Research Scientific Computing Center (NERSC), and found reasonable efficiencies up to about 300 cores. The parallel version enabled studies with very high statistics of nearly 10^9 test particles, such as precise transport vs. hydrodynamics comparisons, and optimizations are underway to improve scaling even further.

1.1.2 Results on thermalization

With MPC/Grid we investigated the influence of pQCD radiative $gg \leftrightarrow ggg$ processes on thermalization and collective flow in heavy-ion reactions. First we calculated scattering rates $R_{2\to n} \equiv (2/n) \int d^3p_1 d^3p_2 f_1 f_2 v_{rel} \sigma_{2\to X}$ in a static, thermal plasma of gluons. Figure 1, left panel, shows the results for strong coupling constant $\alpha_s = 0.3$ and perturbative Debye mass $\mu_D \approx gT$. We found much smaller pQCD rates for radiative $gg \to ggg$ ("LO23") than for elastic $gg \to gg$ ("LO22"), in stark contrast with BAMPS that reported[3] the two rates to be nearly equal. The $gg \to ggg$ rate was slightly higher for Bertsch-Gunion $2 \to 3$ matrix elements ("BG"), which are only approximate and, among other things, do not have proper permutation symmetry over outgoing gluon momenta. A large discrepancy of nearly a factor of 5 (but not 6) still remained even when we used, as BAMPS does, BG with the Landau-Pomerantschuk-Migdal (LPM) interference effect included ("Λ"). (The

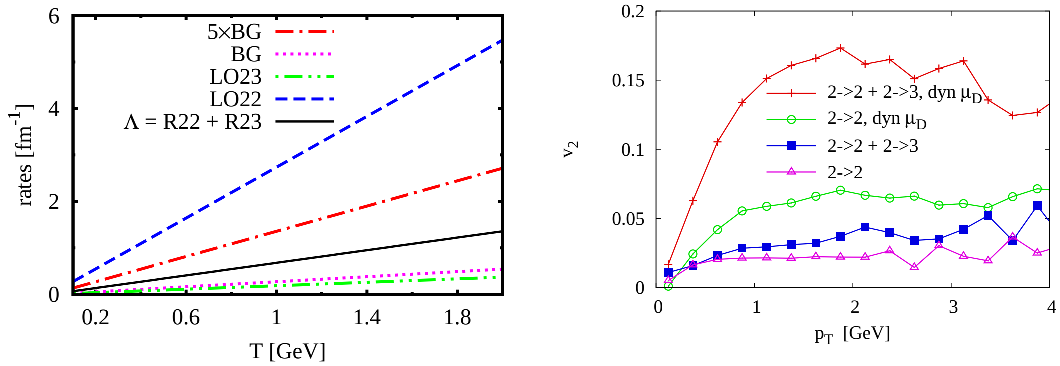

Figure 1: *Left:* $2 \to 2$ and $2 \to 3$ rates in a thermal gluon plasma from MPC/Grid ($\alpha_s = 0.3$). *Right:* final gluon $v_2(p_T)$ in $Au + Au$ at $\sqrt{s_{NN}} = 200A$ GeV, $b = 8fm$ at RHIC from MPC/Grid with perturbative $gg \to gg$ and $gg \leftrightarrow ggg$ cross sections ($\alpha_s = 0.4$).

$gg \to ggg$ rate is even smaller by about 20% if we also include $3 \to 2$ rates in the LPM suppression factor $\Theta(k_T - \Lambda)$).

This was followed up with MPC/Grid studies of thermalization in a uniform, static box, for conditions appropriate for the center of the collision zone in an $Au + Au$ collision at top RHIC energy $\sqrt{s_{NN}} = 200A$ GeV at typical thermalization time in hydrodynamic studies $\tau \sim 0.6$ fm. Starting from a highly nonequilibrium, purely transverse "mini-jet" momentum distribution with particles only above $p_T > 1.2$ GeV, with initial particle density $\sim 25/\text{fm}^3$, we found a roughly $2\times$ speedup in the thermalization rate of the energy distribution of gluons due to radiative processes. This, again, was about 5 times smaller than the almost $10\times$ speedup claimed by BAMPS studies.

Finally, we explored the influence of radiative processes on the "elliptic flow" momentum anisotropy ($v_2 = \langle \cos 2\phi \rangle$) in Au+Au at $b = 8$ fm at RHIC using realistic Woods-Saxon nuclear density profiles (as in Ref. [7]), allowing for free longitudinal and transverse expansion of the system. For perturbative Debye mass $\mu_D = gT$ with $T \sim \tau^{-1/3}$ cooling for an initially longitudinally expanding system, we found only a modest increase in v_2 vs. p_T (cf. Fig. 1, right panel). On the other hand, for self-consistent $\mu_D^2[f] = 3\pi\alpha_s \int d^3p\, \frac{1}{p}\, f(\vec{p})$ from linear response theory[8], elliptic flow was much higher, and the increase in v_2 was more prominent when radiative $2 \leftrightarrow 3$ was included. We traced the origin of this to a high sensitivity of $ggg \leftrightarrow gg$ the rates to the value of the Debye mass, and to how screening and the LPM effect are implemented. For typical scattering energies $\sqrt{s} \sim 4T$ in the plasma, the $2 \to 3$ cross section turned out to decrease very steeply with μ_D. We showed this both numerically, and in some cases also analytically (for example, we derived $\alpha_s^3/\mu_D^2 \times [31.64 \ln(1/z) - 35.80 + ...]$ for the case of Bertsch-Gunion-like but permutation symmetric approximation to the $gg \to ggg$ matrix element). A large part of the v_2 enhancement for dynamical Debye mass occurs because for Boltzmann distribution linear response gives significantly smaller $\mu_D = \sqrt{6/\pi^2}\, gT$, i.e., higher cross sections than $\mu_D = gT$ for Bose statistics. Interestingly, we saw very little change in v_2 when isotropic outgoing momenta were generated instead of the pQCD distribution.

We also found nearly $2\times$ variation in the $2 \to 3$ cross section depending on whether $\sim 1/(p_i p_j)$ collinear divergences were screened via $p_i p_j \to p_i p_j + \mu_D^2$ shift or $\Theta(p_i p_j - \mu_D)$ cut. But neither could compensate for the nearly $5\times$ discrepancy with the scattering rates claimed by BAMPS.

These results have been presented at the Quark Matter Conference 2015 (preliminary results were shown at Quark Matter 2012 and a few smaller workshops), and form a large part of Dustin Hemphill's Ph.D. thesis. After the findings are submitted for publication, the MPC/Grid code will be made publicly available on the Open Standard Codes And Routines (OSCAR) code repository maintained by the PI at Purdue.

1.2 Conversion of viscous fluids to particles

The most common dynamical framework to interpret data from ultrarelativistic heavy-ion ($A+A$) reactions is relativistic hydrodynamics (see, e.g., [9, 10, 11]). A high-priority goal is to utilize dissipative hydrodynamics to extract the transport coefficients of the quark-gluon plasma created in collisions at RHIC or the LHC, such as the shear viscosity to entropy density ratio η/s, from anisotropic flow data[12]. It is often not realized, however, that comparisons of hydrodynamic calculations to experimental data inevitably involve additional model ingredients. For example, hydrodynamic simulations need an initial condition model. They also require the conversion of the fluid (hydrodynamic fields) to particles. Only the particles can be compared directly to experiments, either directly after accounting for decays of unstable resonances ("pure hydro"), or they can be further evolved in a hadron transport model ("hybrid" approach).

The usual approach to "particlization" [13] is a Cooper-Frye conversion on a constant temperature or energy density hypersurface in spacetime[14]. While unambiguous for fluids in perfect local thermal equilibrium, i.e., *ideal* fluids, for *dissipative* fluids an infinite class of nonthermal phase space densities can reproduce the same hydrodynamic fields. This is further exacerbated for mixtures where one can postulate phase space corrections for each particle species almost independently. In practice this ambiguity is commonly ignored, even in state-of-the-art "hybrid" hydro+transport calculations[15]. For example, shear viscous corrections are simply assumed to follow quadratic momentum dependence with a common coefficient for all species, a procedure that the PI has termed "democratic Grad" ansatz[16]. This, however, completely ignores the microscopic dynamics that keeps the hadron gas near local equilibrium.

1.2.1 Self-consistent viscous corrections from linearized transport

The second main achievement of the ECRP was the development of a self-consistent approach for obtaining shear viscous corrections from linearized kinetic theory, and applying it to a mixture of hadrons. This was done by Zack Wolff, who graduated in Dec 2015 with Ph.D. The technique[17, 18, 19] shares a lot of similarity with the calculation of transport coefficients from kinetic theory[20, 21]. One assumes that deviations $\delta f_i = f_i - f_i^{\rm eq}$ from equilibrium phase space densities are small for each species i, linearizes the Boltzmann transport equation (1) in δf_i, and considers late-time behavior (the so-called Navier-Stokes regime) in which derivatives of δf_i on the left hand side of (1) can be neglected together with the time derivative of the equilibrium distribution. This leads to an inhomogeneous linear integral equation for δf_i:

$$\begin{aligned}\mathbf{p}\cdot\nabla f_i^{\rm eq} \;=\; \sum_{jkl}(&C_{22}[\delta f_i, f_j^{\rm eq}, f_k^{\rm eq}, f_l^{\rm eq}] + C_{22}[f_i^{\rm eq}, \delta f_j, f_k^{\rm eq}, f_l^{\rm eq}] \\ &+C_{22}[\delta f_i^{\rm eq}, f_j^{\rm eq}, \delta f_k, f_l^{\rm eq}] + C_{22}[f_i^{\rm eq}, \delta f_j^{\rm eq}, f_k^{\rm eq}, \delta f_l])\ , \end{aligned} \tag{2}$$

where we only included $2\to 2$ processes but still allow for change in particle type $ij\to kl$. For shear viscous corrections, one substitutes equilibrium distributions $f_i^{\rm eq} = g_i(2\pi)^{-3}e^{\mu_i/T - p\cdot u(x)/T}$ with uniform temperature $T = const$ and chemical potentials $\mu_i = const$ but with flow field that has shear

$$\sigma^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - (2/3)\Delta^{\mu\nu}(\partial\cdot u) \neq 0 \tag{3}$$

The integral equation turns out to be equivalent to maximizing a quadratic functional of one-variable functions $\chi_i(|\vec{p}_{LR}|/T)$ of the scaled momentum in the local fluid rest frame, in terms of which the viscous corrections are $\delta f_i(x,\vec{p}) = \chi_i(p_{LR}/T) p_\mu p_\nu \pi^{\mu\nu}(x) f_i^{\rm eq}(x,\vec{p})/2[e(x)+p(x)]T^2(x)$, where $e{+}p$ and $\pi^{\mu\nu}$ are the enthalpy and shear stress of the fluid. The maximization problem is then solved variationally via expansion over a finite basis $\chi_i \approx \sum_{n=1}^{N} c_{i,n}\psi_n$. In the Grad approximation, for each species one takes just one basis fuction with no momentum dependence, i.e., $\delta f_i \sim c_i p^2 f_i^{\rm eq}$. In the "democratic" Grad ansatz, $c_i = const$, thus the coefficient is the same for all species.

The numerical challenges were two-fold. First, whereas for transport coefficients one only needs the maximum value of the functional, for heavy ion observables one must accurately compute the functions χ_i that yield that maximum. In fact, we need χ_i to rather high momenta $p_{LR}/T \sim 30-40$ because most interesting effects in the data are around $p_T \sim 1-3$ GeV $\gg T$, moreover, sources boosted longitudinally by rapidity $\approx \pm 2$ still contribute appreciably (i.e., $p_{LR}/T \sim \gamma p_T/T$). Second, $2 \to 2$ scatterings between any two species in the mixture contribute. Even with elastic collisions only ($k = i$, $l = j$), this is an $\mathcal{O}(S^2)$ enhancement in the number of variational matrix elements needed for a mixture of S species.

First we simplified the quadratic functional to as few integrations as possible (see [17]). For isotropic $2 \to 2$ cross sections, the problem was reduced to integration in 4D, which was then implemented in C++ using deterministic, adaptive GSL routines. Extensive cross-checks were then developed to verify the accuracy of integration, convergence with variational basis size, and to gain analytic insights, especially in the limit of massless and very massive (nonrelativistic) particles. We highlight here only the following: i) In the Grad approximation, we showed that in a mixture of two nonrelativistic species with equal cross sections the heavier species acquires, counter-intuitively, a *smaller* viscous correction χ even though that contributes more to shear viscosity. ii) For massless systems with constant cross section, we reduced the functional to the simplest form possible of just one integration, which enabled analytic calculation of variational matrix elements for a variety of bases, together with analytic bounds on the low- and high-momentum behavior of variational solutions. iii) Through numerical calculations we uncovered an error in the relativistic kinetic theory "bible" [20] for the shear viscosity of a one-component mixture of arbitrary mass, which was then fixed via repeating the entire derivation in that book (in Ch. XI, Sc. 1c, the coefficient 5 in denominator of Eq. (24) should be 15). iv) We also generalized Baym's integration variables (cf. [21]) to massive particles, which are suitable for angle-dependent scattering but require 5D integration even in the isotropic case, and verified that variational matrix elements for isotropic scattering agree from that approach with those from our much faster 4D integration.

1.2.2 Dynamical shear corrections for pion-nucleon gas

Next, we calculated self-consistent shear viscous corrections for a pion-nucleon gas, with effective cross sections $\sigma_{\pi\pi} \approx 30$ mb, $\sigma_{\pi N} \approx 50$ mb, and $\sigma_{NN} \approx 20$ mb fit in the temperature range $100 < T < 160$ MeV to reproduce scattering rates calculated in [22]. Our general finding was that viscous corrections for protons are about a factor of 2 smaller than for pions, a difference driven both by the larger proton mass and the larger $\sigma_{\pi N} > \sigma_{\pi\pi}$.

To see how these correction manifest in elliptic flow, we wrote a new C++ code that includes self-consistent viscous δf_i in Cooper-Frye freezeout. The code reads hydrodynamic hypersurface input in 2+1D (longitudinally boost invariant) AZHYDRO[23, 24, 25] format, and writes hadron spectrum output in the same $p_T - \phi$ format as the hadron spectrum calculator in AZHYDRO. Hydrodynamic solutions were then calculated with AZHYDRO 0.2p2 for $Au + Au$ at $b = 7$ fm at RHIC, with diffuse nuclear profiles and initial (thermalization) time $\tau_0 = 0.6$ fm. This is a version we patched[26] specifically for this calculation, and made available from the OSCAR repository (the

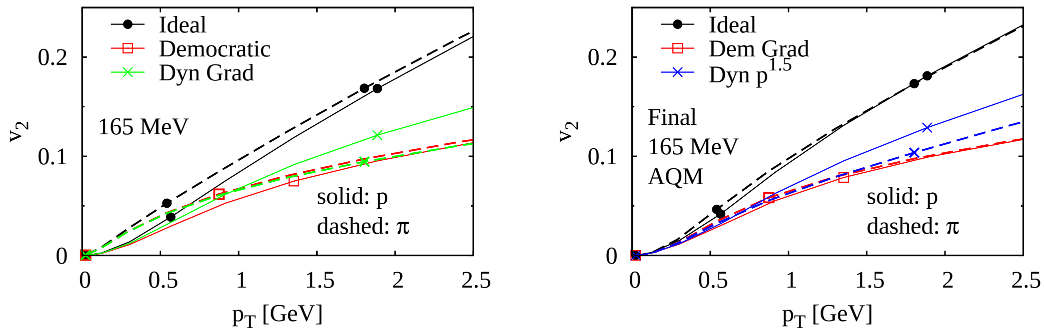

Figure 2: *Left:* direct pion and proton elliptic flow $v_2(p_T)$ in $Au+Au$ at RHIC from fluid-to-particle conversion of a viscous fluid with at $T = 165$ MeV with $\eta/s = 0.1$. *Right:* same as the left plot but for a 49-species hadron gas, with more realistic $\propto p^{1.5}$ viscous corrections and including feeddown from the decay of unstable resonances.

patch fixed crashes on modern 64-bit systems, and included realistic QCD equation of state[27]). Because AZHYDRO is an ideal hydrodynamics solver, it does not follow shear stress evolution. Therefore, shear stress was estimated using the Navier-Stokes relation $\pi^{\mu\nu}_{NS} = \eta\sigma^{\mu\nu}$ (cf. (3)). Flow derivatives were calculated by nearest-neighbor differencing and subsequent interpolation on the AZHYDRO spacetime grid output.

Figure 2 (left) shows the elliptic flow $v_2(p_T)$ of pions (dashed) and protons (solid) in $Au+Au$ at RHIC from self-consistent Cooper-Frye conversion at $T = 165$ MeV, for viscosity to entropy density ratio $\eta/s = 0.1$ (green crosses). Compared to the ideal (non-viscous) case (filled black circles), there is a marked suppression in v_2 at $p_T \sim 1.5-2.5$ GeV due to viscous δf corrections. The suppression is $\sim$ 30% smaller for protons because protons are more equilibrated in the dynamical approach. The pion-proton splitting at high p_T is, however, missed by the "democratic" Grad ansatz (red open boxes). Similar $\sim 30-40\%$ errors were found for conversion at lower $T = 140$ MeV and $T = 120$ MeV as well.

Viscous corrections were also calculated with realistic energy dependent cross sections for $\pi\pi$, $\pi - K$ and $\pi - N$ scattering (based on data in the Particle Data Booklet and also tables in the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) transport code). However, this lead to no qualitative difference at the temperatures considered here.

1.2.3 Self-consistent corrections for a 49-species resonance gas

Finally we investigated viscous corrections for a mixture of all hadrons up to 1.6 GeV in mass (the $\Omega(1600)$). To simplify the calculations, isospin partners were combined into a single species with appropriately enhanced degeneracy, which yielded 49 effective species. Two scenarios were considered: i) identical elastic cross sections between any two species $\sigma_{ij} = const$, and ii) cross sections based on the additive quark model[28, 29, 30, 31] (AQM), in which meson-meson, meson-baryon, and baryon-baryon elastic scatterings scale in proportion to the number of their constituent quarks, i.e., $\sigma_{MM} : \sigma_{MB} : \sigma_{BB} = 4 : 6 : 9$. Besides self-consistent corrections of the quadratic Grad form $\delta f/f^{\rm eq} \propto p^2$, we also investigated linear p^1 and also $p^{3/2}$ dependence, for conversion temperatures $T = 100$, 120, 140, and 165 MeV. In all cases we studied, $p^{3/2}$ dependence was favored variationally (it maximized the functional best). The calculations were done at Community Cluster at the Rosen Center for Advanced Computing (RCAC) at Purdue. The diagonalizations for χ_i were done with routines from Numerical Recipes[32]. Gauss-Jacobi elimination, $L-U$ decomposition, and the conjugate gradient method were used simultaneously to monitor accuracy.

The corrections were then implanted in the $Au+Au$ ideal hydrodynamic calculations used in the pion-nucleon study. The only additional complication is that many of the final observed pions and protons come from decays of unstable resonances. Therefore, after computing the momentum distribution of all species, chains of 2-body, 3-body, and 4-body decays to pions and protons were performed using the RESO program that is part of the AZHYDRO package. To verify the correctness of RESO for the viscous distributions used here, a new C++ code was also written that only performs 2-body decays with a similar algorithm to RESO. Figure 2 (right) shows results for final pion and proton $v_2(p_T)$ from the self-consistent approach with the most realistic $p^{1.5}$ dependence (blue crosses). The viscous suppression relative to the zero-viscosity fluid (filled black discs) is clearly visible, and the "democratic" Grad approach underestimates not only proton v_2 suppression (by $\sim 40-50\%$) but also pion v_2 suppression (by $\sim 10\%$) at high p_T.

As a "by-product" of this work, we also obtained the shear viscosity to entropy density ratio η/s for reach hadron mixture we calculated viscous corrections for. The shear viscosity of the system was found to be largely independent of its composition because it was mainly determined by $\pi-\pi$ interactions. On the other hand, at high temperature the entropy density grows, the more resonances are included. Therefore, while $\eta/s \approx 1$ for the $\pi-N$ gas at $T = 165$ MeV, it is close to 0.1 for the 49-species mixture case at the same temperature.

The results in Section 1.2 have been published in Refs. [17, 18], and highlights were also presented at international conferences and workshops. The viscous suppression of p_T-differential elliptic flow is sensitive, especially for protons, to whether self-consistent corrections or the naive "democratic" Grad ansatz is used. This sensitivity carries over to shear viscosity to entropy density (η/s) ratios extracted from $v_2(p_T)$ data. In particular, calculations based on "democratic" Grad corrections may underestimate η/s by $\sim 10-100\%$. It is important to test the sensitivity in a variety of hydrodynamic and hybrid model calculations. To facilitate this, the self-consistent viscous corrections obtained for the 49-species hadron gas in various scenarios have been disseminated to the community both in tabulated and also parameterized forms[17]. Some very interesting results in Zack Wolff's Ph.D thesis[19] utilizing viscous hydrodynamic solutions for $Pb+Pb$ at the LHC, and on the fourth and sixth flow harmonics[18] ($v_n = \langle \cos n\phi \rangle$ with $n = 4$, 6) at RHIC and LHC are still being written up.

1.3 Causal viscous hydrodynamic response to jets

In the first year and a half of the ECRP we also investigated medium response in causal Israel-Stewart hydrodynamics[33, 34] to see whether exciting azimuthal correlations observed at relative angles $\Delta\phi \approx 120^o$ at RHIC[35] could be due to sound waves[36] (Mach cone) generated by a supersonic fast quark or gluon traversing the hot and dense medium created in the collision. This work was done by graduate student Christina Moody. First we studied the hydrodynamic response to a static Gaussian energy density perturbation over a uniform background medium in spherically symmetric 1+1D geometry. We solved the evolution analytically, in the limit of small perturbations via linearizing the viscous Navier-Stokes equations for an ultrarelativistic ideal gas ($e = 3p$ equation of state) with constant η/s, and compared to numerical solutions of Israel-Stewart hydrodynamics. The latter theory also includes relaxation effects controlled by the shear stress relaxation time τ_π. We established that the Navier-Stokes result is a good approximation when τ_π is smaller than about half the value $6\eta/sT$ predicted from kinetic theory.

We followed up with a study for a Gaussian source moving at constant velocity in linearized Navier-Stokes and Israel-Stewart hydrodynamics (2+1D geometry with axial symmetry only). The primary interest was to see whether ripples generated in the medium will have sufficient time and small enough damping for Mach cones to form and survive. Very modest signals were found unless either η/s was significantly below the conjectured minimal bound[37, 38] of $1/4\pi$, or shear stress relaxation was slower than in $2 \to 2$ kinetic theory. These were still interesting possibilities because there were no reliable estimates for these quantities. Preliminary results were presented at the Fall 2010 DNP Meeting (Nov 2010). However, we chose to discontinue the investigation of viscous hydrodynamics because more and more convincing evidence came to light that these Mach-cone-like correlations likely reflect fluctuations in nucleon positions inside nuclei that give, with appreciable probability, a triangular component to the initial transverse density profile[39], which then imprints a triangular angular pattern in momentum space as the medium expands. (Cristina also switched her degree objective to M.Sc. only.)

1.4 Jet energy loss studies with realistic medium evolution from transport

During the ECRP we also investigated energy loss of high-momentum quarks and gluons (jet energy loss) as they travel through the medium created in heavy-ion collisions. These calculations were performed by graduate student Deke Sun, and often utilized covariant transport solutions to model the bulk medium for jet energy loss studies. Most notably, we calculated a large ensemble of about 600 simulated collision events for conditions expected in $Au + Au$ at $b = 8$ fm at RHIC, with fluctuating initial geometry. Surprisingly strong correlations were found between initial spatial azimuthal anisotropy coefficients $\varepsilon_n = \langle \cos \phi_x \rangle$ and final azimuthal momentum anisotropy coefficients $v_n = \langle \cos \phi_p \rangle$ ($n = 2, 3, 4$) both for the medium particles and for the jets going through the medium. Results on these correlations were presented at the Strange Quark 2013 international conference (July 2013), and reported in the Final Technical Report for the JET Collaboration.

In the last year of the ECRP, we also formulated a covariant approach to heavy quark energy loss, and studied the influence of the covariant treatment on D and B meson observables at RHIC and the LHC. Energy loss in small systems (proton-lead) collisions at the LHC were also investigated. This moderate change in scope was approved by the DOE. Results are reported in a separate Final Technical Report document for Year 5 of the ECRP.

1.5 Presentations and publications

Publications and presentations from the ECRP are listed below.

Conferences/Workshops:

1) invited talk - Denes Molnar, "Hydro paradigm and covariant transport", Workshop on Critical Assessment of RHIC Paradigms, U. of Texas, Apr 14-17, 2010, Austin, TX

2) invited talk - Denes Molnar, "Status of viscosity determination from flow at RHIC", RHIC AGS Users Meeting, Brookhaven National Laboratory, Jun 2010, Upton, NY

3) talk - Denes Molnar, "Testing viscous hydrodynamics formulations using covariant transport", Workshop on Quantifying the properties of hot and dense QCD matter, INT, Jun 14-17, 2010, Seattle, WA

4) invited talk - Denes Molnar, "Overview of hydrodynamics", Hot Quarks 2010 Workshop, Jun 20-26, 2010, La Londe-les-Maures, France

5) talk - Denes Molnar, “Viscous fluid and particles”, Midwest Theory Get-Together, Argonne Nat. Lab., Sep 10, 2010, Argonne, IL

6) invited talk - Denes Molnar, “Extracting the properties of strongly-interacting plasmas from RHIC data”, Fall 2010 APS DNP Meeting, Nov 2-6, 2010, Santa Fe, NM

7) talk - Cristina Moody, “Causal viscous hydrodynamic response to high-momentum jets”, Fall 2010 APS DNP Meeting, Nov 2-6, 2010, Santa Fe, NM

8) talk - Denes Molnar, “From a viscous fluid to particles”, 10th Zimányi Winter School on Heavy Ion Physics, KFKI/RMKI, Nov 29 - Dec 3, 2010, Budapest, Hungary

9) invited summary - Denes Molnar, “Summary and perspective”, Heavy Flavor Workshop, Purdue U., Jan 4-6, 2011, West Lafayette, IN

10) talk - Denes Molnar, “Identified particles from viscous hydrodynamics”, Quark Matter 2011 Int. Conf., May 23-28, 2011, Annecy, France

11) talk - Denes Molnar, “Realistic medium-averaging in radiative energy loss”, Hard Probes 2012 Int. Conf., May 27 - Jun 1, 2012, Cagliari, Italy

12) talk - Denes Molnar, “From (on-shell) transport to hydro and back”, NeD/TURIC Workshop 2012, Jun 25-30, 2012, Hersonissos, Greece

13) poster - Deke Sun, “Realistic medium-averaging in radiative energy loss”, Quark Matter 2012 Int. Conf., Aug 12-18, 2012, Washington DC

14) poster - Dustin Hemphill, “The Effect of $3 \leftrightarrow 2$ Rates on Thermalization in Covariant Transport“, Quark Matter 2012 Int. Conf., Aug 12-18, 2012, Washington DC

15) poster - Zack Wolff, “Viscous Corrections to Hadron Phase Space Distributions from linearized Boltzmann Equation”, Quark Matter 2012 Int. Conf., Aug 12-18, 2012, Washington DC

16) invited talk - Denes Molnar, “Hard and soft responses from parton transport”, Workshop on Jet Quenching at RHIC vs LHC in Light of Recent dAu vs pPb controls, Brookhaven National Laboratory, Apr 15-17, 2013, Upton, NY

17) invited talk - Denes Molnar, “Multiparticle reactions in a parton transport approach”, Workshop on Transport Theory in Heavy-Ion Collisions, Jul 15-17, 2013, Schmitten, Germany

18) invited talk - Zack Wolff, “Viscous corrections to the distribution function on the transition hypersurface“, Workshop on Sampling Particles on the Cooper-Frye Transition Surface, Jul 18-20, 2013, Schmitten, Germany

19) talk - Denes Molnar, “Event-by-event correlation between medium and jet flow“, Strange Quark Matter Int Conf., U. of Birmingham, Jul 21-27, 2013, Birmingham, UK

20) talk - Mridula Damodaran, ”Parallelization of Covariant Transport”, XXVIth Midwest Theory Get-Together, Argonne National Laboratory, Sep 6-7, 2013, Argonne, IL

21) talk - Zack Wolff, “Self-consistent Cooper-Frye freezeout of a viscous fluid to particles”, Winter Workshop on Nuclear Dynamics (WWND2015), Apr 2014, Galveston, TX

22) poster - Zack Wolff, “Self-consistent Cooper-Frye freeze-out of a viscous fluid to particles”, Quark Matter 2014 Int. Conf., May 19-24, 2014, Darmstadt, Germany

23) poster - Dustin Hemphill, “Radiative $ggg \leftrightarrow gg$ transport and thermalization”, Quark Matter 2015 Int. Conf., Sep 27 - Oct 3, 2015, Kobe, Japan

Seminars:

1) colloquium - Denes Molnar, “Hot, dense, and nearly perfect - a new form of matter probed in heavy-ion collisions”, Purdue U., Aug 26, 2010, West Lafayette, IN

2) Denes Molnar, “Getting particles from viscous hydrodynamics”, U. of Frankfurt, Jun 21, 2012, Frankfurt, Germany

3) Denes Molnar, “Converting a viscous fluid to particles”, McGill U., Nov 22, 2012, Montreal, Canada

4) Denes Molnar, “GLV, viscous δf, radiative transport”, Columbia U., Feb 2, 2013, New York, NY

5) Denes Molnar, “Radiative $gg \leftrightarrow$ ggg transport and thermalization”, Brookhaven National Laboratory, Oct 18, 2013, Upton, NY

6) Zack Wolff, “Self-consistent Cooper-Frye particlization of a viscous fluid and its applications to LHC heavy-ion physics”, McGill U., Apr 16, 2015, Montreal Canada

7) Mridula Damodaran, ”From Fluid to Particles in Heavy Ion Collisions”, Purdue Graduate Student Seminar, Purdue U., Mar 30, 2016, West Lafayette, IN

Refereed publications from ECRP:

1) D. Molnar, “Identified particles from viscous hydrodynamics,” J. Phys. G **38**, 124173 (2011) [arXiv:1107.5860 [nucl-th]].

2) D. Molnar and Z. Wolff, “Self-consistent conversion of a viscous fluid to particles,” arXiv:1404.7850 [nucl-th], to appear in Phys. Rev. C

3) Z. Wolff and D. Molnar, “Self-consistent Cooper-Frye freeze-out of a viscous fluid to particles,” J. Phys. Conf. Ser. **535**, 012020 (2014) [arXiv:1407.6413 [nucl-th]].

Other publications from ECRP:

4) Z. Wolff, “Self-consistent conversion of a viscous fluid to particles and heavy-ion physics applications” (Ph.D. thesis), Purdue University, Dec 2015

5) D. Hemphill, “Radiative transport in heavy-ion collisions” (Ph.D. thesis), Purdue University, in preparation (exp. Aug 2016)

6) D. Molnar, “Generating random thermal momenta,” arXiv:1212.1853 [nucl-th]

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