

# Correlations and photon statistics in nanocavity emitters

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**Motivation for nanocavity emitters**

**Modeling approaches**

**Research:**

**a)  $\beta = 1$  and thresholdless lasing**

**b) Single-photon sources and photon statistics**

**Thanks to:**

**Christopher Gies and Frank Jahnke, Bremen University**

**Sandia's Laboratory Directed Research & Development (LDRD) Program**

# Why nano-emitter research?

## ① Save energy

Data centers and optical communication

- Reducing energy per bit
- At limits for electrical approaches
- Optical interconnects: laser energy consumption  $\propto$  volume

## ② Safe communication

- Quantum key distribution (QKD)
- Single-photon sources
- Types of light

Laser (random)



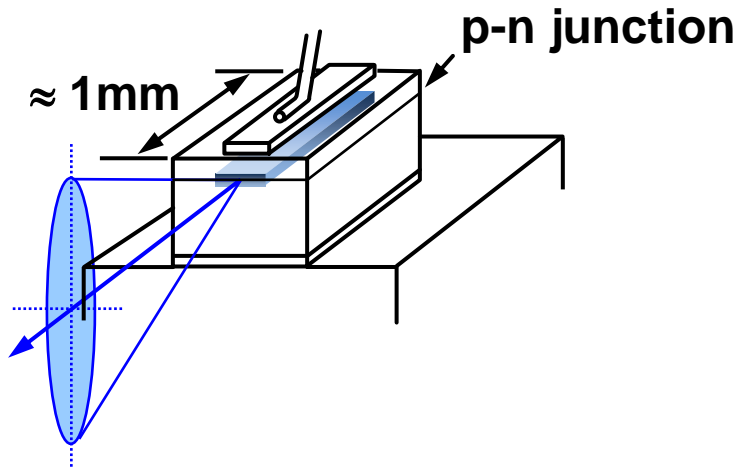
Single-photon (antibunched)



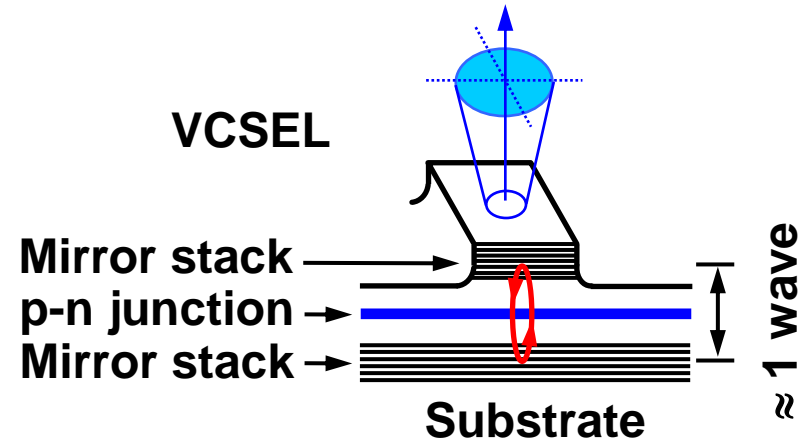
Time

# Towards smaller and smaller lasers

## Edge - Emitting Laser

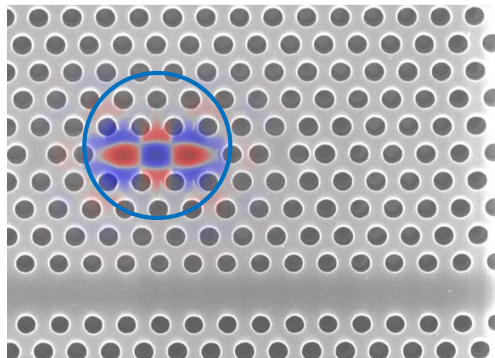


## Vertical-Cavity Surface-Emitting Laser



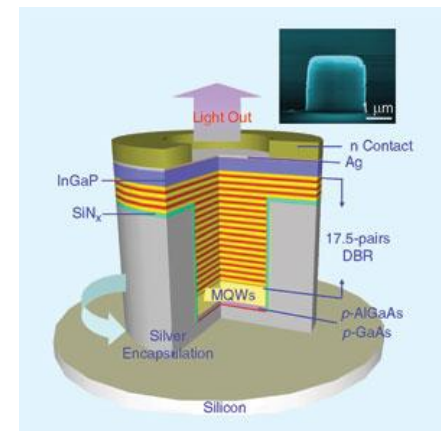
## Nanolasers

### Photonic crystal



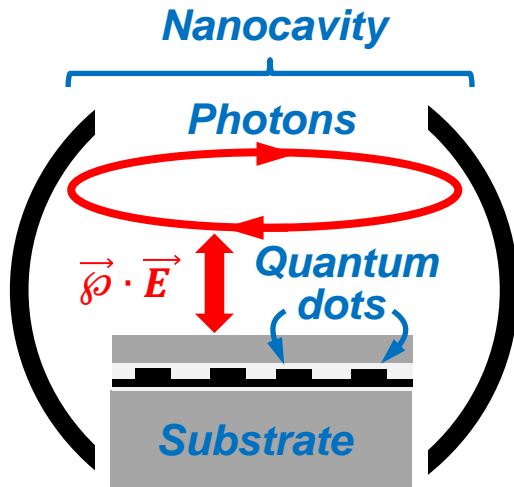
(Courtesy of Willie Luk, Sandia National Labs)

### Nanocavity

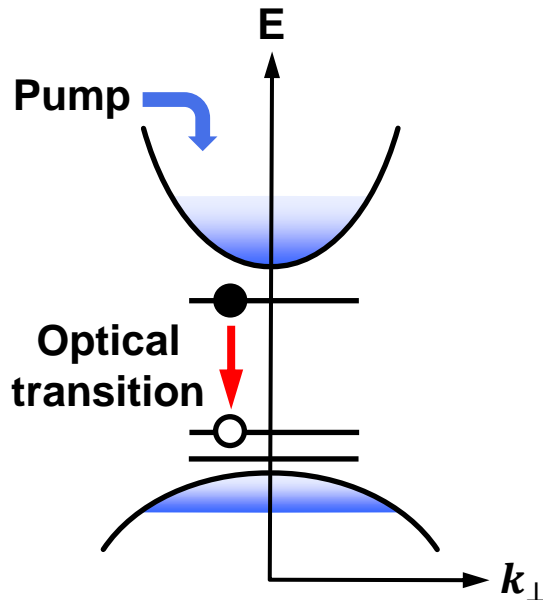


(Adapted from a figure by Lu et al., UIUC)

# Hamiltonian: physics entering into the theory



Electronic structure



$$H = \sum_{\alpha} \epsilon_{\alpha}^e c_{\alpha}^{\dagger} c_{\alpha} + \sum_{\beta} \epsilon_{\beta}^h b_{\beta}^{\dagger} b_{\beta} + \hbar \omega \left( a^{\dagger} a + \frac{1}{2} \right) \quad \text{Single-particle}$$

$$- \hbar \sum_{\alpha} (g_{\alpha} b_{\alpha}^{\dagger} c_{\alpha}^{\dagger} a - g_{\alpha}^{*} a^{\dagger} c_{\alpha} b_{\alpha}) \quad \text{Light-carrier interaction}$$

$$g_{\alpha} = \sqrt{\frac{v}{\hbar \epsilon_b V}} W(R_{QD}) \sum_n c_{\alpha}(R_n) V_{\alpha}(R_n)$$

$$\left. \begin{aligned} &+ \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} c_{\sigma} + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} b_{\beta}^{\dagger} b_{\eta} b_{\sigma} \\ &- \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\eta} b_{\sigma} \end{aligned} \right\} \quad \text{Carrier-carrier}$$

Matrix element of  $\frac{e^2}{4\pi\epsilon_b|r-r'|}$

$$+ \hbar \sum_{\alpha\beta q} G_q (c_{\alpha}^{\dagger} c_{\beta} + b_{\alpha}^{\dagger} b_{\beta}) (d_q + d_q^{\dagger}) \quad \text{Carrier-phonon}$$

# Dynamical behavior of a quantum system

## Schrödinger Picture

$$\frac{\partial}{\partial t} \varrho = -\frac{i}{\hbar} [H, \varrho]$$

**Density operator:**  $\sum_{\psi} P_{\psi} |\psi\rangle\langle\psi|$

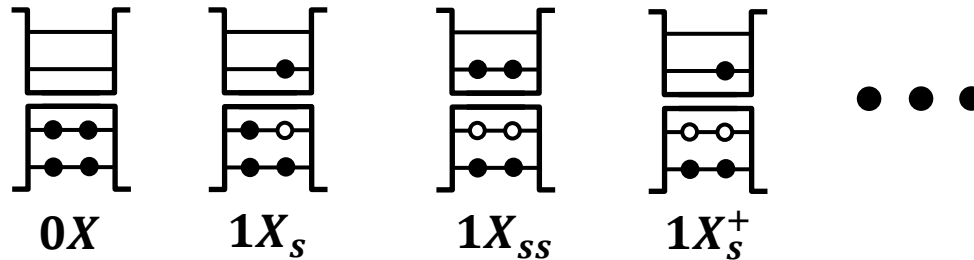
**Wave function:**  $\psi(r) \equiv \langle r | \psi \rangle$

**Quantum-dot states (configurations)**

**Basis:**

$|j, n\rangle$

Photon  
number



**Observable**

**Expectation values:**  $\langle A \rangle = \text{Tr}\{\varrho A\} = \sum_{n,j,n',j'} \langle n, j | \varrho | j', n' \rangle \langle n', j' | A | j, n \rangle$

**Photon statistics:**  $P_n = \langle n, 0X | \varrho | 0X, n \rangle + \langle n, 1X_s | \varrho | 1X_s, n \rangle + \langle n, 2X_{ss} | \varrho | 2X_{ss}, n \rangle + \dots$

# Dynamical behavior of a quantum system

## Heisenberg Picture

System operator

$$\frac{\partial}{\partial t} A = -\frac{i}{\hbar} [A, H]$$

Populations and correlations:

$$\langle c_{\alpha}^{\dagger} c_{\alpha} \rangle, \langle b_{\alpha}^{\dagger} b_{\alpha} \rangle, \langle c_{\alpha}^{\dagger} b_{\alpha}^{\dagger} a \rangle, \langle a^{\dagger} a \rangle$$

$$\langle a^{\dagger} a^{\dagger} a a \rangle, \langle c_{\alpha}^{\dagger} c_{\sigma}^{\dagger} c_{\sigma} c_{\alpha} \rangle, \langle c_{\alpha}^{\dagger} c_{\alpha} a^{\dagger} a \rangle$$

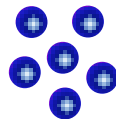
Single  
particles

Correlated  
pairs

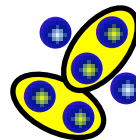
Correlated  
3-particle clusters

Cluster expansion:

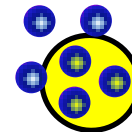
$$\langle \hat{N} \rangle =$$



+

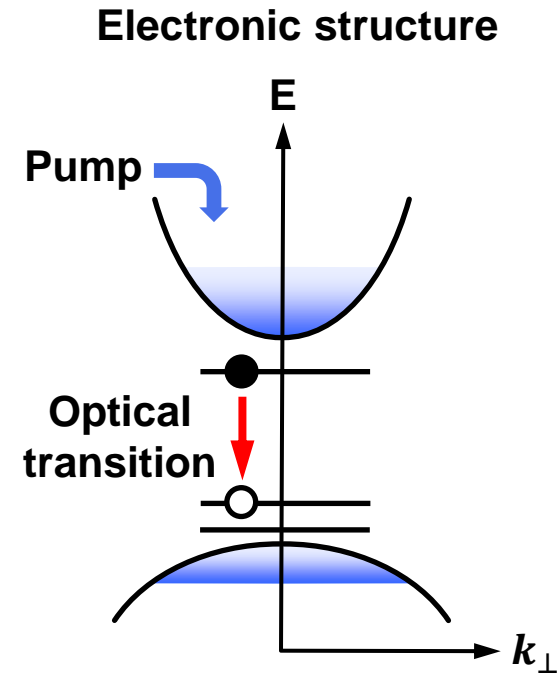
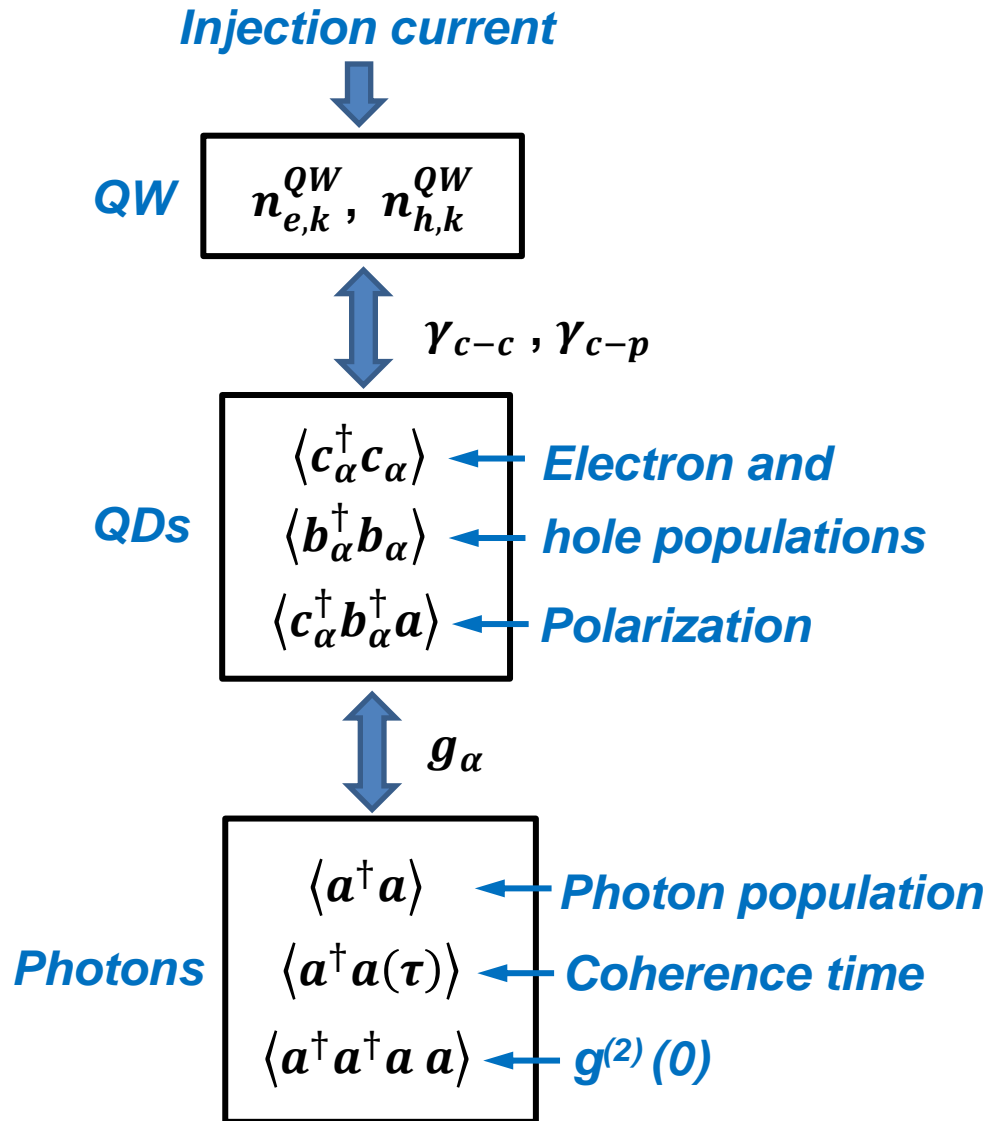


+



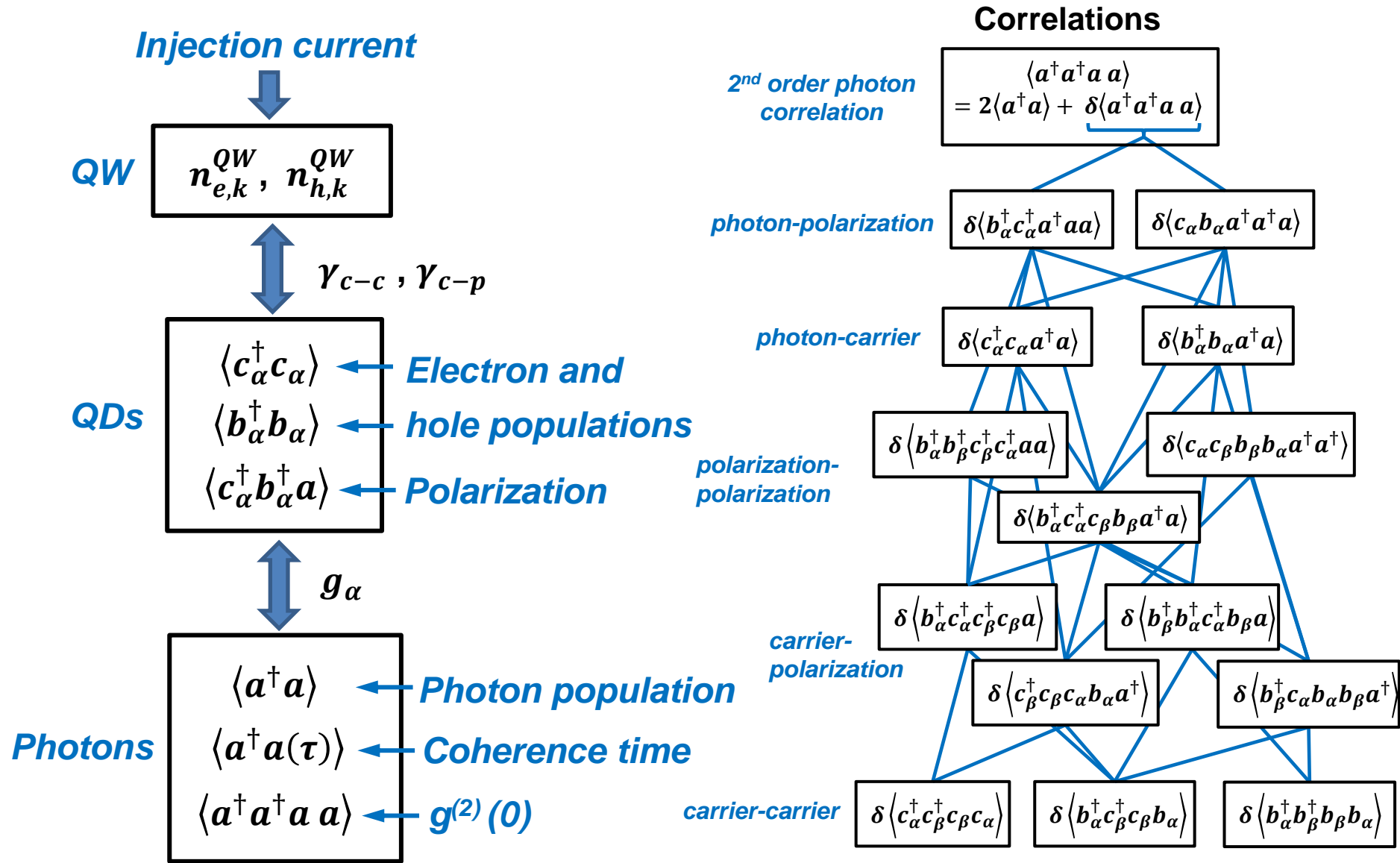
+ ...

# Nano-emitter model: population dynamics and correlations



For now emphasis is on correlations involving light-matter interaction instead of Coulomb interaction

# Nano-emitter model: population dynamics and correlations

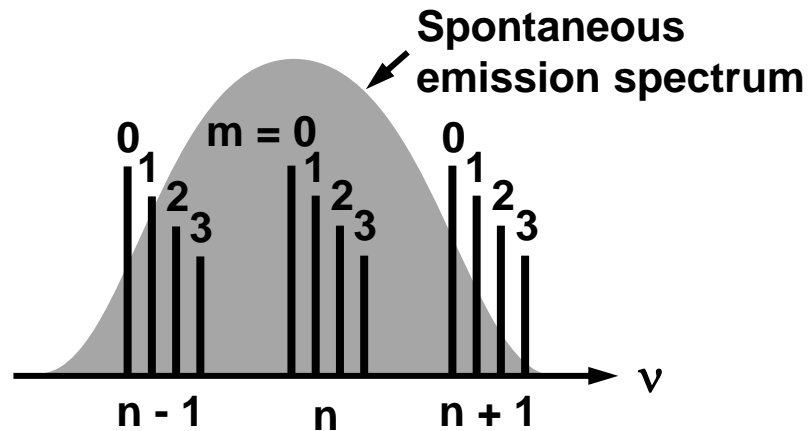




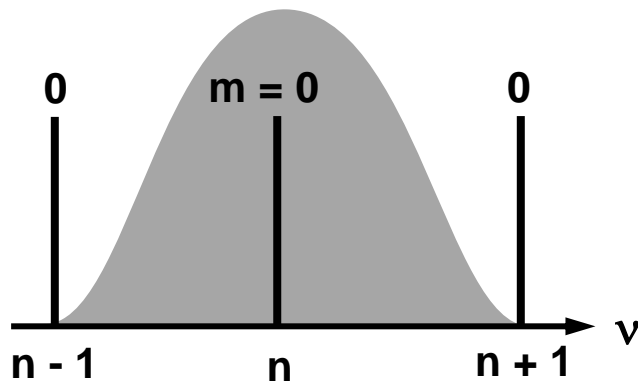
# Interesting physics with nanolasers

## Example 1: Laser threshold and thresholdless lasing

Most lasers  $\beta \ll 1$



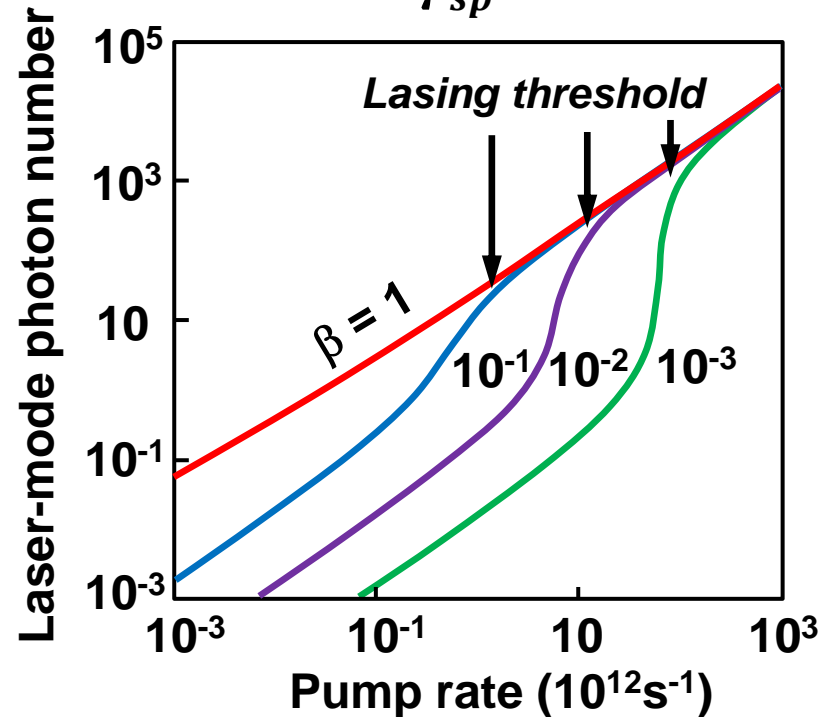
Some nanolasers  $\beta = 1$



**All emission into single resonator mode**

Spontaneous emission factor

$$\beta = \frac{\gamma_l}{\gamma_{sp}}$$



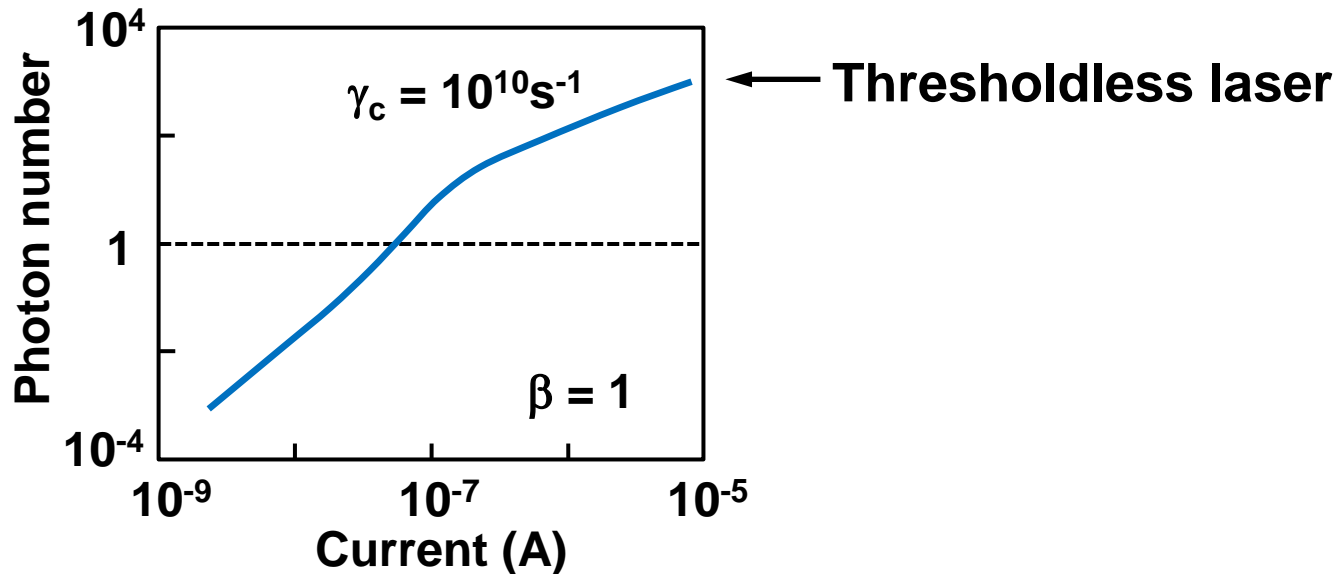
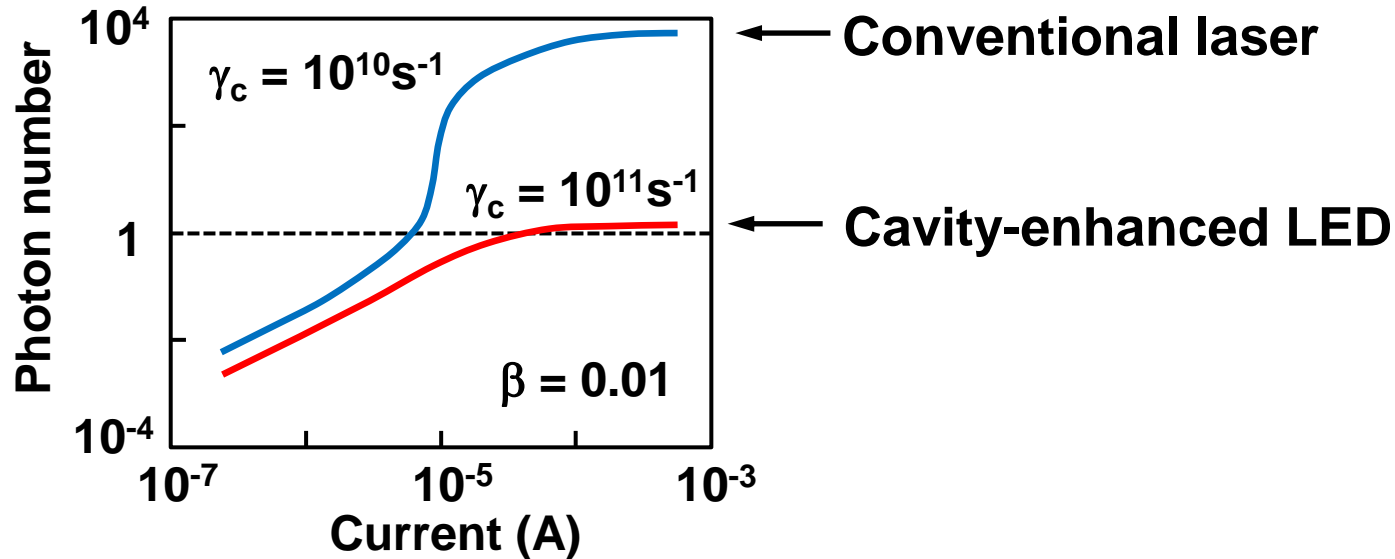
Questions:

- 1) Is thresholdless lasing real?
- 2) What is lasing?

# Criterion for lasing

$$N_{\text{QD}} = 50, \Delta_{\text{inh}} = 20\text{meV}$$

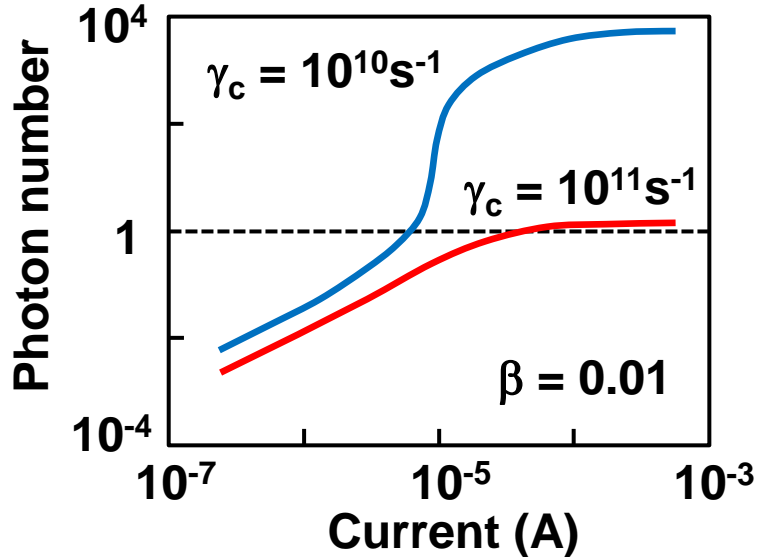
## Input/Output



# Criterion for lasing

$$N_{\text{QD}} = 50, \Delta_{\text{inh}} = 20\text{meV}$$

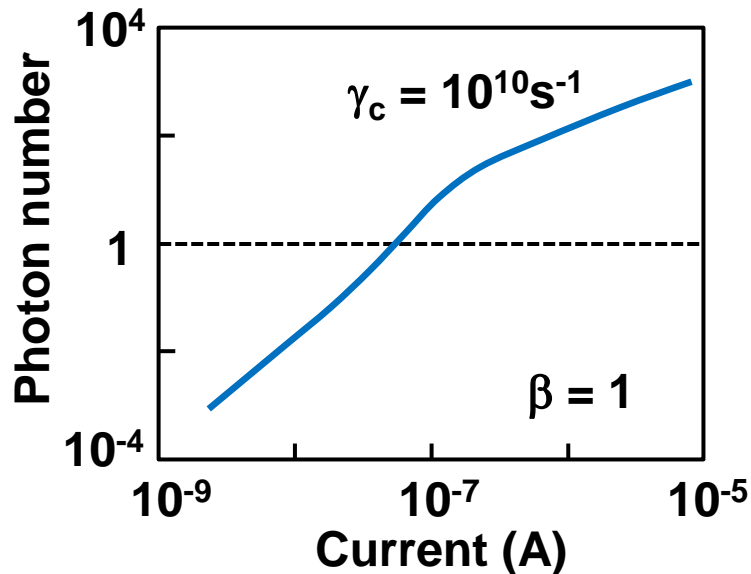
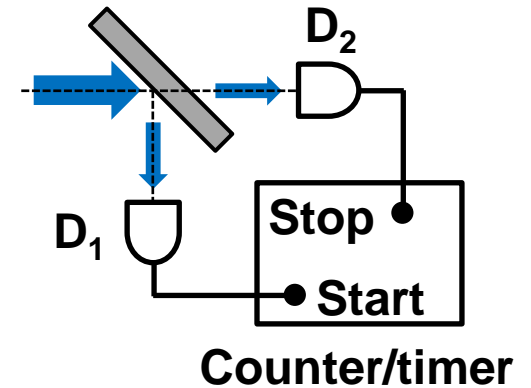
## Input/Output



## Second-order intensity correlation function

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$

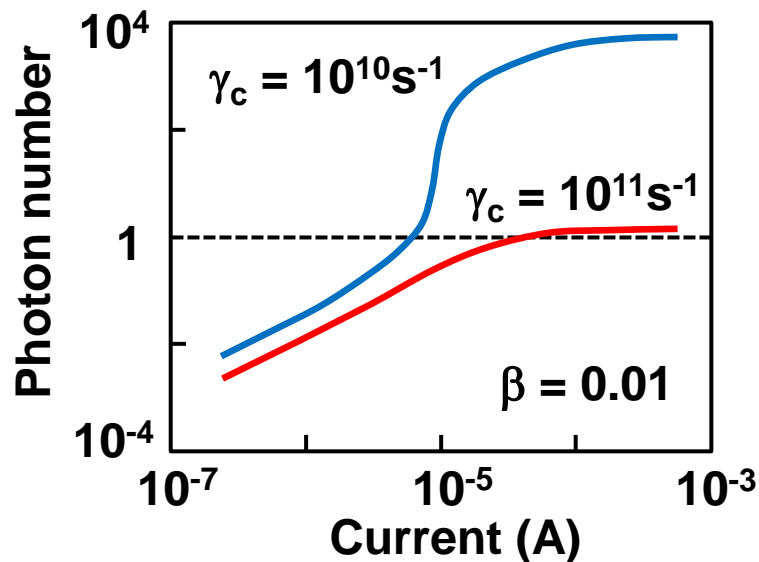
## Hanbury-Brown-Twiss experiment



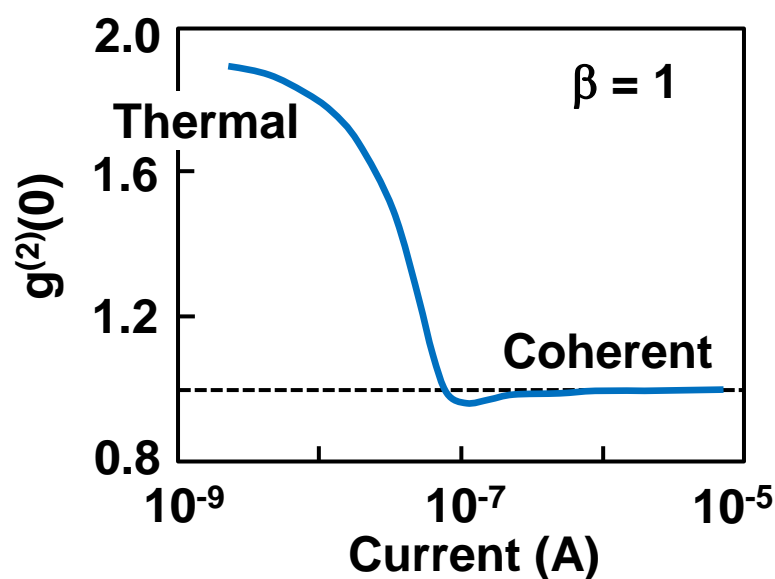
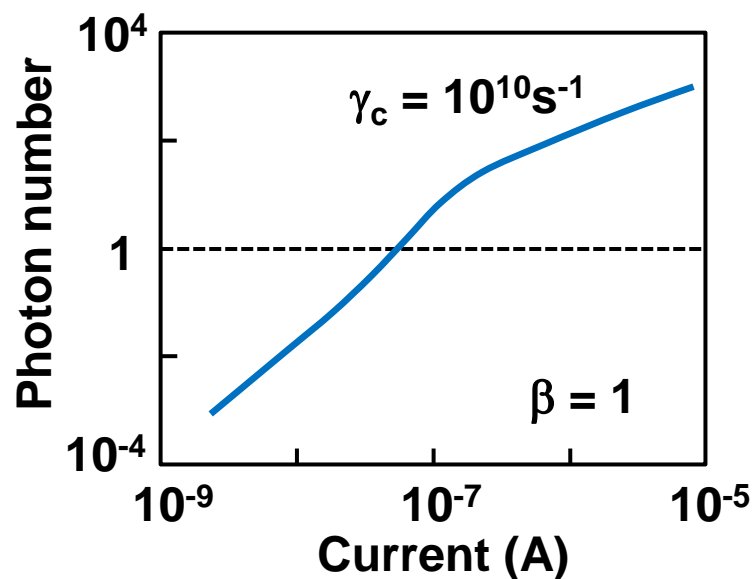
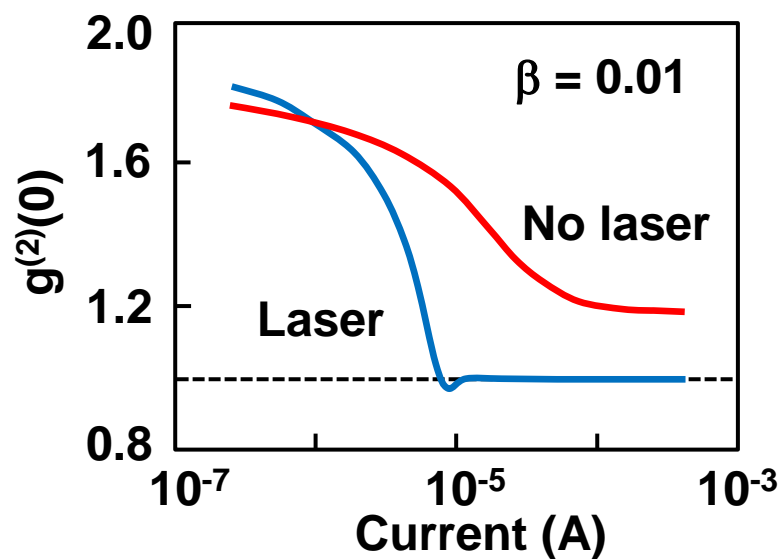
# Criterion for lasing: $g^{(2)}(0)$

$$N_{\text{QD}} = 50, \Delta_{\text{inh}} = 20\text{meV}$$

Input/Output



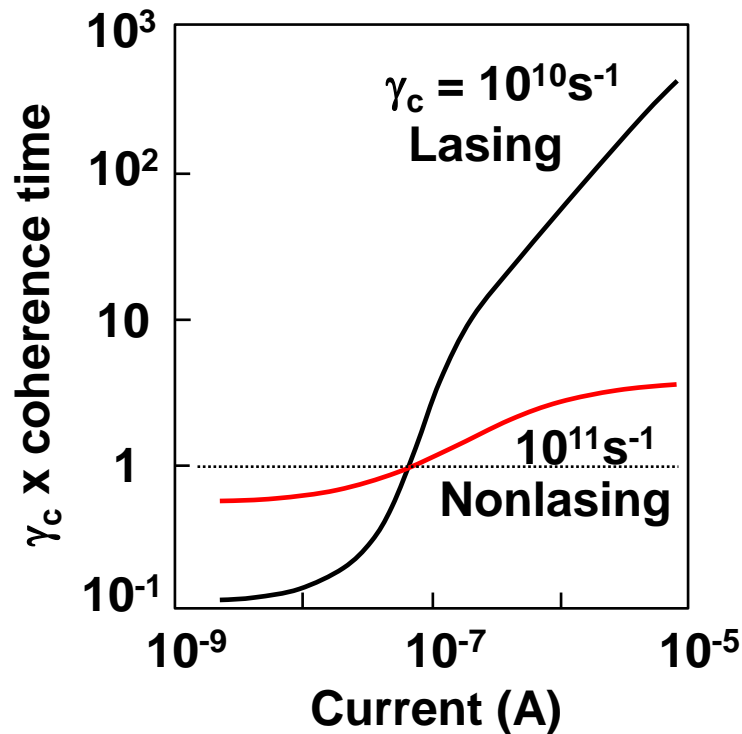
Photon correlation



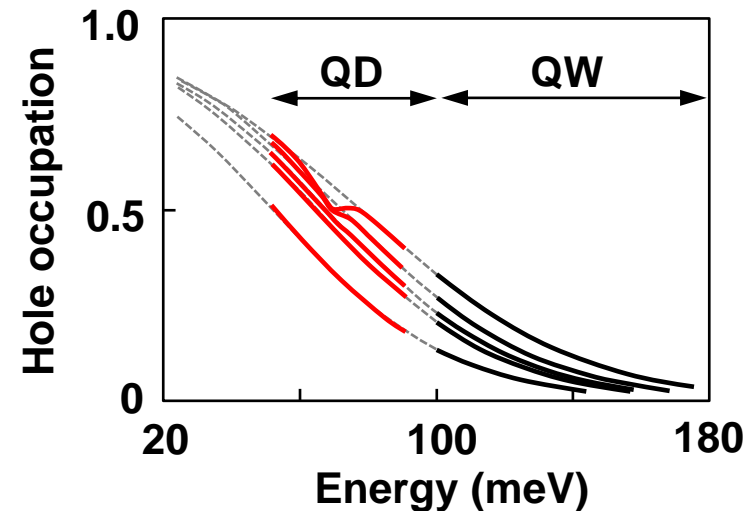
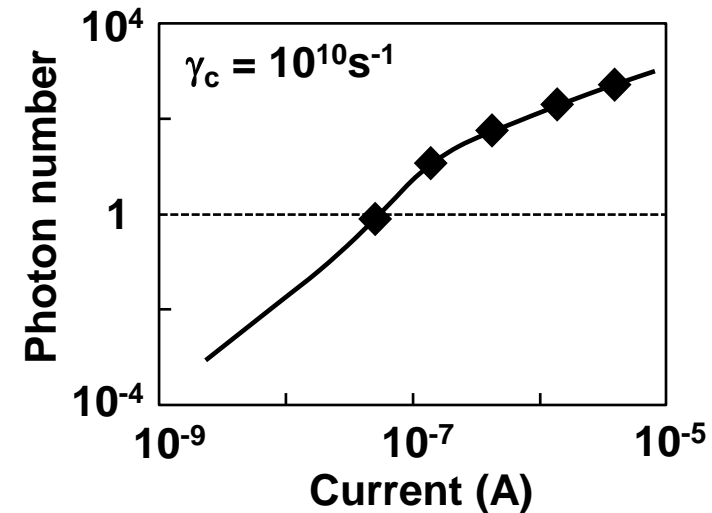
# Other criteria for lasing

## Coherence time

$$\tau_c = 2 \int_{-\infty}^{\infty} d\tau \left| \frac{\langle a^\dagger a(\tau) \rangle_{ss}}{\langle a^\dagger a \rangle_{ss}} \right|^2$$

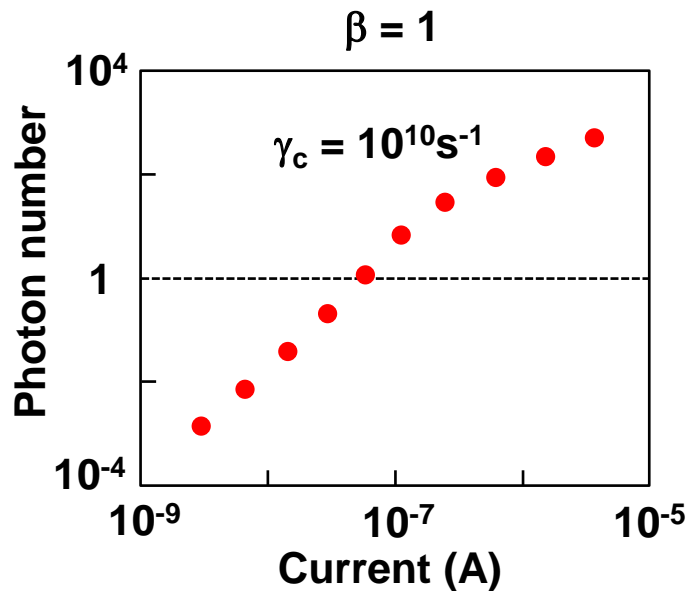
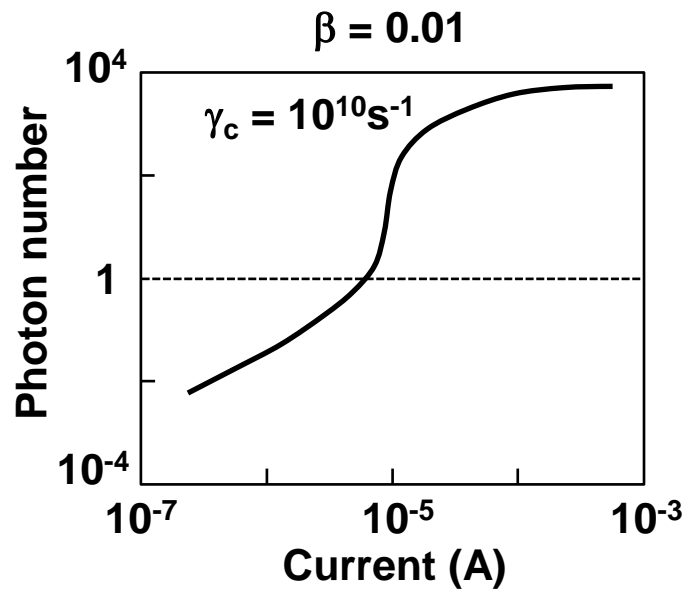


## Population clamping and hole burning



$$\beta = 1, N_{\text{QD}} = 50, \Delta_{\text{inh}} = 20 \text{ meV}$$

# Other criteria for laser: stimulated emission



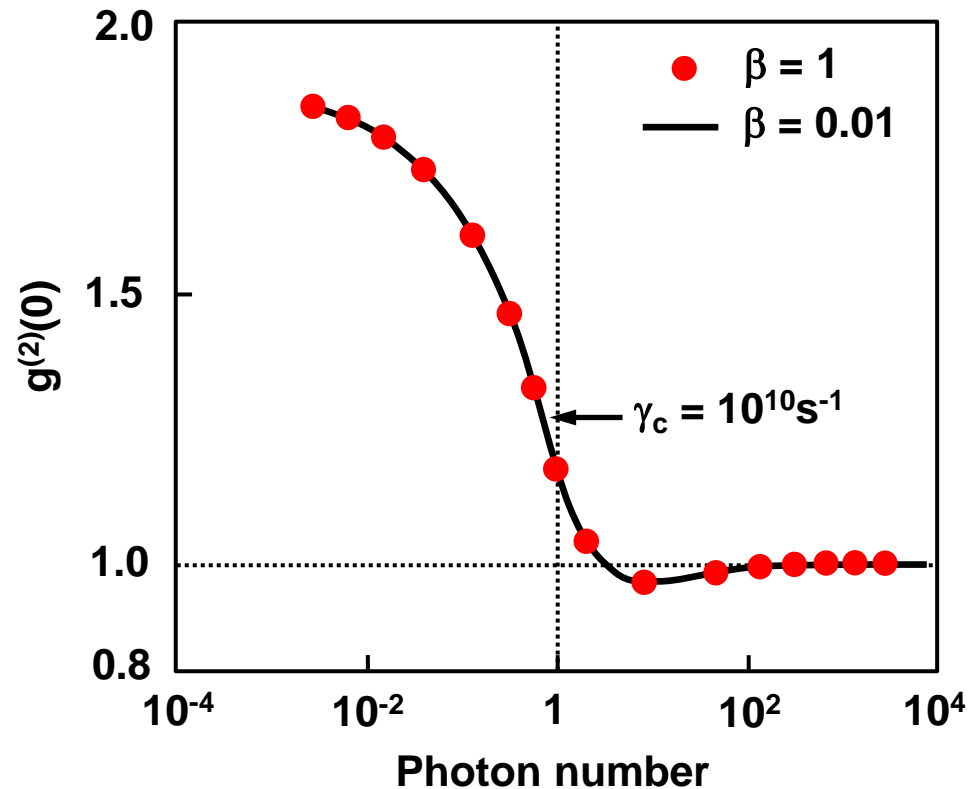
$N_{\text{QD}} = 50, \Delta_{\text{inh}} = 20 \text{ meV}$

↓  
Light amplification by stimulated emission of radiation

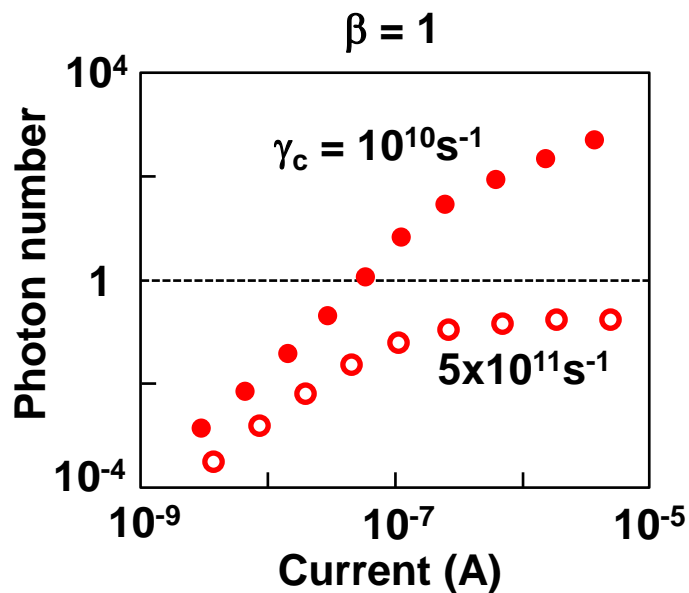
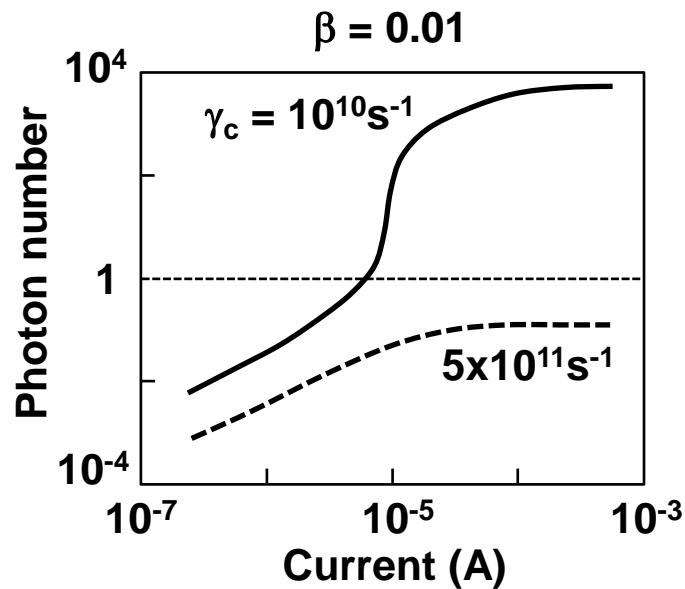
$$\frac{dP_a}{dt} = -\gamma_l(n + 1)$$

Stimulated emission

Spontaneous emission



# Other criteria for laser: stimulated emission



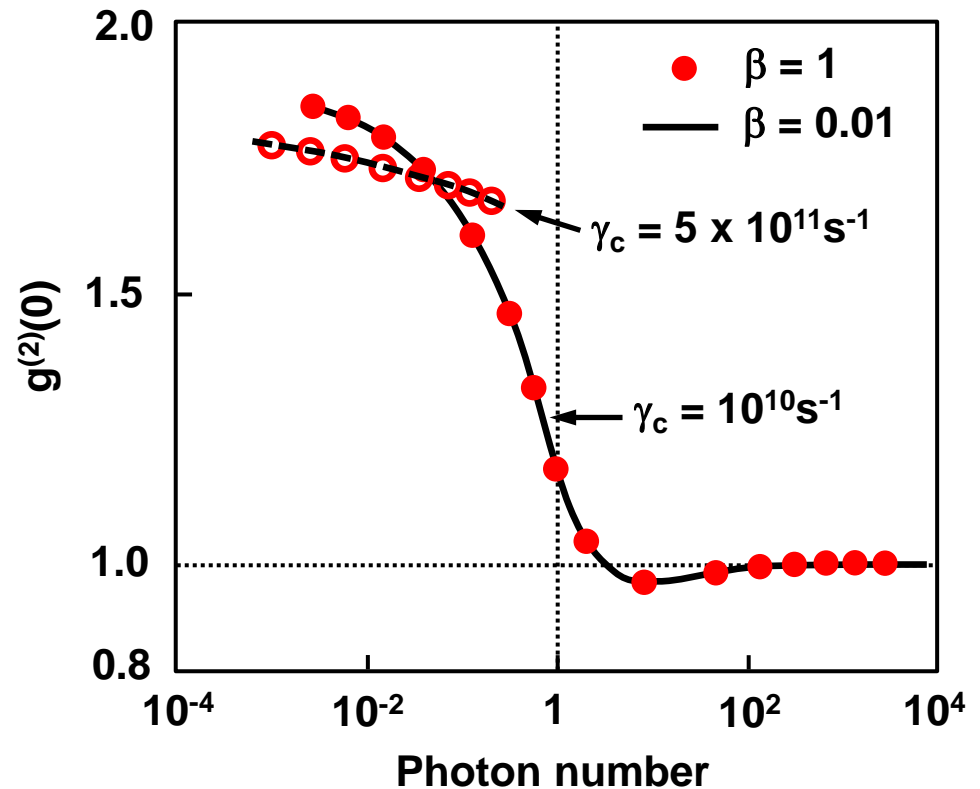
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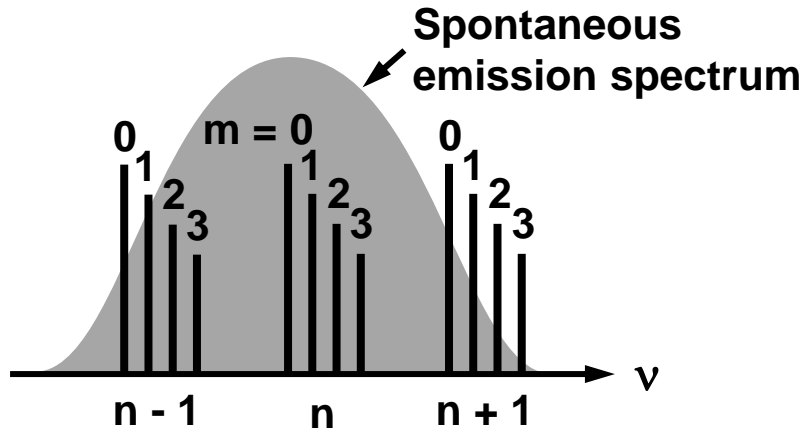


# Interesting physics with nanolasers

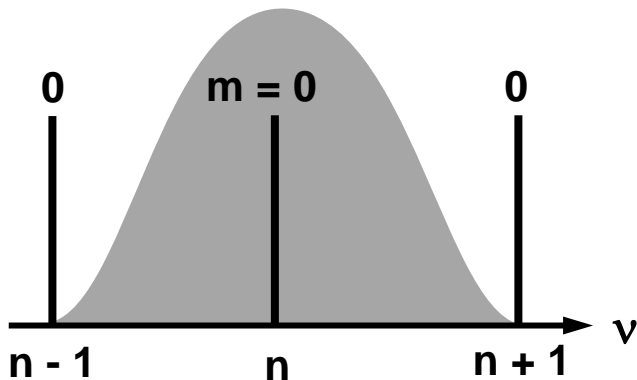
## Example 1

### Thresholdless lasing

Most lasers  $\beta \ll 1$



Some nanolasers  $\beta = 1$

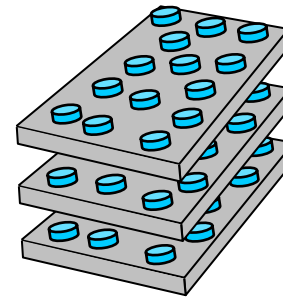


**All emission into single resonator mode**

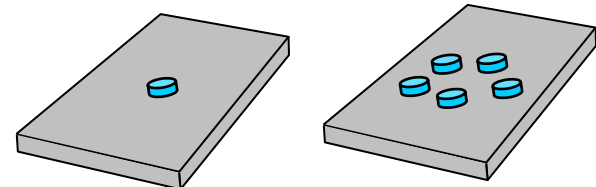
## Example 2

### Single-photon generation

Most QD-laser active regions



Few- QD active regions



**Nonclassical light**

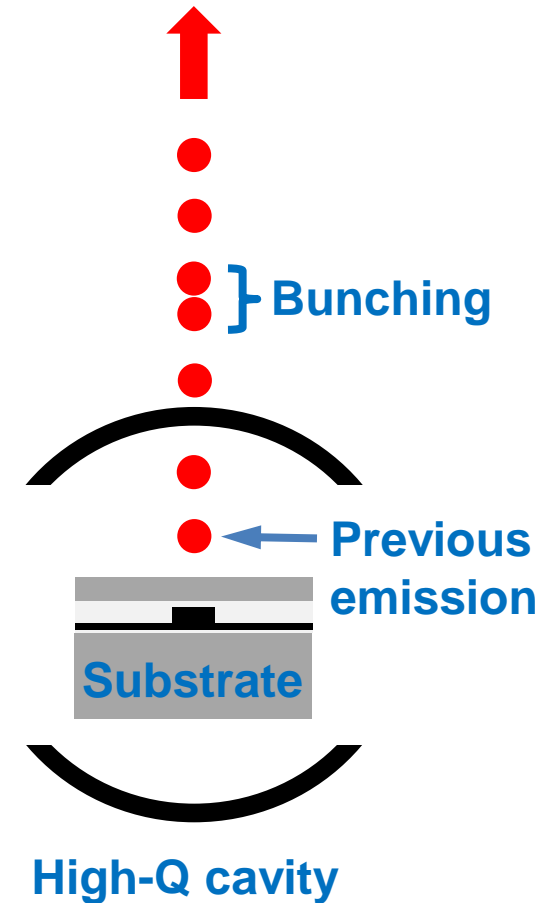
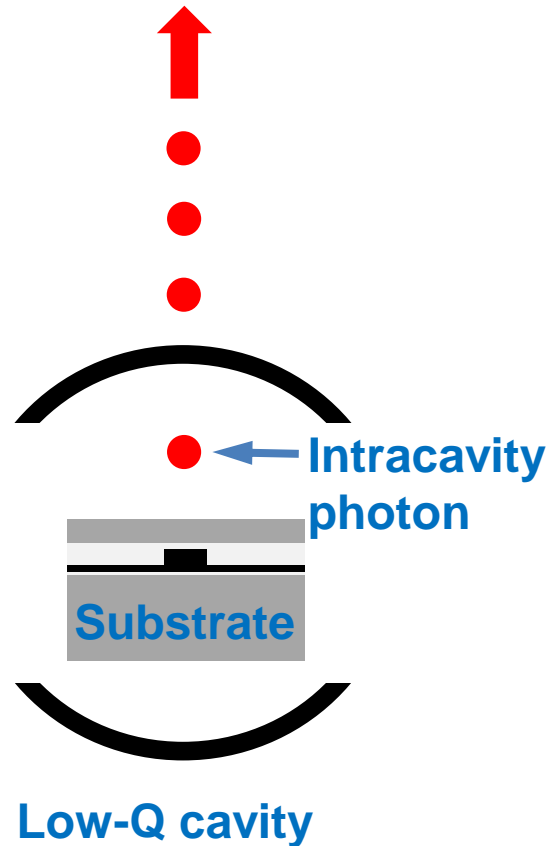
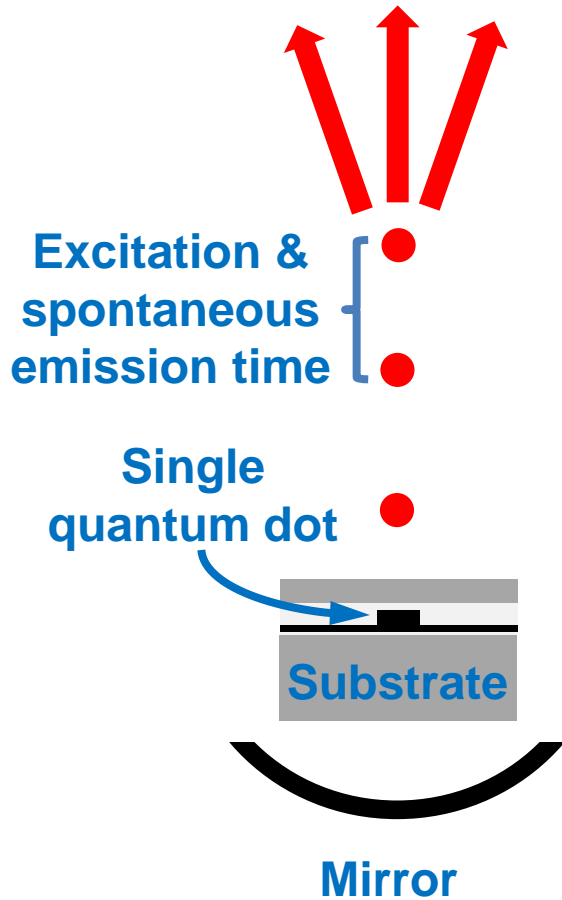


# Single-photon source

Error-free but slow

Cavity enhancement:  
Directionality and Purcell

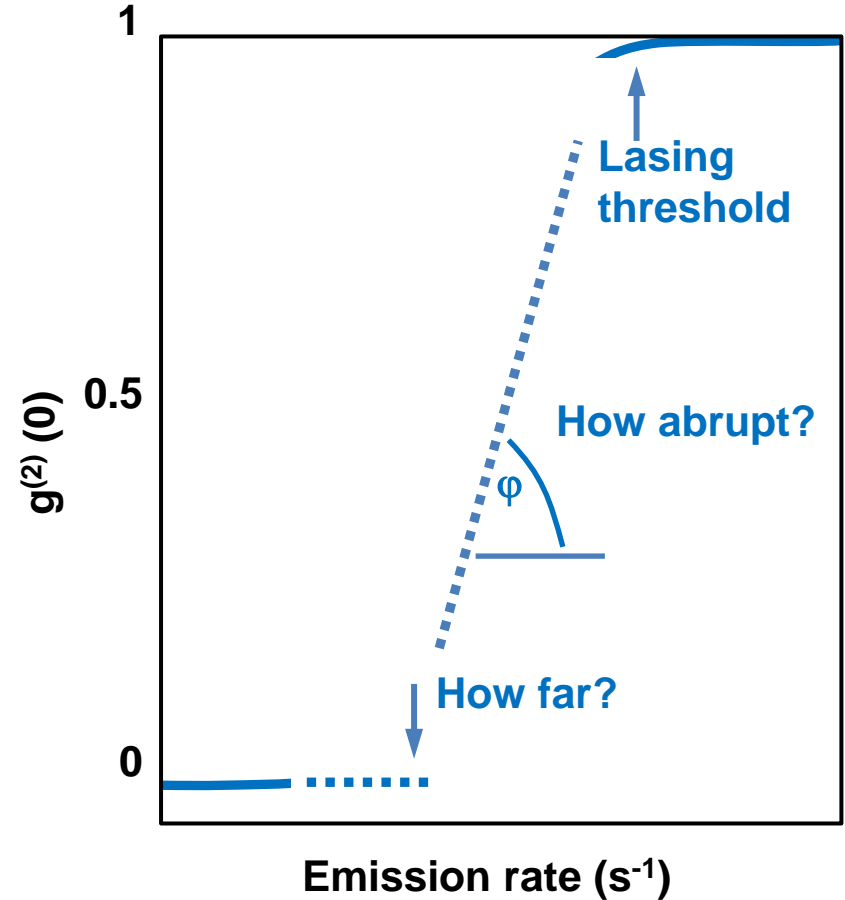
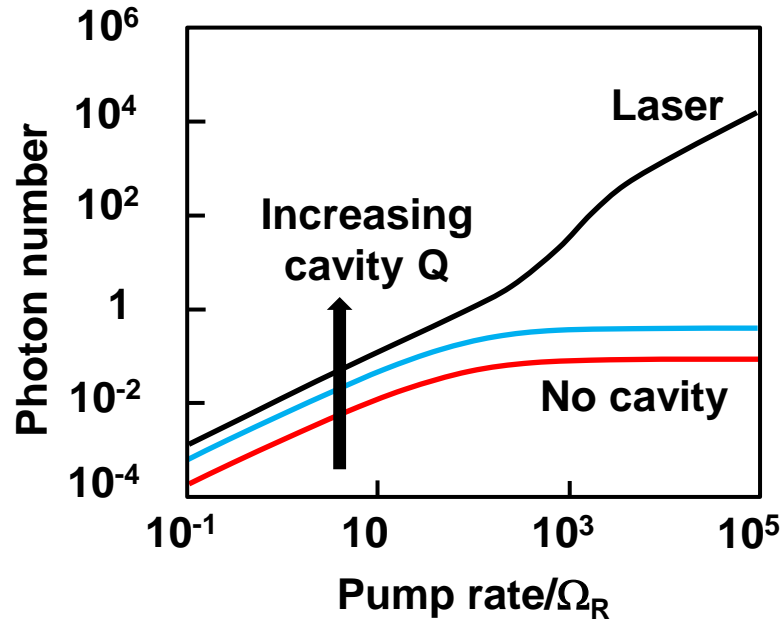
Too much cavity



What is the right Q?

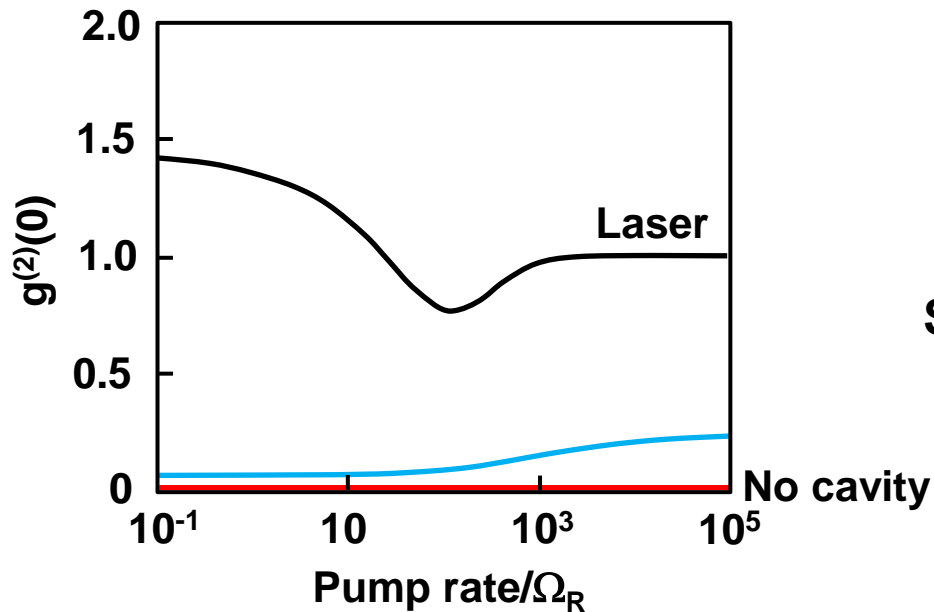
Fundamental limit to efficiency, rate and error?

# Simulations



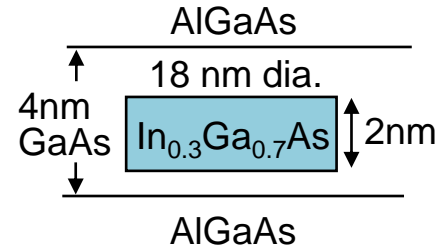
Second-order intensity correlation

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# Single-photon purity and emission rate

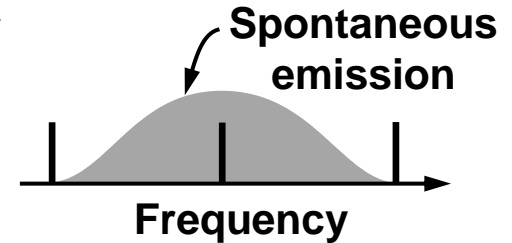
## ① Shallow quantum dot



*Only s-shell transition*

## ② Nanocavity

$$\beta = \frac{\gamma_l}{\gamma_{sp}} = 1$$

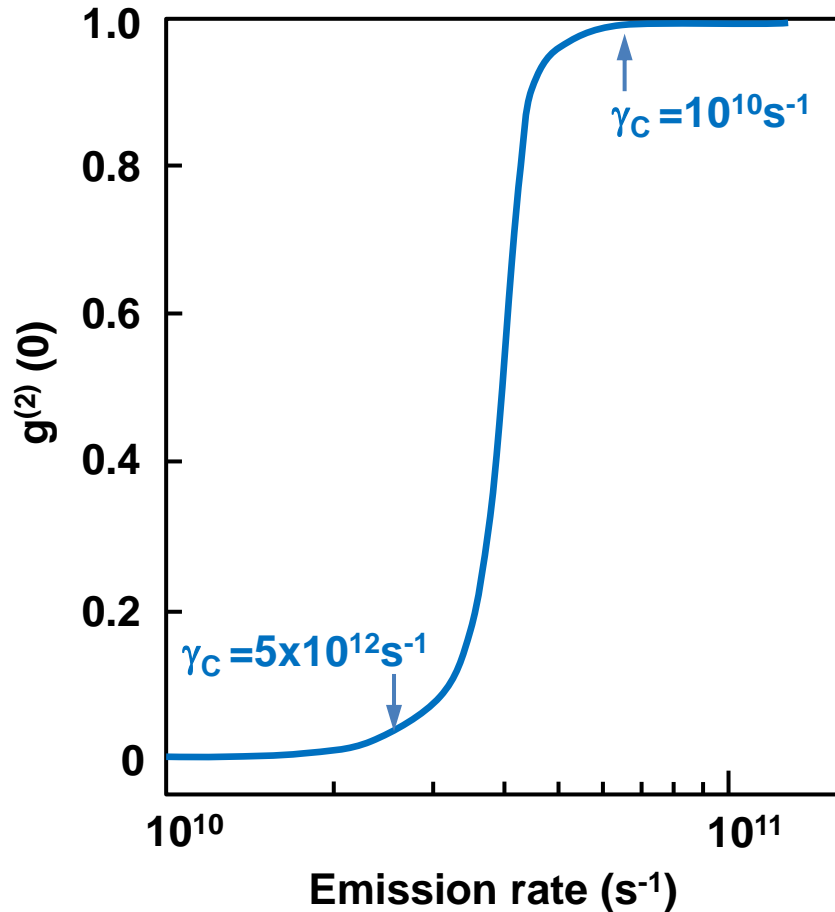


*All emission into single resonator mode*

## ③ Scaling with electron-light coupling

$$\underbrace{\frac{1}{\sqrt{\hbar \epsilon_b V}}}_{\text{Mode volume}} \underbrace{W(R_{QD})}_{\text{Confinement factor}} \underbrace{\sum_n C(R_n) V(R_n)}_{\text{Electron-hole envelope overlap}}$$

$g^{(2)}(0)$  vs. emission rate  
(by increasing cavity-Q)

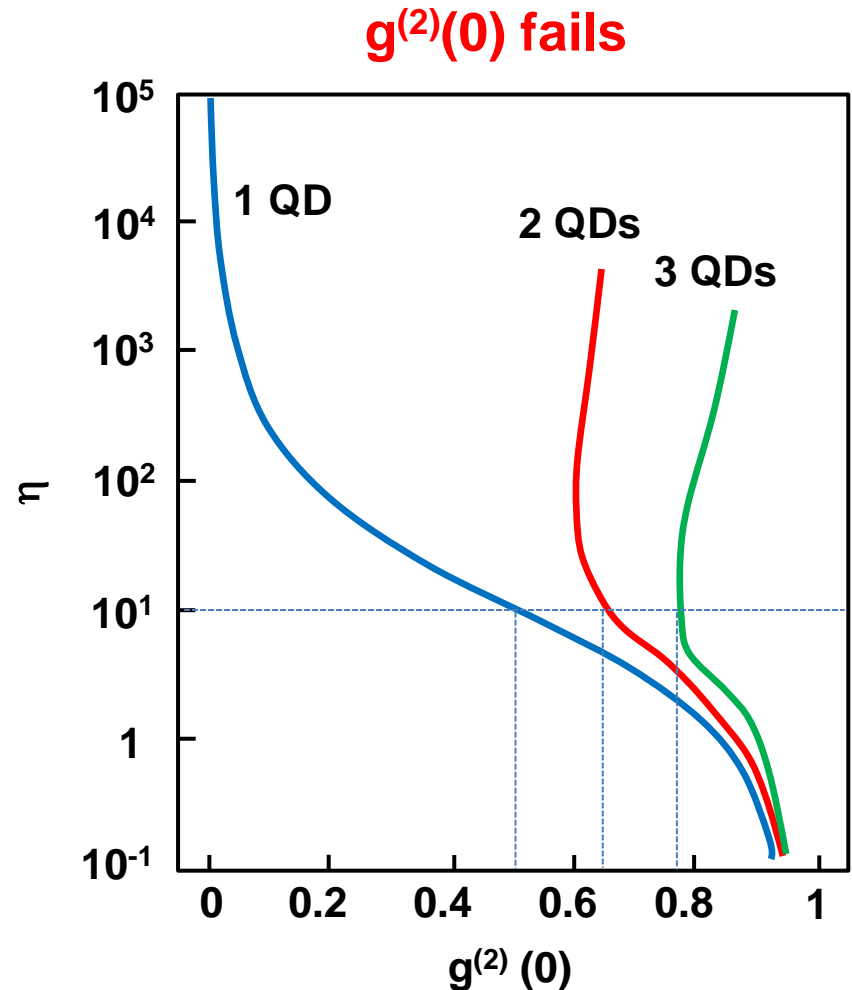
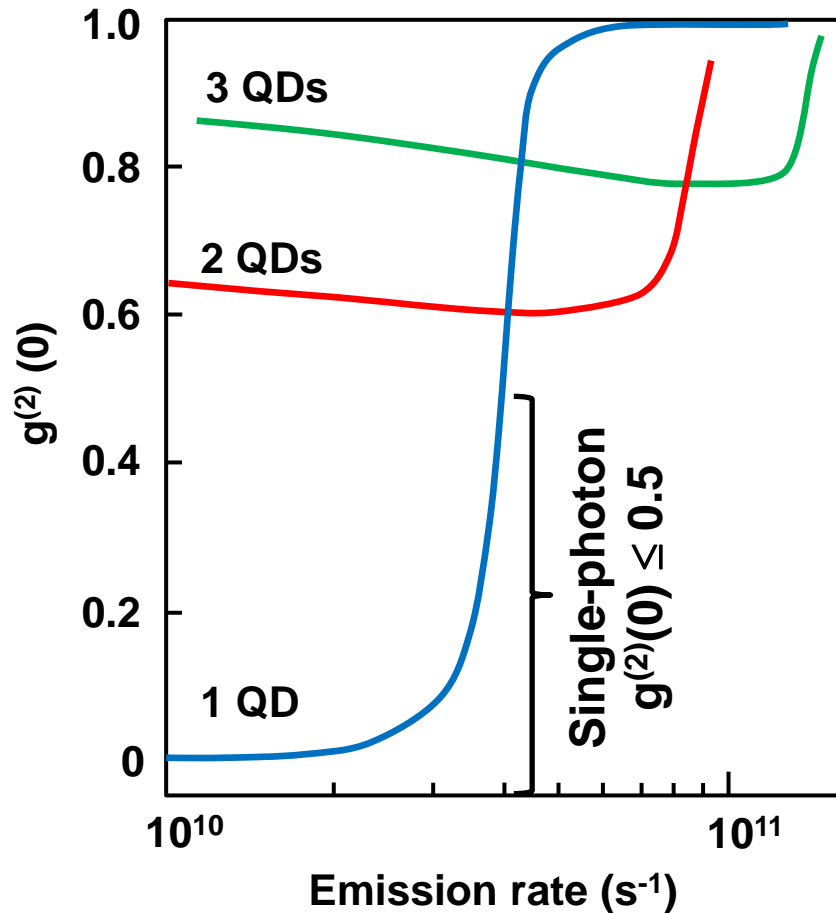


Cavity-enhanced rate  $\sim 10^9 \text{ s}^{-1}$  (expt)

# Concern: $g^{(2)}(0)$ as measure of error

Single-photon purity:  $\eta = \frac{\text{Single-photon emission probability}}{\text{Multi-photon emission probability}}$

$g^{(2)}(0)$  vs. emission rate  
(by increasing cavity-Q)

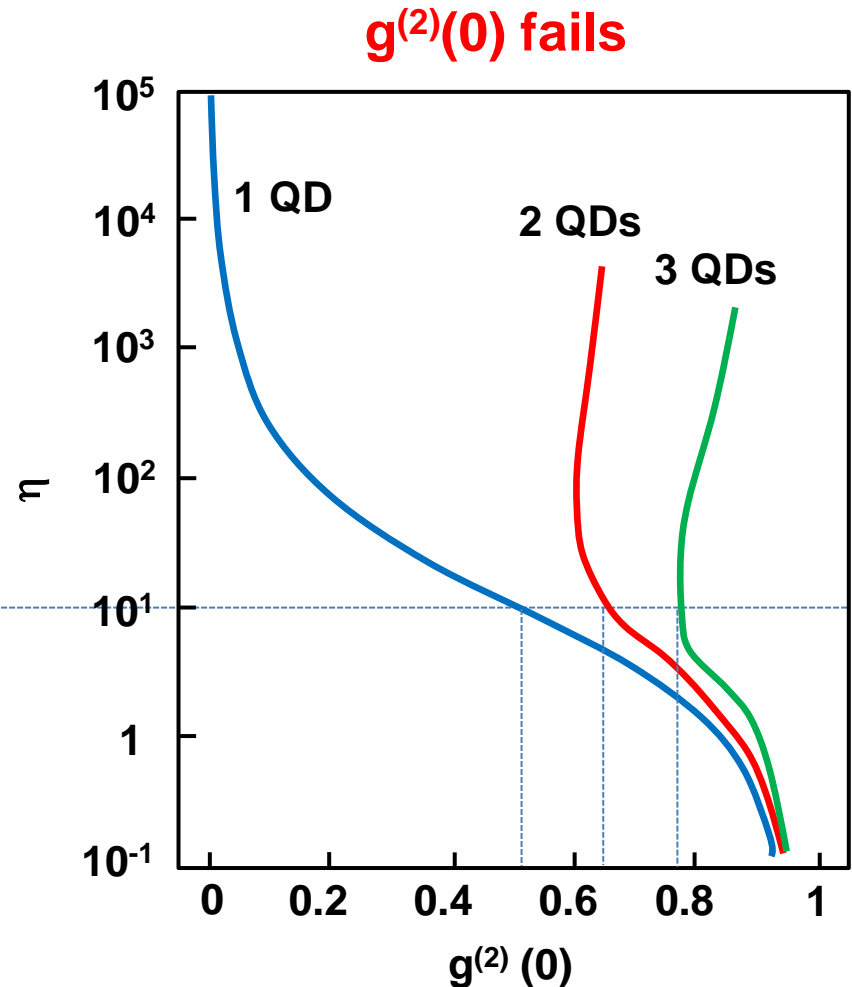
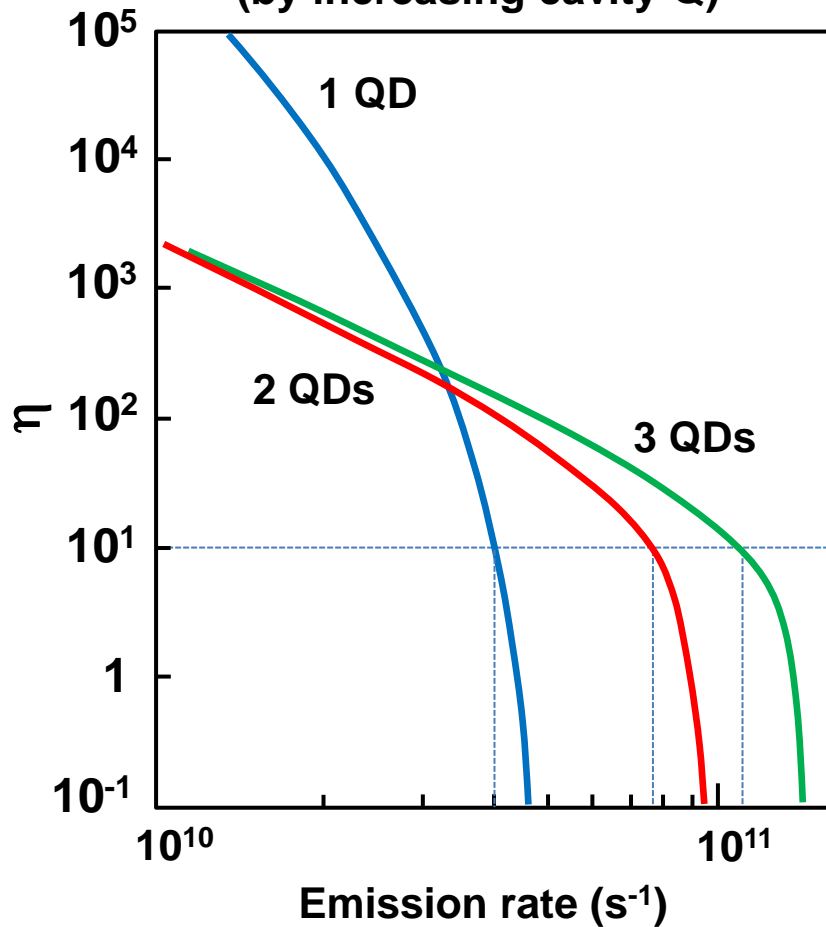


From calculating photon statistics  
Gies, Jahnke, Chow (submitted)

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Purity vs. emission rate  
(by increasing cavity-Q)

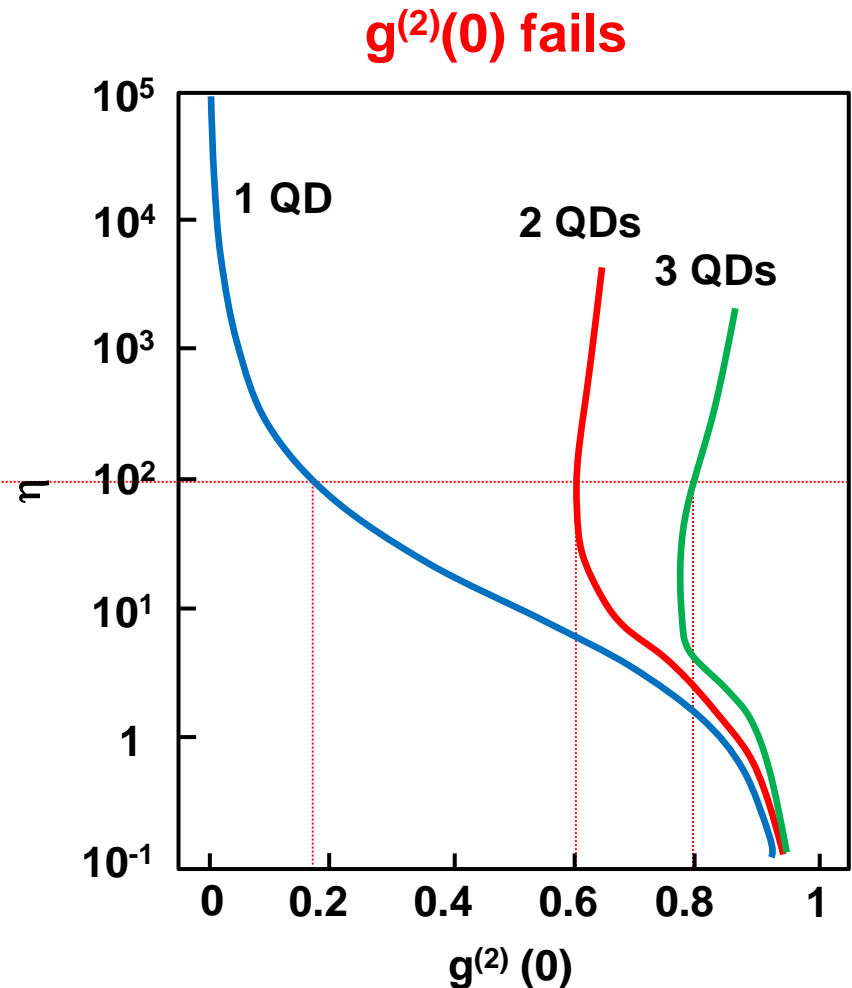
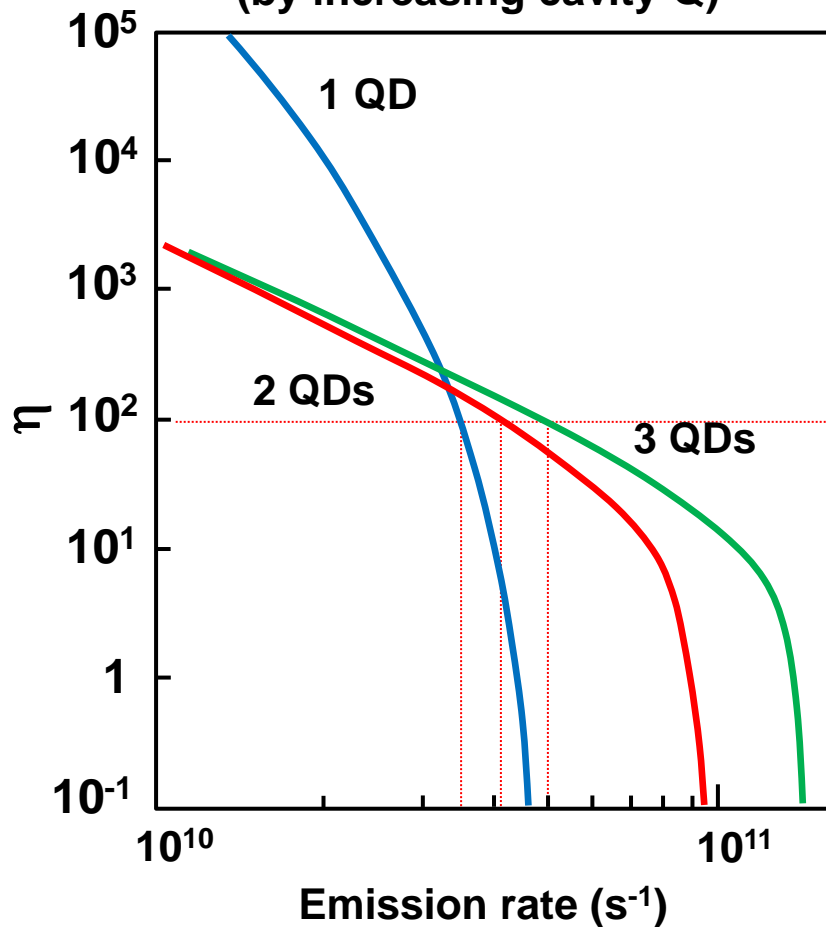


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From calculating photon statistics  
Gies, Jahnke, Chow (submitted)

# Correlations and photon statistics in nanocavity emitters

## Approach

- Quantized light and carriers
- Consistent account of light-carrier correlations

## Nanolasers

- Combination of intensity &  $g^{(2)}(0)$  gives definitive description of lasing
- There is no thresholdless lasing


## Single-photon sources

### Quantum communications

Disconnect:

Ideal  
Single-photon

Applications  
Dimmed laser

$$|n\rangle \quad e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad |\alpha|^2 \ll 1$$


Bridge: tradeoff among efficiency, rate and error

- Challenges in fabrication and modeling
- Questions concerning present measure of performance

# Other applications of modeling approach

## Gain medium engineering

Chow, Lorke & Jahnke, 'Will Quantum Dots Replace Quantum Wells As the Active Medium of Choice in Future Semiconductor Lasers?' IEEE J. Selected Topics in Quantum Electron. **17**, 1349 (2011).

**Frank Jahnke, Bremen University**

Liu, Chow, Gossard, Bowers, "Extraction of inhomogeneous broadening and nonradiative losses in InAs quantum-dot lasers,' (in preparation)

**John Bowers, UC Santa Barbara**

## Solid state lighting

Chow, *Novel LED Model Offers New Insights*, Compound Semiconductor Magazine, July, 2014.

**EFRC**

## Quantum optomechanics

Carmelet, Kabuss & Chow, 'Highly detuned Rabi oscillations for a quantum dot in a microcavity,' Physical Review B **87**, Rapid Communication, 041305 (2013).

**Andreas Knorr: TU-Berlin**

## BEC and Atomtronics

Chow, Straatsma & Anderson, 'An engineering design tool for atomtronic circuits' (submitted PRA).

**Dana Anderson, U Colorado and JILA**

**Stephan Koch, Philipps University, Marburg**