

## HARD Solids

## Micro-Scale Inverse Modeling

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## Goals and Objectives

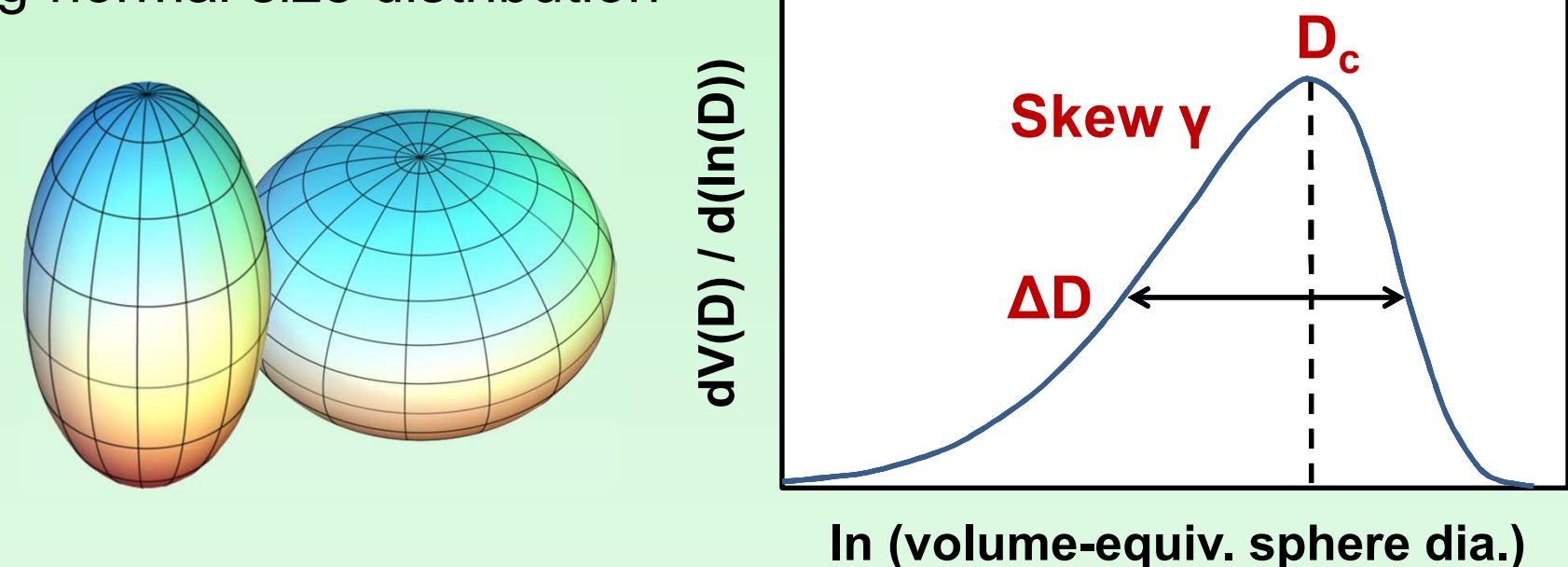
- Develop an invertible model for the reflectivity/emissivity of a surface chemical deposited on a substrate
- Model will incorporate fundamental physical properties ( $n, k$ ) and account for morphological characteristics – particle size distribution (PSD), shape, and packing density
- Model should demonstrate agreement with reflectivity/emissivity measurements...
- ...while the parameters extracted via model inversion (e.g., the PSD) should demonstrate agreement with independent measurements of related properties/characteristics

## Introduction

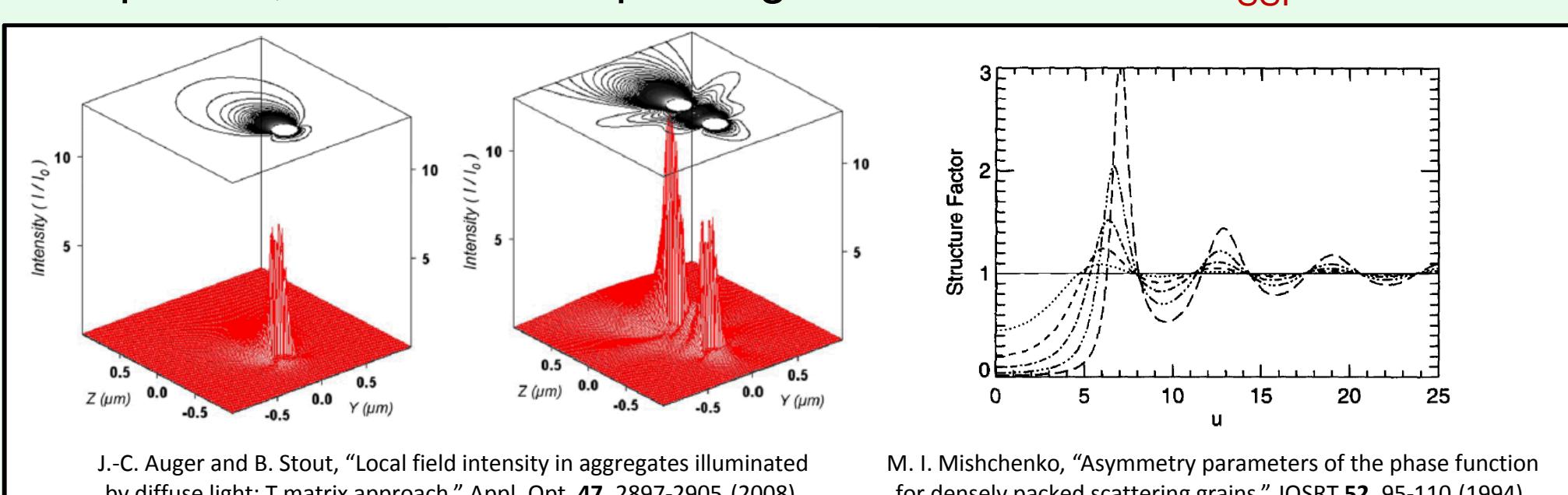
- Model inversion addresses the question, "Given a model, what do the data say about it?"
- Optimizing the match between model and data sheds light on model shortcomings → Leads to improved model fidelity
- Framework is synergistic with SA/UQ

## Methods

- Initially implement a solution for the reflectivity of an optically thick dilute particulate layer
  - Implement Ambartsumian invariant imbedding numerical solution (open-source code provided by NASA GISS)
- Allow for mixture of prolate/oblate spheroids with a skewed volume log-normal size distribution



- Calculate scattering/extinction properties for size/shape mixture
  - For  $D \sim \lambda$ , calculate properties via T-matrix
  - For  $D > \lambda$ , calculate properties via ray tracing
- Invoke static structure factor (SSF), based on equivalent-volume spheres, to account for packing effects → fill factor  $f_{\text{SSF}}$



- Invoke effective media approximation (EMA) to account for "sub-spheroid" morphological features → fill factor  $f_{\text{EMA}} = f_{\text{air}}$

$$\varepsilon_{\text{eff}} = \varepsilon_{\text{mat}} \left[ \frac{3f_{\text{air}} \left( \frac{\varepsilon_{\text{air}} - \varepsilon_{\text{mat}}}{\varepsilon_{\text{air}} + 2\varepsilon_{\text{mat}}} \right)}{1 + \frac{1 - f_{\text{air}} \left( \frac{\varepsilon_{\text{air}} - \varepsilon_{\text{mat}}}{\varepsilon_{\text{air}} + 2\varepsilon_{\text{mat}}} \right)}{1}} \right]$$

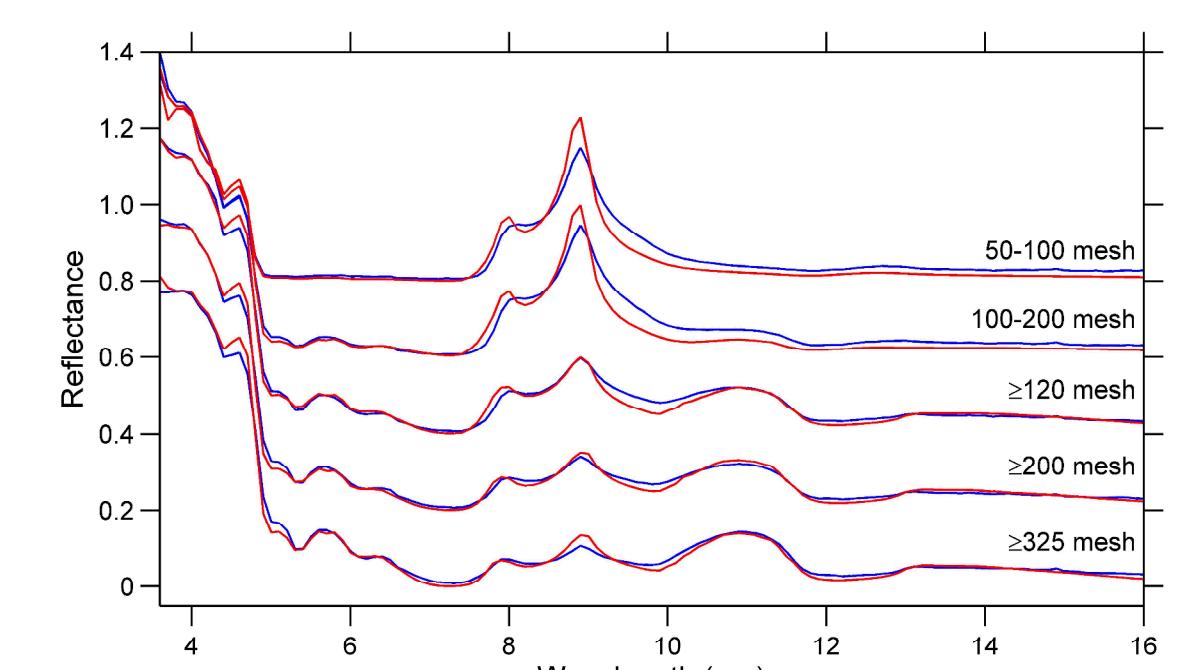
- 6 parameters:  $D_c$ ,  $\Delta D$ ,  $\gamma$ ,  $f_{\text{SSF}}$ ,  $f_{\text{EMA}}$ , and angle of incidence  $\theta$  (accounting for surface roughness)
- Vary these parameters to match model to data
  - Match optimized per forward-difference method in Sandia's Dakota package
  - 125  $\lambda$ -specific calculations distributed over 64 processors



## Results / Major Findings

## Spectral fitting

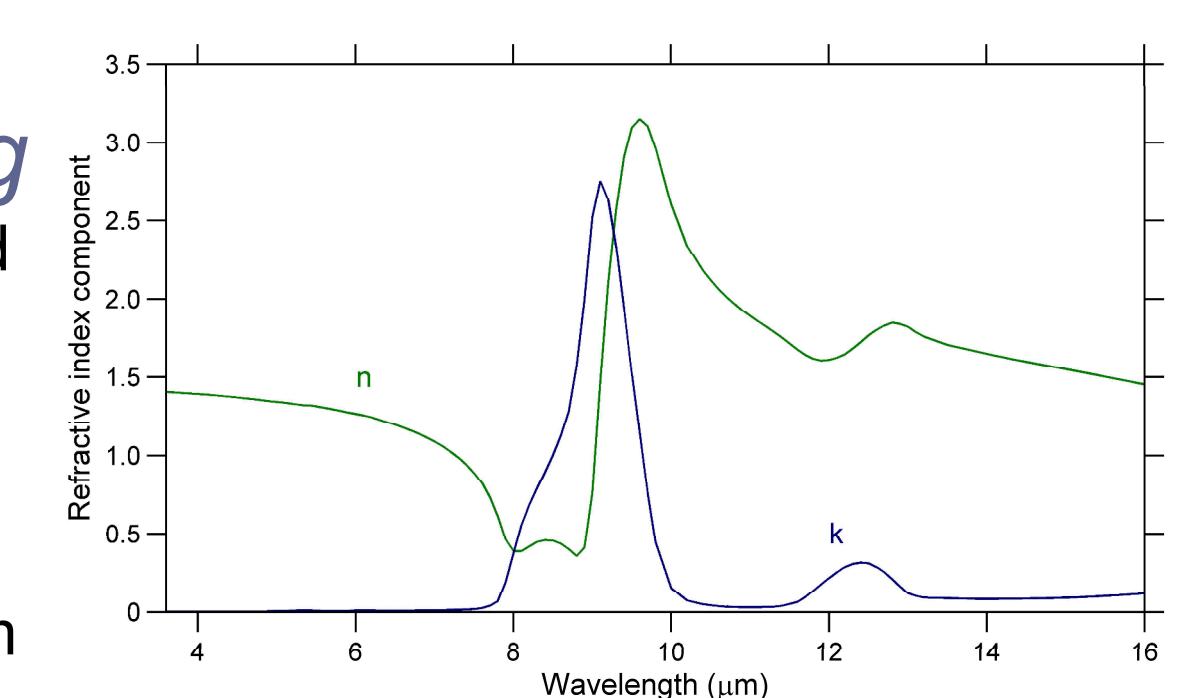
Size-dependent features, quite evident in the reflectance data of model systems (see "Micro-Scale Measurements" poster), are effectively captured by the reflectance model



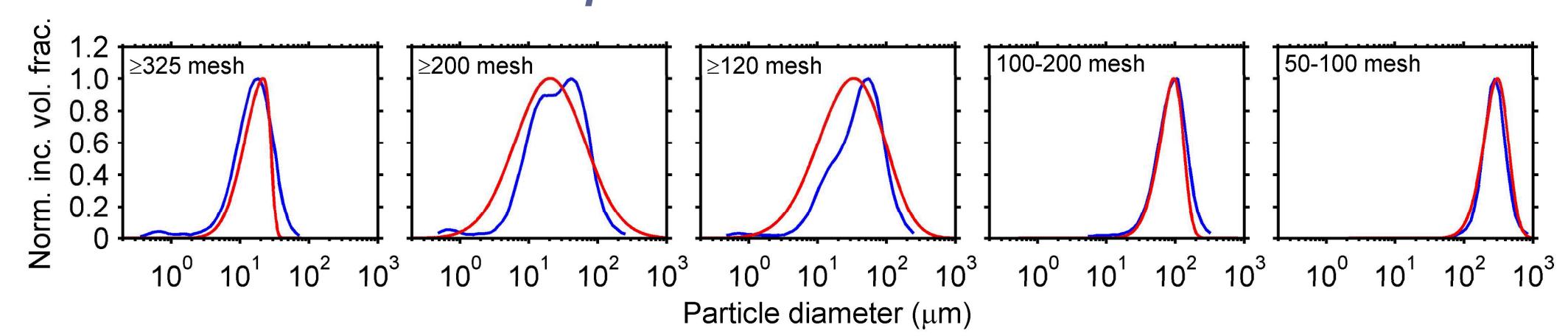
## n/k spectra: Surface vs. volume scattering

$k$  large: Reflectance dominated by field from particle surfaces

$k$  small,  $n$  moderate: Reflectance dominated by field propagating into particulate medium



## Particle size comparison



- PSDs measured by laser diffraction instrumentation
- Model-extracted PSDs exhibit good agreement with measurements

## Packing density comparison

Extracted volume fractions deviate from those gravimetrically measured for finer particles.

- Significant approximations in applying the SSF and EMA

Volume Fraction		
Mesh size	Gravimetrically measured	Model extracted
50-100	0.56	0.59
100-200	0.51	0.53
≥120	0.47	0.72
≥200	0.47	0.60
≥325	0.38	0.52

## Discussion / Next Steps

- Given the approximations in the model, it nevertheless matches reflectance measurements of model systems
- Model developed on silica – extreme range of  $n$  (<1) and  $k$  (~3)
- As  $n, k$  measurements are performed on additional materials (see "Lab Measurements of Optical Constants" poster), the reflectance model can be applied to those materials as well
- Next: Perform SA/UQ on current model

## Conclusion and Relevance to Program Objectives

- The numerically invertible model provides the means to account for particle size effects in the reflectance spectra of particulate media.
  - Based on fundamental physical properties ( $n, k$ )
  - Extracted parameters can be compared to auxiliary measurements → PSD agreement is especially encouraging
- Assessment via numerical inversion can continue to be applied as more advanced models become available (see "Micro-Scale Forward Modeling" poster)

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