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Uncertainty Quantification Methods for Electrical Simulation

Sandia National Laboratories

Eric Keiter and Laura Swiler

Sandia National Laboratories, New Mexico 87185

Problem

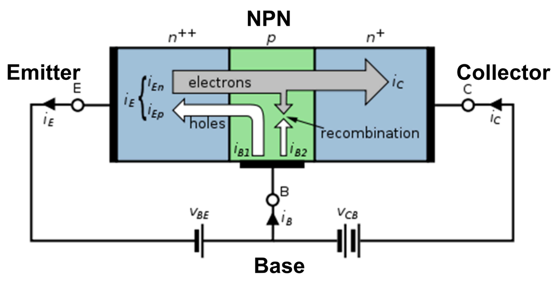
To efficiently characterize and model sources of variability based on electrical test data. This can be thought of as a type of inverse problem, or an optimization, in which the inner "forward solve" is some type of forward UQ calculation.

Why is this hard? The original sources of variability can be hard to measure, and are not well represented in circuit level ("compact") models.

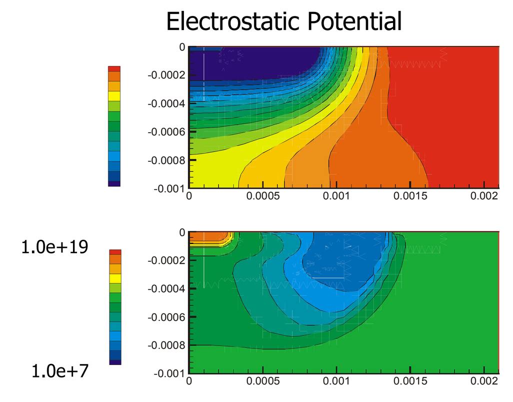
Circuit simulation makes use of ``compact models'' of basic circuit electrical components such as diodes and transistors to provide current-voltage (I-V) relationships. They are very useful for representing I-V data, often are not a good candidate to represent the true sources of uncertainty.

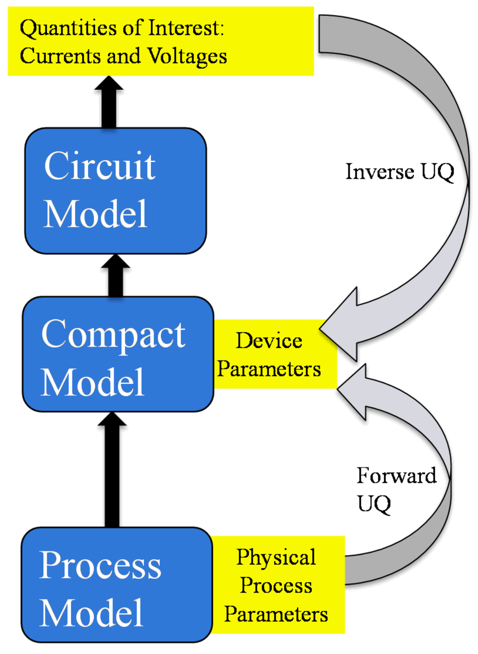
Sources of variability (which may or may not be known):
- manufacturing variations (doping levels, oxide thickness, etc.)
- measurement uncertainty

Bipolar Junction Transistor (BJT)



TCAD (process) BJT Model



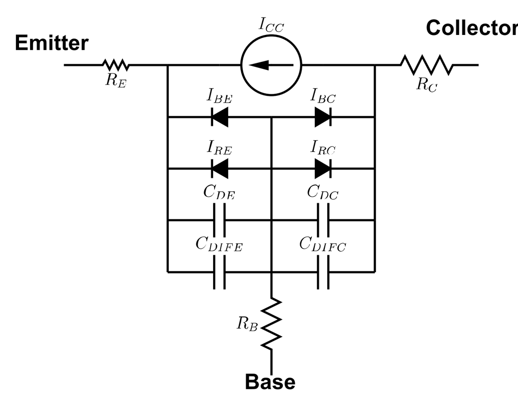


Compact model parameters:
- Function of process parameters
- Highly correlated

$$I_s \propto \frac{(L_e+\Delta e)(W_e+\Delta e)}{N_A t_b}$$

$$V_{af} \propto \frac{N_A t_b}{\sqrt{N_C}}$$

Compact BJT Model



Approach

Backward Propagation of Variance (BPV)
- Method adapted from circuit simulation literature (McAndrew et al.)
- Similar to reliability based design optimization, where the reliability method may be as simple as a Mean Value method
- Match model means and variances of several output measures to test data, by varying means and variances of input distributions.

$$p_i \sim N(\bar{p}_i, \sigma_i)$$

$\bar{\mathbf{p}}$ = mean parameters

$M_{Rk}(\bar{\mathbf{p}})$ = Xyce output

$\left(\frac{\partial M_{Rk}}{\partial p_i}(\bar{\mathbf{p}})\right)$ = Xyce sensitivity

Outer Optimization Loop: Find $\bar{p}_i, \sigma_i$

Inner loop: Calculate Model Moments (RHS)

$$\mu_{Rk} = M_{Rk}(\bar{\mathbf{p}})$$

$$\sigma_{Rk}^2 = \sum_{i=1}^{P} \sigma_i^2 \left(\frac{\partial M_{Rk}}{\partial p_i}(\bar{\mathbf{p}})\right)^2$$

From device tests

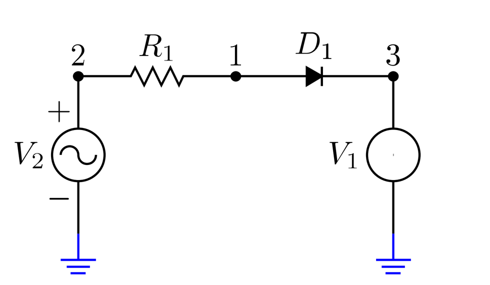
From the Xyce circuit model.

Enabling Technologies:
- Reliability Methods: in Dakota
- Nesting of Optimization and Uncertainty Quantification (UQ): in Dakota
- Direct and Adjoint Sensitivities in **Xyce**
- Sacado Automatic Differentiation (AD)

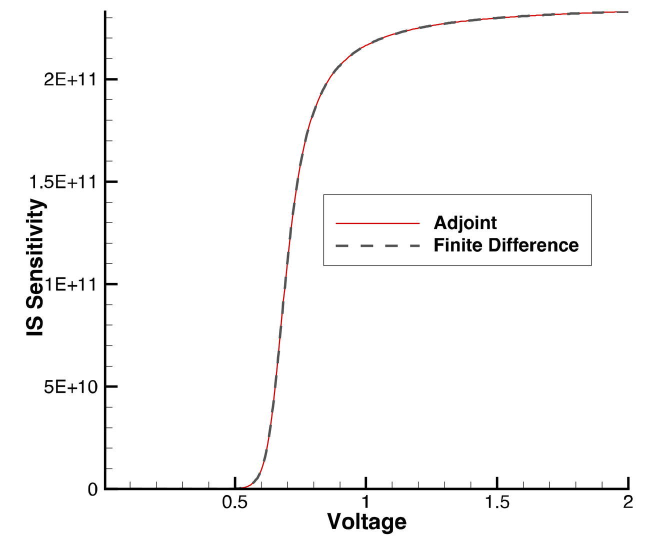
General $$\frac{dO}{dp} = \frac{dO}{dx}\left(\frac{dF}{dx}\right)^{-1}\frac{dF}{dp} + \frac{dO}{dp}$$

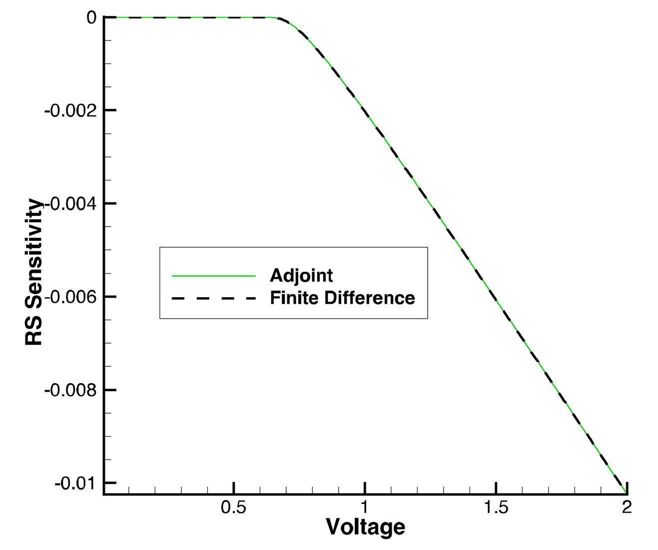
Direct $$\frac{dO}{dp} = \frac{dO}{dx}\left[\left(\frac{dF}{dx}\right)^{-1}\frac{dF}{dp}\right] + \frac{dO}{dp}$$

Adjoint $$\frac{dO}{dp} = \left[\frac{dO}{dx}\left(\frac{dF}{dx}\right)^{-1}\right]\frac{dF}{dp} + \frac{dO}{dp}$$



Sensitivity test circuit
IS and RS are parameters of the diode model D1





Results

Case Study: III-V HBT (heterojunction bipolar transistors)

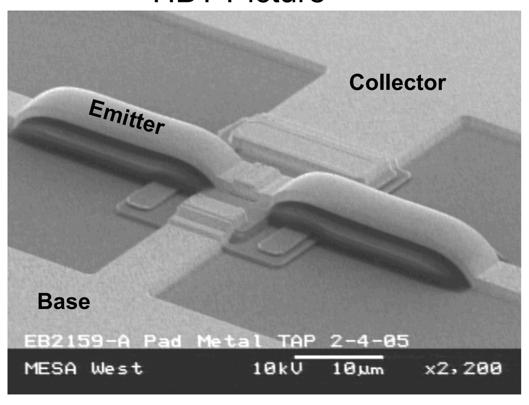
Approach: Generate "truth" data using TCAD, varying process parameters such as doping levels, etc. This results in an ensemble of IV curves.

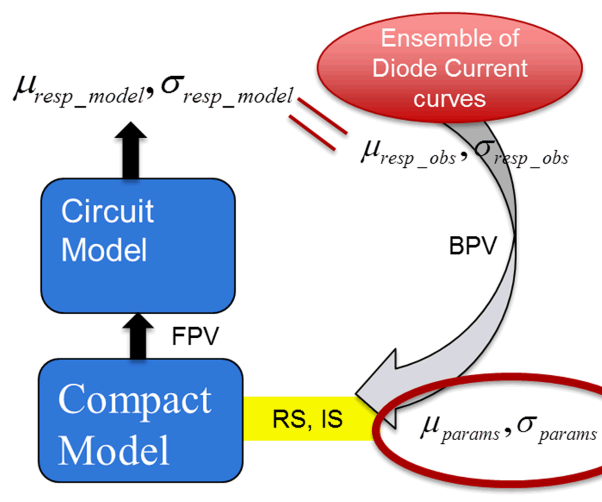
Calibrate each IV curve from this ensemble, to have an ensemble of compact model parameter sets.

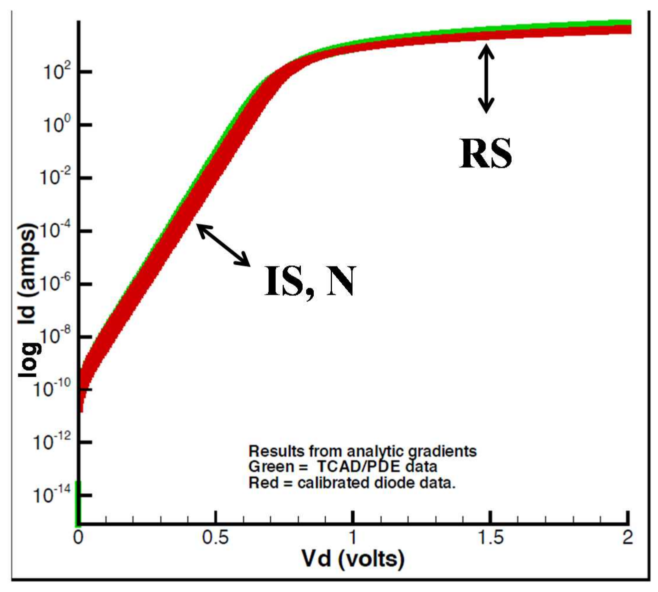
Compare the ensemble of parameter sets to the parameter moments generated by BPV and other methods.

The compact model parameters used in the study are saturation current (IS) and series resistance (RS). Both of these should have an inverse relationship with doping levels.

HBT Picture

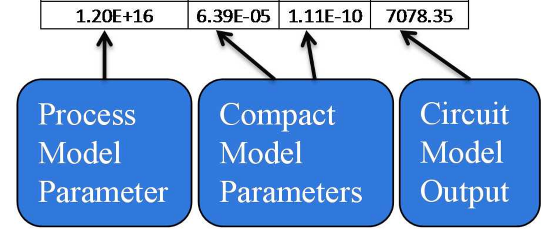






Ensemble of IV curves

DOPING LEVEL	RS	IS	Id(2)
1.21E+16	6.36E-05	1.10E-10	7091.76
1.15E+16	6.53E-05	1.16E-10	7021.17
1.48E+16	5.74E-05	8.98E-11	7364.88
1.07E+16	6.80E-05	1.25E-10	6912.58
8.09E+15	7.95E-05	1.66E-10	6478.43
9.45E+15	7.27E-05	1.41E-10	6725.50
8.05E+15	7.97E-05	1.67E-10	6470.51
1.10E+16	6.71E-05	1.22E-10	6949.32
1.46E+16	5.76E-05	9.05E-11	7354.43
1.20E+16	6.39E-05	1.11E-10	7078.35



"True" values

	MEAN	STD. DEV.
RS	7.32E-05	1.30E-05
IS	1.45E-10	4.87E-11

Reliability Method: 17 fn. evals

	MEAN	STD. DEV.	EST. of Id(2)	TRUE Id(2)
RS	7.24E-05	1.05E-05	6725.25	6711.5
IS	1.43E-10	4.11E-11	431.9	412

Sampling Method: 4450 fn. evals

	MEAN	STD. DEV.	EST. of Id(2)	TRUE Id(2)
RS	7.06E-05	1.13E-05	6738.57	6760
IS	1.17E-10	3.37E-11	438.2	447.6

Significance

If we can successfully relate ensemble electrical data back to the original process variations, such as doping, electrical designers could:

- Calibrate an ensemble, rather than individual parts of the ensemble
- Perform forward UQ by varying the original source of uncertainty
- Interpolate between UQ sample points
- Honor correlations between all the sources of uncertainty.

This would dramatically simplify the current process in which designers are forced to calibrate 100's or more individual, nominally identical parts. This will result in time and cost savings, allowing better designs.

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