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Unified Creep Plasticity Damage (UCPD) Model for Rigid Polyurethane Foams

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ABSTRACT

Experiments were performed to characterize the mechanical response of several different rigid polyurethane foams to large deformation. In these experiments, the effects of load path, loading rate, and temperature were investigated. Results from these experiments indicated that rigid polyurethane foams exhibit significant volumetric and deviatoric plasticity when they are compressed. Rigid polyurethane foams were also found to be extremely strain-rate and temperature dependent. These foams are also rather brittle and crack when loaded to small strains in tension or to larger strains in compression. Thus, a phenomenological Unified Creep Plasticity Damage (UCPD) model was developed to describe the mechanical response of these foams to large deformation at a variety of temperatures and strain rates. This paper includes a description of recent experiments and experimental findings. Next, development of a UCPD model for rigid, polyurethane foams is described. Finite element simulations with the new UCPD model are compared with experimental results to show behavior that can be captured with this model.

KEYWORDS: Polyurethane Foam, Cellular Solid, Constitutive Model, Fracture, Unified Creep Plasticity

INTRODUCTION

Polyurethane foams are often used in packaging to protect sensitive components from accidental impact events. These foams are designed to absorb energy during impact events by undergoing large inelastic deformation. Thus, constitutive models that describe foam response to large deformation at various rates and temperatures are needed for use in finite element analyses of impact events.

Rigid, closed-cell, polyurethane foam consists of nearly spherical voids (Figure 1) with a typical diameter of 100 to 300 microns. The closed cells are separated by a polymer matrix that forms cells. Voids are less spherical and walls between neighboring cells are often very thin or even ruptured in rigid polyurethane foams with densities of 192 kg/m3 (12 pcf) or less. In higher density foams with densities of 320 kg/m3 (20 pcf) or greater, cells are more spherical and walls between neighboring cells are typically intact.

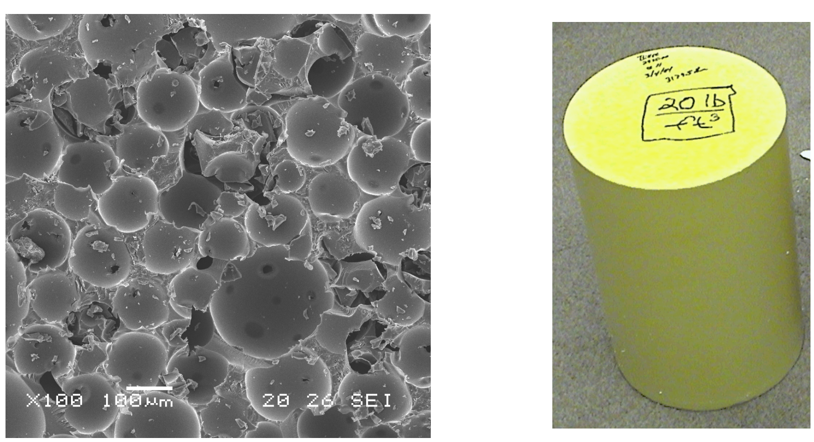


Figure 1. 320 kg/m^3 (20 pcf) rigid polyurethane foam cell geometry and 12 inch tall billet.

EXPERIMENTAL OBSERVATIONS

When rigid, closed-cell polyurethane foam is compressed, it exhibits an initial elastic regime followed by a plateau regime in which the load needed to compress the foam remains nearly constant (Figure 2). In the elastic regime, the foam sample is uniformly deformed. In the plateau regime, cell walls are plastically deformed and large permanent volume changes are generated. When additional load is applied, cell walls are compressed against neighboring cell walls (Figure 3), and the stiffness and strength of the foam increases. In Figure 2, uniaxial stress and strain are plotted as positive for both compression and tension. When rigid polyurethane foam is loaded in tension, it exhibits only a very small amount of plastic deformation before it fractures. Fracture surfaces generated by uniaxial tension are oriented such that the loading axis is normal to the fracture surface (Figure 4). The mechanical response of polyurethane foam is also very sensitive to changes in either loading rate or temperature. The plateau strength of rigid polyurethane foam subjected to uniaxial compression decreases significantly with increases in temperature. The plateau strength is also observed to increase significantly with increases in loading rate. When rigid polyurethane foam is subjected to hydrostatic compression, it exhibits a pressure versus volume strain curve that is similar in shape to its uniaxial stress-strain curve (Figure 2). There is again an initial elastic regime followed by a plateau regime and finally a lock-up regime.

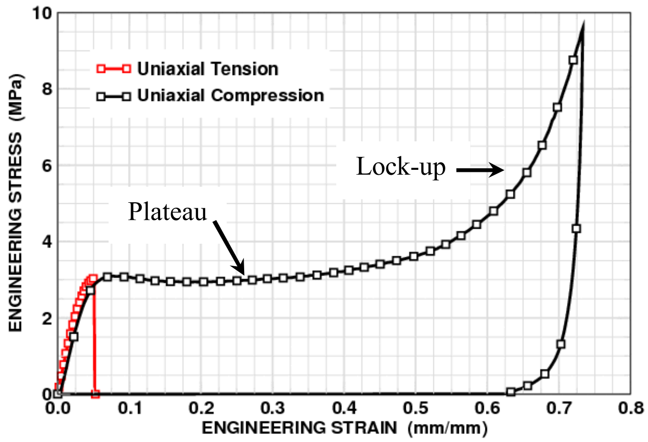


Figure 2. Typical stress-strain curves for 176 kg/m^3 (11 pcf) polyurethane foam subjected to either uniaxial compression or uniaxial tension.

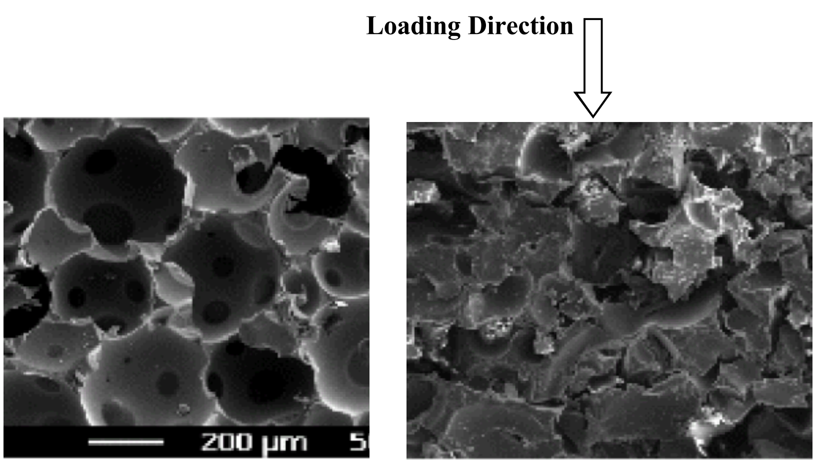


Figure 3. Cell walls compressed against neighboring cell walls when 176 kg/m^3 (11 pcf) polyurethane foam is compressed into the lock-up regime and then unloaded.

In addition to uniaxial and hydrostatic compression, FR3712 rigid polyurethane foam was also subjected to a variety of triaxial compression load paths in which the sample was initially subjected to hydrostatic compression and then the confining pressure was maintained while additional stress was applied in the axial direction only. Results from this series of triaxial compression experiments were then used to generate a plot of the initial yield surface for the foam in a von Mises effective stress versus mean stress space (Figure 5). The experimental results (blue symbols in Figure 5) indicate that the initial yield surface for the FR3712 foam could be described as an ellipse in this two dimensional space (solid line in Figure 5) or as an ellipsoid about the hydrostat in three-dimensional principal stress space.

UNIFIED CREEP PLASTICITY DAMAGE (UCPD) MODEL

From the experimental results presented in the previous section, it is clear that a metal plasticity model which includes only deviatoric (shape-changing) plasticity would not be adequate for describing the mechanical behavior of rigid polyurethane foams. Constitutive models for foams were previously developed by a number of researchers [e.g. 1-6]. Neilsen et al. [1] developed a plasticity model for polyurethane foams with a yield surface that has a cubic shape based on the use of a principal stress yield criterion. Deshpande and Fleck [2] developed a plasticity model for metal foams with a yield surface that is an ellipsoid about the hydrostat. Deshpande and Fleck [3] subsequently developed a yield surface for polymeric foams with a yield surface that is the inner envelope of the ellipsoidal surface previously developed for metal foams and a surface based on a minimum (compressive) principal stress criterion.

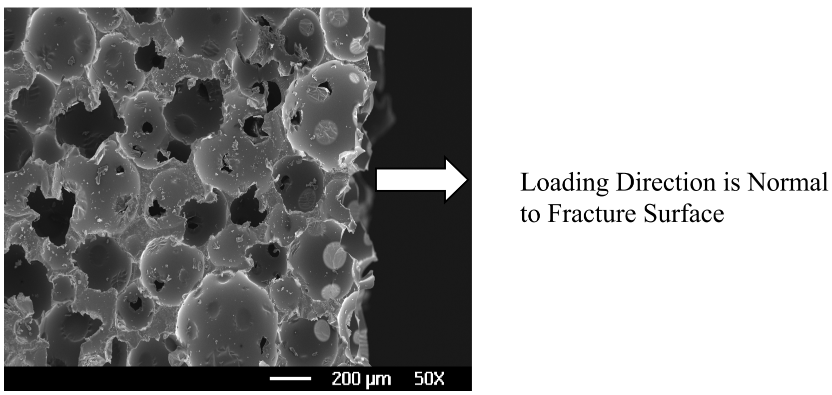


Figure 4. Tensile failure surface generated by uniaxial tension of 176 kg/m^3 (11 pcf) polyurethane foam in the indicated direction.

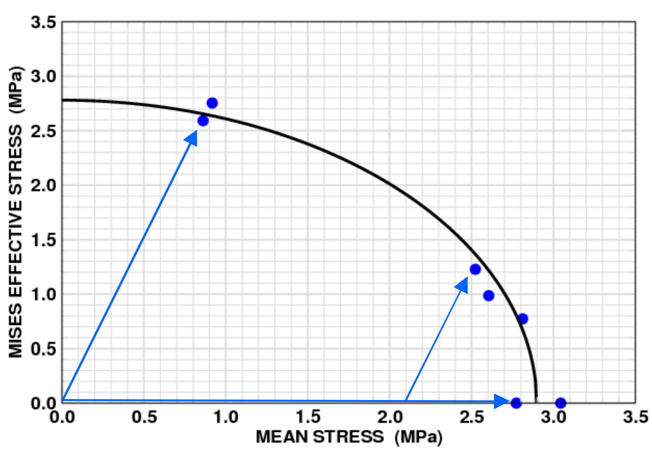


Figure 5. Yield surface obtained from a series of uniaxial compression, hydrostatic compression and triaxial compression experiments on 192 kg/m^3 (12 pcf) FR3712 foam. Each blue symbol represents the result from one experiment.

The model developed here is similar to many existing foam models [e.g. 1-6]. Our current implementation in SIERRA uses the unrotated Cauchy stress, $\boldsymbol{\sigma}$, and unrotated deformation rate, $\dot{\boldsymbol{\varepsilon}}$ [7, 8]. For small elastic strains, the total strain rate, $\dot{\boldsymbol{\varepsilon}}$, can be additively decomposed into elastic, $\dot{\boldsymbol{\varepsilon}}^{e}$, and inelastic, $\dot{\boldsymbol{\varepsilon}}^{in}$, parts as follows

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^{e} + \dot{\boldsymbol{\varepsilon}}^{in} \tag{1}$$

We also assume that the elastic response is linear and isotropic such that the stress rate is given by the following equation

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}}^{e} = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in}) \tag{2}$$

where **E** is the fourth-order isotropic elasticity tensor. Based on the experimental results shown in Figure 5, the initial yield surface is an ellipsoid about the hydrostat described by the function

$$\varphi = \frac{\bar{\sigma}^{2}}{a^{2}} + \frac{p^{2}}{b^{2}} - 1.0 \tag{3}$$

where *a* and *b* are state variables that define the current deviatoric and volumetric strengths of the foam. $\bar{\sigma}$ is the von Mises effective stress, a scalar measure of the deviatoric stress and is given by

$$\bar{\sigma} = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}} \tag{4}$$

p is the pressure or mean stress and is given by

$$p = \frac{1}{3}\boldsymbol{\sigma} : \mathbf{i} \tag{5}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress and **i** is the second-order identity tensor. **s** is the second-order deviatoric stress tensor

$$\mathbf{s} = \boldsymbol{\sigma} - p\,\mathbf{i} \tag{6}$$

Puso and Govindjee [5] and Zhang et al. [6] developed strain rate dependent models for foam that have the foam's inelastic rate given as a power-law function of stress. For the model developed here, we start with the yield function, Equation (3), rewritten as follows

$$\varphi = \sigma^{*} - a \tag{7}$$

where the effective stress, σ^{*}, is given as a function of the vonMises effective stress, $\bar{\sigma}$, and pressure, *p*, as follows

$$\sigma^{*} = \sqrt{\bar{\sigma}^{2} + \frac{a^{2}}{b^{2}} p^{2}} \tag{8}$$

Next, using a Perzyna-type formulation, the following expression for the inelastic rate, $\dot{\boldsymbol{\varepsilon}}^{in}$, is developed

$$\dot{\boldsymbol{\varepsilon}}^{in} = \begin{cases} e^{h}\left\langle \dfrac{\sigma^{*}}{a} - 1 \right\rangle^{n} \mathbf{g} & when \quad \dfrac{\sigma^{*}}{a} - 1 > 0 \\ \mathbf{0} & when \quad \dfrac{\sigma^{*}}{a} - 1 \le 0 \end{cases} \tag{9}$$

where **g** is a symmetric, second-order tensor that defines the orientation of the inelastic flow. This type of model is sometimes referred to as an overstress model because the inelastic rate is a power-law function of the overstress (distance outside the yield surface). For associated flow, **g** is simply normal to the yield surface and is given by

$$\mathbf{g}_{associated} = \frac{\dfrac{\partial \varphi}{\partial \boldsymbol{\sigma}}}{\left|\dfrac{\partial \varphi}{\partial \boldsymbol{\sigma}}\right|} = \frac{\dfrac{3}{a^2}\mathbf{s} + \dfrac{2}{3b^2} p\,\mathbf{i}}{\left|\dfrac{3}{a^2}\mathbf{s} + \dfrac{2}{3b^2} p\,\mathbf{i}\right|} \tag{10}$$

When lower density foams are subjected to a simple load path like uniaxial compression, the inelastic flow direction, at least prior to lock-up, is nearly uniaxial. In other words, the flow direction is given by the normalized stress tensor as follows

$$\mathbf{g}_{radial} = \frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}|} = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{\sigma} : \boldsymbol{\sigma}}} \tag{11}$$

This type of flow is referred to as radial flow. The UCPD model has a parameter, β, which allows for the flow direction to be prescribed as a linear combination of associated and radial flow directions as follows

$$\mathbf{g} = \frac{(1-\beta)\ \mathbf{g}_{associated} + \beta\, \mathbf{g}_{radial}}{\left|(1-\beta)\ \mathbf{g}_{associated} + \beta\, \mathbf{g}_{radial}\right|} \tag{12}$$

Rigid polyurethane foams have little ductility when they are subjected to tensile stress and behave more like elastic brittle materials for this load path. Even for uniaxial compression, these foams often exhibit cracking. The damage surfaces for the UCPD model are simply 3 orthogonal planes with normals given by the positive principal stress axes in principal stress space as shown in Figure 6 and are described by the following equation

$$\varphi^{i}_{Damage} = \sigma^{**i} - c(1-w) = 0,\ i = 1,3 \tag{13}$$

where σ^{**i} is a principal stress, c is the initial tensile strength which is a material parameter, and w is a scalar measure of the damage. Damage has an initial value of 0.0 and is limited to a maximum value of 0.99. As damage occurs, the damage surface will collapse toward the origin and the foam will have very little tensile strength. The foam will, however, still have compressive strength. Foam that is completely damaged can be removed using element death based on the damage variable reaching a value equal to 0.99 but removal of fully damaged elements is not required.

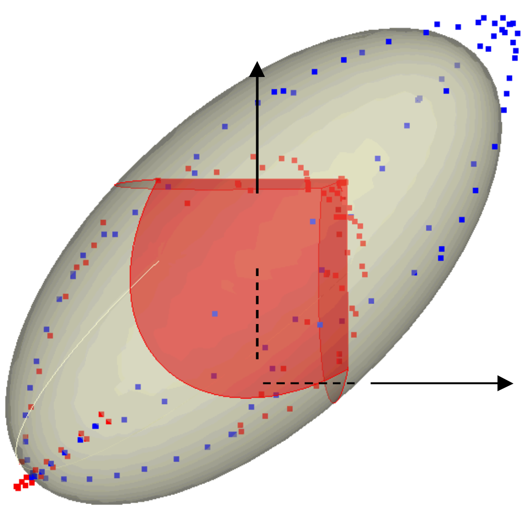


Figure 6. Yield (white) and damage (red) surfaces in principal stress space. Symbols represent results from either experiments or cell-level simulations on a representative volume of foam.

Damage is given as a monotonically-increasing, user-prescribed function of damage strain, ε_{dam}, and damage strain is a function of the maximum tensile strain, $\varepsilon_{\max}$, and the plastic volume strain, ε_{vol}^{p}, as follows

$$w = w(\varepsilon_{dam}) = w\left(a_{dam}\varepsilon_{\max} + b_{dam}\,\varepsilon_{vol}^{p}\right) \tag{14}$$

where a_{dam} and b_{dam} are positive material parameters which allow the user to control the rate at which damage is generated in tension and compression. Note that in compression the plastic volume strain obtains a negative value so the maximum tensile strain needed to generate damage is larger. Damage is never allowed to decrease even if the maximum tensile strain or plastic volume strain decrease which means that once foam is damaged, healing is not allowed.

To fully capture temperature, strain-rate, and lock-up effects several material parameters are no longer simply material constants but are instead functions of temperature, θ, and/or the maximum volume fraction of solid material obtained during any prior loading, ϕ, which depends on the volume strain. Material parameters defining the foams elastic response, Young's modulus and Poisson's ratio, are functions of both temperature, θ, and ϕ. To be more specific, the current Young's modulus and Poisson's ratio used in a simulation are given by

$$\begin{aligned} E &= E(\theta)\cdot E(\phi) \\ v &= v(\theta)\cdot v(\phi) \end{aligned} \tag{15}$$

The natural log of the reference flow rate, *h*, and the power law exponent, n, in Equation 9 are also functions of temperature

$$\begin{aligned} h &= h(\theta) \\ n &= n(\theta) \end{aligned} \tag{16}$$

State variables that define current deviatoric and volumetric strengths, *a* and *b*, are user-prescribed functions of ϕ. Also in the UCPD model, the parameter β which defines the fraction of associated and radial flow is a user-prescribed function of ϕ.

Material parameters for a 192 kg/m^3 (12 pcf) FR3712 foam at temperatures between -53.9 ^{o}C and 73.9 ^{o}C, and quasi-static (0.001 per second) to dynamic (200 per second) strain rates are given in Table 1 and Figure 7. The first step in the generation of these material parameters was to determine the initial volume fraction of solid material in the foam. Since the foam has a density of approximately 192 kg/m^3 and solid rigid polyurethane has a density of 1200 kg/m^3, the foam has an initial volume fraction of solid material, ϕ_0, equal to 0.16 (0.16 = 192/1200).

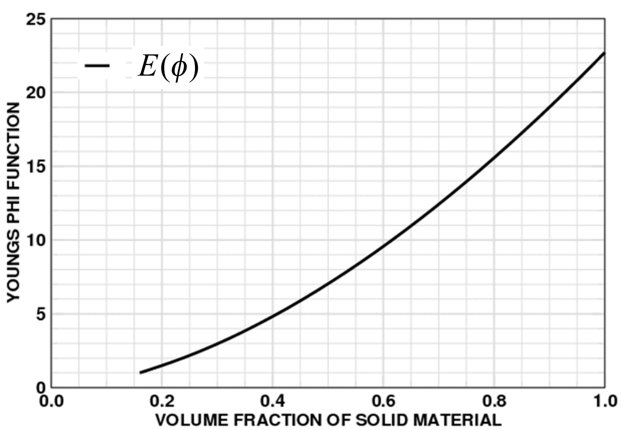
The next step in the fitting process was to plot the crush strength measured during uniaxial and hydrostatic compression experiments as a function of the current volume fraction of solid material. If we assume that the change in volume of the solid material is negligible compared with the change in volume of the foam, then the current volume fraction of solid material, ϕ, is related to the initial volume fraction, ϕ_0, of solid material as follows

$$\phi = \frac{\phi_0 V_0}{V} \tag{17}$$

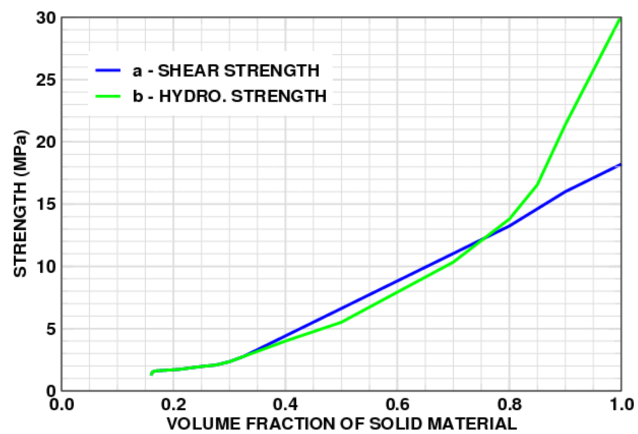
where V_o is the initial volume of foam and V is the current volume of foam. Note that equations that are function of ϕ actually use maximum volume fraction of solid material obtained during any prior loading in simulations; however, for monotonic compressive loading the current volume fraction of solid material is the maximum value obtained during any prior loading. The consequence of this assumption in simulations is that once the foam is compressed it cannot be re-expanded to have the same mechanical properties it had prior to compression. Material parameter functions which were found to provide a good fit to the experimental data are shown in Figure 7.

Table 1. Foam Damage Model Parameters for 192 kg/m^3 (12 pcf) foam, FR3712

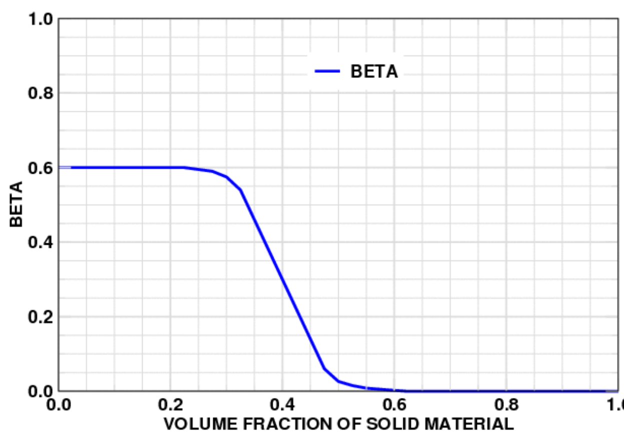
Parameter	Units	Value	Value	Value
Temperature	C	-53.9	18.3	73.9
Young's Modulus $E(\theta)$	MPa	79.7	79.6	63.7
Poisson's Ratio v	-	0.250		
Initial Volume Fraction Solid ϕ_0	-	0.160		
Flow Rate $h(\theta)$	-	-10.0	2.60	7.50
Power Exponent $n(\theta)$	-	18.0	14.0	6.00
Tensile Strength c	MPa	1.931		
Adam a_{dam}	-	1.00		
Bdam b_{dam}	-	1.00		
Thermal Expansion Coefficient	1/C	60.0 x 10^{-6}		



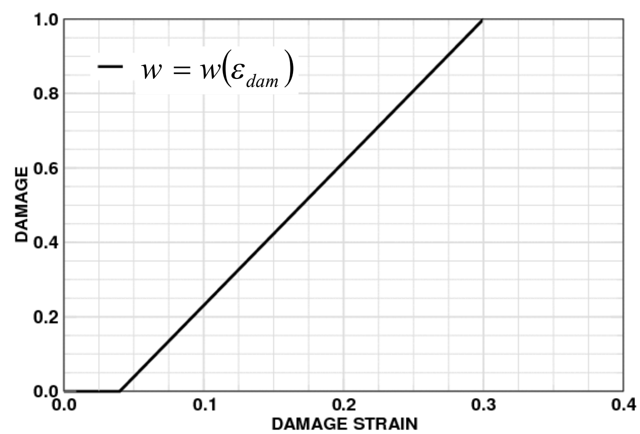
(a) $E(\phi)$ effect of compaction on Young's Modulus



(b) shear and hydrostatic strengths increase with compaction



(c) flow direction more associated with compaction



(d) damage as a function of damage strain

Figure 7. Material parameter functions for FR3712.

Uniaxial and hydrostatic compression experiments on FR3712 were simulated. The stress-strain curves generated by these simulations are compared with the experimental data in Figures 8 and 9. For all temperatures and strain rates the fit is good. The hydrostatic compression experiment was then simulated. Pressure applied to the finite element model was increased at a constant rate of 0.1 MPa/sec (14.5 psi/second) to match the experiment. The UCPD Model prediction matched the experiment well (Figure 10). Unfortunately, there was no uniaxial tension data available for this foam so the damage parameters selected for FR3712 were simply based on experience with other rigid polyurethane foams with similar density.

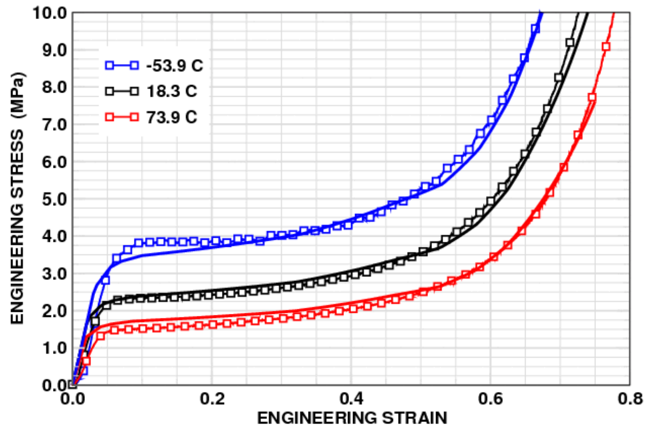


Figure 8. FR3712 uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and a constant engineering strain rate of 0.001 per second.

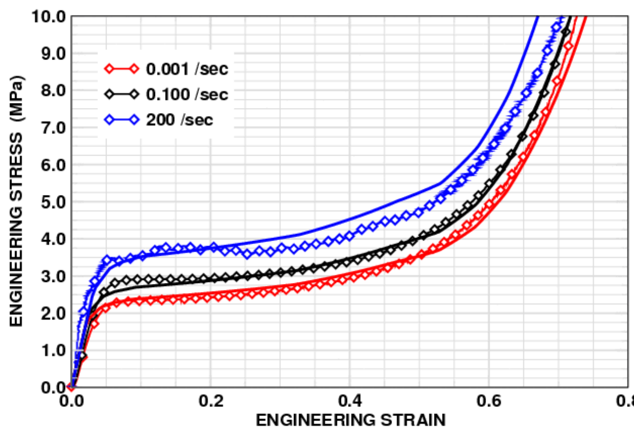


Figure 9. FR3712 uniaxial compression experiments (symbols) and simulations (solid lines) at three different engineering strain rates and a constant temperature of 18.3 ^{o}C.

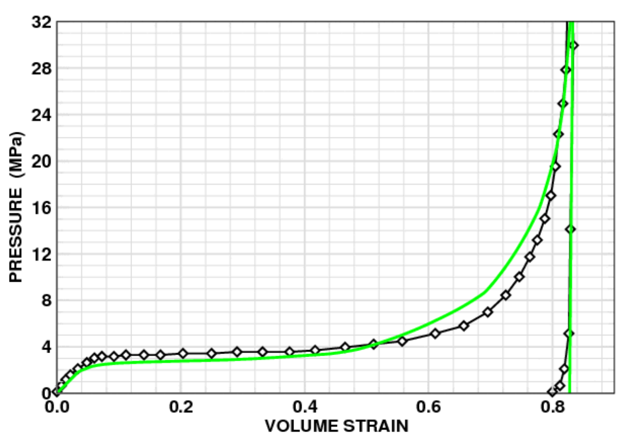


Figure 10. FR3712 hydrostatic compression experiment (symbols) and simulation (solid green line) at room temperature.

Parameters were also generated for other foams and the experiments used to generate those parameters were then simulated to show that the model could capture both the inelastic deformation and cracking exhibited by the foam. For example, Figure 11 shows a comparison of experiments and deformed model shapes from simulations of uniaxial tension and compression of a 320 kg/m^3 (20 pcf) foam. The model is able to predict cracking from both uniaxial tension and uniaxial compression.

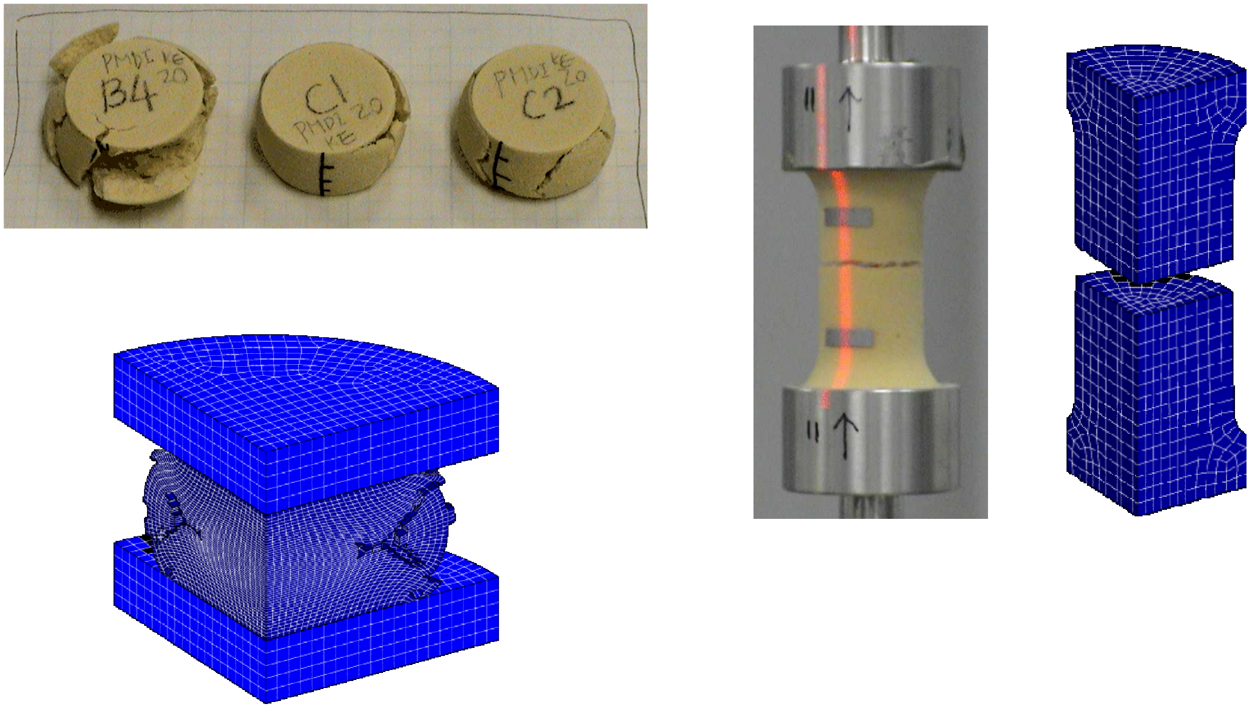


Figure 10. Additional simulations showing UCPD models ability to capture foam cracking exhibited during uniaxial compression and uniaxial tension experiments on a 320 kg/m^3 (20 pcf) foam.

SUMMARY

A new UCPD model was developed to describe the mechanical response of rigid polyurethane foams to loading experienced during accidental impact events. This model captures the effects of load path, strain rate, and temperature on mechanical response. A series of experiments was performed to characterize the mechanical response of several different rigid polyurethane foams to large deformation. In these experiments the effects of load path, loading rate, and temperature were investigated. Results from these experiments indicated that, as expected, these foams exhibit significant volumetric and deviatoric plasticity when they are compressed. Mechanical response of these foams is also significantly affected by changes in either loading rate or temperature. The new UCPD model captures both temperature and strain rate effects. This model also captures cracking exhibited by these rather brittle foams subjected to either uniaxial tension or compression. Investigation of the models ability to capture inelastic deformation and fracture for any load path are in progress.

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