Correlations and photon statistics in nanocavity emitters

SAND2015-1821PE

Weng Chow, Sandia National Laboratories

Motivation for nanocavity emitters

Modeling approaches

Applications:

a) β = 1 and thresholdless lasing

b) Single-photon sources and photon statistics

Thanks to:

Christopher Gies and Frank Jahnke, Bremen University

Sandia's Laboratory Directed Research & Development (LDRD) Program

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Why nano-emitter research?

① Save energy

> **Talk: Attojoule optoelectronics – why and how**
>
> David Miller, Stanford University
> IEEE Photonics Summer Topicals 2013
>
> **Information communication and processing growth:**
>
> - Energy per bit has to reduce
> - At limits for electrical approaches

Lasers: can still reduce required electrical energy by reducing volume

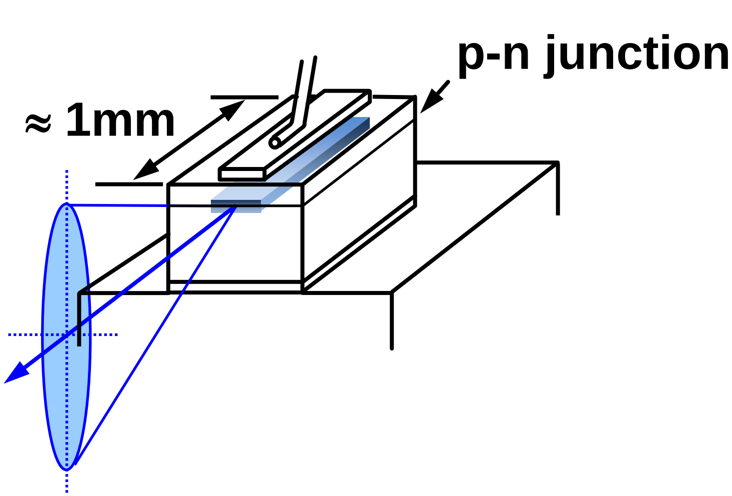
② Safe communication and quantum computing: Single-photon sources

Types of light

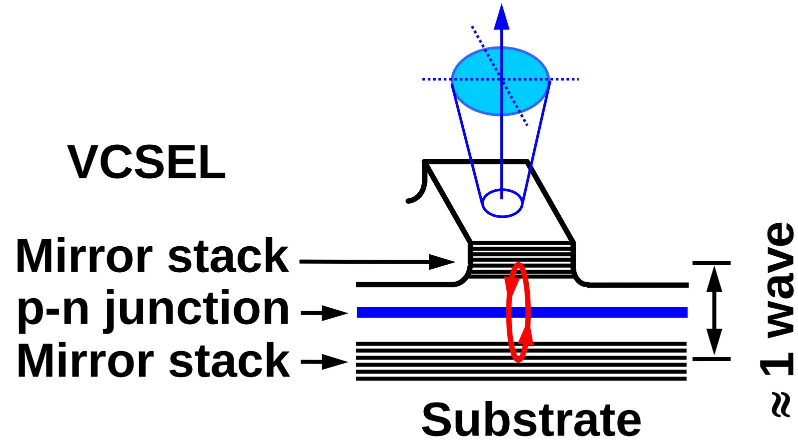
Laser (random) **Single-photon (antibunched)**

Towards smaller and smaller lasers

Edge - Emitting Laser

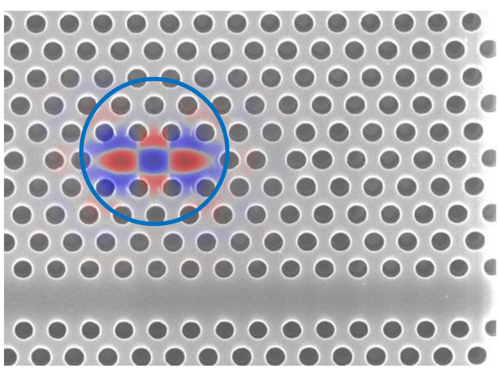


Vertical-Cavity Surface-Emitting Laser



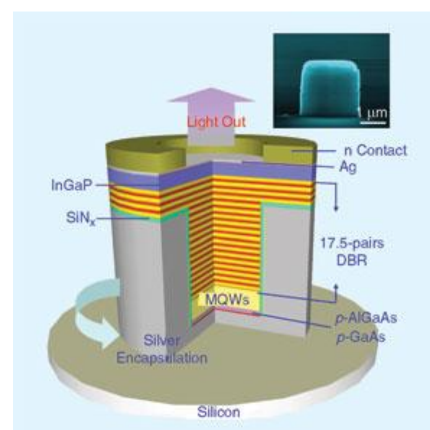
Nanolasers

Photonic crystal



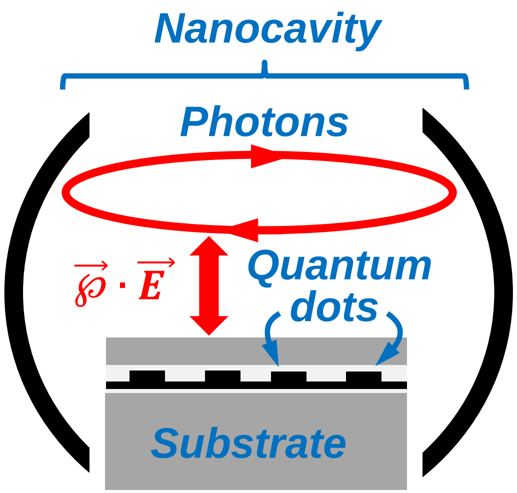
(Courtesy of Willie Luk, Sandia National Labs)

Micro- or nano-cavity

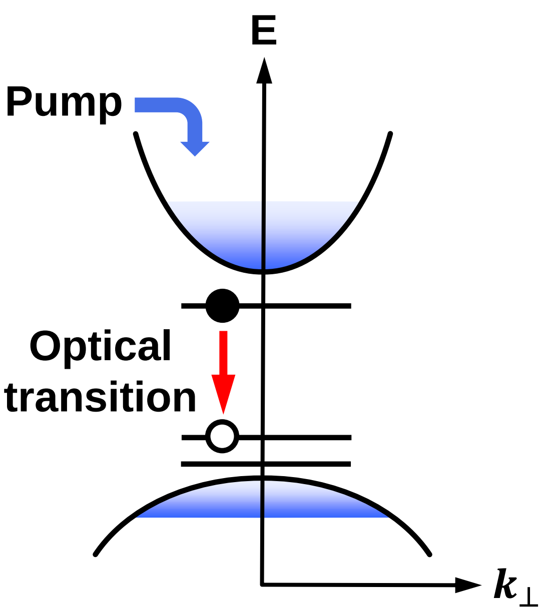


(Adapted from a figure by Lu et al., UIUC)

Model setup: Hamiltonian



Electronic structure



$$H = \sum_{\alpha} \varepsilon^{e}_{\alpha}\, c^{\dagger}_{\alpha} c_{\alpha} + \sum_{\beta} \varepsilon^{h}_{\beta}\, b^{\dagger}_{\beta} b_{\beta} \ + \hbar\omega\left(a^{\dagger}a + \frac{1}{2}\right)$$

Single-particle

$$+\frac{1}{2}\sum_{\alpha\beta\sigma\eta} W^{\alpha\beta}_{\sigma\eta}\, c^{\dagger}_{\alpha} c^{\dagger}_{\beta} c_{\eta} c_{\sigma} + \frac{1}{2}\sum_{\alpha\beta\sigma\eta} W^{\alpha\beta}_{\sigma\eta}\, b^{\dagger}_{\alpha} b^{\dagger}_{\beta} b_{\eta} b_{\sigma}$$

$$-\sum_{\alpha\beta\sigma\eta} W^{\alpha\beta}_{\sigma\eta}\, b^{\dagger}_{\alpha} c^{\dagger}_{\beta} c_{\eta} b_{\sigma}$$

Matrix element of $\dfrac{e^2}{4\pi\varepsilon_b |r - r'|}$

Carrier-carrier

$$+\hbar \sum_{\alpha\beta q} G_q \left(c^{\dagger}_{\alpha} c_{\beta} + b^{\dagger}_{\alpha} b_{\beta}\right)\left(d_q + d^{\dagger}_q\right)$$

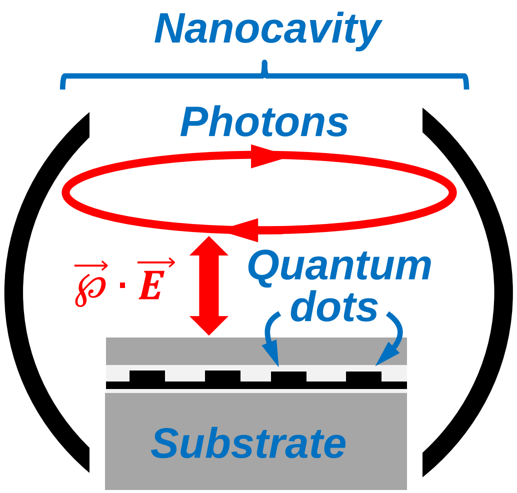
Carrier-phonon

$$-\hbar \sum_{\alpha} \left(g_{\alpha} b^{\dagger}_{\alpha} c^{\dagger}_{\alpha} a - g^{*}_{\alpha} a^{\dagger} c_{\alpha} b_{\alpha}\right)$$

Light-carrier

$$\wp \sqrt{\frac{\nu}{\hbar \epsilon_b V}}\, W(R_{QD}) \sum_n C_{\alpha}(R_n)\, V_{\alpha}(R_n)$$

Model setup: Hamiltonian



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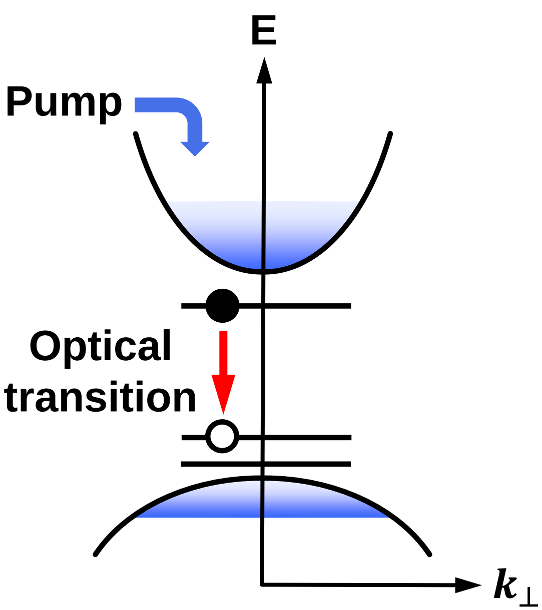


Approaches

Schrödinger Picture

$$\frac{\partial}{\partial t}\varrho = -\frac{i}{\hbar}\left[H_0 + H_{L-C}, \varrho\right] + \sum_X L_X(\varrho)$$

Heisenberg Picture

$$\frac{\partial}{\partial t}A = -\frac{i}{\hbar}\left[A, H\right]$$

Approaches

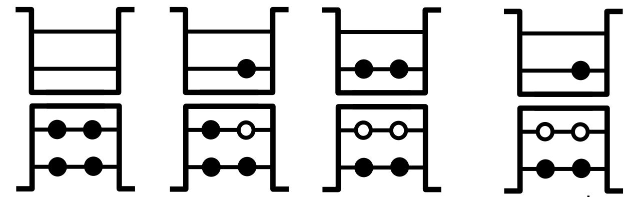
Schrödinger Picture

$$\frac{\partial}{\partial t}\varrho = -\frac{i}{\hbar}\left[H_0 + H_{L-C}, \varrho\right] + \sum_X L_X(\varrho)$$

Basis: $|j, n\rangle$

Photon number

Quantum-dot configurations



Expectation values: $\langle A\rangle = Tr\{\varrho A\} = \sum_{n,j,n'j'} \langle n, j|\varrho|j', n'\rangle\langle n', j'|A|j, n\rangle$

Photon statistics: $P_n = \langle n, 0X|\varrho|0X, n\rangle + \langle n, 1X_s|\varrho|1X_s, n\rangle + \langle n, 2X_{ss}|\varrho|2X_{ss}, n\rangle + \ldots$

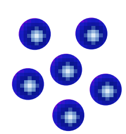
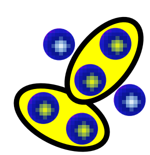
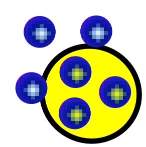
Heisenberg Picture

$$\frac{\partial}{\partial t}A = -\frac{i}{\hbar}\left[A, H\right]$$

Populations and correlations: $\langle c_\alpha^\dagger c_\alpha\rangle, \langle b_\alpha^\dagger b_\alpha\rangle, \langle c_\alpha^\dagger b_\alpha^\dagger a\rangle, \langle a^\dagger a\rangle$

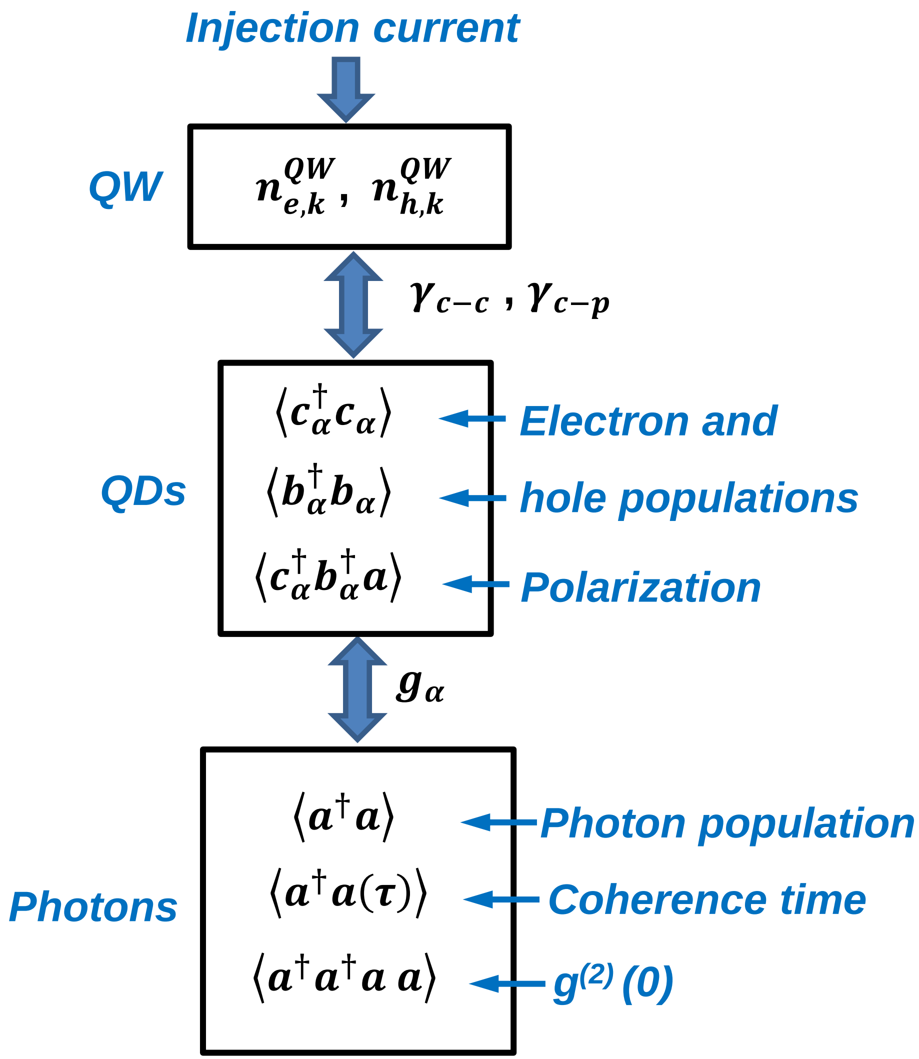
$\langle a^\dagger a^\dagger a a\rangle, \langle c_\alpha^\dagger c_\sigma^\dagger c_\sigma c_\alpha\rangle, \langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle$

Single particles — Correlated pairs — Correlated 3-particle clusters

Cluster expansion: $\langle \hat{N} \rangle =$  +  +  + ...

Overview: Chow & Jahnke, Progress in Quantum Electronics 37, 109 (2013)

Nano-emitter model: population dynamics and correlations



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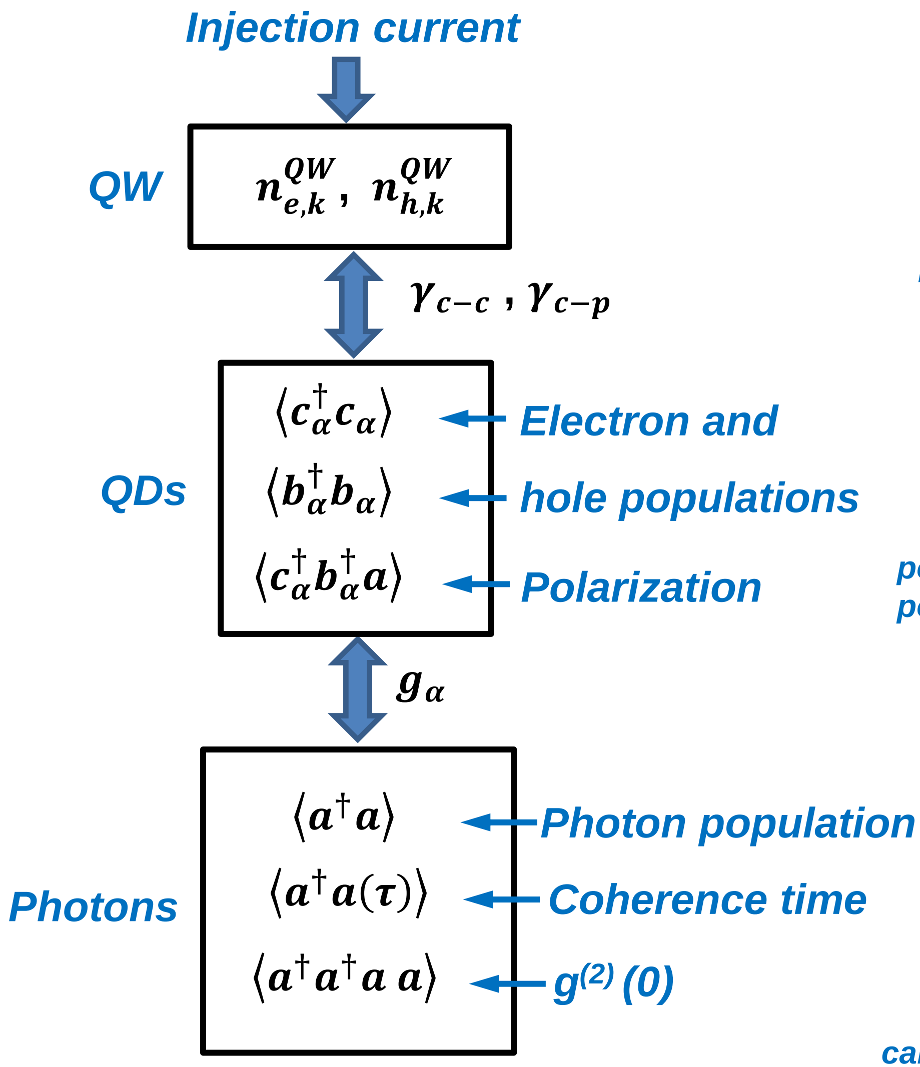
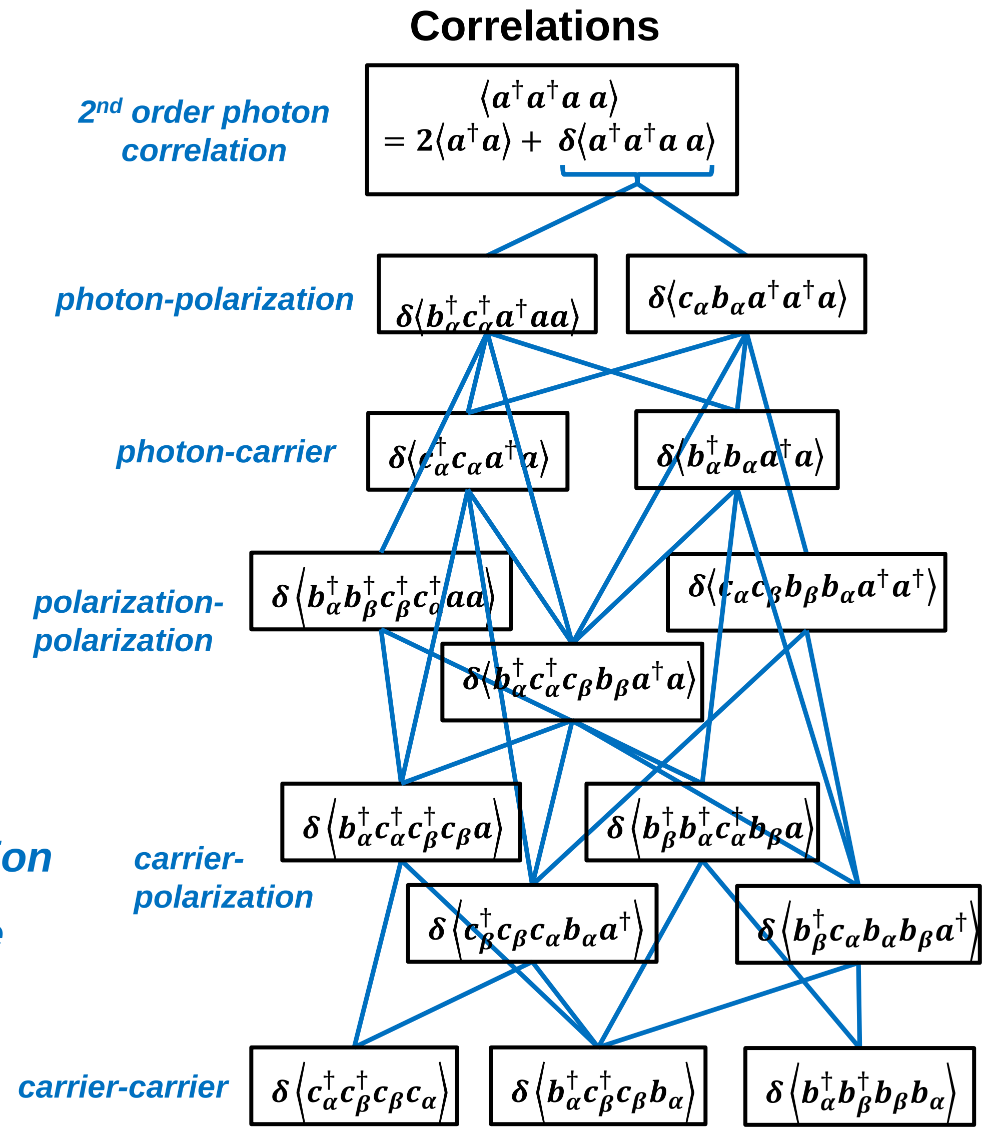




Emphasis now is on correlations involving light-matter interaction instead of Coulomb interaction

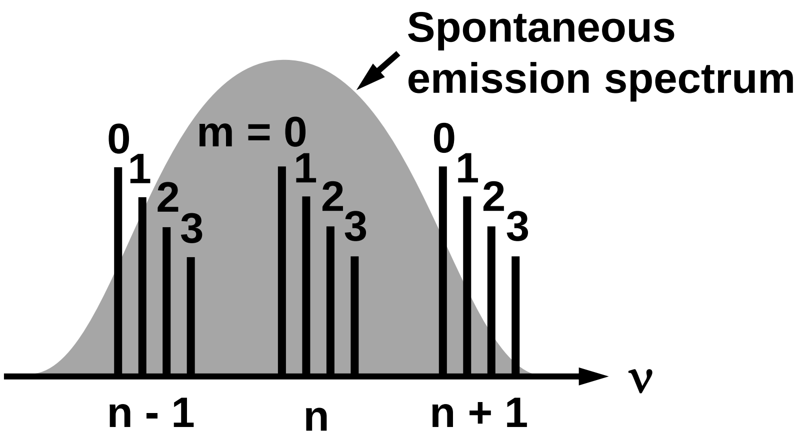
Interesting physics with nanolasers

Example 1: Laser threshold and thresholdless lasing

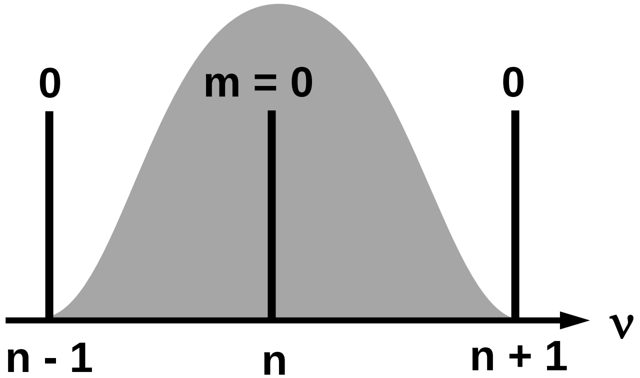
Spontaneous emission factor

$$\beta = \frac{\gamma_l}{\gamma_{sp}}$$

Most lasers $\beta \ll 1$



Some nanolasers $\beta = 1$



All emission into single resonator mode

Interesting physics with nanolasers

Example 1: Laser threshold and thresholdless lasing

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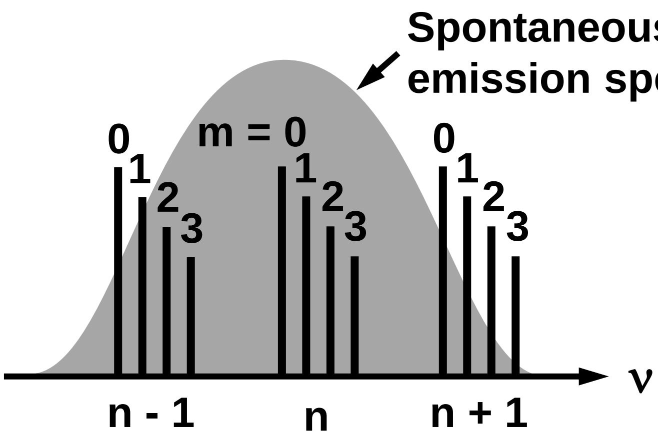

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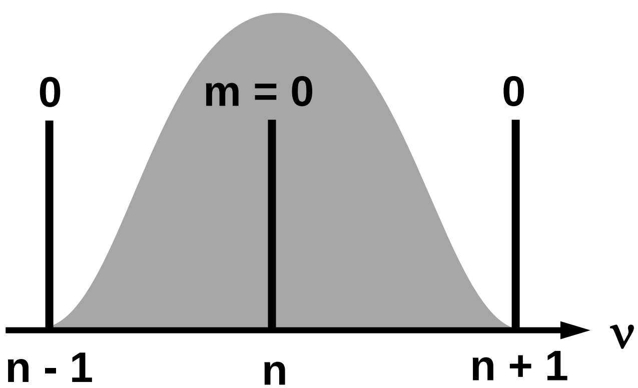

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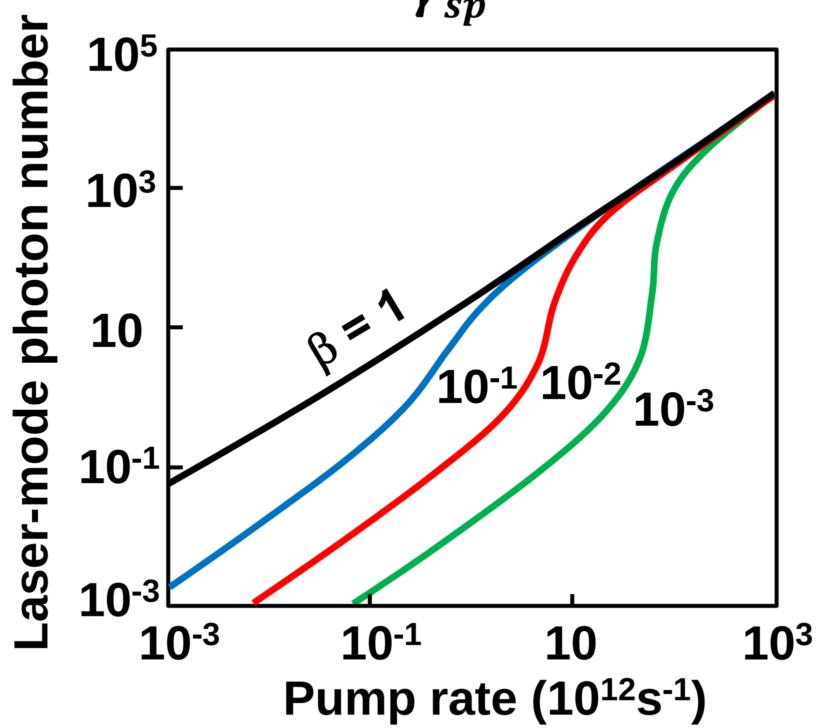


Questions:

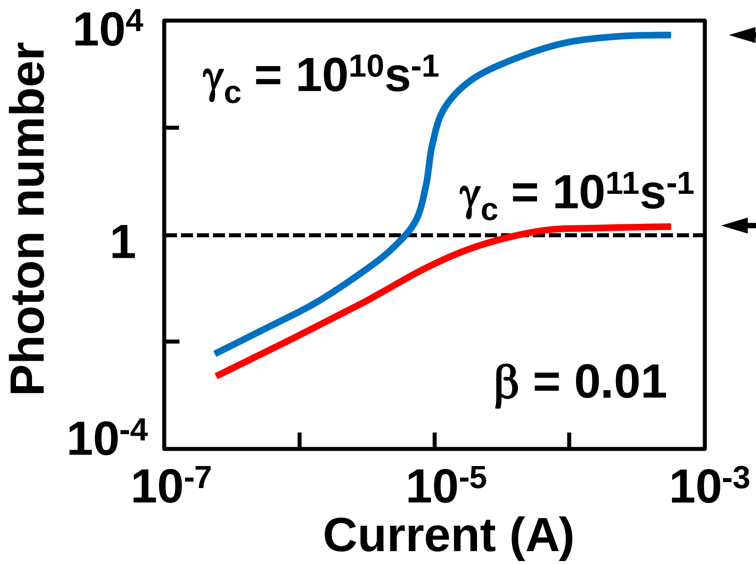
1) Is thresholdless lasing real?
2) What is lasing?

Early answer in Jin, Boggavarapu, Sargent, Meystre, Gibbs, Khitrova, Phy Rev A 49, 1994

Criterion for lasing

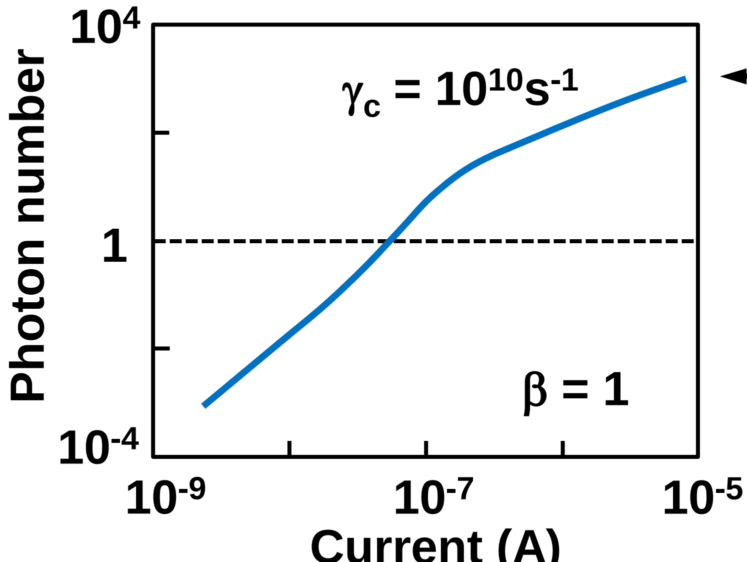
N_{QD} = 50, Δ_{inh} = 20meV

Input/Output



Conventional laser

Cavity-enhanced LED



Thresholdless laser

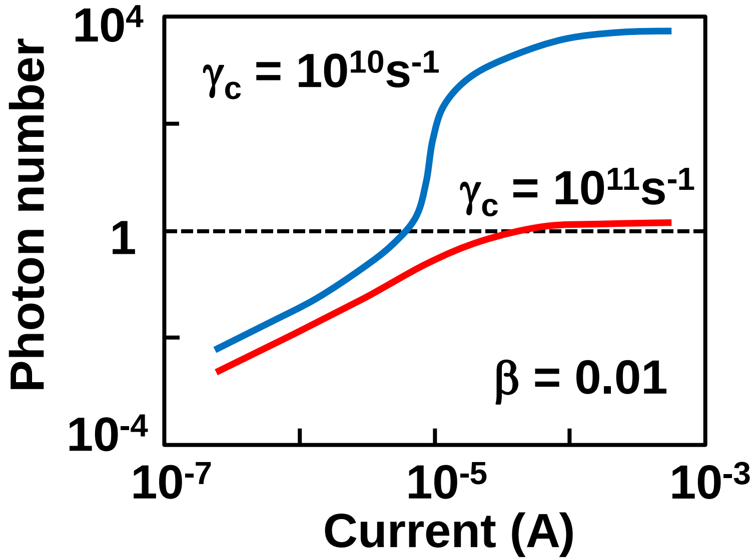
Additional question:

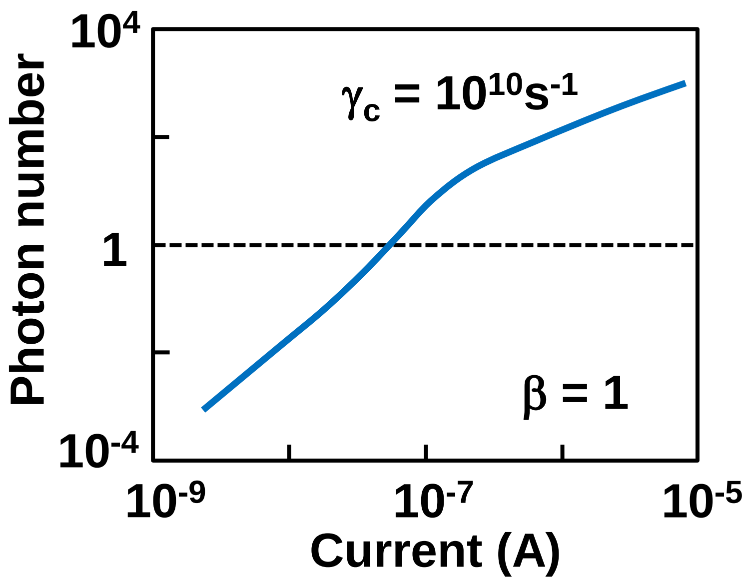
How to tell difference between thresholdless lasing and non-lasing with $\beta < 1$ (with y-axis in arbitrary units)?

Criterion for lasing

$N_{QD} = 50$, $\Delta_{inh} = 20$ meV

Input/Output

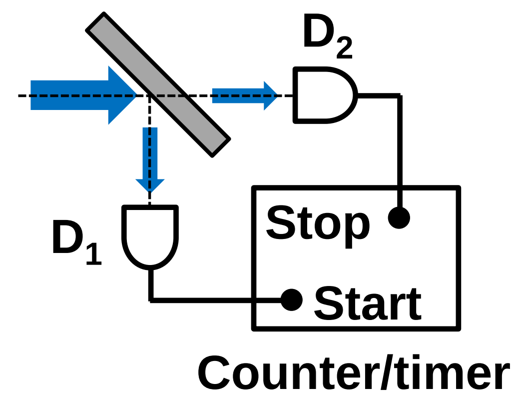




Second-order intensity correlation function

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$

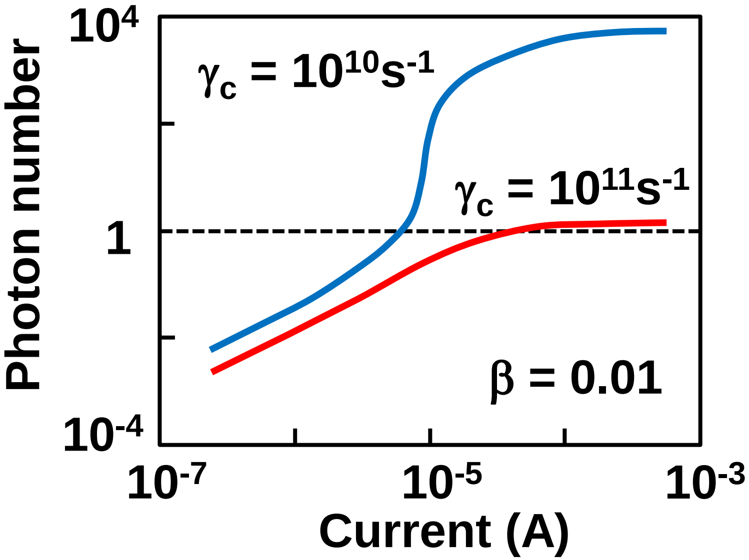
Hanbury-Brown-Twiss experiment



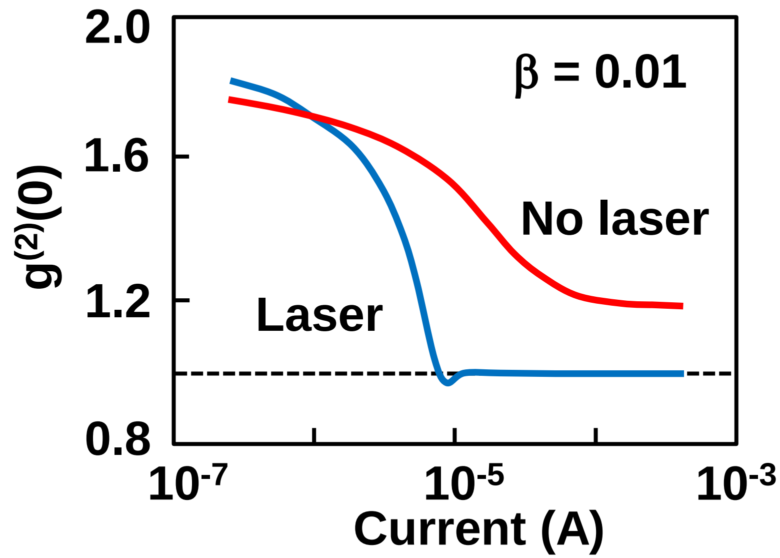
Criterion for lasing: $g^{(2)}(0)$

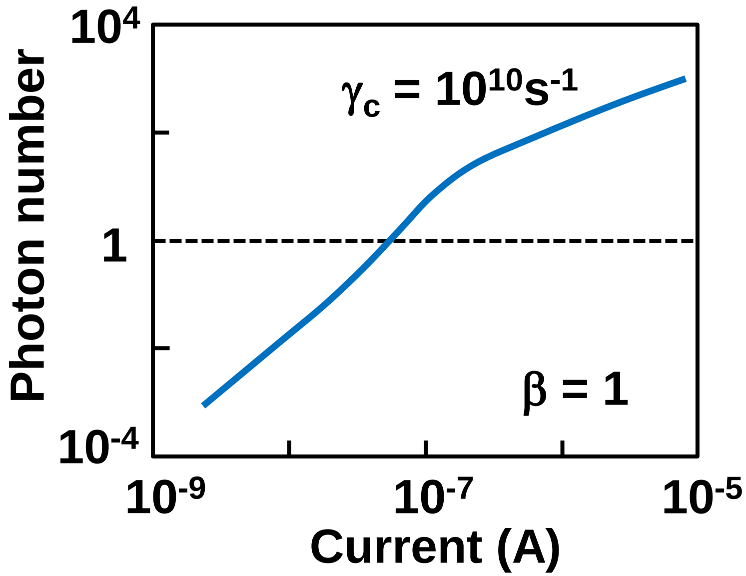
$N_{QD} = 50$, $\Delta_{inh} = 20$meV

Input/Output



Photon correlation





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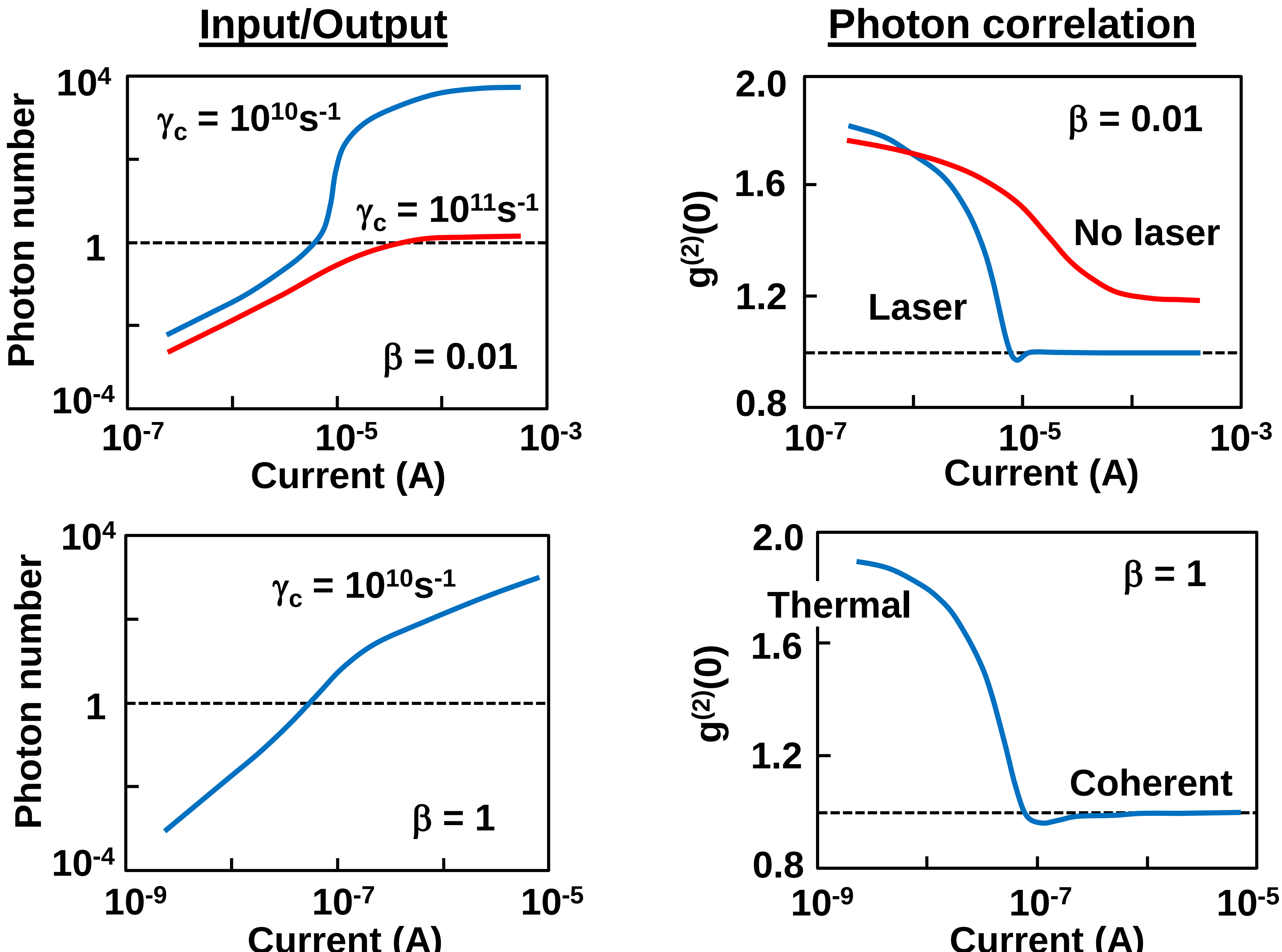
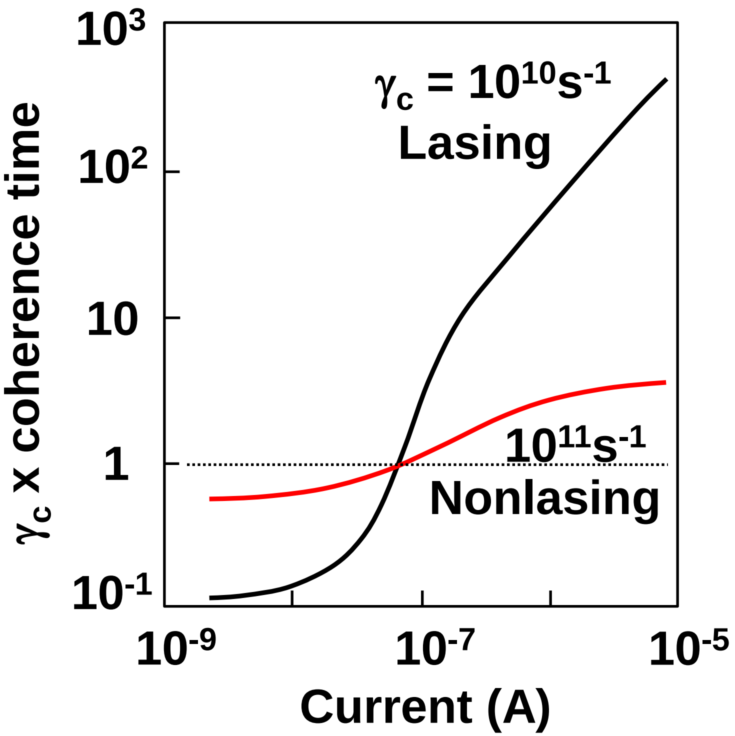


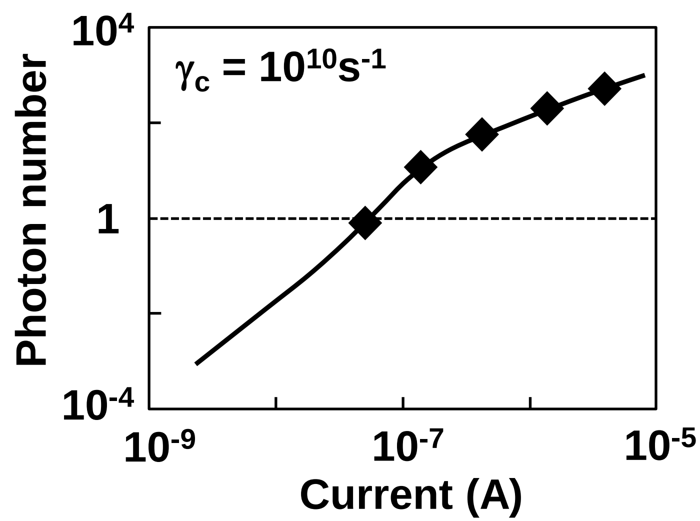
Other criteria for lasing

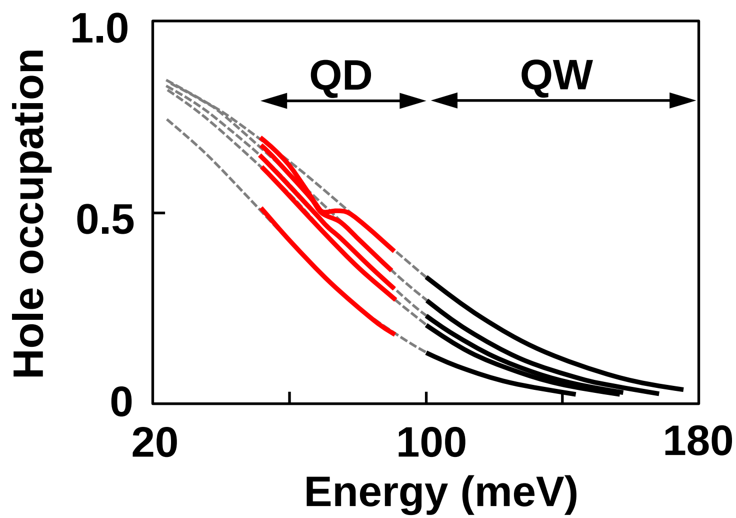
Coherence time

$$\tau_c = 2 \int_{-\infty}^{\infty} d\tau \left| \frac{\langle a^\dagger a(\tau) \rangle_{ss}}{\langle a^\dagger a \rangle_{ss}} \right|^2$$



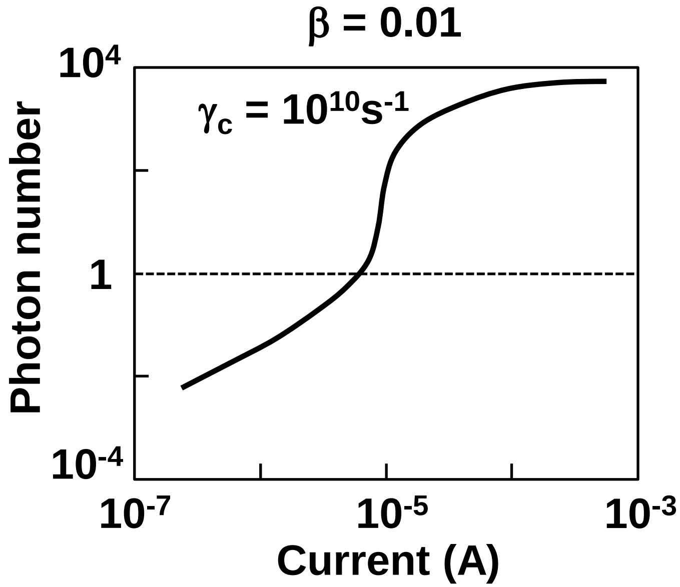
Population clamping and hole burning

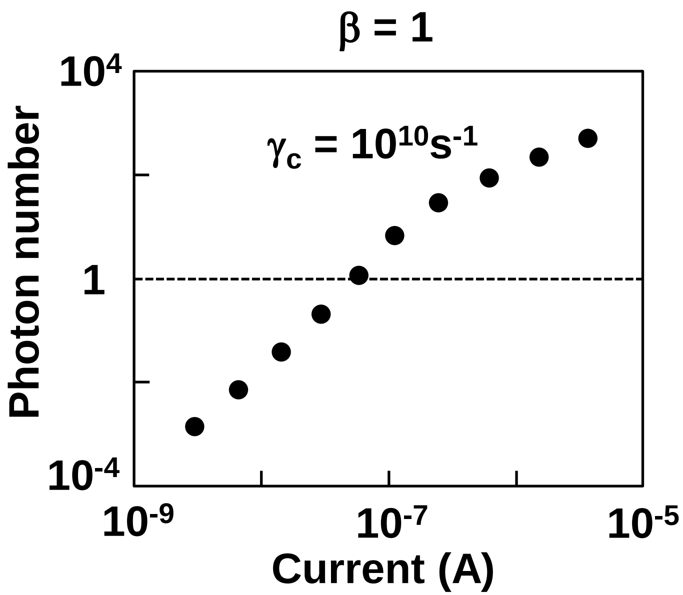




$\beta = 1$, $N_{QD} = 50$, $\Delta_{inh} = 20\text{meV}$

Other criteria for laser: stimulated emission



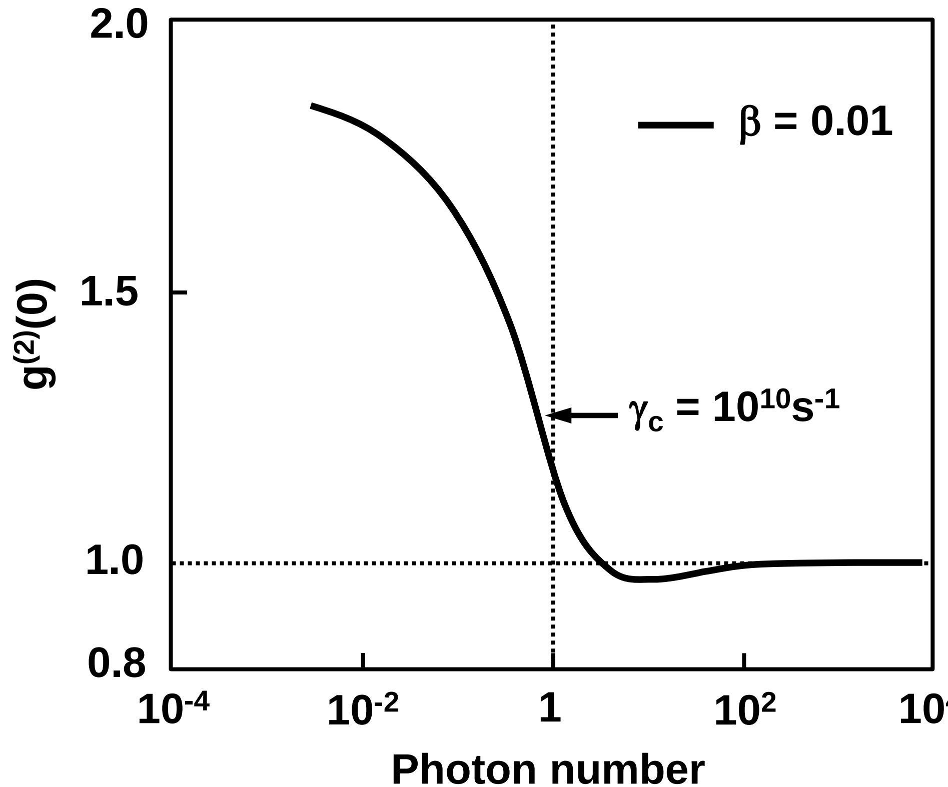


$N_{QD} = 50, \Delta_{inh} = 20 meV$

Light amplification by stimulated emission of radiation

$$\frac{dP_a}{dt} = -\gamma_l(n + 1)$$

Stimulated emission (n) — Spontaneous emission (1)



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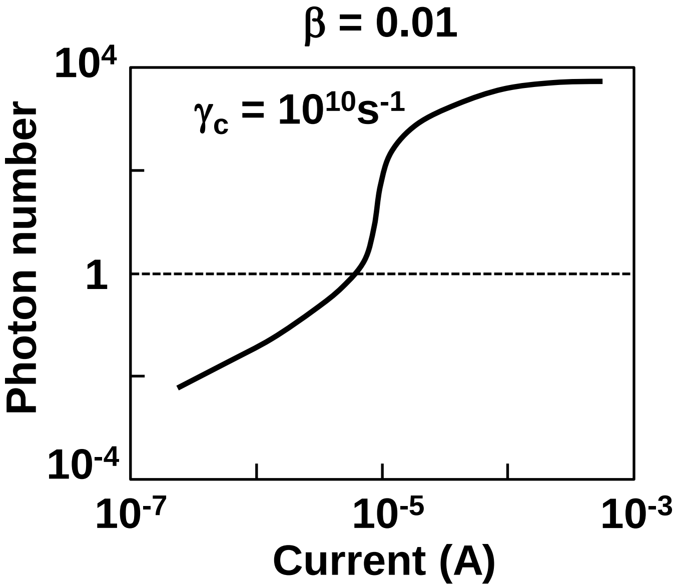

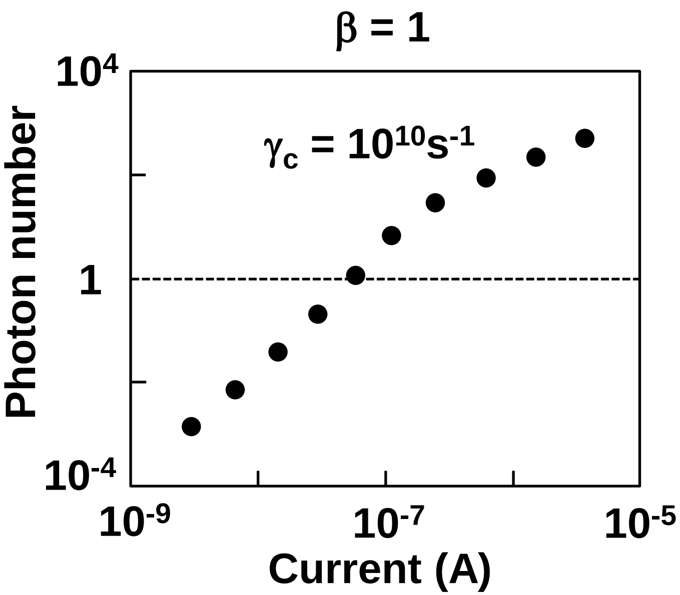

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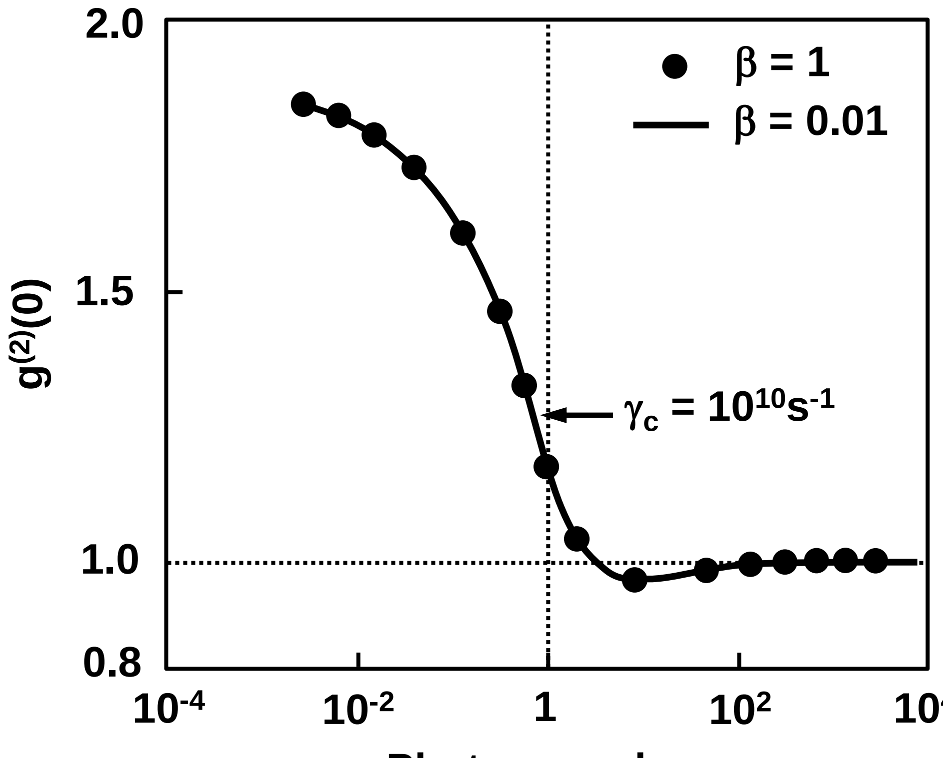

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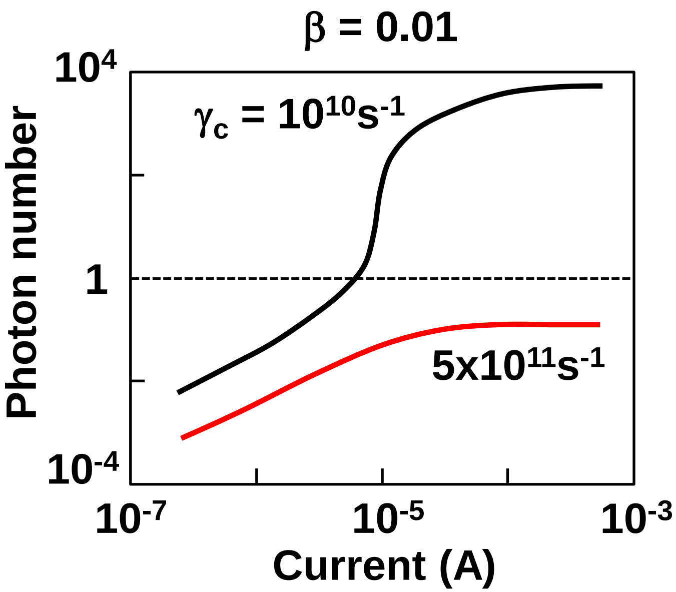

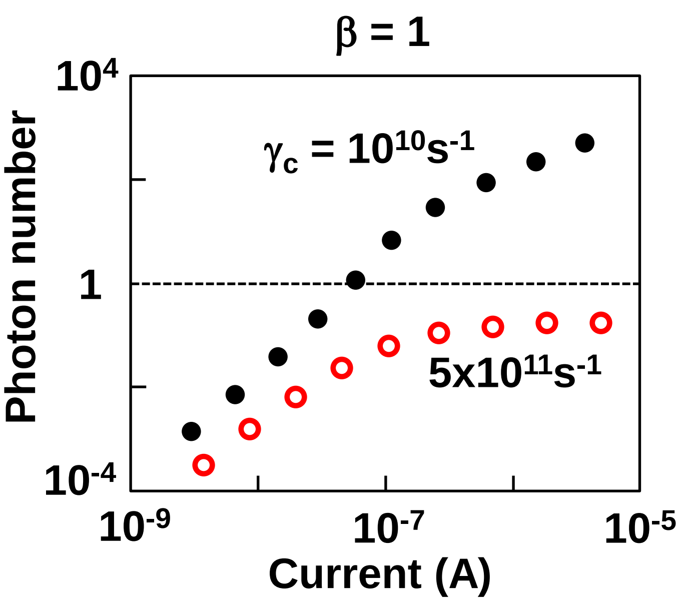

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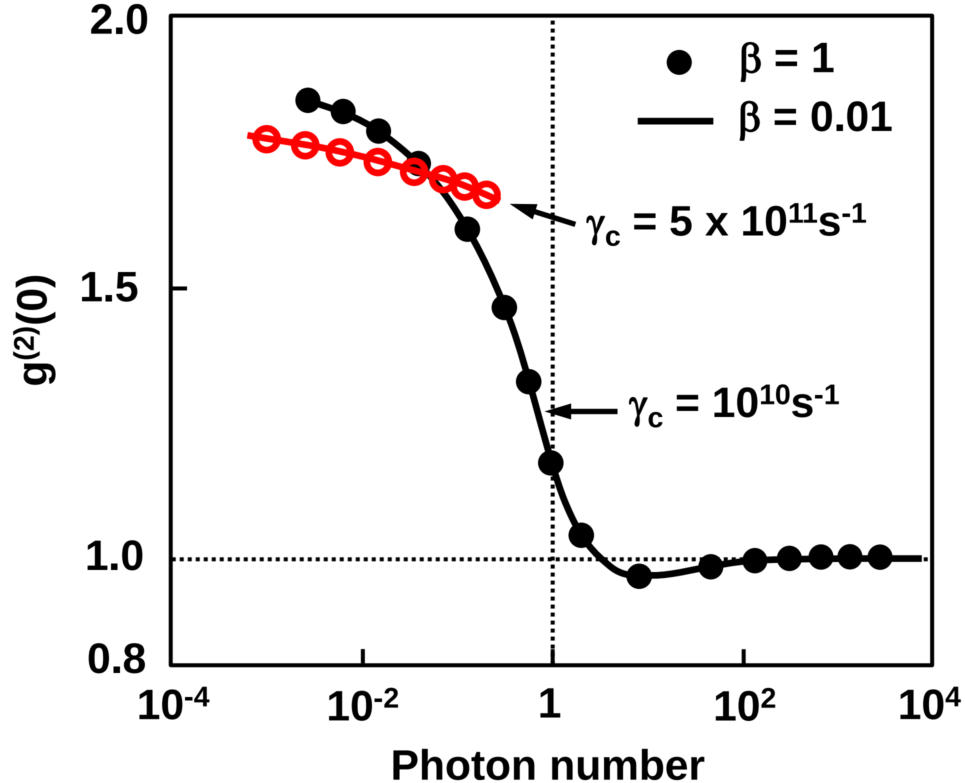
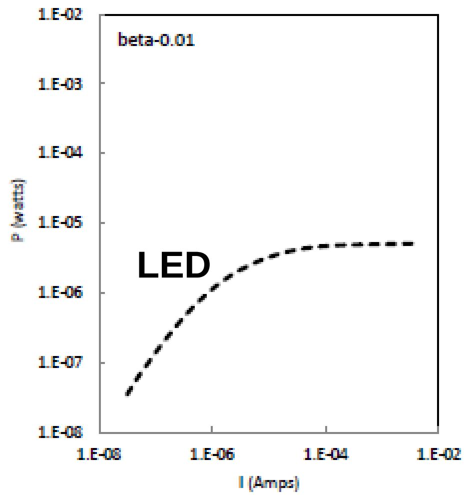
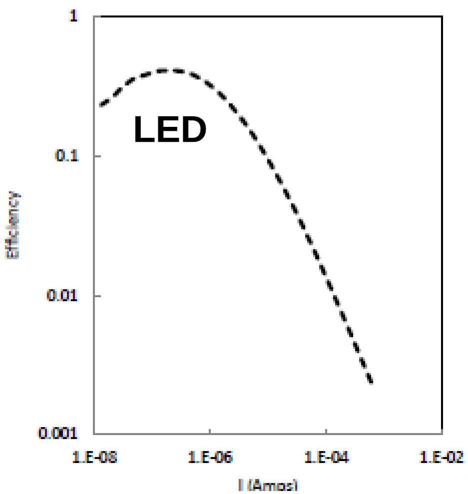


Light: Science and Applications, online 29 August, 2014

Efficient, high power SSL via Auger mitigation with quantum nanophotonics (or how to make a semiconductor dim-able headlight)

Conventional laser





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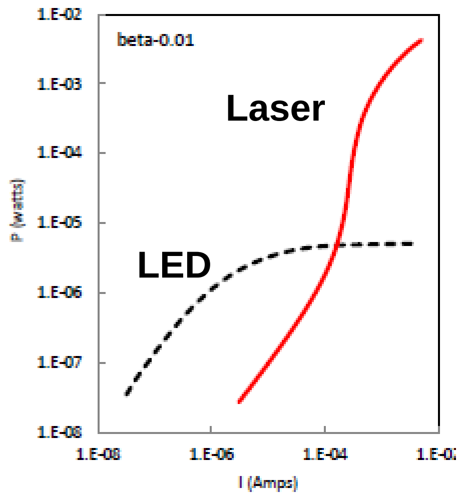

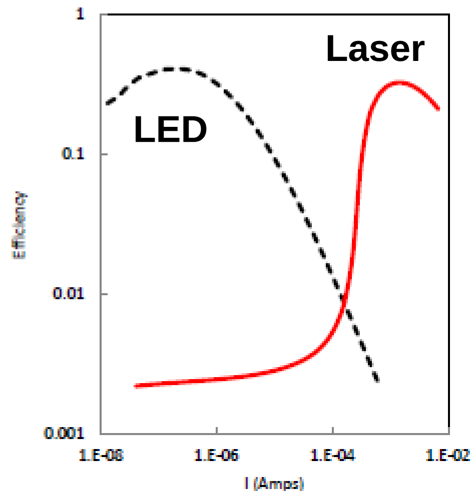

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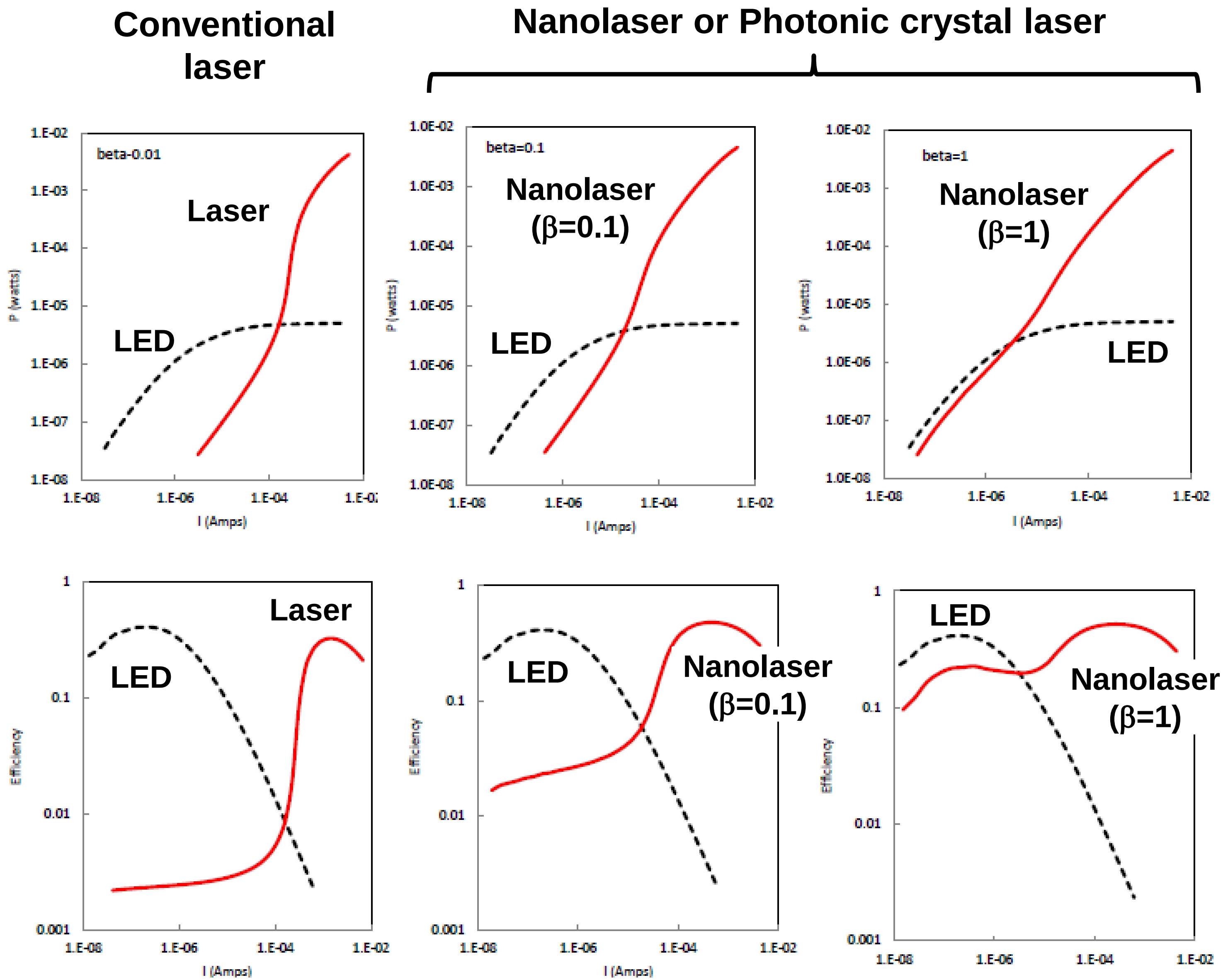
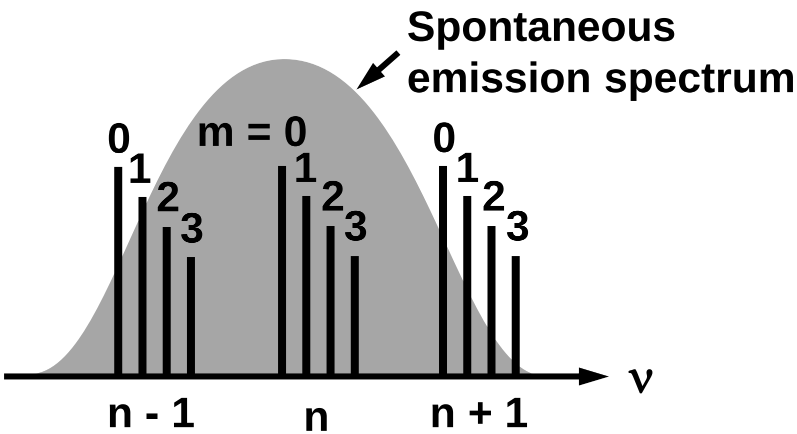


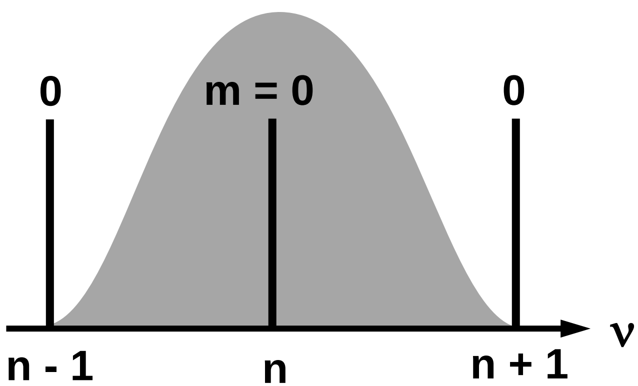
Interesting physics with nanolasers

Example 1
Thresholdless lasing

Most lasers **β << 1**



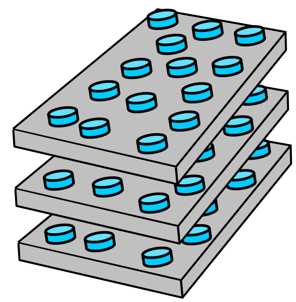
Some nanolasers **β = 1**



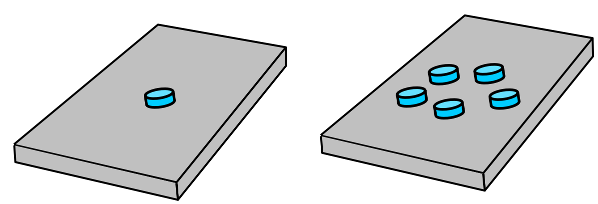
All emission into single resonator mode

Example 2
Single-photon generation

Most QD-laser active regions



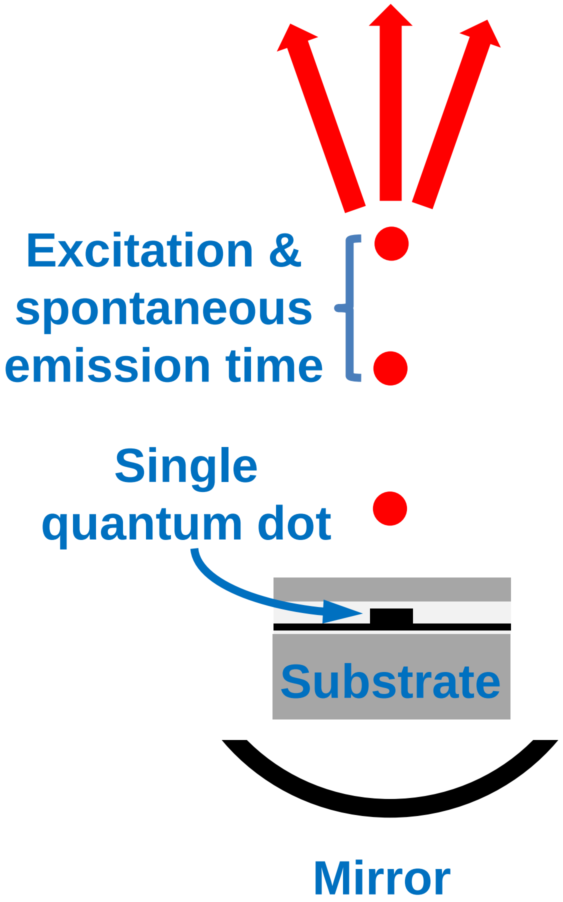
Few- QD active regions



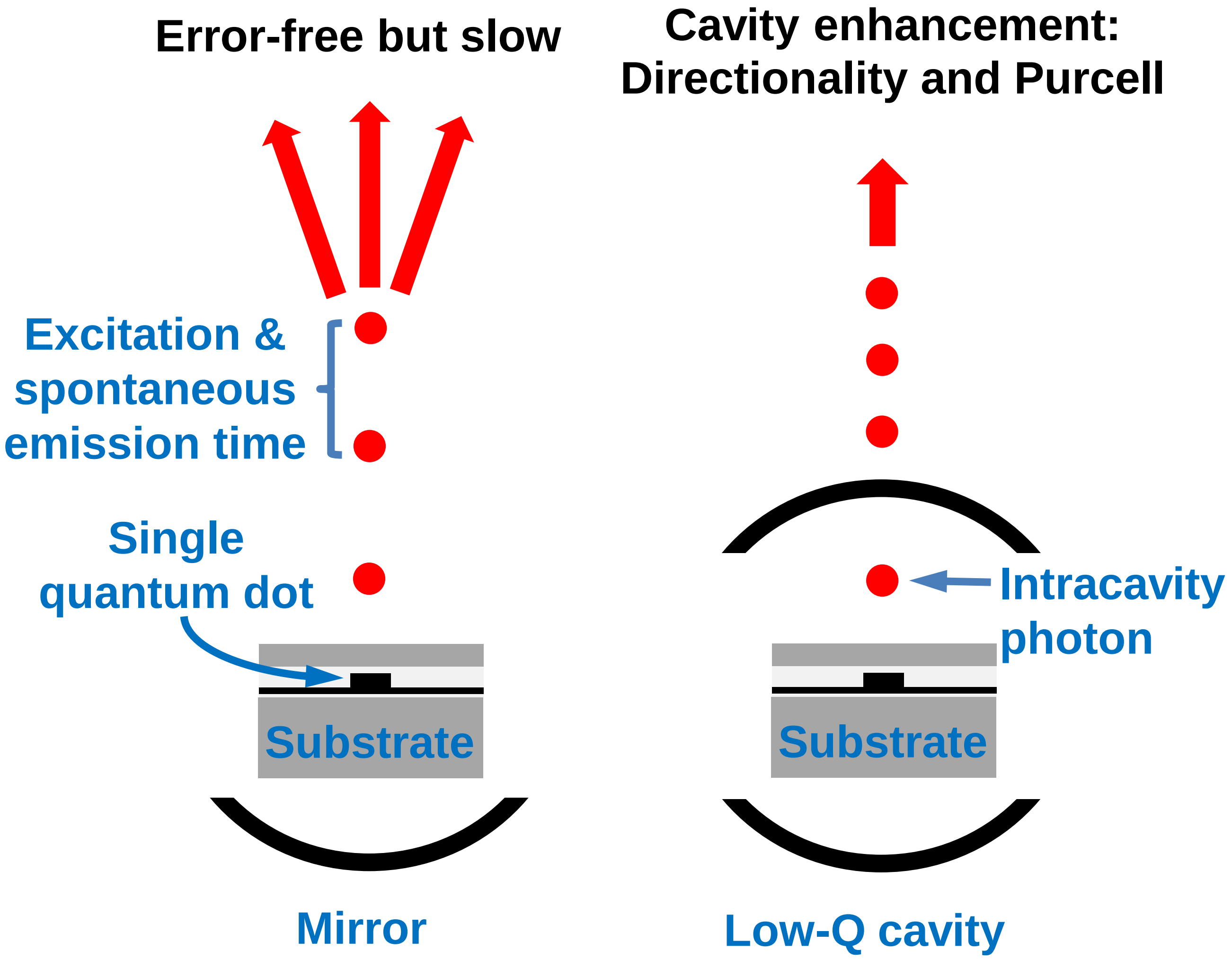
Nonclassical light

Single-photon source

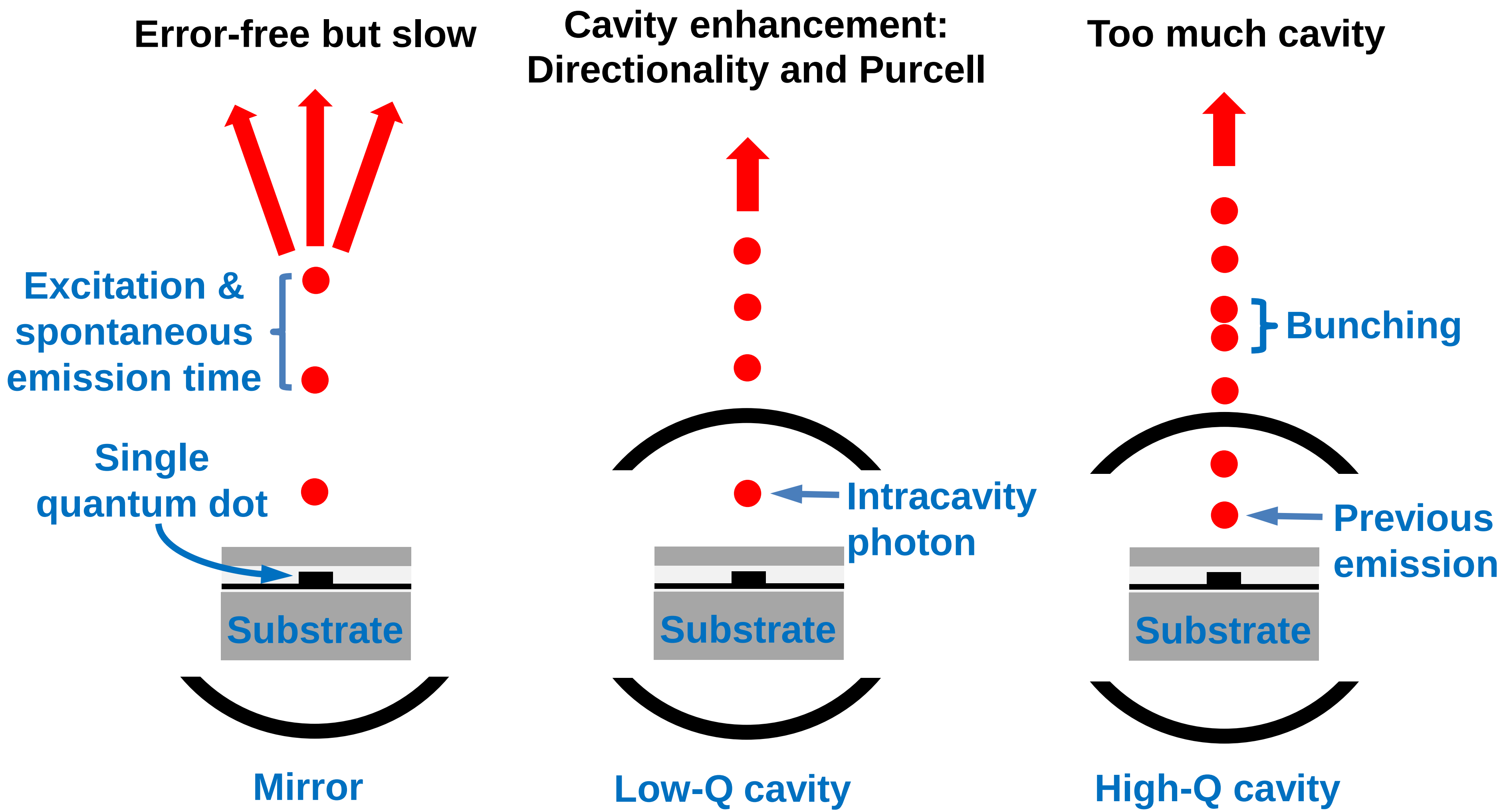
Error-free but slow



Single-photon source



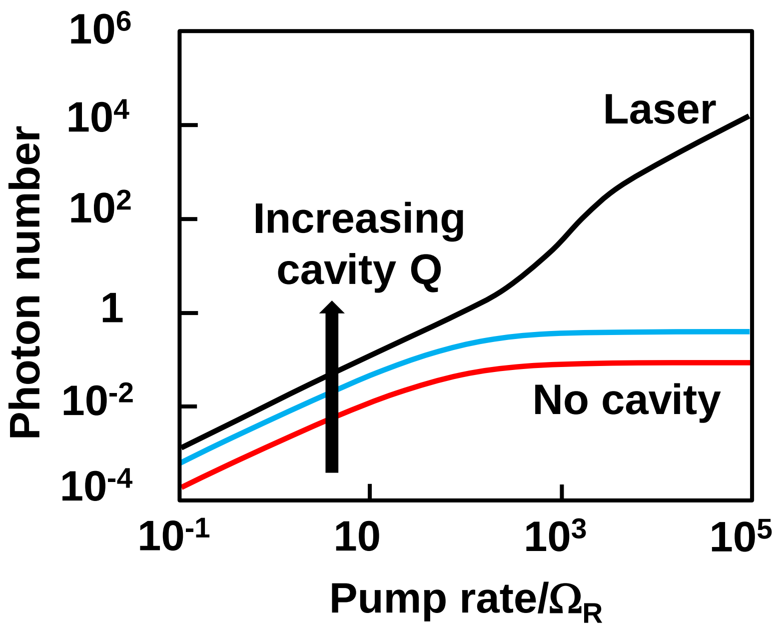
Single-photon source

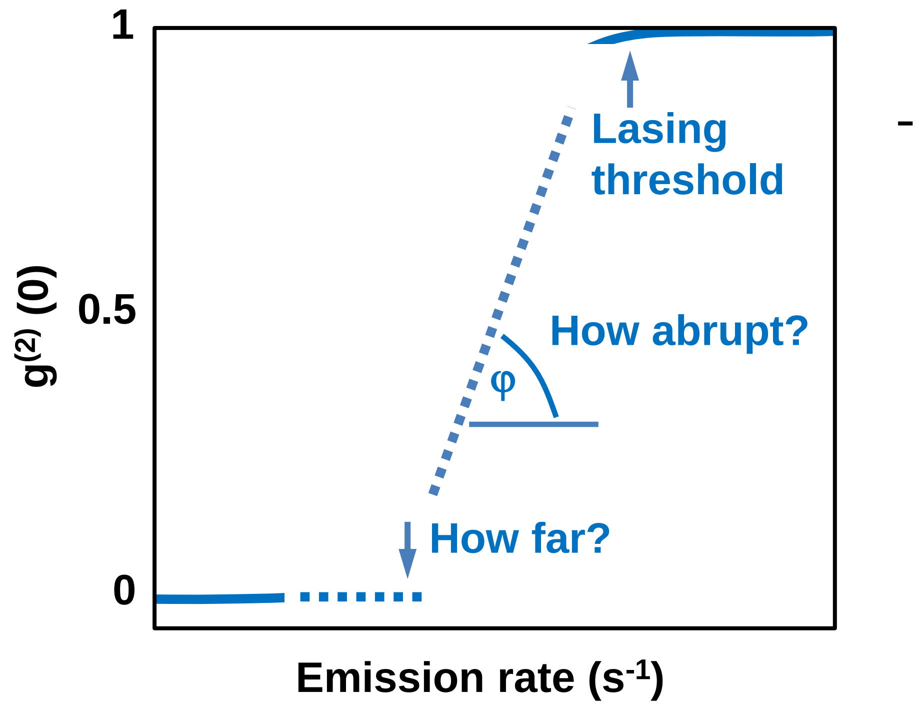


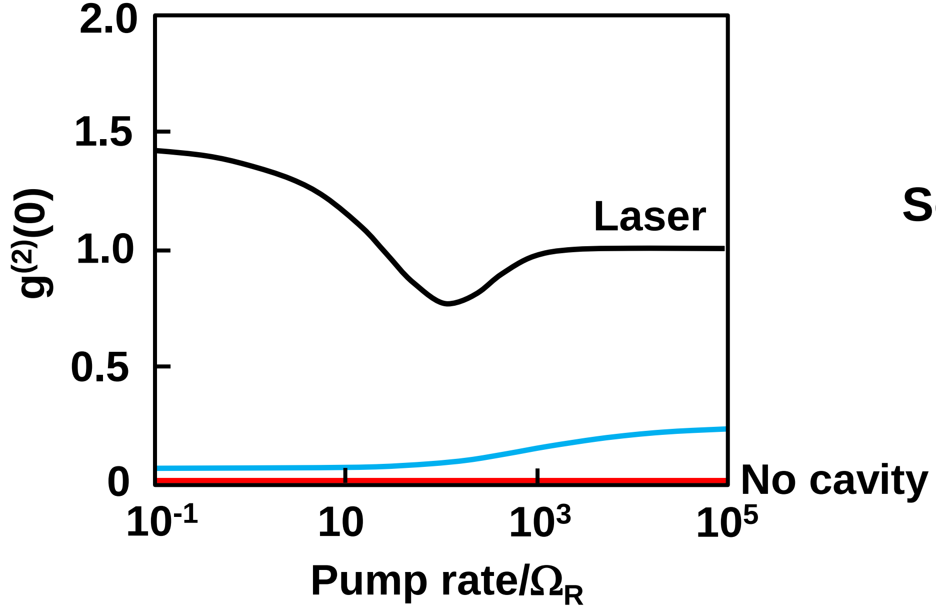
What is the right Q?

Fundamental limit to efficiency, rate and error?

Simulations







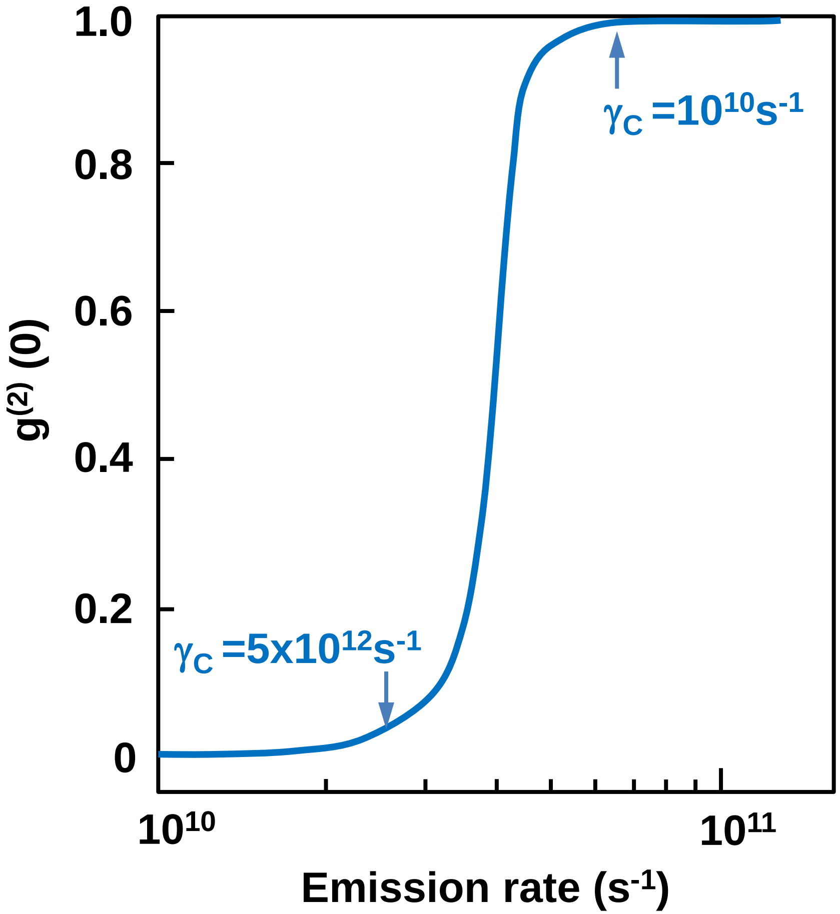
Second-order intensity correlation

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

Single-photon purity and emission rate

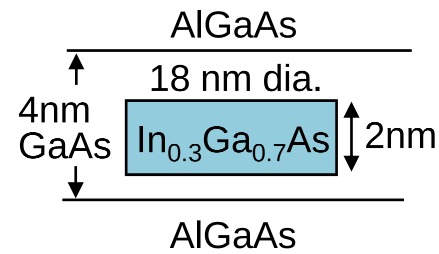
$g^{(2)}(0)$ vs. emission rate

(by increasing cavity-Q)



Cavity-enhanced rate ~ $10^9 s^{-1}$ (expt)

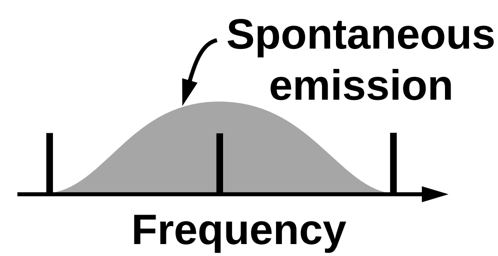
① Shallow quantum dot



Only s-shell transition

② Nanocavity

$$\beta = \frac{\gamma_l}{\gamma_{sp}} = 1$$



All emission into single resonator mode

③ Scaling with electron-light coupling

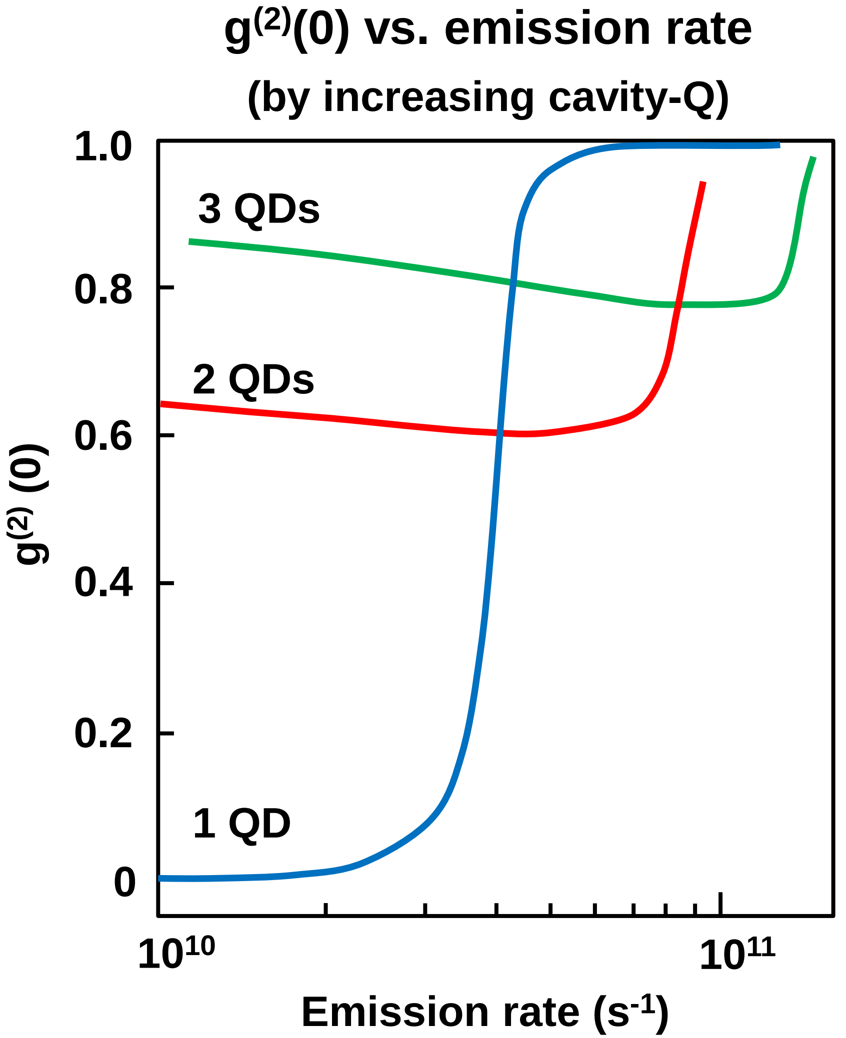
$$\wp \sqrt{\frac{\nu}{\hbar \epsilon_b V}}\, W(R_{QD}) \sum_n C(R_n)\, V(R_n)$$

Mode volume

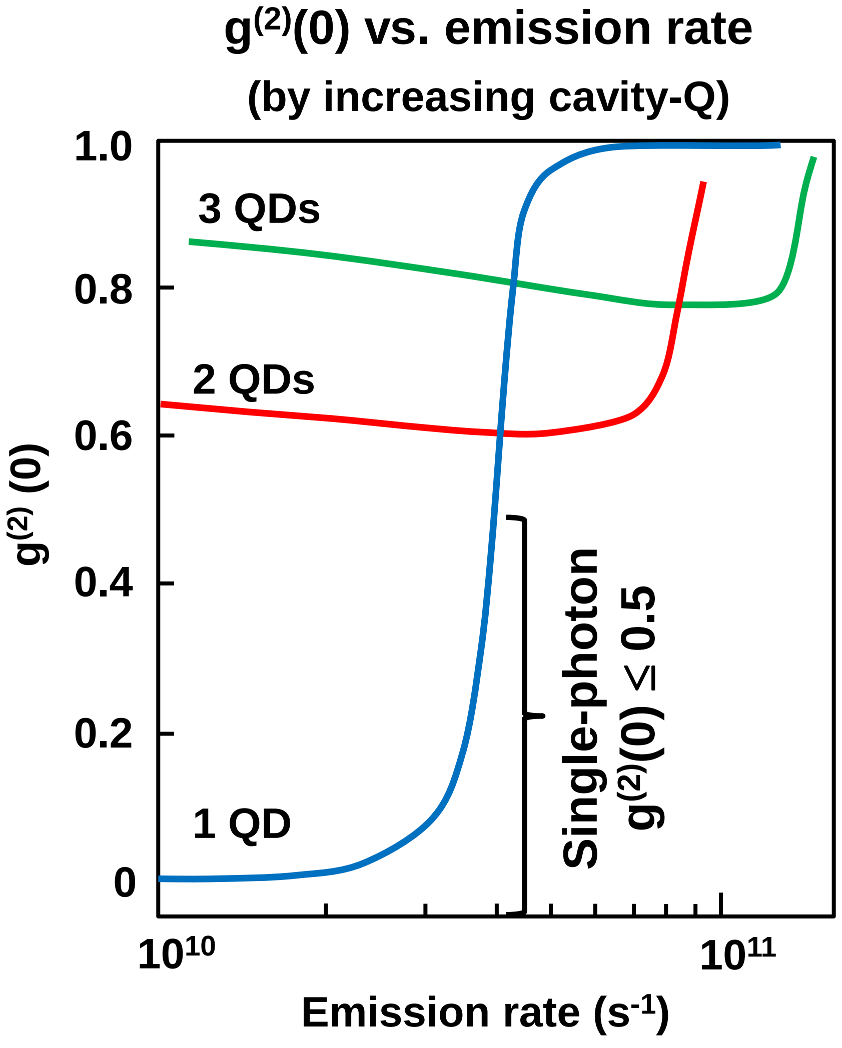
Confinement factor

Electron-hole envelope overlap

Concern: Extraneous quantum dots

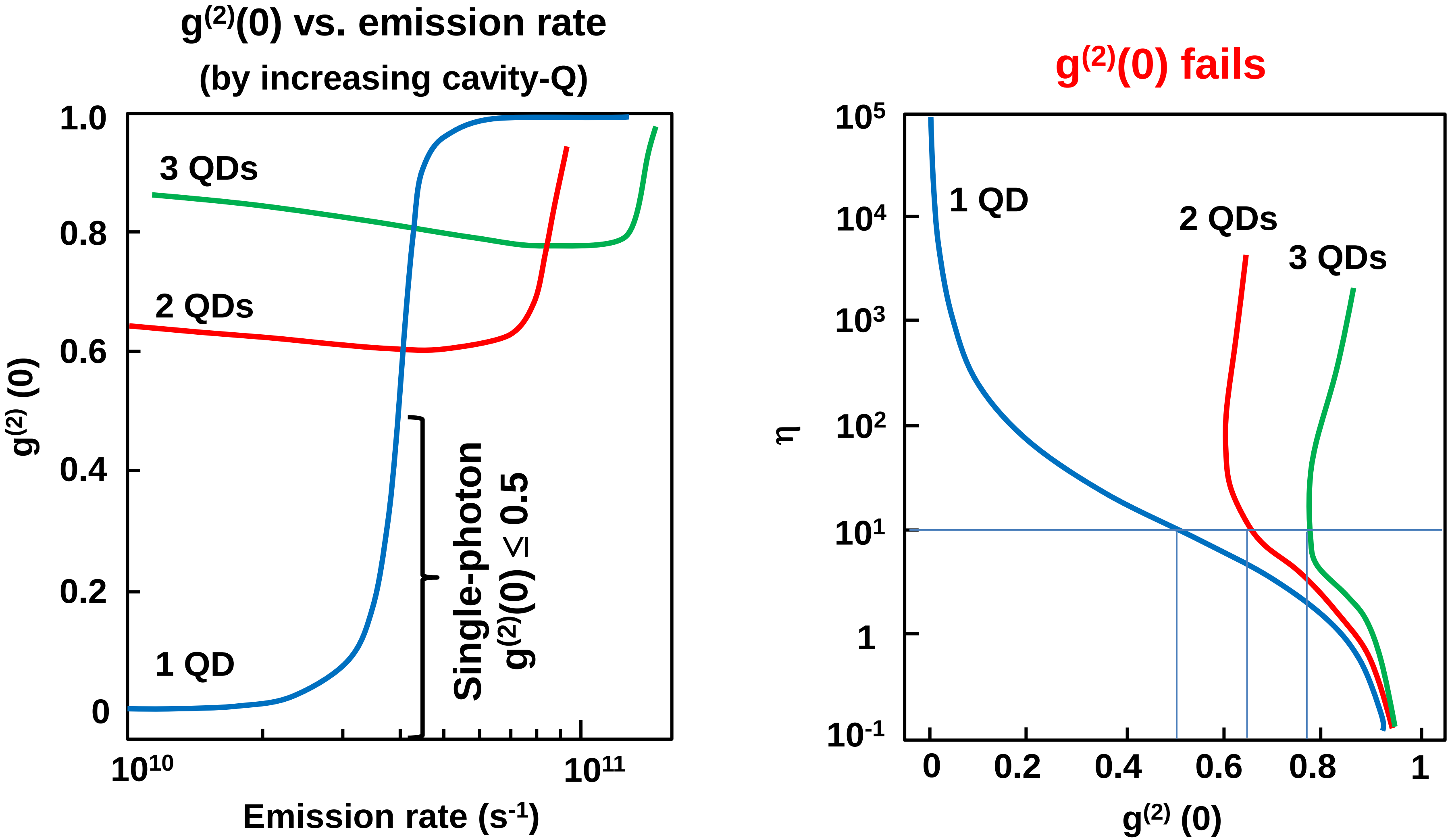


Concern: $g^{(2)}(0)$ as measure of error



Concern: $g^{(2)}(0)$ as measure of error

Single-photon purity: $\eta = \dfrac{\text{Single-photon emission probability}}{\text{Multi-photon emission probability}}$



From calculating photon statistics
Gies, Jahnke, Chow (submitted)

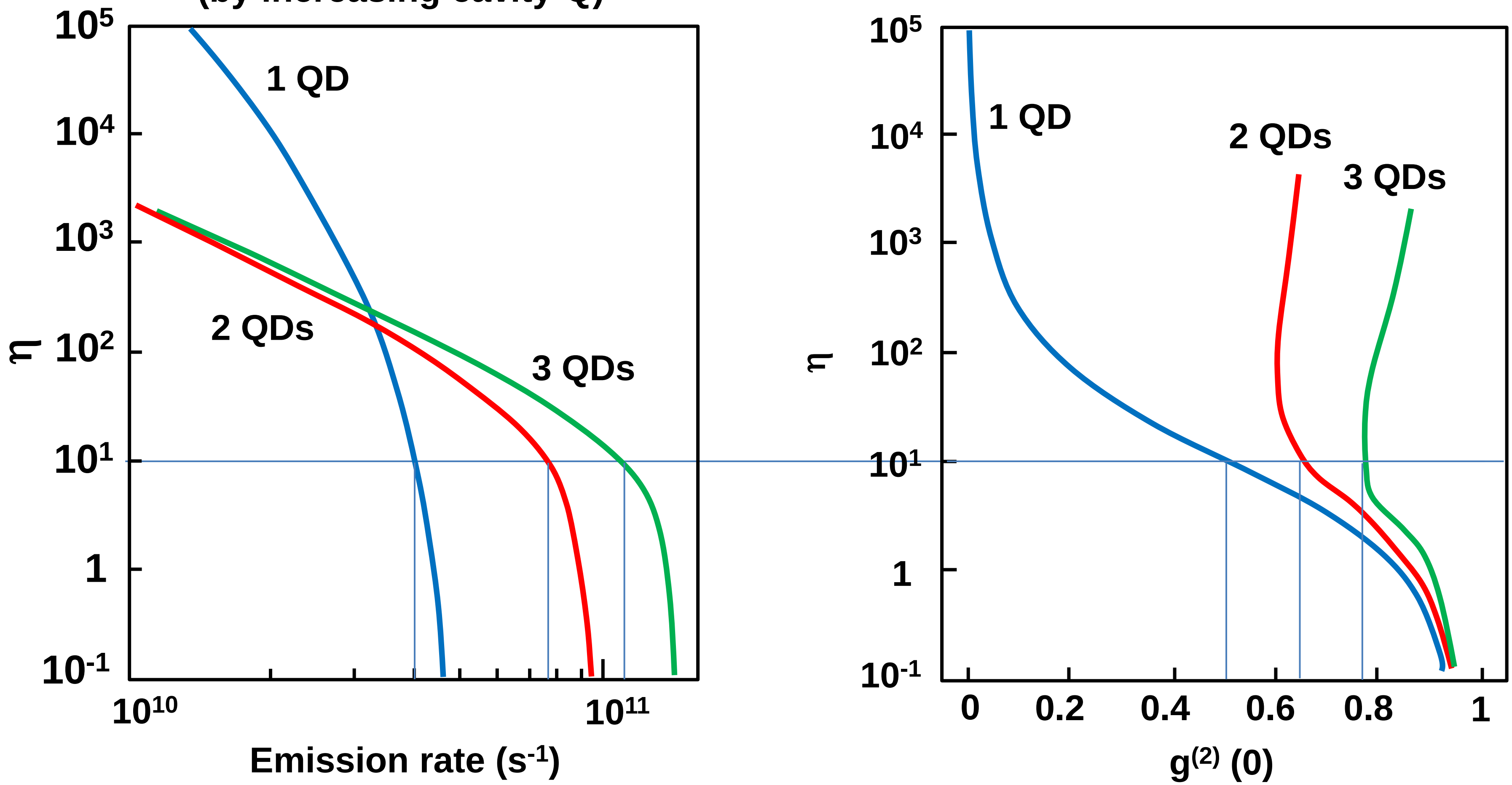
> 1 QD in cavity

Single-photon purity: $\eta = \dfrac{\text{Single-photon emission probability}}{\text{Multi-photon emission probability}}$

Purity vs. emission rate

(by increasing cavity-Q)

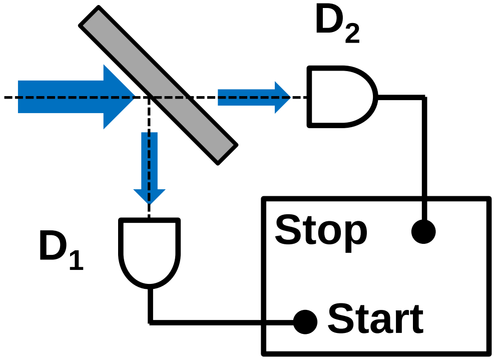
$g^{(2)}(0)$ fails



From calculating photon statistics
Gies, Jahnke, Chow (submitted)

Conversion from $g^{(2)}(0)$ to η : nontrivial

Hanbury-Brown-Twiss

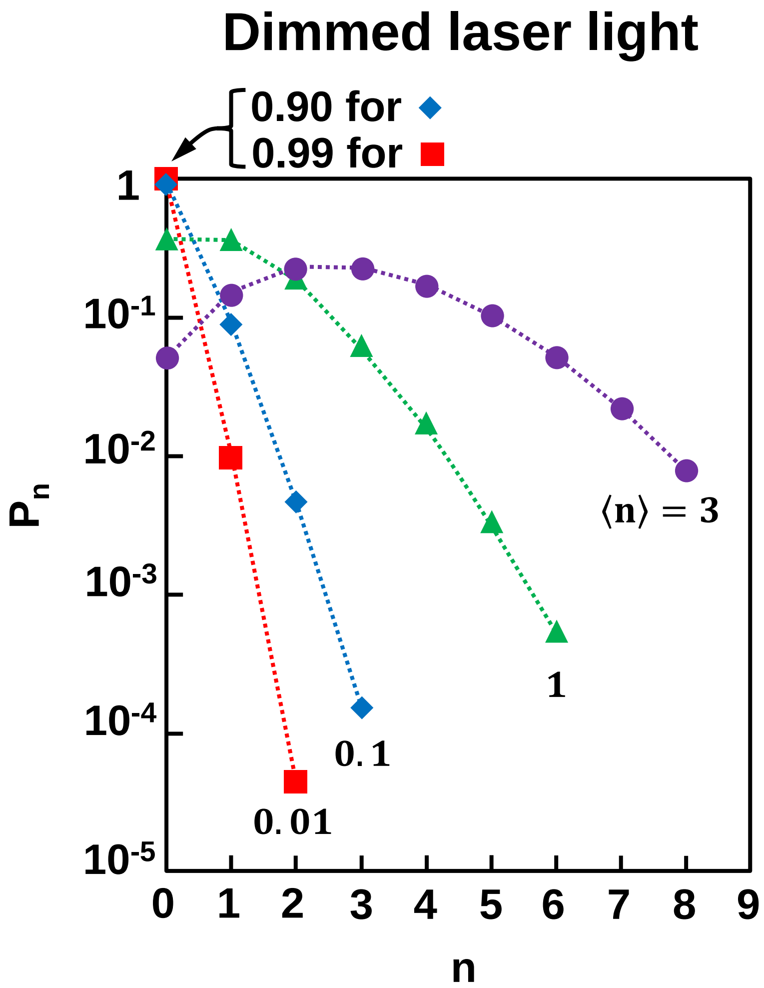


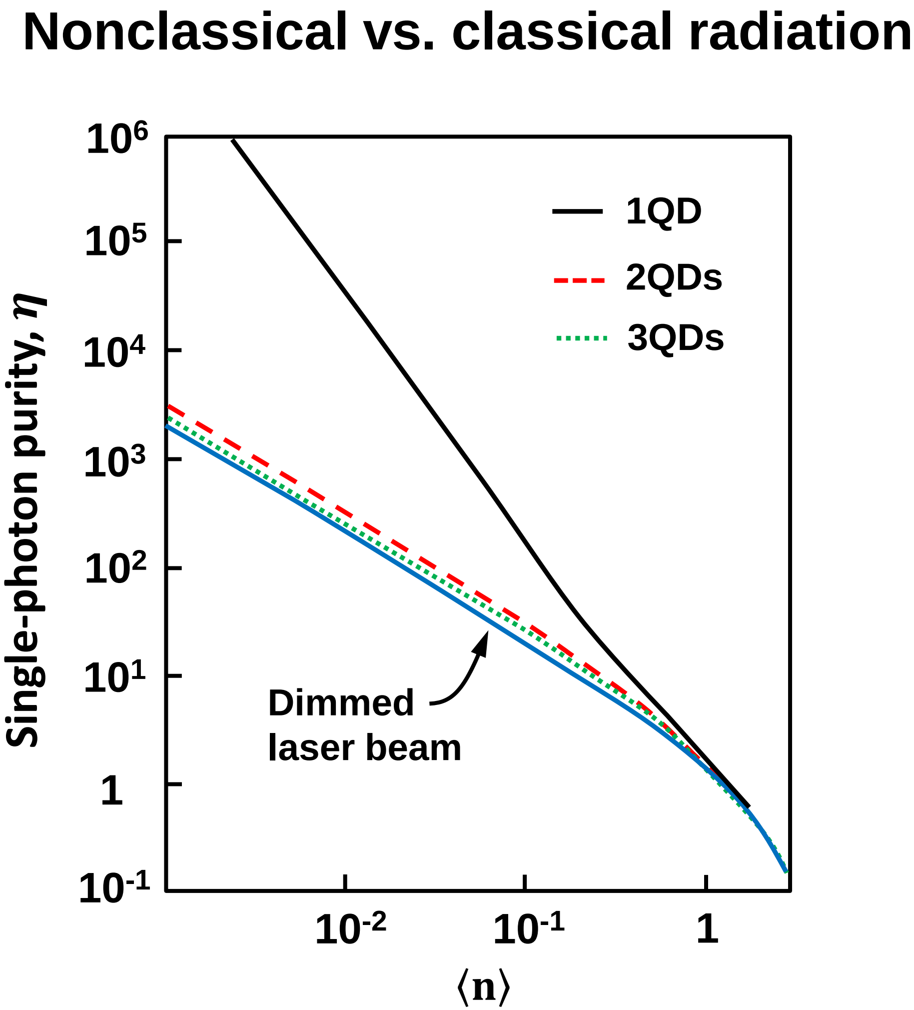
$g^{(2)}(0)$: $\langle a^\dagger a^\dagger a a \rangle$

η : Photon statistics

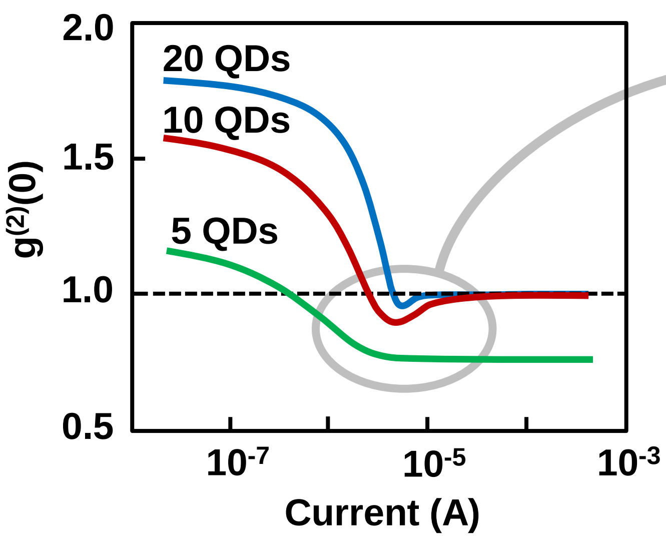
$$P_n = \langle n, 0X|\varrho|0X, n\rangle + \langle n, 1X_s|\varrho|1X_s, n\rangle + \langle n, 2X_{ss}|\varrho|2X_{ss}, n\rangle + \ ...$$

Dimmed laser beam versus single-photon source





? Increasing single-photon production rate with few-emitter active region?

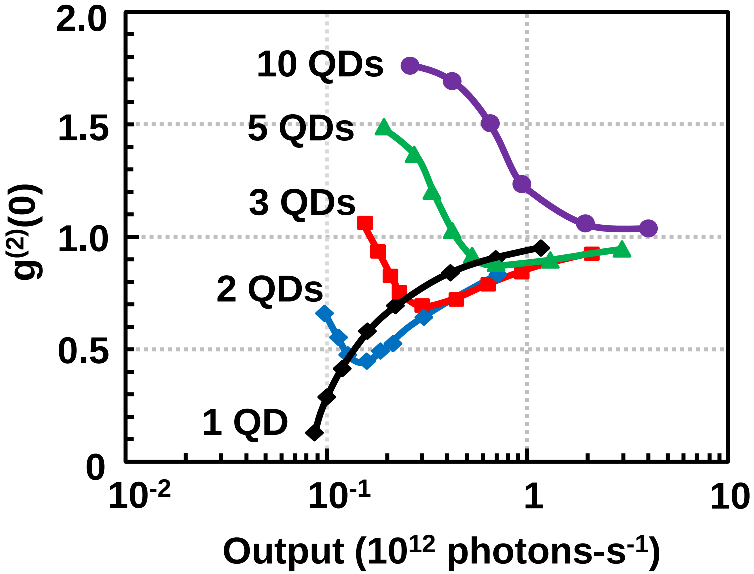


Light-carrier correlation

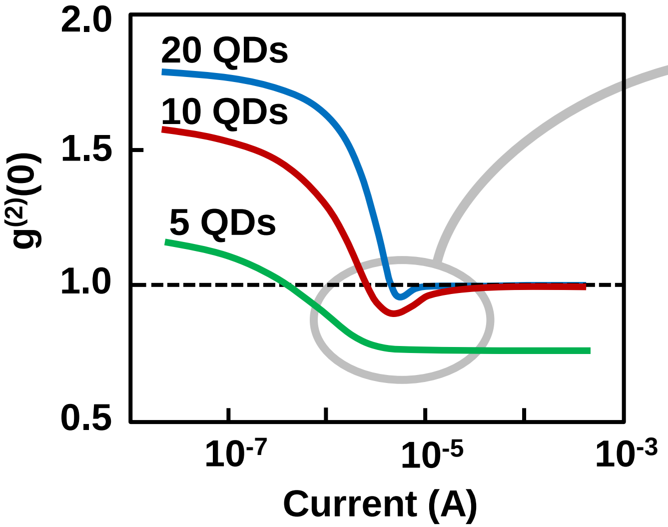
Leading terms: $\delta\langle c^\dagger c a^\dagger a\rangle, \delta\langle b^\dagger b a^\dagger a\rangle, \delta\langle b^\dagger c^\dagger a^\dagger a a\rangle$

$$\frac{d\,\delta\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle}{dt} = -(\gamma_e + 2\gamma_c)\delta\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle - (g_\alpha p_\alpha + g_\alpha^* p_\alpha^*)(n_p + n_e^\alpha) - 2Re[g_\alpha \delta\langle b_\alpha^\dagger c_\alpha^\dagger a^\dagger a a\rangle] - \sum_\sigma 2\,Re[g_\sigma \delta\langle b_\sigma^\dagger c_\sigma^\dagger c_\alpha^\dagger c_\alpha a\rangle]$$

γ_e calculated from carrier-carrier and carrier phonon interactions



? Increasing single-photon production rate with few-emitter active region?

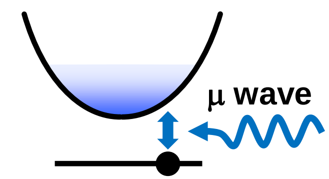


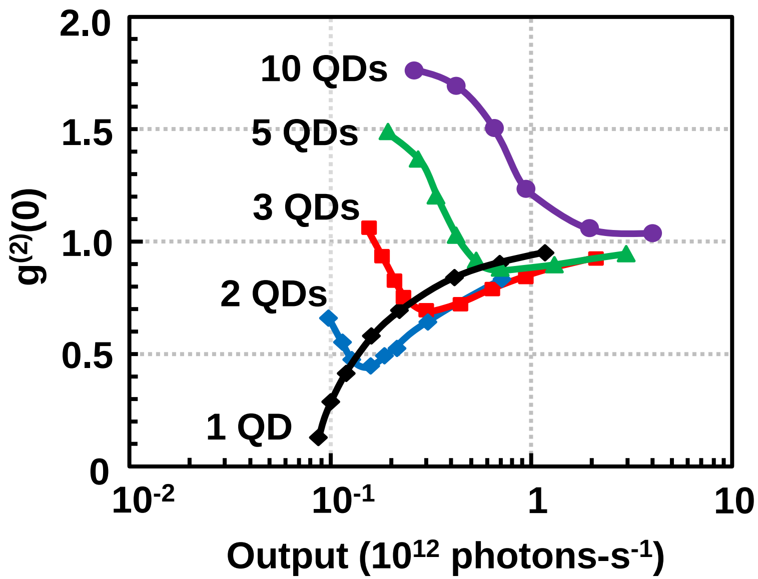
Light-carrier correlation

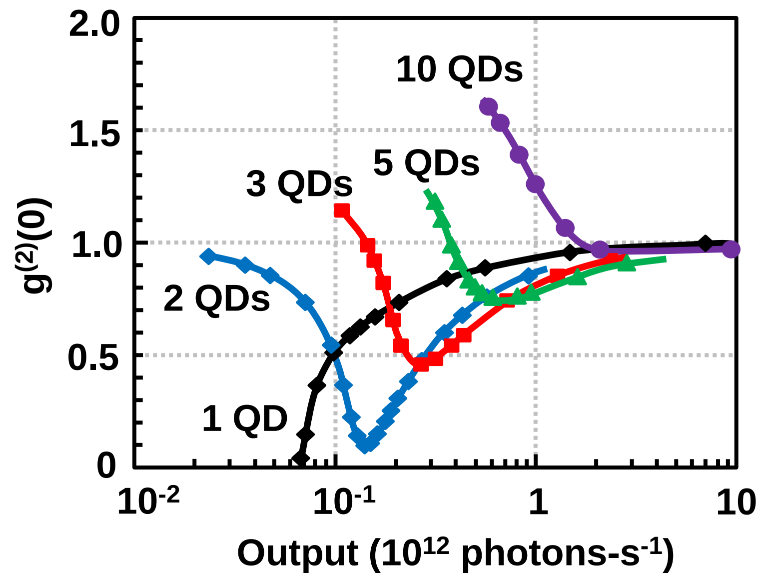
Leading terms: $\delta\langle c^\dagger c a^\dagger a\rangle, \delta\langle b^\dagger b a^\dagger a\rangle, \delta\langle b^\dagger c^\dagger a^\dagger a a\rangle$

$$\frac{d\,\delta\langle c^\dagger_\alpha c_\alpha a^\dagger a\rangle}{dt} = -(\gamma_e + 2\gamma_c)\delta\langle c^\dagger_\alpha c_\alpha a^\dagger a\rangle - (g_\alpha p_\alpha + g^*_\alpha p^*_\alpha)(n_p + n^\alpha_e) - 2Re\left[g_\alpha \delta\langle b^\dagger_\alpha c^\dagger_\alpha a^\dagger a a\rangle\right] - \sum_\sigma 2\,Re\left[g_\sigma \delta\langle b^\dagger_\sigma c^\dagger_\sigma c^\dagger_\alpha c_\alpha a\rangle\right]$$

γ_e calculated from carrier-carrier and carrier phonon interactions ➡ $\gamma_e^{eff} < \gamma_e$ **"by hand" ?**







Correlations and photon statistics in nanocavity emitters

Approach
- Quantized light and carriers
- Consistent account of light-carrier correlations

Nanolasers
- Combination of intensity & $g^{(2)}(0)$ gives definitive description of lasing
- There is no thresholdless lasing

Single-photon sources

Quantum communications

Disconnection:

	Ideal	Applications
	Single-photon	Dimmed laser
	$\|n\rangle$	$e^{-\frac{\|\alpha\|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} \|n\rangle \; , \quad \|\alpha\|^2 \ll 1$

Bridge: tradeoff among efficiency, rate and error
- Challenges in fabrication and modeling
- Questions concerning present measure of performance

Chow & Jahnke, Progress in Quantum Electronics 37, 109 (2013)

Chow, Gies & Jahnke, Light: Science and Applications, online 29 August, 2014

Other applications of modeling approach

Gain medium engineering

Chow, Lorke & Jahnke, 'Will Quantum Dots Replace Quantum Wells As the Active Medium of Choice in Future Semiconductor Lasers?' IEEE J. Selected Topics in Quantum Electron. **17**, 1349 (2011).

BEC and Atomtronics

Chow, Straatsma & Anderson, 'An engineering design tool for atomtronic circuits' (in preparation).

Quantum optomechanics

Carmele, Kabuss & Chow, 'Highly detuned Rabi oscillations for a quantum dot in a microcavity,' Physical Review B **87**, Rapid Communication, 041305 (2013).

Solid state lighting

Chow, *Novel LED Model Offers New Insights,* Compound Semiconductor Magazine, July, 2014.

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Frank Jahnke, Bremen University

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Dana Anderson, U Colorado and JILA

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Andreas Knorr: TU-Berlin

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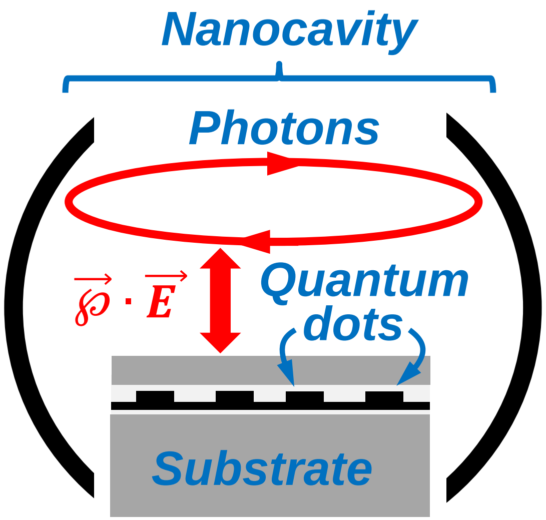
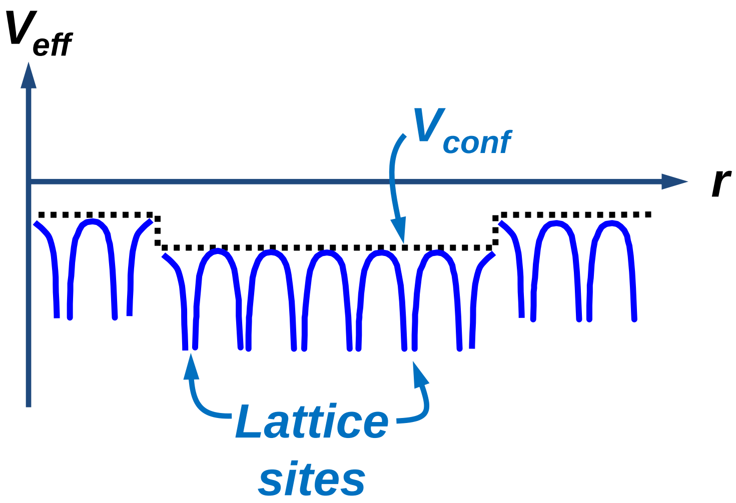
EFRC

Stephan Koch, Philipps University, Marburg

Hamiltonian for semiconductor quantum dots in nanocavity





$$H = \sum_i \left[\frac{p_i^2}{2m_0} - \sum_j \frac{Ze^2}{4\pi\varepsilon_b |r_i - R_j|} + V_{conf}(r_i) + \sum_{j \neq i} \frac{e^2}{4\pi\varepsilon_b |r_i - r_j|} - \vec{\wp}_i \cdot \vec{E} \right]$$

Single-particle energy & carrier-phonon interaction

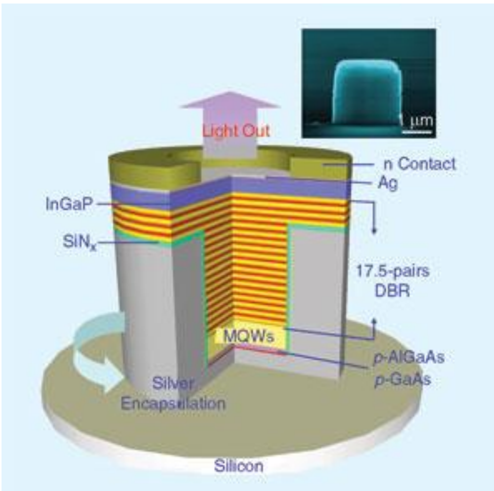
Many-body carrier-carrier interaction

Light-matter interaction (Dipole approx.)

Experimental setup

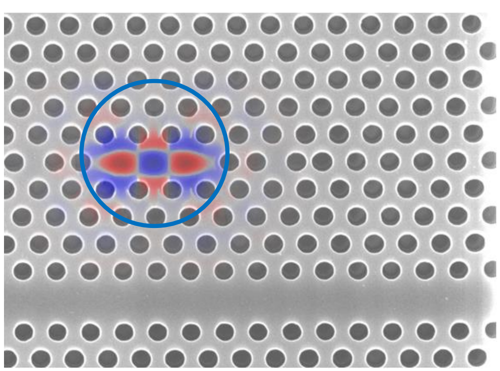
Optical cavity

Micro- or nano-cavity



(Adapted from a figure by Lu et al., UIUC)

Photonic crystal

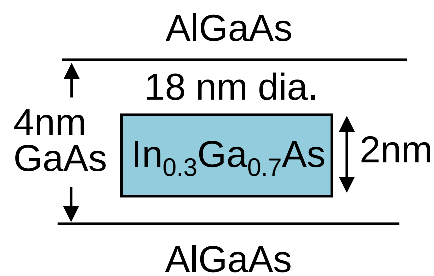


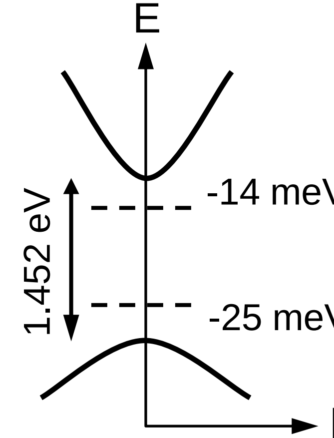
(Courtesy of Willie Luk, Sandia National Labs)

Active region

Shallow quantum dot

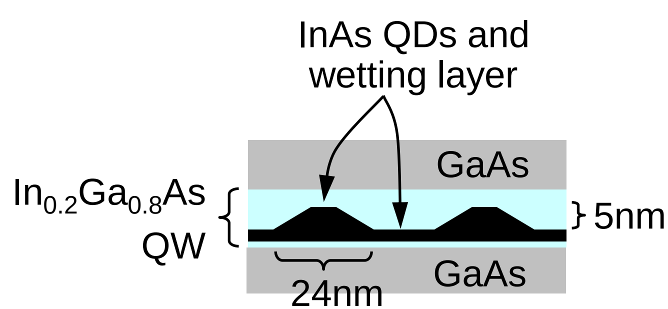
950 nm emission

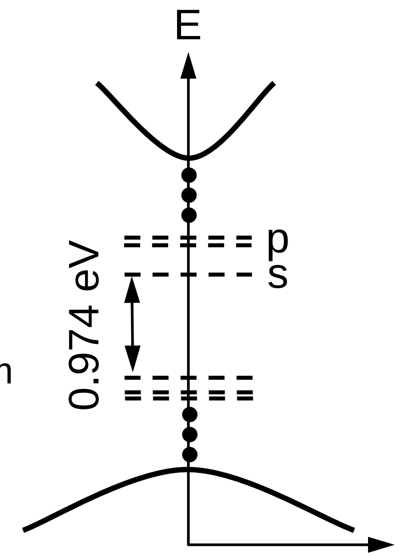




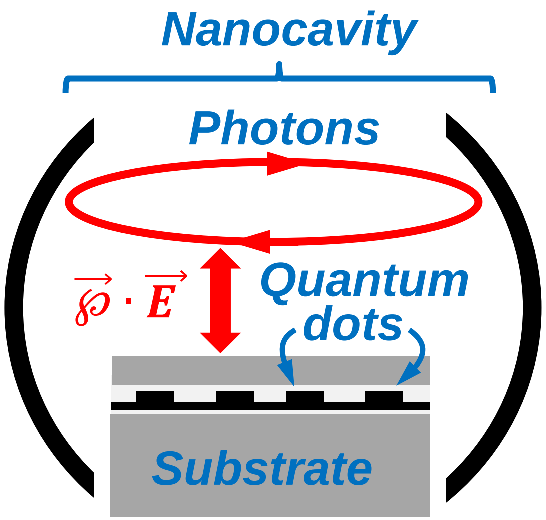
Deep quantum dot

1.3 – 1.5 μm emission

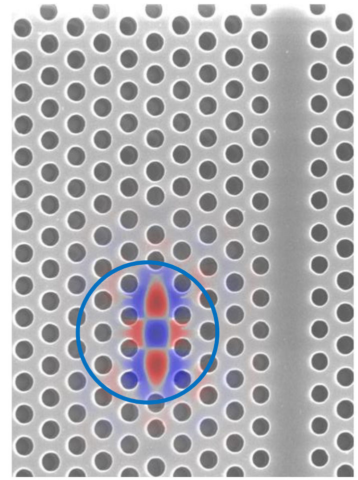




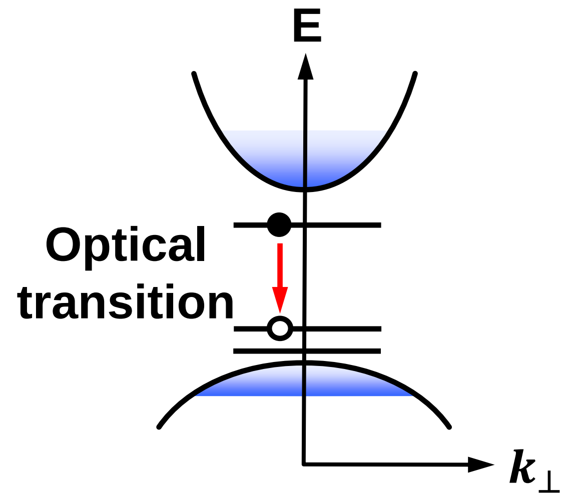
Nano-emitter model – setup: time-independent part



Optical mode



Electronic structure



Second quantization

Radiation field

$$E(\mathbf{r}) = \hat{\epsilon}\sqrt{\frac{\hbar\nu}{2\epsilon_b V}}\, W(\mathbf{r})\left(a + a^\dagger\right)$$

Photon annihilation and creation operators

Carriers

$$\psi_e(\mathbf{r}) = C(\mathbf{r})\left\langle \mathbf{r}\middle|\tfrac{1}{2}, s_z\right\rangle c_e$$

$$\psi_h(\mathbf{r}) = V(\mathbf{r})\langle \mathbf{r}|m\rangle\, c_h$$

+ Adjoint

Hole and electron annihilation operators