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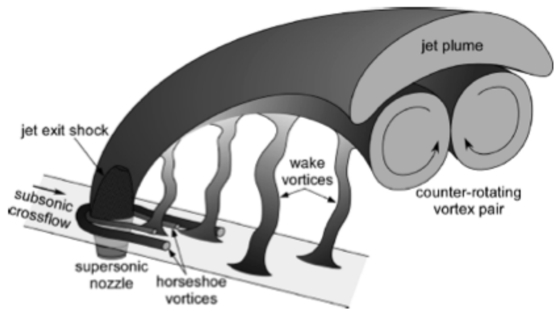

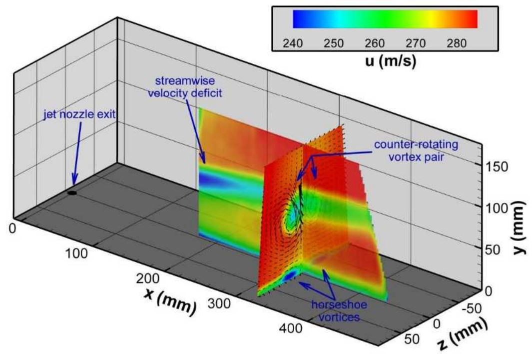

Eddy viscosity model selection for transonic turbulent flows using shrinkage regression

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U.S. DEPARTMENT OF ENERGY

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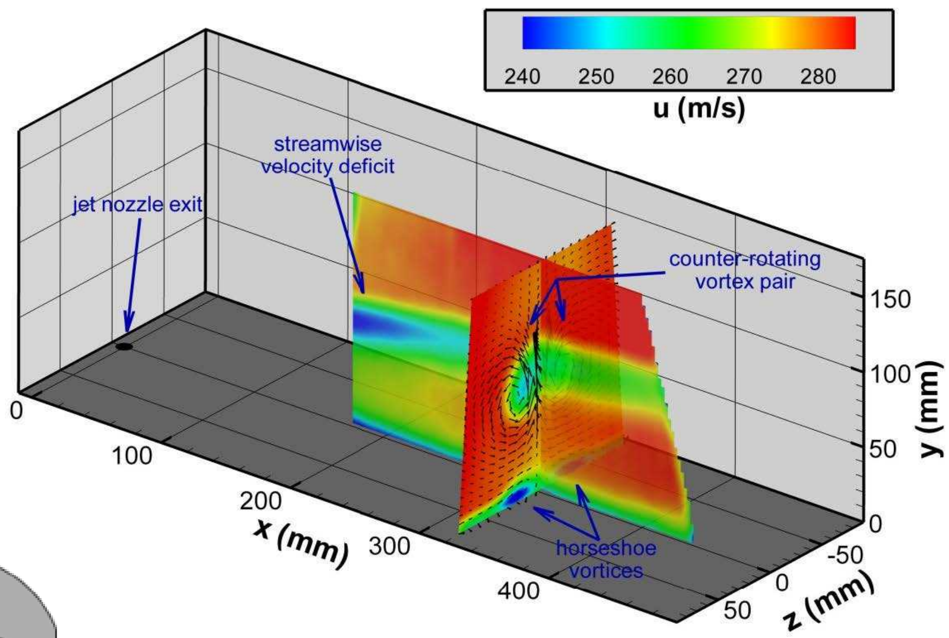
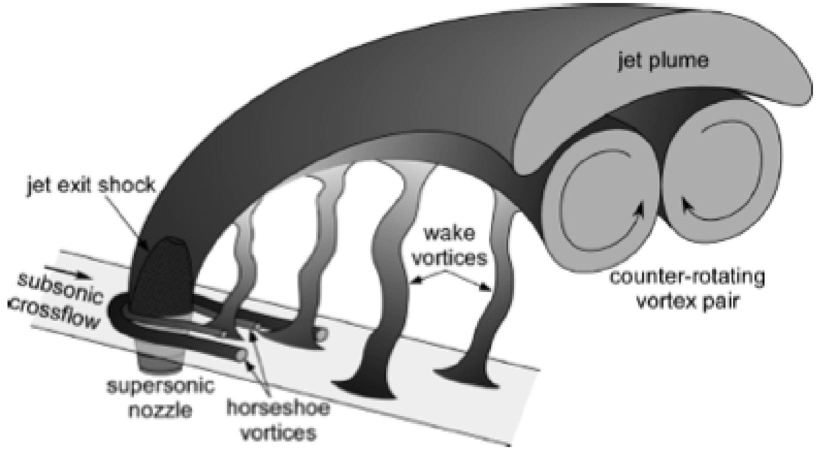
Introduction

- Aim: Develop a principled way of enriching a turbulence model to reduce model-form error
  - Needed for a predictive RANS simulator for transonic jet-in-crossflow
- Drawback: RANS simulations are simply not predictive
  - They have "model-form" error i.e., missing physics
  - They use parameters derived from canonical flows quite unlike jet-in-crossflow interactions.
- Hypothesis
  - Once a RANS model has been calibrated to a jet-in-crossflow experiment, any lack of predictive skill is due to model-form uncertainty i.e., shortcomings of the linear eddy viscosity model (LEVM)
  - If the LEVM can be enriched with higher-order terms and re-calibrated, we could reduce the error further

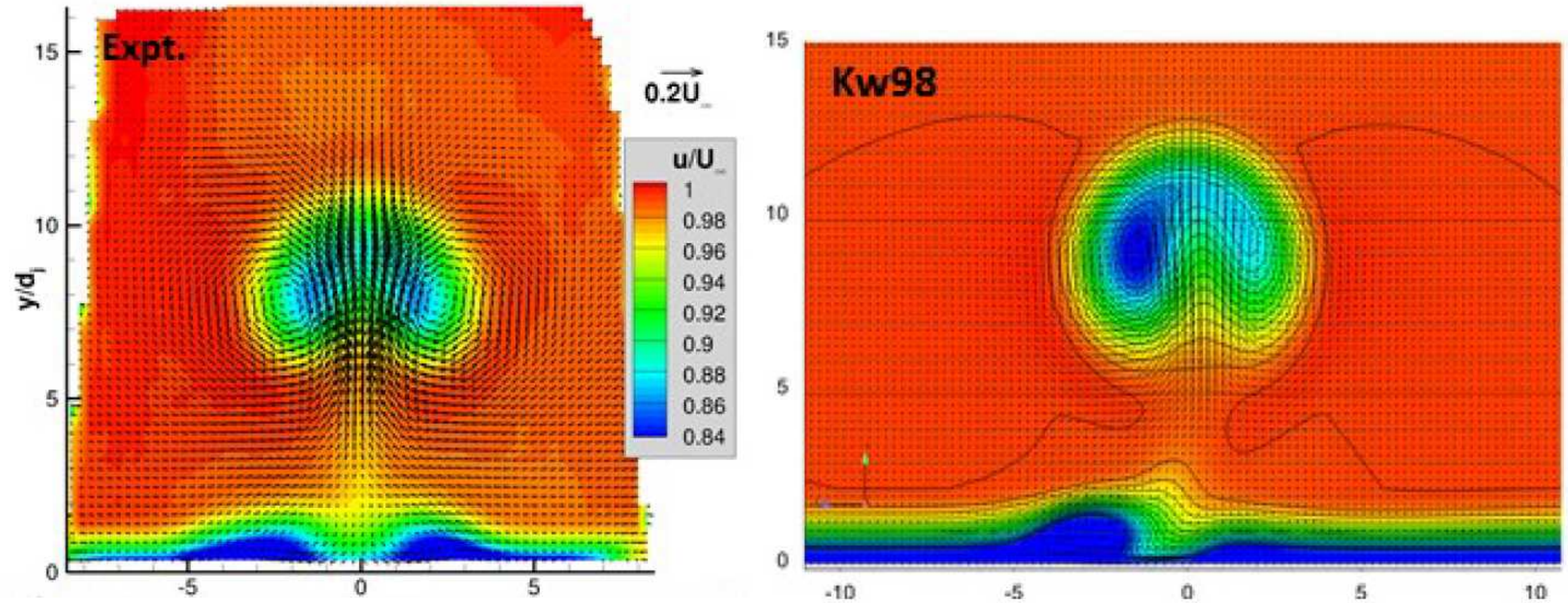
Target problem - jet-in-crossflow

- A canonical problem for spin-rocket maneuvering, fuel-air mixing etc.
- We have experimental data (PIV measurements) and corresponding RANS simulations
- The RANS simulations have stability problems





RANS (k-ω) simulations - crossplane results



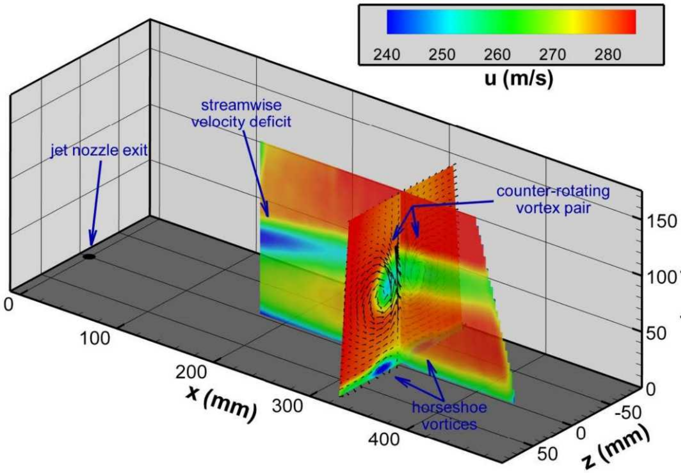
- Crossplane results for stream
- Computational results (SST) are too round; Kw98 doesn't have the mushroom shape; non-symmetric!
- Less intense regions; boundary layer too weak

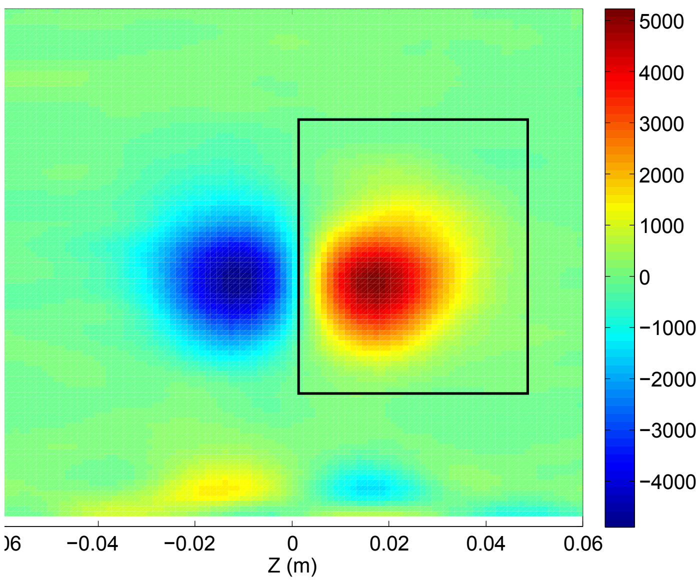
Sources of error in the RANS model

- There are multiple sources of error
  - Bad parameters: the k-ε model has parameters, specifically, $\{C_\mu, C_{\varepsilon 2}, C_{\varepsilon 1}\}$, whose values are obtained from canonical flow
    - Can be fixed by calibration to jet-in-crossflow data
  - Shortcomings of the LEVM in the RANS model
    - Can be fixed by using a quadratic or cubic EVM
  - Shortcomings of the k-ε model itself
    - Use explicit algebraic stress model or LES
- We addressed the problem of bad parameters by calibrating the RANS model to jet-in-crossflow (JinC) data
  - Performed using Bayesian inference and surrogate models of the RANS simulator
  - Resulted in a PDF for the parameters in question

Bayesian calibration

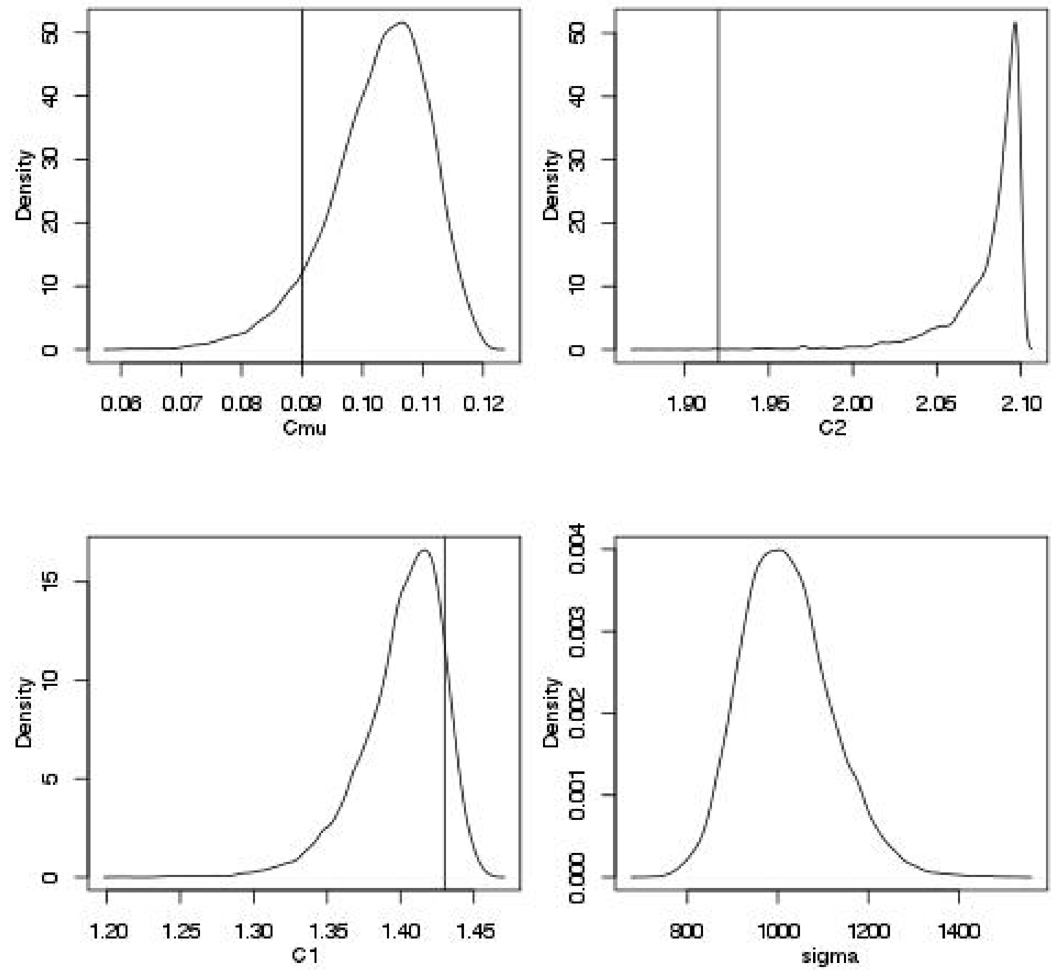
- We have velocity measurements on the crossplane
  - We computed a vorticity field
  - And used that (in a window) as the calibration variable
- We create a training set of 2744 3D RANS simulations by sampling in the (C_μ, $C_{\varepsilon 2}$, $C_{\varepsilon 1}$) space
- We create statistical models for $\omega_i = \omega_i(C_\mu, C_{\varepsilon 2}, C_{\varepsilon 1})$ using polynomials
  - ω_i is the streamwise vorticity in grid-cell I
- The statistical models were used in Bayesian inversion, in lieu of the RANS simulator



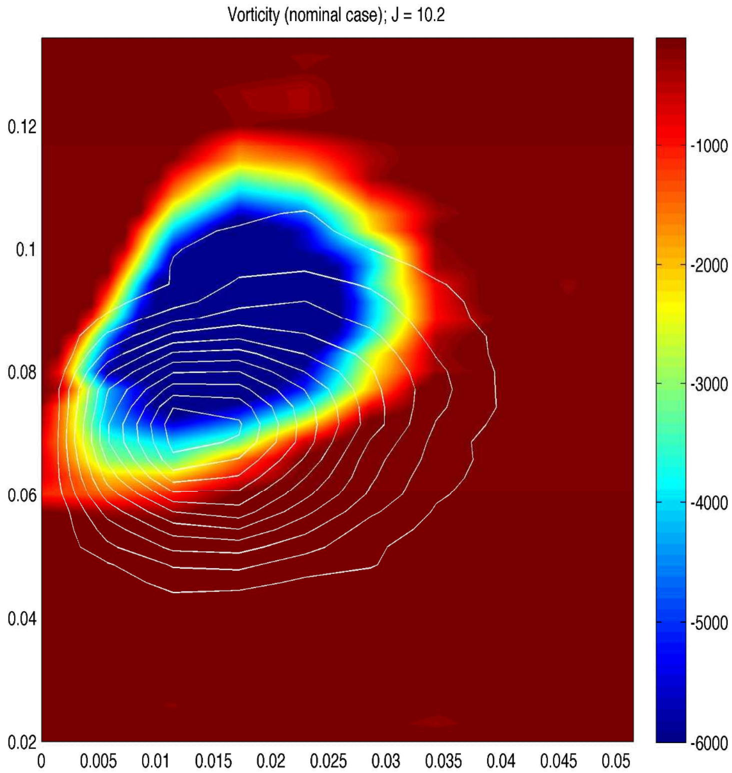


PDF of (C_μ, $C_{\varepsilon 2}$, $C_{\varepsilon 1}$)

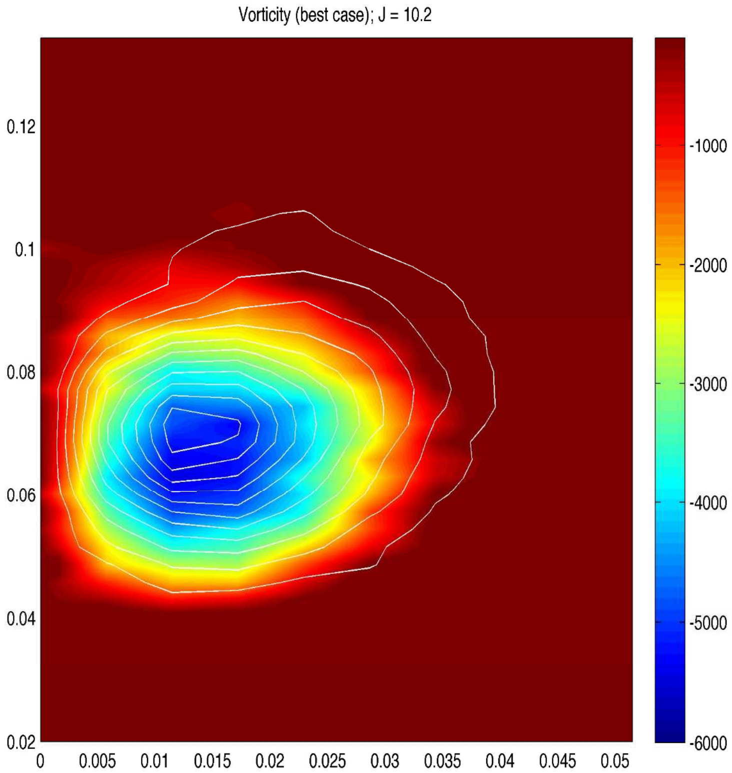
- Marginalized versions of the 3D PDF shown here
  - Vertical lines are the "nominal" values of the parameters
- We sampled 100 (C_μ, $C_{\varepsilon 2}$, $C_{\varepsilon 1}$) realizations from the PDF
  - Generated 100 realizations of the crossplane vorticity field using the RANS simulator
  - Also found the best (C_μ, $C_{\varepsilon 2}$, $C_{\varepsilon 1}$) combination my matching the experimental vorticity field



Crossplane predictions



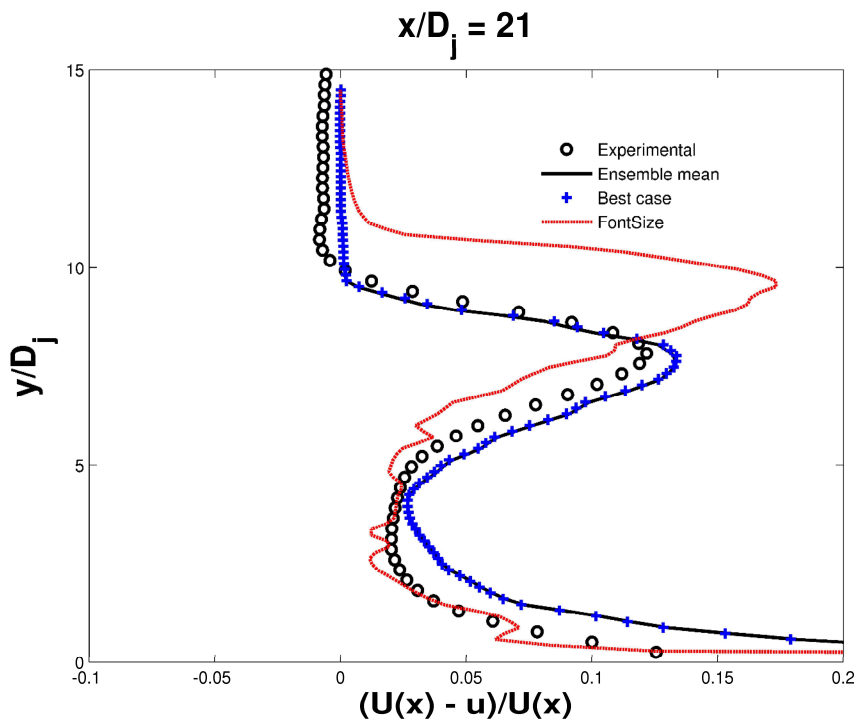
With nominal parameter values

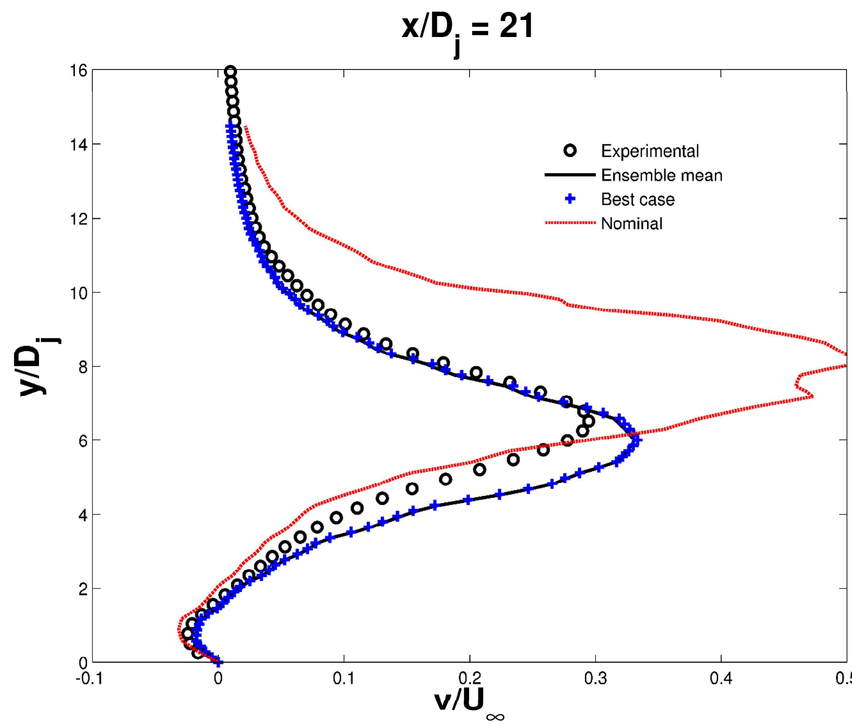


With calibrated parameter values

- Experimental vorticity in contours
- Stunning improvement in vorticity predictions

Mid-plane predictions



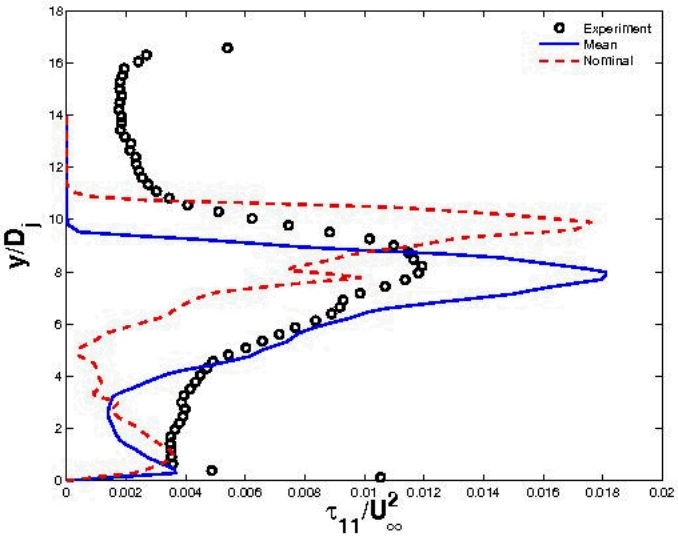


Velocity deficit

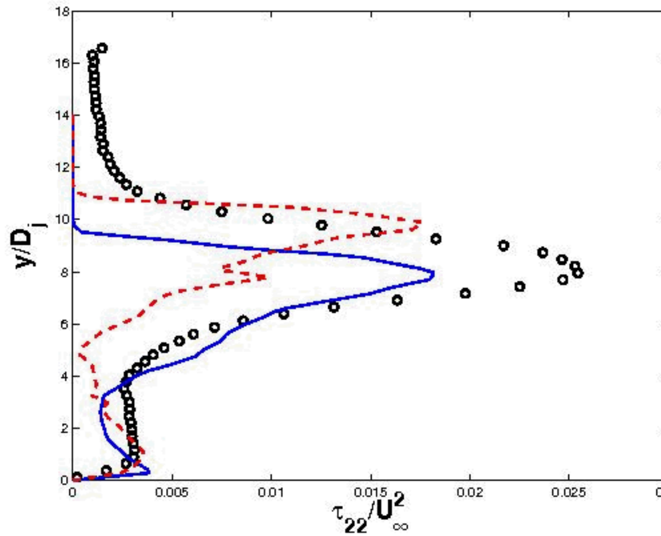
V-velocity

- Stunning improvement in vertical velocity predictions

Prediction of turbulent stresses

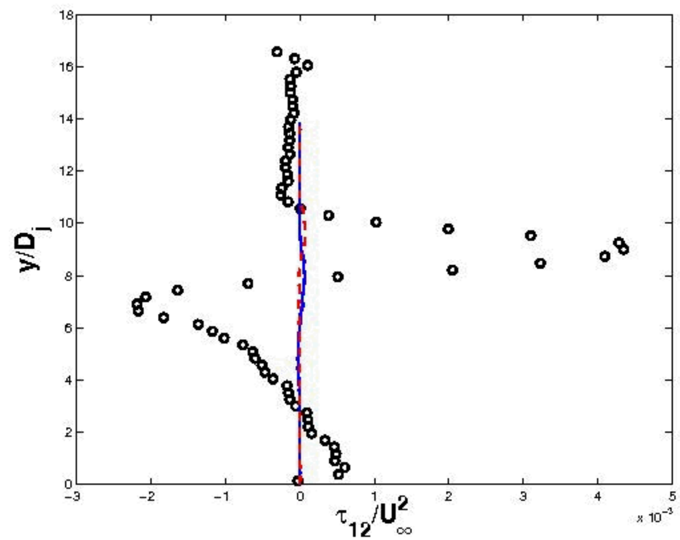


τ_{11}



τ_{22}

- M=0.8, J=10.2
- Not very good agreement; LEVM is deficient
- Improve it



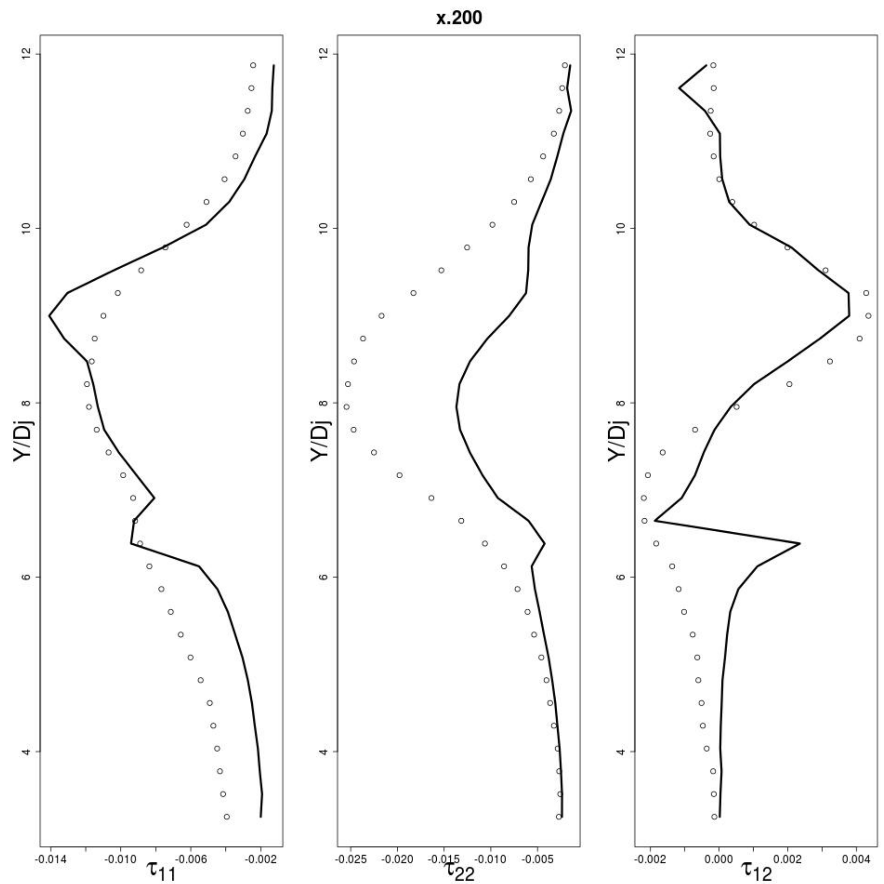
τ_{12}

High-order eddy-viscosity model

- Craft 95 [3] describes a cubic eddy viscosity (CEVM) model
  - $\tau_{ij} = -2/3k\ \delta_{ij} + C_\mu F(S_{ij}, \varepsilon) + c_1 f_1(S_{ij}, \Omega_{ij}, \varepsilon) + c_2 f_2(S_{ij}, \Omega_{ij},\varepsilon)\ \ldots\ldots\ c_7 f_7(S_{ij}, \Omega_{ij}, \varepsilon)$
  - $F(S_{ij})$ is linear in Sij, $f_1(:, :, :)$ - $f_3(:, :, :)$ are quadratic in S_{ij} & Ω_{ij}
  - $f_4(:, :, :) – f_7(:, :, :)$ are cubic in S_{ij} & Ω_{ij}
- Our experimental data, on the midplane, consists of:
  - S_{ij} & Ω_{ij} obtained from the measured velocity field
  - τ_{ij} and k, also measured
  - ε (dissipation rate of turbulent KE, k) cannot be measured
    - It is approximated by assuming equilibrium of production and dissipation of turbulent KE.
- Craft's model prescribes $\{c_1 \ldots c_7\}$
  - Parameter value obtained from a simple, incompressible turning flow
  - May not be valid for transonic JinC interaction

Reasons for hope

- The default CEVM reproduces τ_{11} and τ_{12} OK, but not τ_{22}
  - We could estimate better $\{c_1 \dots c_7\}$
  - We have about 60 useable probe locations with τ_{11}, τ_{12} & τ_{22} measurements
- Estimation can be reduced to a Y = Ax problem
  - Y contains the measured τ_{ij}
  - x contains $\{c_1, \dots, c_7\}$
  - A contains $C_\mu F(S_{ij}, \varepsilon)$ (the LEVM) and $f_l(S_{ij}, \Omega_{ij}, \varepsilon)$, l = 1…7



Experimental data & Craft model predictions

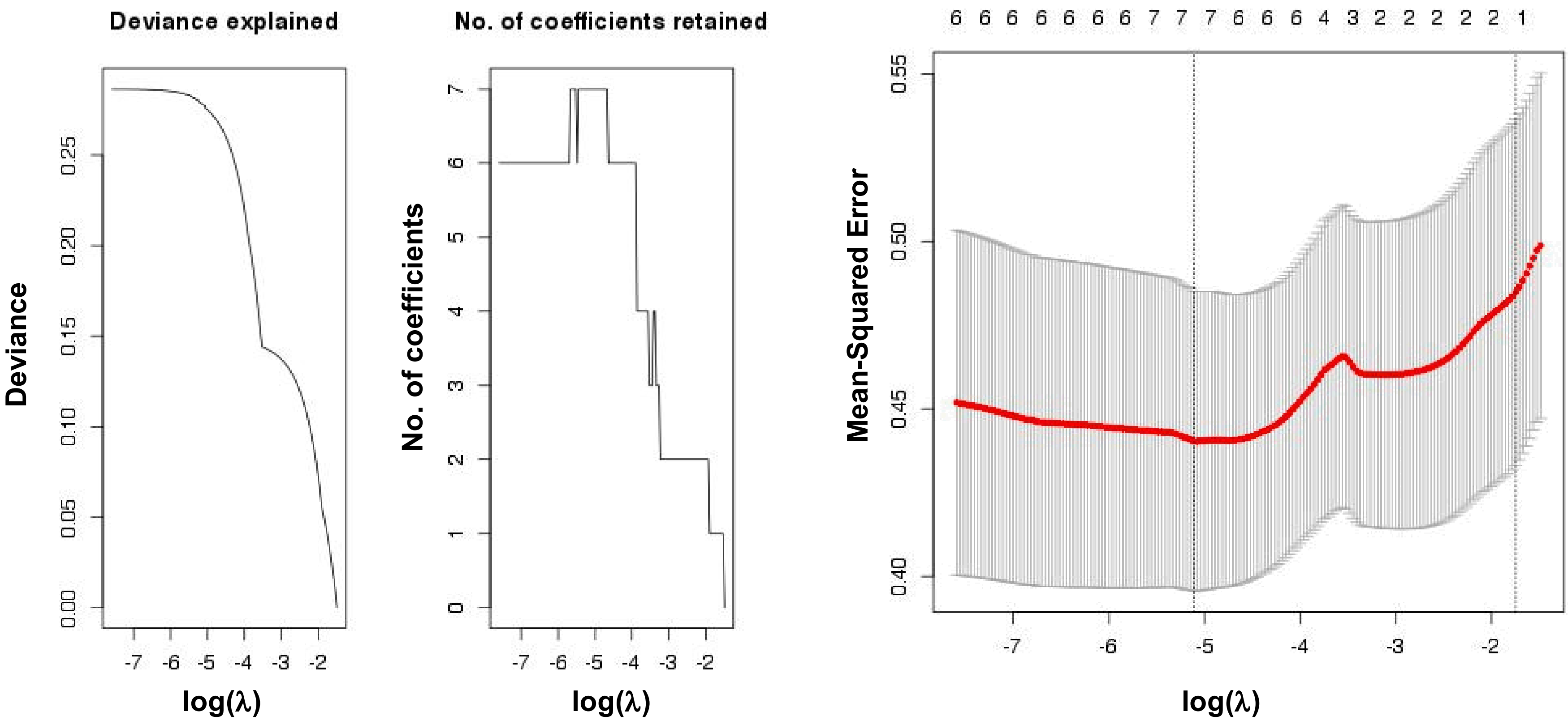
Estimation of CEVM parameters

- It may not be possible to estimate c_1, .. c_7 from the 180 data points
  - We'll estimate it using LASSO

$$\boxed{\min_{x} \quad \left\| Y - Ax \right\|_2^2 + \lambda \left\| x \right\|_1}$$

  - The first half estimates x = $\{c_i\}$ that provide CEVM predictions near Y
  - The second half – the λ penalty – tries to set as many c_i to zero
- Estimating a good x means estimating a good λ
  - If penalty too small wrt information content of Y, x estimated from subsets of Y (cross-validation) will vary wildly (overfitting)
    - As will the the deviance $\eta = || Y - Ax||_2$
  - So choose λ, perform cross-validation, compute mean & variance of η
  - Do this for a range of λ and pick λ_{min}
  - Also compare with the case of λ = 0 (risk overfitting; call it 'LM')

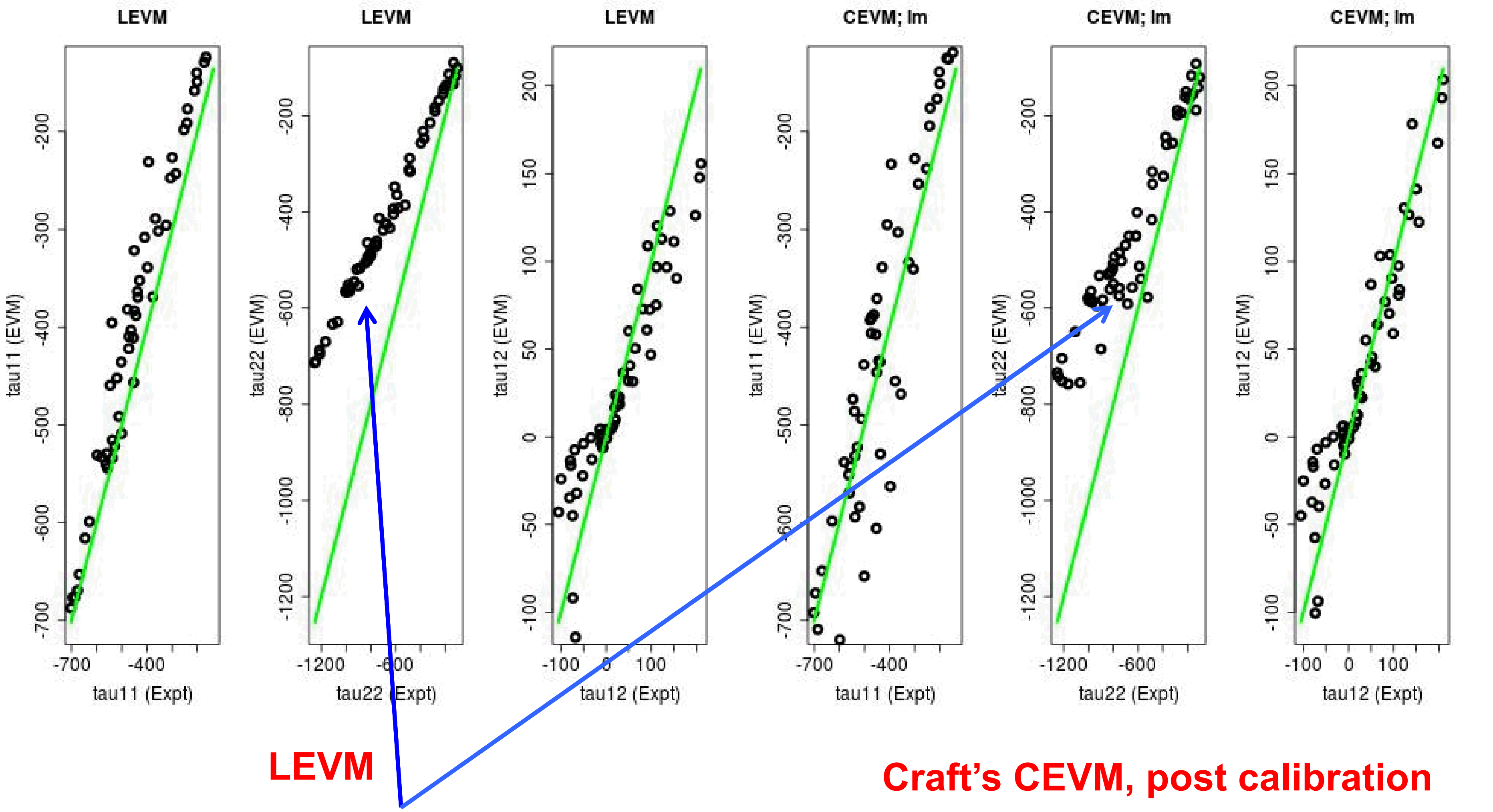
LASSO results



Looking for a good λ

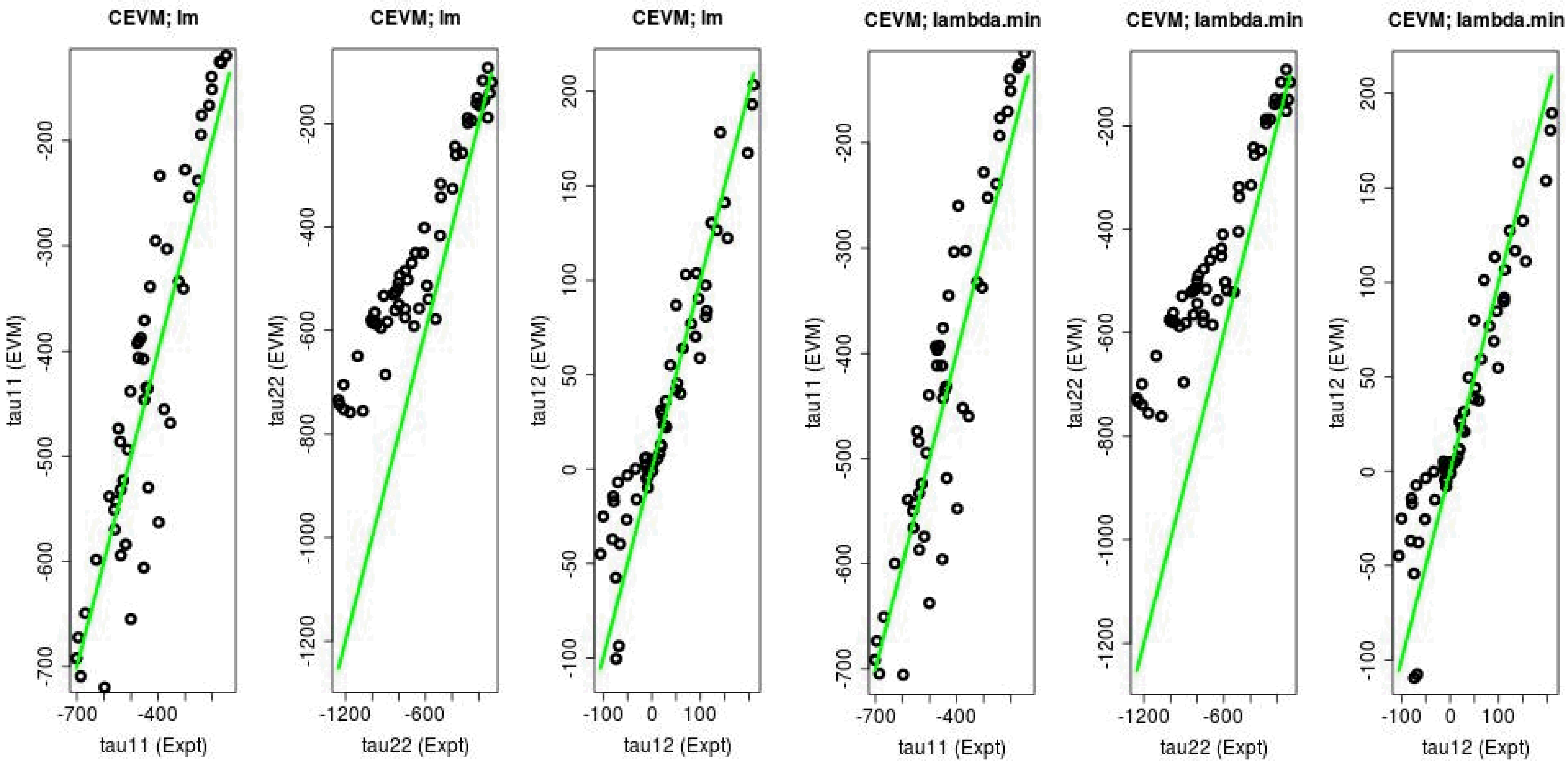
- Craft explains around 28% of deviance
- As $\log(\lambda)$ increases and # of terms retained decreases, CEVM worsens
- One gets λ_{min} and λ_{1se}

Calibrate all h.o. terms. LEVM v/s LM



- τ_{22} improved. No overfitting?

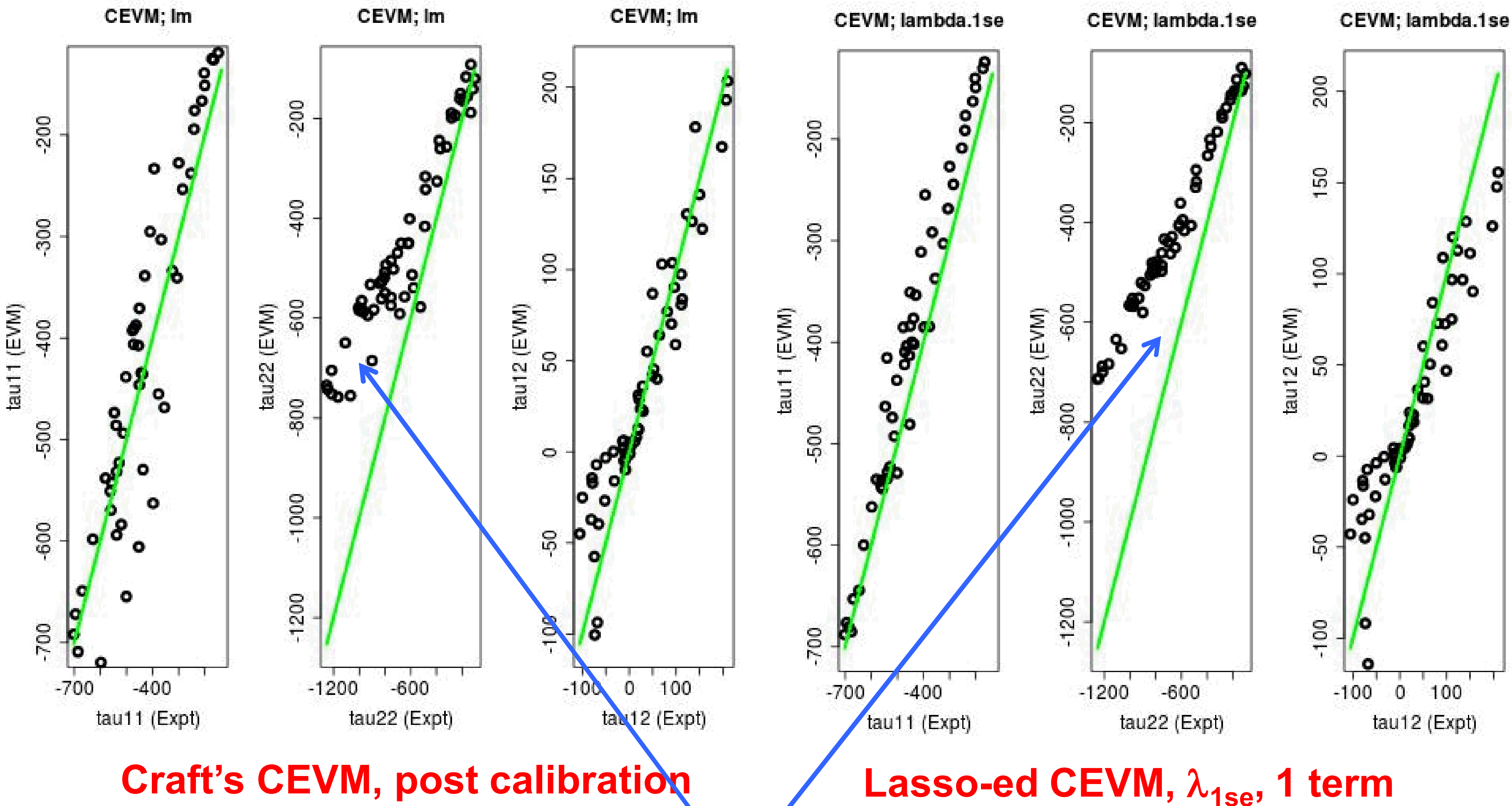
Start penalizing. LM v/s LASSO-ed CEVM, λ_{min}



Craft's CEVM, post calibration **Lasso-ed CEVM, λ_{min}, 7 terms**

- Change? Not to the naked eye.

Penalize more. LM v/s LASSO-ed CEVM, λ_{1se}



- Change? Worsens τ_{22} a bit

Tabulate coefficients and MSE

Method	c_1	c_2	c_3	c_4	c_5	c_6	c_7	MSE
Craft	-0.1	0.1	0.26	-10	0	-5	5	0.662
LM	-0.0789	-0.149	2.02	-5.88	0	6.68	-11.87	0.382
λ_{min}	-0.065	-0.103	1.68	-4.02	5.7	5.4	-3.64	0.386
λ_{1se}	0.0	0.0	0.455	0.0	0	0	0	0.483

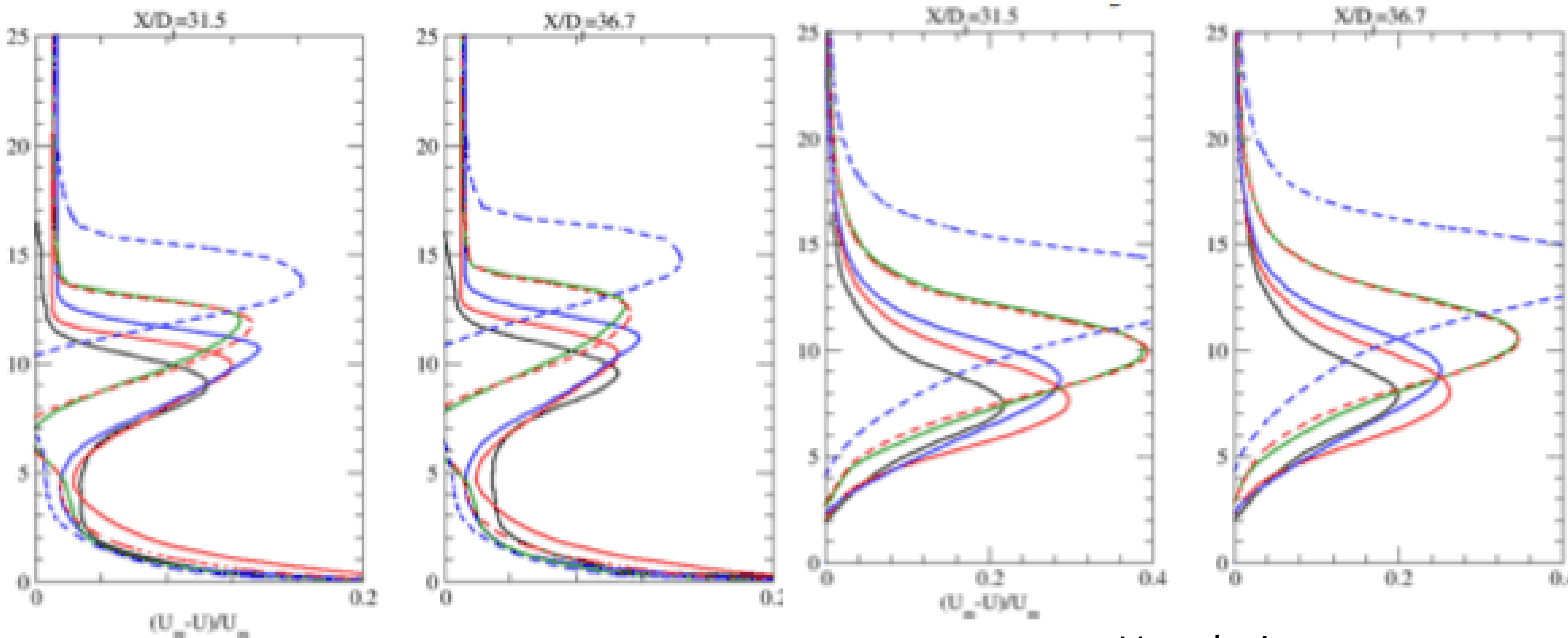
- $\ln(\lambda_{min}) = -5.11$, $\ln(\lambda_{1se}) = -1.75$
- Craft's default parameters are changed when we regress it to data
  - Results called 'LM'
- When we LASSO the model using λ_{1se}, we're left with just 1 quadratic term
  - But the model loses much accuracy
- Bottomline: Can't simplify Craft's model (remove terms), but have to live with uncertain values of $\{c_1, .. c_7\}$ due to shortcomings of our dataset

Conclusions

- We have performed a full calibration of a RANS model for transonic jet-in-crossflow simulator
  - First, we calibrated the parameters, when using a linear eddy viscosity model, and isolated the model-form error (due to LEVM)
  - Then we explored enriched versions of LEVM i.e. Craft's cubic eddy viscosity model, and calibrated them to data
- We found, using LASSO, that our dataset could only support enrichment by 1 term (for sure)
  - But that does not prevent us from estimating all 7 terms in the Craft cubic eddy viscosity model using Bayesian inversion
  - Since the estimation problem is linear, this admits a simple analytical solution
    - See backup slides if interested

BACKUP SLIDES

RANS (k-ω) simulations – midplane results

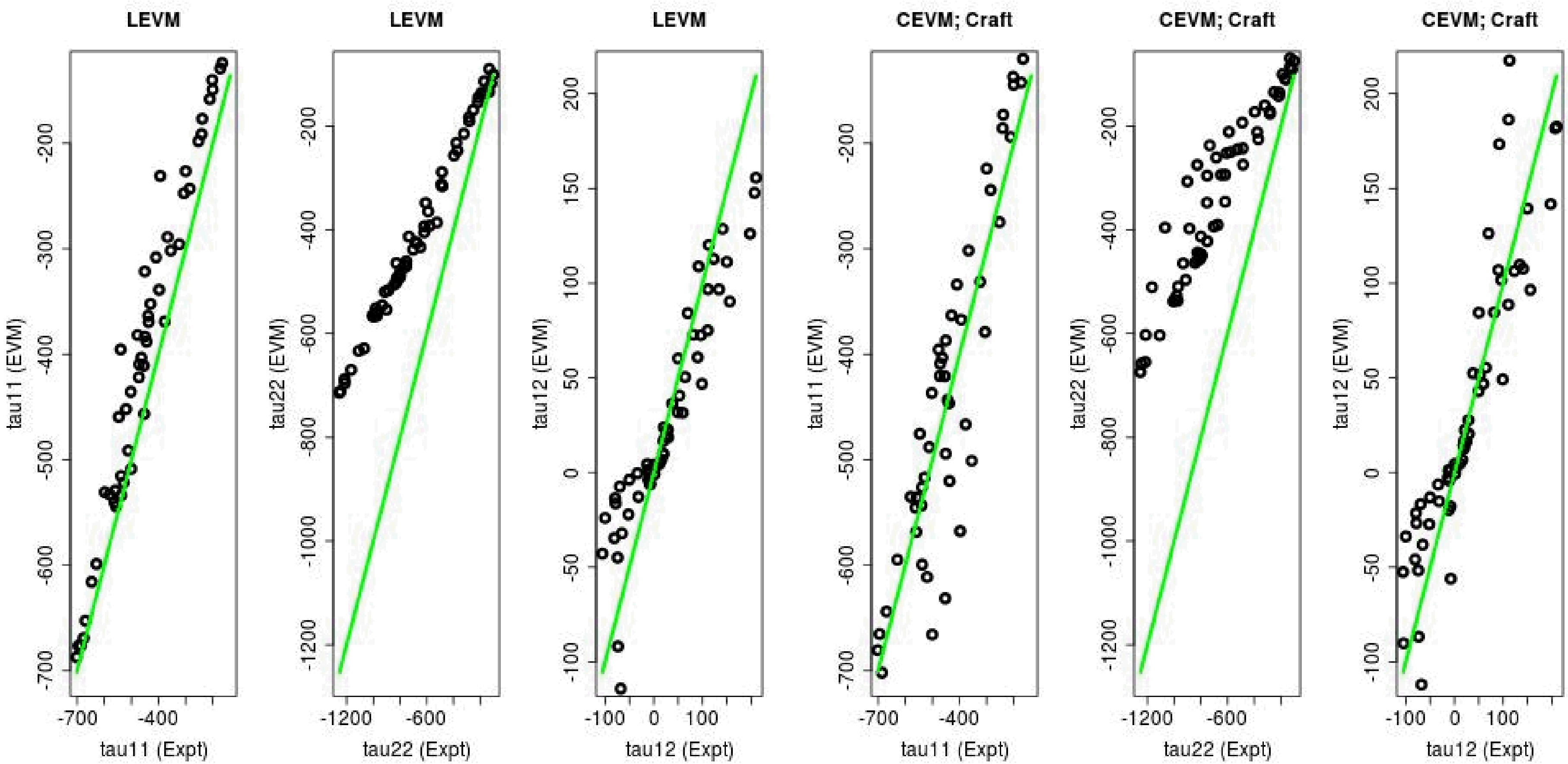


U-defect

V - velocity

- Experimental results in black
- All models are pretty inaccurate (blue and red lines are the non-symmetric results)

Do high-order terms help? LEVM v/s CEVM



LEVM **Craft's CEVM**

- LEVM seems to be the same as Craft. So calibrate Craft.

Estimate of CEVM parameters

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
	Posterior Mean						
	-7.89e-2	-1.49e-1	2.02e+0	-5.88e+0	0	6.68e+0	-1.19e+1
	Posterior Covariance						
c_1	2.125e-03	-7.036e-04	-4.551e-03	-5.893e-02		-1.873e-02	5.265e-02
c_2	-7.036e-04	5.477e-03	1.037e-04	2.153e-01		-4.765e-03	-1.948e-02
c_3	-4.551e-03	1.037e-04	2.682e-01	4.167e-01		7.767e-02	-3.821e-02
c_4	-5.893e-02	2.153e-01	4.167e-01	1.359e+01		7.544e-01	-6.672e-01
c_5							
c_6	-1.873e-02	-4.765e-03	7.767e-02	7.544e-01		1.891e+00	-3.360e+0
c_7	5.265e-02	-1.948e-02	-3.821e-02	-6.672e-01		-3.360e+00	7.643e+0