Coherence Model for Building Layover in Interferometric SAR

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ABSTRACT

The complex coherence function describes information that is necessary to create maps from interferometric synthetic aperture radar (InSAR). This coherence function is complicated by building layover. This paper presents a mathematical model for this complex coherence in the presence of building layover and shows how it can describe intriguing phenomena observed in real interferometric SAR data.

Keywords: IFSAR, InSAR, antenna, building, layover.

1 INTRODUCTION

The complex coherence function provides information important to forming the interferograms used in interferometric synthetic aperture radar (InSAR) mapping. Various documents discuss the coherence and issues affecting the coherence[1,2]. This document derives a mathematical model for the coherence due to building layover for a two-antenna system with a single pass or two pass InSAR, and discusses implications for the phenomena observed in coherence data. In particular, we show that although typically the magnitude of the coherence is used solely for assessing the quality of the interferograms, it actually also contains target geometry information.

The mathematical model and observed phenomena for this layover near building edges was first presented by Bickel, et al [3]. In this paper, we discuss additional implications exposed by the model. We present a couple of examples that illustrate the effect. Simulation that validated the model was presented by Yocky, et al[4]. This paper presents the detailed development of the coherence model for building layover regions.

2 MATHEMATICAL MODEL

The development of the model follows from the application of the van-Cittert Zernike (VCZ) theorem from optics[5,6]. The use of VCZ theorem has been recognized as applicable to the InSAR coherence problem[7]. The usual assumption in the VCZ theorem is that the target areas are incoherent. We will assume that the target areas are uncorrelated circularly Gaussian clutter areas. Although this is the case, an interesting relationship will be shown to the point target case with the results below.

Figure 1 shows the geometry used for this formulation. Without loss of generality, a perpendicular baseline geometry is assumed with the angular separation of the building, $\Delta\theta$, bisected by the boresight to the baseline. The building and the ground scattering are both presumed to be uniform with backscatter coefficients of σ_{roof} and σ_{ground}, respectively. Under these conditions, the complex coherence using the VCZ theorem is given by:

$$\mu = \frac{e^{-\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sigma(\xi,\eta)\exp\left\{j2k\left(\frac{B_{\xi}\xi + B_{\eta}\eta}{r}\right)\right\}d\xi d\eta}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\sigma(\xi,\eta)\,d\xi d\eta}. \tag{1}$$

where

ψ - is the InSAR phase to the center of the resolution cell, which we will assume is zero from here on

σ - is backscatter as a function of position

k - is the wavenumber

r - is the range

ξ - is the coordinate in the baseline direction

η - is the coordinate orthogonal to the baseline direction (into the page)

B_i - is baseline component in the *i*-direction

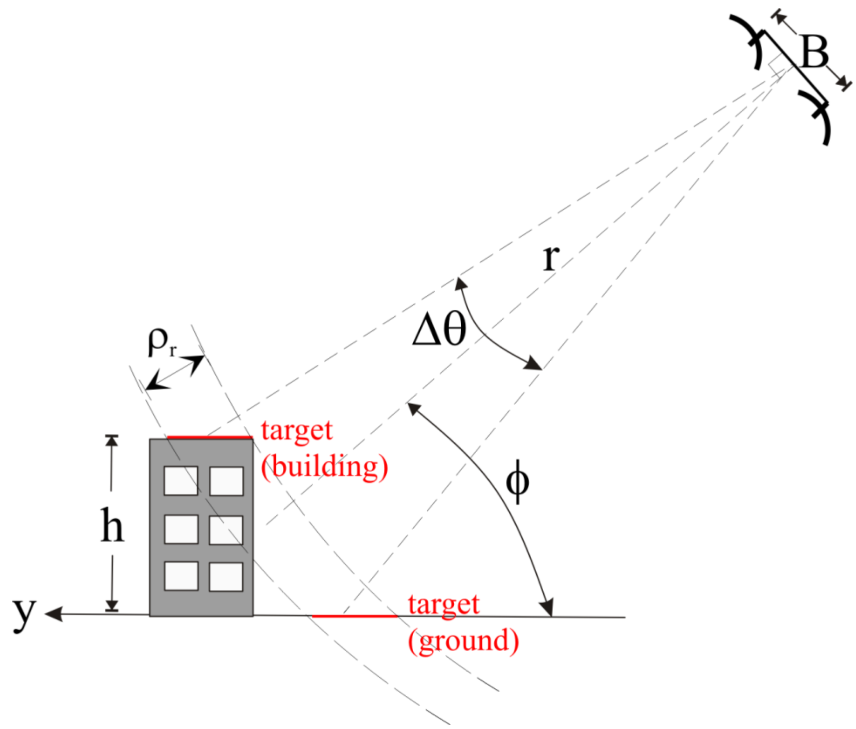

Figure 1. Geometry considered in this paper.

For completeness, we included baseline component in the range direction in equation (1), but we have assumed that $B_\eta = 0$ in the rest of this paper. Also, the factor of 2 in the exponential in the equation assumes that we multiplex ("ping-pong") transmission for the single pass InSAR case, or we use two pass InSAR. The integration is limited over a resolution cell.

Let the component of the range resolution cell, ρ_r, in the direction of the baseline be given as $\delta = \rho_r \tan\phi$. The component of the layover separation in the direction of the baseline is given as $\Delta\xi = h/\cos\phi$. Under these conditions, the VCZ integral from the above equation becomes:

$$\mu = \frac{\Delta\eta\sigma_{roof}\int_{\Delta\xi/2-\delta/2}^{\Delta\xi/2+\delta/2}\exp\left\{j\frac{2kB_\xi\xi}{r}\right\}d\xi + \Delta\eta\sigma_{ground}\int_{-\Delta\xi/2-\delta/2}^{-\Delta\xi/2+\delta/2}\exp\left\{j\frac{2kB_\xi\xi}{r}\right\}d\xi}{\left(\sigma_{roof}+\sigma_{ground}\right)\delta\Delta\eta}. \quad (2)$$

Which leads to:

$$\begin{aligned}\mu = &\frac{\sigma_{roof}}{\left(\sigma_{roof}+\sigma_{ground}\right)}\exp\left\{j\frac{kB_{\xi}h}{r\cos\phi}\right\}\left[\frac{\sin\left(\frac{kB_{\xi}\rho_{r}\tan\phi}{r}\right)}{\left(\frac{kB_{\xi}\rho_{r}\tan\phi}{r}\right)}\right] \\ &+\frac{\sigma_{ground}}{\left(\sigma_{roof}+\sigma_{ground}\right)}\exp\left\{-j\frac{kB_{\xi}h}{r\cos\phi}\right\}\left[\frac{\sin\left(\frac{kB_{\xi}\rho_{r}\tan\phi}{r}\right)}{\left(\frac{kB_{\xi}\rho_{r}\tan\phi}{r}\right)}\right]\end{aligned}. \tag{3}$$

Let:

$$X = \frac{kB_{\xi}\rho_{r}\tan\phi}{\pi r}. \tag{4}$$

$$\beta = \frac{\sigma_{roof}}{\left(\sigma_{roof}+\sigma_{ground}\right)}. \tag{5}$$

$$\alpha = \frac{2kB_{\xi}}{r\cos\phi}. \tag{6}$$

Then we arrive at:

$$\mu = \left(\frac{\sin\pi X}{\pi X}\right)\left[\beta\exp\left\{j\alpha\frac{h}{2}\right\}+\left(1-\beta\right)\exp\left\{-j\alpha\frac{h}{2}\right\}\right]. \tag{7}$$

which is the result we desire.

3 IMPLICATIONS OF THE BUILDING LAYOVER MODEL

There are numerous implications of the coherence model from equation (7) that we address in this section. First, it is interesting to note that the equation follows the product form of coherence[1,2]. The first term in the equation (i.e., the sinc function term) is the traditional geometrical coherence term. The second term (i.e., the term in brackets) is the new coherence term caused by the building layover. Interestingly, this latter term is the coherence due to two spatially separated point targets.

As noted above, if the building has no height, i.e., if there were effectively no building, then the equation becomes:

$$\mu = \left(\frac{\sin\pi X}{\pi X}\right). \tag{8}$$

which is the standard result for geometric coherence for InSAR (refer to the references). The interferometric phase is zero in this case because we have assumed that the target is located at the boresight of the baseline. In general, this would be the standard interferometric phase found in literature.

Next, if the backscatter coefficient between the roof and the ground is equal, then, we get the result:

$$\mu = \left(\frac{\sin \pi X}{\pi X} \right) \cos\left(\alpha \frac{h}{2} \right). \tag{9}$$

The presence of the building adds a (co)sinusoidal component to the coherence, which means that the coherence can fade in and out as a function of the product of the baseline length and building height. We will come back to this, but this indicates that the magnitude of the coherence contains geometry information. The mean interferometric phase for this case indicates the height at the middle of the building. The apparent mean direction-of-arrival to the target is at a lower angle than the roof. Since the range is fixed and known from the radar timing (see Figure 1), in the layover region the apparent target location for the combination of the roof and building is pulled forward and at a lower height than the roof. This can yield the deceptive "front-porch" at the front of buildings[3,8], particularly in orthorectified maps. This case of equal returns from the roof and ground yields the lowest coherence which implies the greatest phase noise case, as well.

The next case to consider is for $\beta = 1$ which means that the only return is from the roof. In this case we get:

$$\mu = \left(\frac{\sin \pi X}{\pi X} \right) \exp\left\{ j\alpha \frac{h}{2} \right\}. \tag{10}$$

The magnitude of this coherence term is same as in equation (8). The difference is the phase of the coherence. If we note that the relative height is given by a scale factor times the phase, where the scale factor is given by[9]:

$$\frac{r\cos\phi}{2kB_{\xi}}. \tag{11}$$

so that the height estimate is $h/2$. This corresponds to the roof height.

The analog case is for $\beta = 0$ which means the only return is from the ground. The coherence becomes:

$$\mu = \left(\frac{\sin \pi X}{\pi X} \right) \exp\left\{ -j\alpha \frac{h}{2} \right\}. \tag{12}$$

Again, the magnitude of the coherence matches equation (8). However, the estimated height is $-h/2$ in this case which corresponds to the ground location.

The general case of equation (7) can be rewritten as:

$$\mu = \left(\frac{\sin \pi X}{\pi X} \right) \sqrt{2\beta(\beta-1)\left[1-\cos(\alpha h)\right]+1}\, \exp\left\{ j(2\beta-1)\tan\left(\alpha \frac{h}{2} \right) \right\}. \tag{13}$$

This version of the equation separates out the magnitude and phase portions of the coherence due to building. Figure 2 shows a plot of the magnitude and the apparent height from the InSAR phase of this coherence term as a function of the ratio of the backscatter from the roof to the ground, $\sigma_{roof} / \sigma_{ground} = \beta / (1-\beta)$ in deciBels for differing building heights. For the plots we assume that X is small so that $\sin(\pi X) / \pi X \approx 1$.

From Figure 2 we verify what we have said above, namely that as $\beta \to 1$ the height estimated approaches the height of the building and the magnitude of the coherence approaches the coherence of standard terrain. Similarly, as $\beta \to 0$, the estimated height approaches the height of the ground and the coherence also approaches that for standard terrain. When the backscatter of the roof and ground are similar, the coherence is reduced. When the backscatter is similar, the coherence is very sensitive to the product of the baseline times the building height. Although we did not choose the parameters to show it, we recall that the coherence is actually sinusoidal. As far as the InSAR phase, or estimated height, goes we see a hysteresis with the ratio of the backscatter power. This hysteresis is also a sinusoidal function of the baseline times the building height. In fact, an increase in hysteresis corresponds directly to the loss in coherence.

4 EXAMPLES FROM DATA

In this section we show a couple of examples from InSAR data that show some of the effects above. The first example, shown in Figure 3, is an image from a Sandia National Laboratories Ku-band InSAR system of the Washington monument in Washington D. C. We see that the SAR image is illuminated from the top by the radar system as evidenced by the shadow of the monument pointing to the bottom of the image. The important point in this figure is that due to direct returns all along the side of the monument laying over on top of direct ground returns, it creates the effect of apparently varying "building" heights. This layover is not readily visible in the SAR image, with the exception of the bright dot near the top of the image from the monument viewing ports. The result is that we observe the cyclical coherence response in the coherence image that is predicted by the model above.

In Figure 4, we show several examples of the "front porch" layover effect from the orthorectified digital surface model (DSM) from the Rapid Terrain Visualization system (RTV)[10]. This effect appears as an extra lip on the layover side of the radar image which is to the left in the figure.

5 ADDITIONAL COMMENTS

An important point in this paper is that there is additional valuable information contained in the magnitude of the coherence that is often neglected. It is well known that with a single pass using two antennas, or two passes with a single antenna, we get a single direction of arrival. However, there are actually two pieces of information that we derive from the two samples, namely, the magnitude and the phase of the coherence. As we see from the equation (7) and corresponding figures, there is target geometry information in the magnitude of the coherence as well as in the phase. The phase of the coherence yields a weighted average of the mean location of the entire building; whereas, the magnitude contains information about the building height span. We can find similarities to monopulse in this respect.

Another related point is that the magnitude of the coherence provides an indicator of when layover is occurring[3]. Another way to state this is that the magnitude of the coherence is an indicator of when multiple targets are present within the given range-Doppler cell. This fact can be used to advantage in processing InSAR data[11]. This is associated with the discussion in the following reference[12]. It is also related to detection of volume scattering in InSAR[13]. An intriguing relationship to this is discussed in a forthcoming report on spatial nulls.

All of these comments can be extended to more passes.

6 SUMMARY & CONCLUSIONS

A mathematical model for building layover is presented. The model and example data illustrate that the magnitude of the coherence, as well as the phase contains geometry information. In theory this information can be used to support interferometric processing, including building geometry and detecting potential building layover areas.

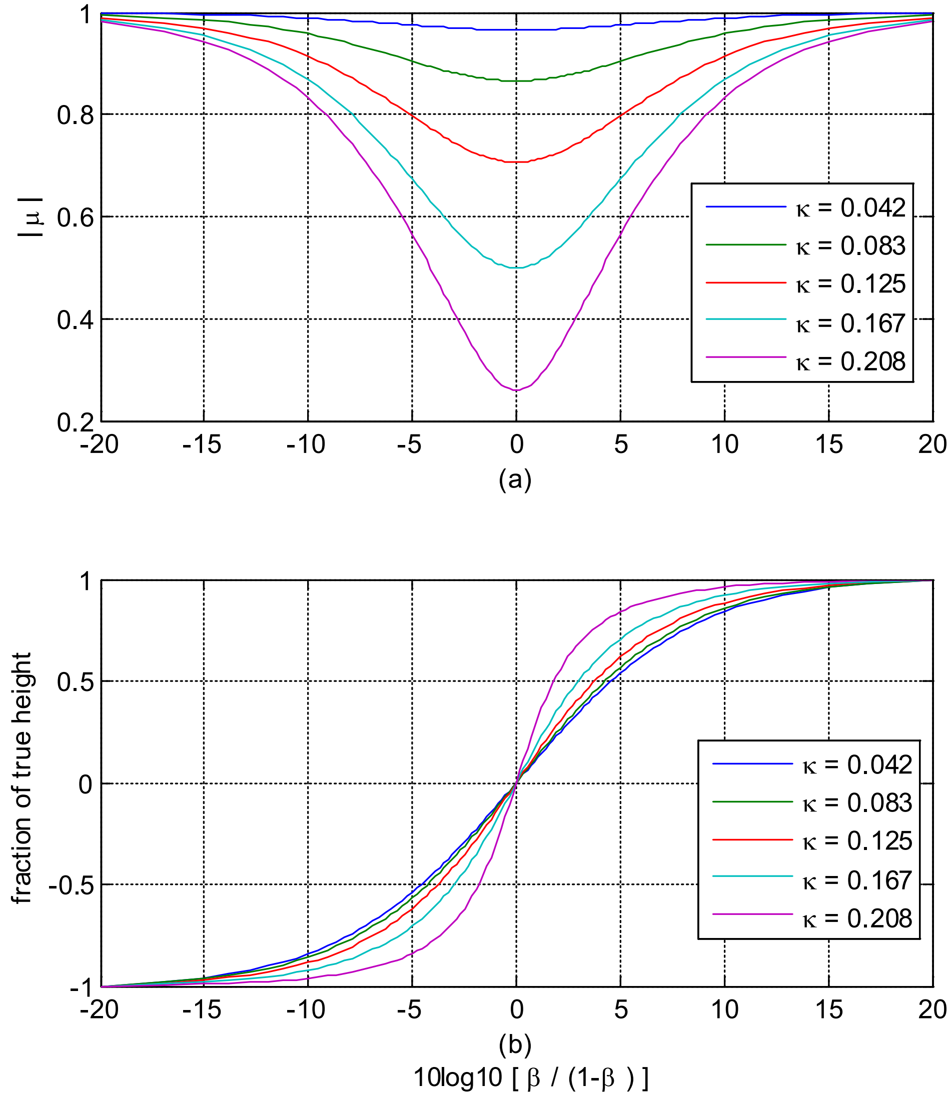

Figure 2. (a) magnitude of the coherence from building (b) estimated height as fraction of true height for building versus ratio of roof to ground backscatter (with $\kappa = \alpha h / 2\pi$)

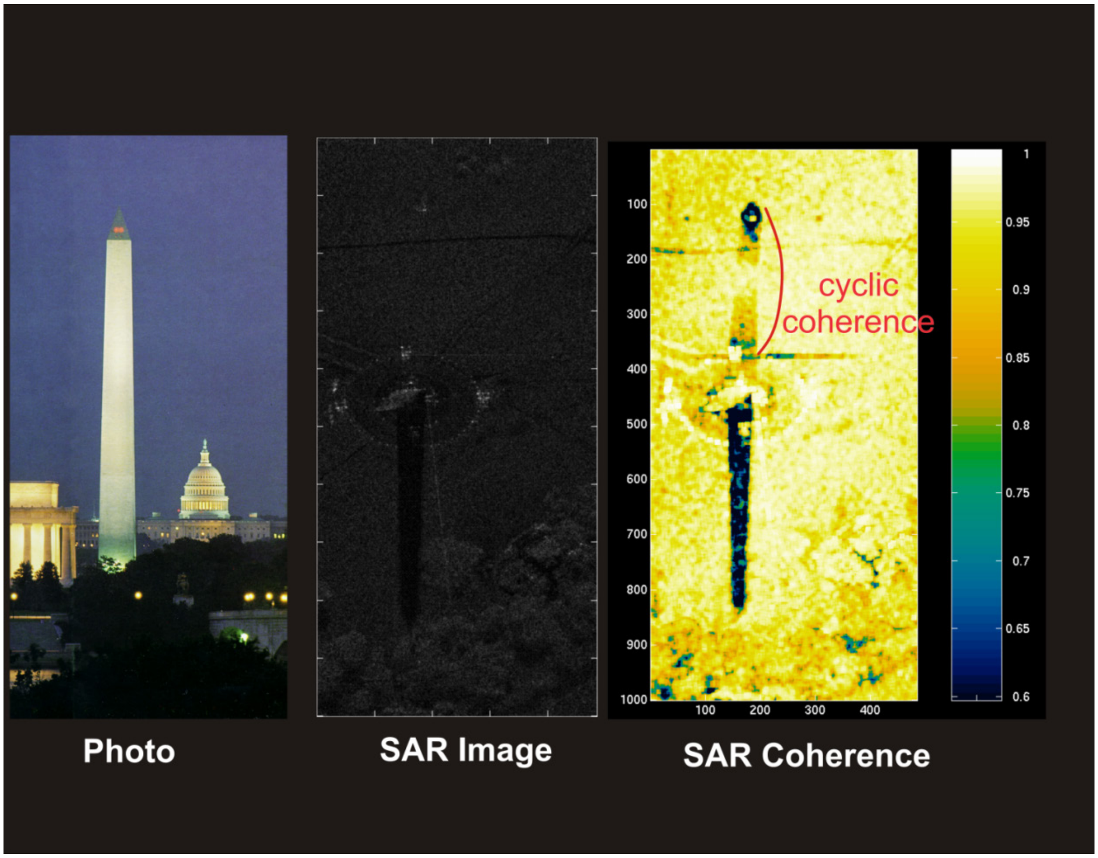

Figure 3. Example of cyclical nature of coherence from layover off the side of the Washington monument.

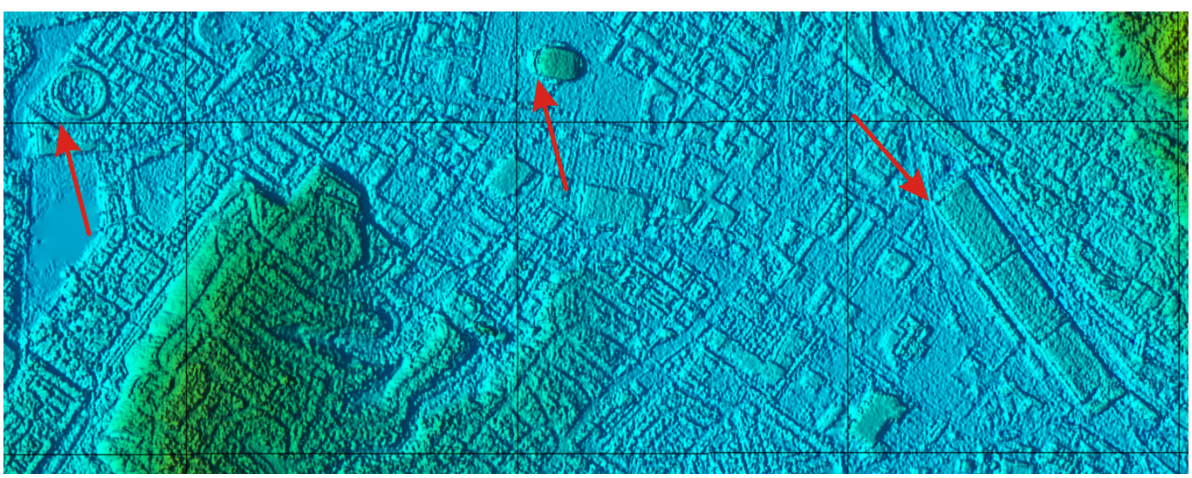

Figure 4. Examples of "front-porch" effect in DSM of San Diego from RTV system

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