SAND2015-2279PE

NST 560: Surety and Reliability Analysis Techniques that Estimate Uncertainty

Part 2: Bayesian Approach

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February 1, 2015

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Objective

- Transfer expertise to Sandia staff on how to estimate uncertainty for complicated real-world problems addressed by Sandia, with emphasis on applications that involve a great deal of State of Knowledge (SOK) uncertainty
  - Surety (safety, security, use control)
  - Reliability
  - Counter terrorism

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Course Structure

- Three parts
  1. Classical probability and statistical inference
  2. Bayesian approach
  3. Belief / Plausibility measure
- Each part
  – Three day in-class session
    - Basics and mathematical development
    - Simple examples
    - Complex real-world examples (some classified)

  – Homework
- Final exam

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Why The Three Parts?

- Techniques to address state of knowledge uncertainty build on classical probability and statistical inference
- For some applications we can use classical techniques
- Many event and fault tree tools consider uncertainty in the probability of basic events in a Bayesian context
- So we will start with Classical, move to Bayesian, then address Belief and Plausibility

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Miscellaneous

- NST460 “Extensions to Conventional Surety Analysis” desired but not required prerequisite
- Important references noted
- Numerous computer tools discussed and referenced

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Fidelity of Information Available Drives Selection of Best Technique

How to Treat Uncertainty

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Information Available

Quantitative	Quantitative, some Qualitative	Qualitative
Lightning strikes at Pantex	New strong link	Abnormal environments Terrorist attacks
Objective Aleatory Uncertainty Classical Probability Classical Statistics	Bayesian Concepts	Subjective Epistemic Uncertainty Belief/Plausibility Fuzzy Sets

Progression of Course Topics →

Traditional Techniques | Advanced Techniques

Classical Probability and Statistical Inference: Review of Important Points

- Summary of key points from part 1 pertinent to the Bayesian Approach
- See part 1 “Classical Probability and Statistical Inference” lecture notes for details

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Sample Space, Event, Random Variable

- Sample space is set of all outcomes
  - Outcomes are mutually exclusive
- Event is a subset of the sample space
- Random variable is a mapping of outcomes to reals

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Classical Probability is Objective

- Classical probability is a specific value (one value)
  - Event E, N identical trials
  - Probability of Event: P(E)
    - P(E) = $\lim_{N \to \infty}$ (number of time E occurs / N)
    - P(E) is fixed but perhaps unknown with certainty
    - To know P(E) precisely requires infinite number of identical trials
  - Classical probability is an Objective concept
    - Probability is a Frequency
      - Not a physical rate but a dimensionless ratio
- Can infer parameters of probability distributions using statistics

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Probability Measure

Probability Measure, P, for Sample Space, S:
Kolmogorov axioms
(the mathematics for probability)

1. For any event E, $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. For any set of mutually exclusive events $\{E_1, E_2, E_3, \ldots E_n\}$ the Probability of the union (logical OR) of all the events is the sum of the probabilities of each event

   $P(E_1 \text{ or } E_2 \text{ or } E_3 \text{ or } \ldots \text{ or } E_n\} = P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n)$

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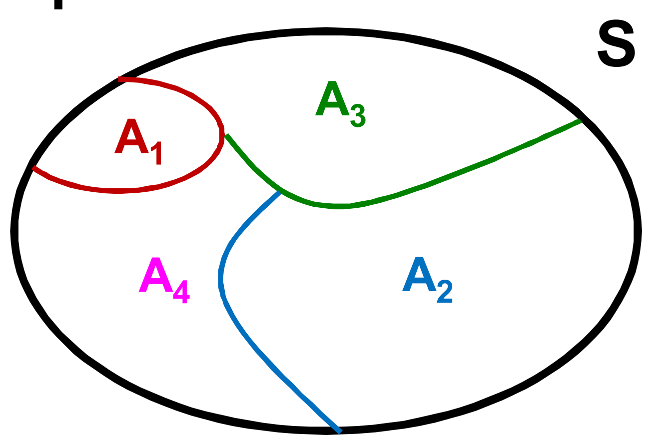
Partition

- Partition is a set of mutually exclusive events that covers the sample space
  - $\{A_i\ i = 1, 2, 3, \ldots\}$ is a partition over S if: $\cup_{\text{all } i} A_i = S$ and

    $P(A_j \cap A_k) = 0$ for any A_j and A_k in $\{A_i\}$
  - The set of all outcomes is one partition
  - A notional partition



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Conditional Probability

- **Probability of event A given event B: P(A|B)**

$$P(A|B) = P(A \cap B) / P(B)$$

(if P(B) not zero)

If A and B are independent
$P(A \cap B) = P(A)*P(B)$ and
$P(A|B) = P(A)$

Note: $P(A \cap B) = P(B \cap A)$

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Law of Total Probability

- S a sample space
- B any event in S
- $\{A_i \; i = 1, 2, 3, \ldots\}$ a partition over S

- $P(B) = \sum_{k \text{ over all } i} P(B|A_k) * P(A_k)$

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Classical Statistical Inference

- Assume the population is characterized by an appropriate probability distribution
  - Binomial
  - Hypergeometric
  - Exponential
  - …
- The probability distribution has parameters that are inferred from a random sample of the population; Parameters are fixed but not precisely known
  - Probability of failure is a parameter of the binomial distribution
  - Failure rate is a parameter of the exponential distribution
  - …
- Population has parameters
- Sample has statistics used to estimate parameters
- From the sample establish confidence intervals to represent uncertainty in the parameters of the population
  - Part 1 discussed one sided upper confidence level
  - Parameter is either within or not within the confidence interval with a certain confidence level

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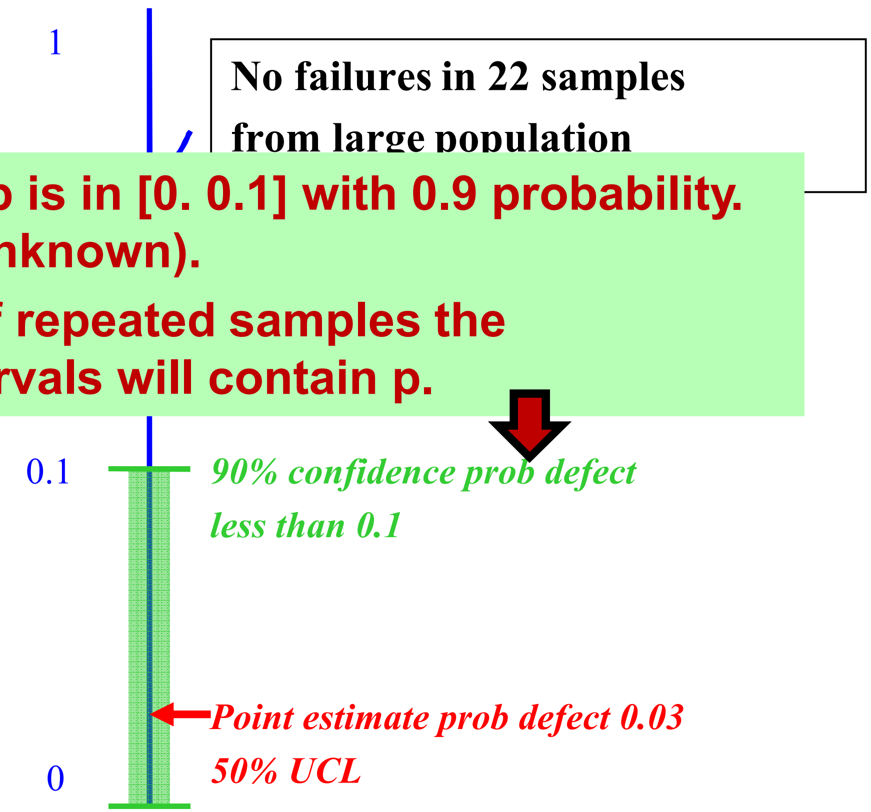
Classical Statistical Inference

- **Weapon Component Failures: SNL Point Estimates**
  - **Sample n of N and observe x failures**

This does NOT mean that p is in [0. 0.1] with 0.9 probability. p is a specific value (but unknown).

This means that for 90% of repeated samples the calculated confidence intervals will contain p.

- **x = 0 from 2 samples has same estimate as x = 0 from 10^6 samples**
- **If x is 0 use 50% UCL for point estimate to consider that larger n provides less uncertainty**



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Part #2: Bayesian Approach

- **Day 1: Basics and mathematical development**
- **Day 2: Simple Examples**
- **Day 3: Real world examples (some classified)**

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Part 2 Lecture: Day 1

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Conditional Probability

- Conditional probability
  - Probability of event A *given* event B: P(A|B)
  - P(A|B) = P(A and B) / P(B)
  - P(B|A) = P(B and A) / P(A)

- Since P(A and B) = P(B and A)
  - P(A|B) = P(B|A) * P(A) / P(B)

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Bayes Theorem

- Bayes theorem
  - S a sample space
  - $\{A_1, A_2, A_3, \ldots, A_n\}$ a partition over S
    - The A's are mutually exclusive and their union is S
  - B any event in S with non-zero probability
    - Law of total probability

      $P(B) = \sum_{k = 1 \text{ to } n} P(B|A_k)*P(A_k)$
  - A_i an event of interest in the partition
- $P(A_i|B) = P(B|A_i) * P(A_i) / \sum_{k = 1 \text{ to } n} P(B|A_k)*P(A_k)$

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Conditional Probability and Bayes Theorem

- Example
  - Test for disease is 99% accurate given you have the disease
  - Test has 10^{-4} false positive (falsely says you have disease)
  - 1 in 10^{6} people have the disease

  - You test positive
    - Probability you have the disease is 0.99?
    - NO

  - P(T|D) = 0.99 is probability Test T is positive given you have the Disease D
  - P(T|ND) = 10^{-4} is probability Test T is false positive given you do not have the disease ND
  - P(D) = 10^{-6} is probability an individual selected at random has the disease

  - P(D|T) is the probability you have the disease given you test positive
  - $P(D|T) = P(T|D) * P(D) / \{P(T|D)*P(D) + P(T|ND)*P(ND)\} =$
    $0.99 * 10^{-6} / \{0.99 * 10^{-6} + 10^{-4} * (1 - 10^{-6})\} \approx 10^{-6} / 10^{-4} = 0.01$

  - Probability you have the disease given you test positive is 0.01, not 0.99

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Bayesian Approach

So far we have just used the properties of conditional probability.

The Bayesian approach is revolutionary in its *interpretation* of conditional probability.

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Bayesian Concept of Subjective Probability

- $P(A_i|B) = P(B|A_i) * P(A_i) / \sum_{k=1 \text{ to } n} P(B|A_k)*P(A_k)$
- Let $P(A_i)$ be our initial probability distribution for event A_i
  - $P(A_i)$ is our *prior* probability distribution for A_i before updating with new information
- Let event B be new information
- $P(A_i|B)$ is our updated probability distribution for A_i given the new information B
  - $P(A_i|B)$ is our *posterior* probability distribution for A_i after updating with information B
- Technique to update given new information

Probability is SUBJECTIVE based on your state of knowledge. Totally different from classical, objective concept of probability.

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Bayesian Approach

- You want to know the probability of event A_i
- $P(A_i)$ is your initial estimate (the prior) before obtaining more information
- Event B is the new information you use to refine (update) your estimate of the probability of event A_i
- Your updated estimate (the posterior) of the probability of event A_i given information B is $P(A_i|B)$
- $P(A_i|B) = P(B|A_i) * P(A_i) / \sum_{k = 1 \text{ to } n} P(B|A_k)*P(A_k)$
  - Bayes equation for discrete events

$P(A_i|B)$ is the probability the event of interest A_i occurs given the event B occurred.

Think about this!

$P(B|A_i)$ is the probability event B would occur given the event of interest A_i occurred.

$\sum_{k = 1 \text{ to } n} P(B|A_k)*P(A_k)$ is $P(B)$

Normalization: Same for $P(A_i|B)$ for each A_i

Simple Example

- Prize behind one of three doors A_1, A_2, or A_3
- You guess $P(A_1) = P(A_2) = P(A_3) = 1/3$ (equal probability)
- Open any one door, say A_3, and prize not there
- You update your guess to $P(A_1) = 1/2$, $P(A_2) = 1/2$, $P(A_3) = 0$

- Here is the Bayesian approach
- $\{A_1, A_2, A_3\}$ is a partition for "location of prize"
- Priors
  - $P(A_1) = P(A_2) = P(A_3) = 1/3$
- Event B is "prize is not behind door A_3"
- Posteriors to be calculated:
  - $P(A_1|B) = P(A_1) * P(B|A_1) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*$
  - $P(A_2|B) = P(A_2) * P(B|A_2) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*$
  - $P(A_3|B) = P(A_3) * P(B|A_3) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*$

Probability the information that you obtained (event B) would have occurred if event in the partition did occur

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Simple Example continued

- A_1 is "prize is behind door A_1"
- A_2 is "prize is behind door A_2"
- A_3 is "prize is behind door A_3"
- B is "prize is not behind door A_3"

Understanding this is a key to understanding the Bayesian update approach

- $P(B|A_1)$ is P(prize is not behind door A_3 given the prize is behind door A_1) = 1.0
- $P(B|A_2)$ is P(prize is not behind door A_3 given the prize is behind door A_2) = 1.0
- $P(B|A_3)$ is P(prize is not behind door A_3 given the prize is behind door A_3) = 0.0

- $\sum_{k = 1 \text{ to } 3} P(B|A_k)*P(A_k) = 1(1/3) + 1(1/3) + 0(1/3) = 2/3$

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Simple Example continued

- $P(A_1|B) = P(A_1) * P(B|A_1) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*P(A_k) =$
  $(1/3) * 1 / (2/3) = 1/2$

- $P(A_2|B) = P(A_2) * P(B|A_2) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*P(A_k) =$
  $(1/3) * 1 / (2/3) = 1/2$

- $P(A_3|B) = P(A_3) * P(B|A_3) / \sum_{k = 1 \text{ to } 3} P(B|A_k)*P(A_k) =$
  $(1/3) * 0 / (2/3) = 0$

We will look at this example again tomorrow with more definitive information B:
The "Let's Make a Deal" situation

Posterior distrib

Probability prize behind A_1 = ½
Probability prize behind A_2 = ½
Probability prize behind A_3 = 0

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Bayesian Approach for Parameter of a Distribution: Use Continuous Distribution for Parameter

Two random variables:

X the one of concern has PDF f(x)

Θ the parameter of the probability distribution that describes X has PDF g(θ)

Our prior PDF for g is g(θ)

Given information x we update our PDF for g: g(θ|x). This is our posterior PDF for g

- **Here the parameters themselves are considered to be random variables**

$$g(\theta|x) = \frac{g(\theta)\, f(x|\theta)}{\int f(x|\theta)\, g(\theta)\, d\theta}$$

$$f(x) = \int f(x|\theta)\, g(\theta)\, d\theta$$

Note: it is the parameter θ that is not known

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Bayesian Approach for Parameter of a Distribution: Use Continuous Distribution for Parameter

$$g(\theta|x) = \frac{g(\theta)\, f(x|\theta)}{\int f(x|\theta)\, g(\theta)\, d\theta}$$

**f(x) is a PDF you select for the random variable X.
f(x) has a parameter θ. θ is not known and is considered a specific value of a random variable Θ.**

g(θ) is prior PDF you assume for Θ

Normalization: Same for g(θ|x) for every θ

x is an observed value of the random variable X

f(x|θ) is the PDF at X equal to the value x given a specific θ. Called the likelihood function.

Understanding this is a key to understanding the Bayesian update approach

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Bayesian Approach for Parameter of a Distribution: Use Discrete Distribution for Parameter

$$g(\theta|x) = \frac{g(\theta)\, f(x|\theta)}{\sum f(x|\theta)\, g(\theta)}$$

$$f(x) = \sum f(x|\theta)\, g(\theta)$$

f(x|θ) is the PDF at X equal to the value x given a specific θ. Called the likelihood function.

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Bayesian Approach Example for Parameter

- **Assume the appropriate PDF for time to failure is exponential distribution. Time to failure is a random variable T.**
  - **PDF is f(t) = $\lambda e^{-\lambda t}$ where parameter λ is failure rate**
- **Treat λ as specific value of random variable Λ**
- **Assume PDF for prior g(λ) is uniform over [0, 0.01]: g(λ) is 100 in this interval, 0 elsewhere**
- **Observe one failure at 0.0072 hours**

for λ in [0, 0.01]

$$g(\lambda|T=\mathbf{0.0072}) = \frac{f(\mathbf{0.0072}|\boldsymbol{\lambda})\ \boldsymbol{g}(\boldsymbol{\lambda})}{\int_0^{0.01} f(\mathbf{0.0072}|\boldsymbol{\lambda})\boldsymbol{g}(\boldsymbol{\lambda})\ \boldsymbol{d\lambda}} = \frac{\lambda \boldsymbol{e}^{-\mathbf{0.0072}\lambda} * 100}{100\int_0^{0.01} \lambda \boldsymbol{e}^{-\mathbf{0.0072}\lambda}\ \boldsymbol{d}\lambda}$$

$$= \frac{\lambda \boldsymbol{e}^{-\mathbf{0.0072}\lambda}}{\mathbf{5.0\ x10^{-5}}}$$

for λ > 0.01, g(λ) is 0; so g(λ |T) is 0

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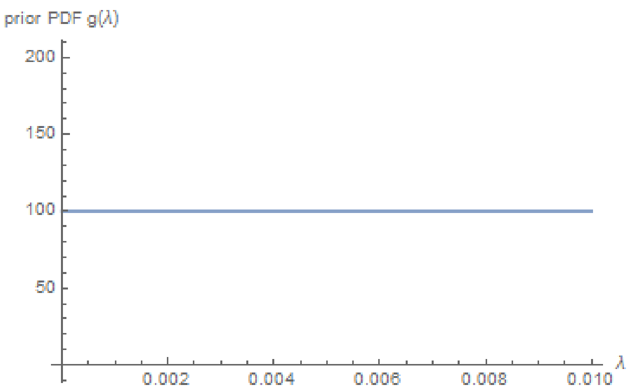
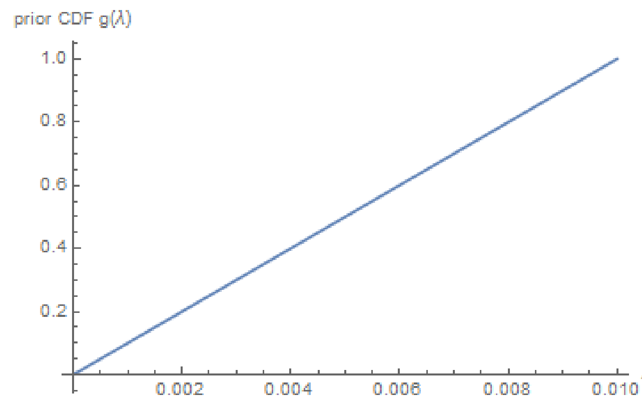
Bayesian Approach Example for Parameter

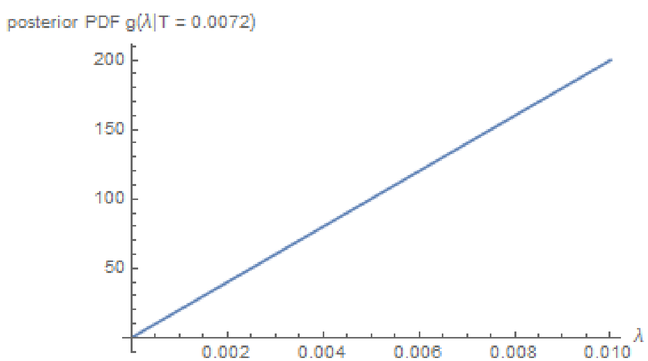


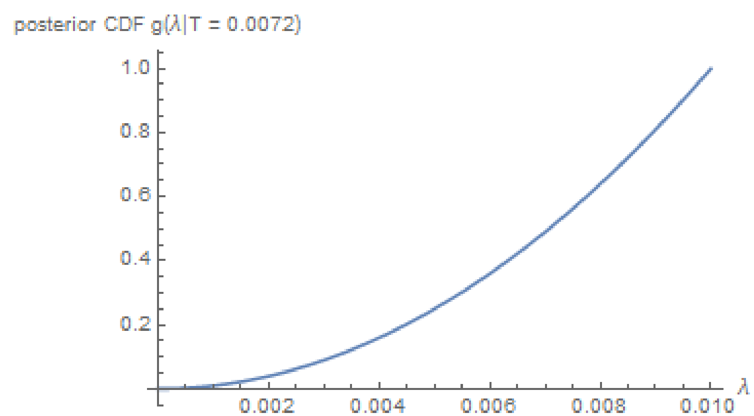


Prior g(λ)

Prior mean

0.005





Posterior g(λ|T = 0.0072)

Posterior mean

0.0067

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Bayesian Approach Example

- So far we have updated the parameter λ
- How do we use the updated parameter for the probability of failure of the component?
- Assume exposure time of concern is 24 hours
- Using exponential distribution Probability fail in 24 hours is

$$P(24) \;=\; \int_0^{24} \lambda\, e^{-\lambda t}\, dt \;=\; 1 - \; e^{-24\,\lambda}$$

- λ has uncertainty
- Treat P(24) as a function of the random variable Λ

$$P(24) \;=\; 1 - \; e^{-24\,\lambda}$$

where λ is a specific value of Λ. Λ has a PDF; prior and posterior

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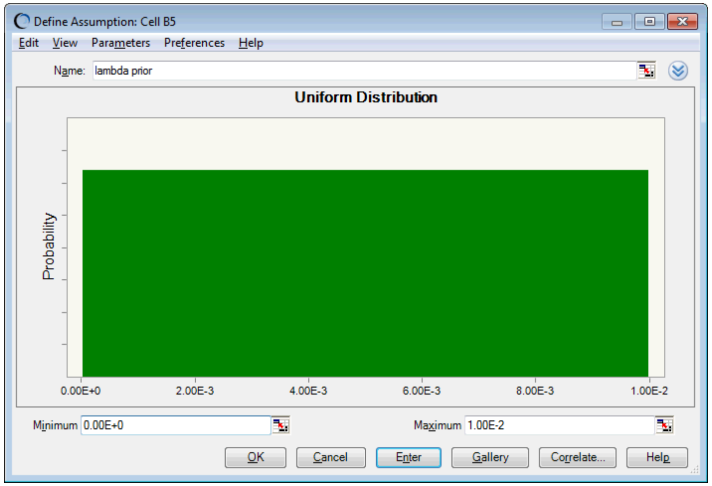
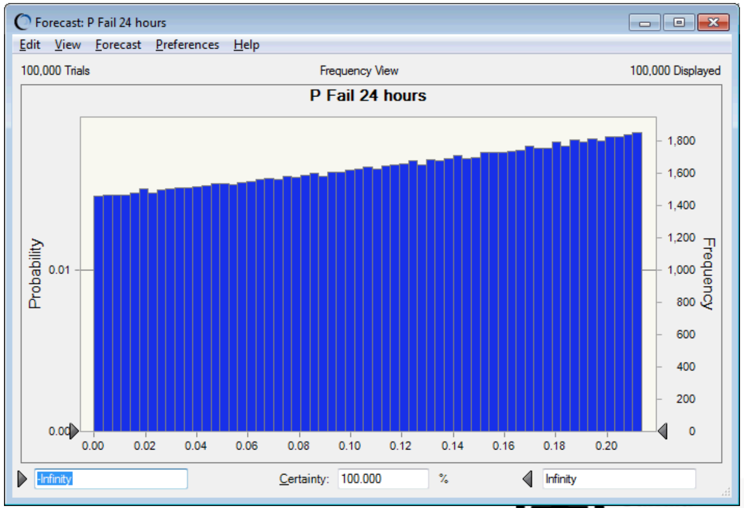
Bayesian Approach Example Solve with Sampling using Crystal Ball using Prior for g

- g(λ) = 100 over [0, 0.01], mean 0.005



- Solve for $P(24) = 1 - e^{-24\,\lambda}$ by sampling, mean 0.11



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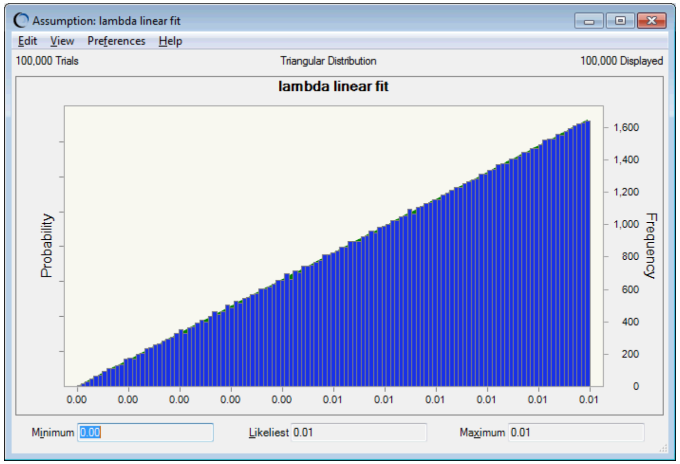
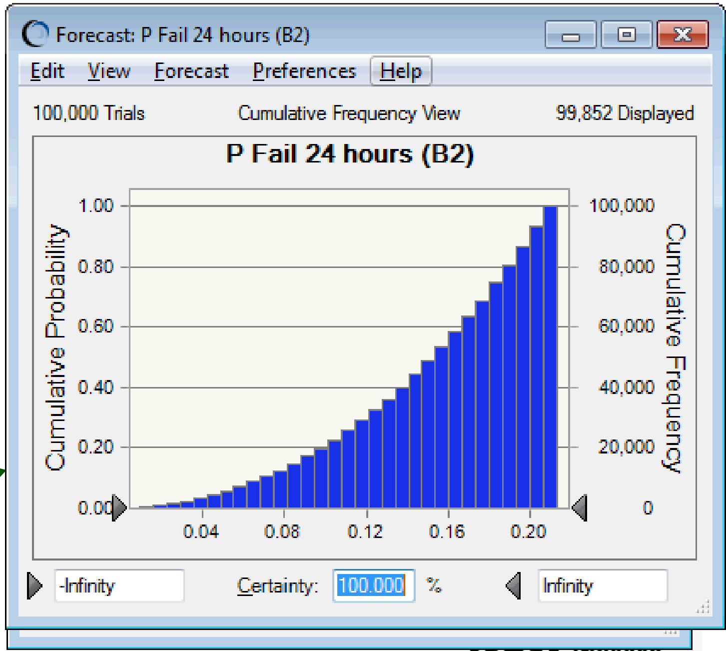
Bayesian Approach Example Solve with Sampling using Crystal Ball using Posterior for g

- g(λ|T = 0.0072) **is essentially linear, mean 0.0067**



- **Solve for** $P(24) = 1 - e^{-24\lambda}$ **by sampling, mean 0.15**



CDF for P fail in 24 hours using Bayesian updated PDF for failure rate

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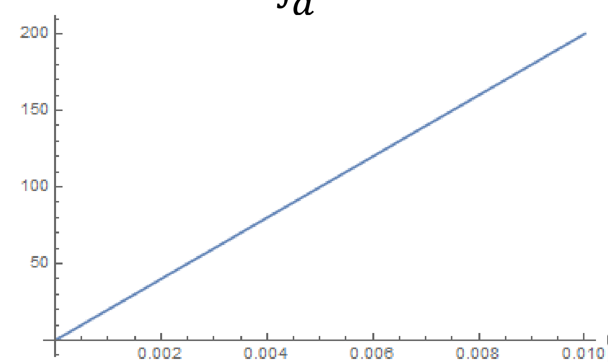
Bayesian Approach Example using Posterior for g Standard Solution

- **Standard solutions exist for updating parameters for commonly used distribution**
  - **See Martz and Waller reference**
- **Exponential distribution is a commonly used distribution**
  - **If prior for parameter λ is uniform over [a, b]**
  - **Posterior for parameter λ given T = t_1 is** $$\frac{\lambda\, e^{-\lambda t_1}}{\int_a^b \lambda\, e^{-\lambda t_1}\, d\lambda}$$
  - **For t_1 of 0.0072 posterior PDF is for parameter λ**

    (same as obtained earlier)



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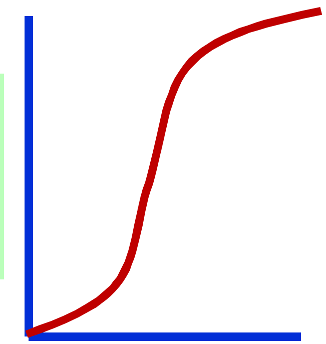
Subjective Approach: Parameters and Probability Itself Treated as a Random Variable

- Wish to estimate uncertainty for a probability
- Classical approach:
  - Probability is fixed but not pr
  - Produce a Confidence Interv
- Subjective approach
  - Probability itself is a random variable
  - Produce a PDF
  - PDF for P(E) the objective probability for event E
    - $P(E) = \lim_{N \to \infty}$ (number of time E occurs / N)
    - P(E) is a random variable with a subjective PDF
      - p(E) a specific value of P(E)

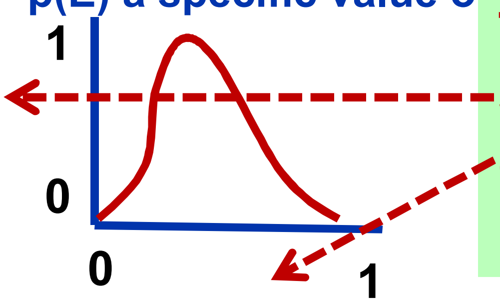
CDF for
P(E)

Prob (P(E) ≤ p(E))



p(E)

PDF for
P(E)



Two different probabilities:
Subjective
Objective
Both obey Kolmogorov Axioms

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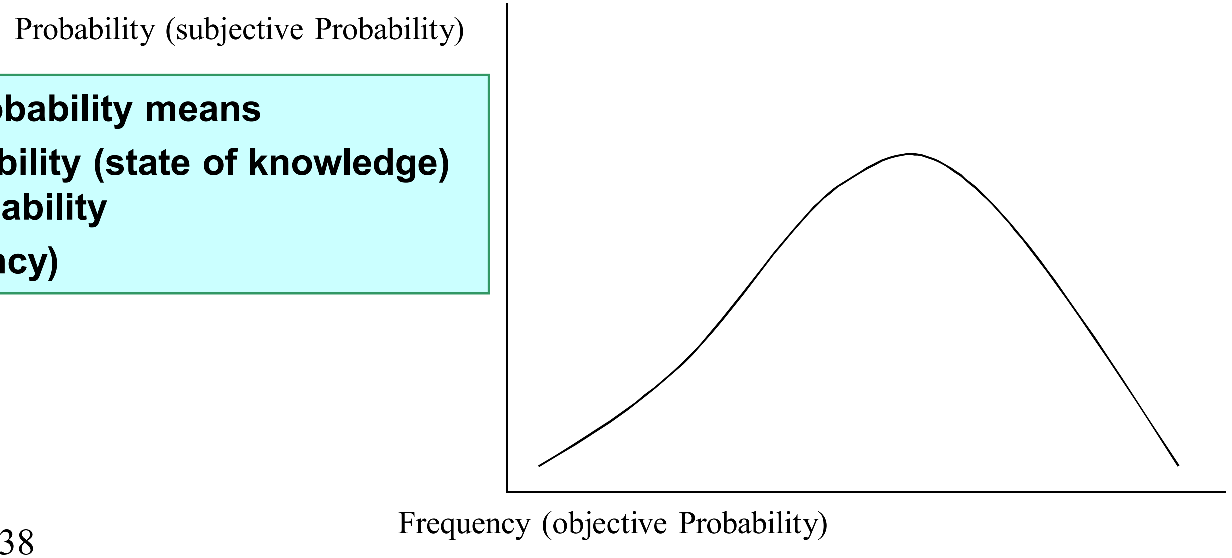
Bayesian Approach is Subjective

- Probability is a state of knowledge and can be estimated even without sufficient data to evaluate the classical frequency
  - Subjective concept of probability
- Treat probability itself as a random variable instead of a fixed, but perhaps unknown, frequency
- Probability of a Probability means
  - Subjective probability of the objective probability (the frequency)
  - Confusion is that the name Probability used to mean two different concepts
    - Both concepts obey Kolmogorov axioms
- See reference: Kaplan and Garrick 1981 paper in <u>Risk Analysis</u>
- Update P(E) with information: P(E | Information) as discussed earlier

Probability of Probability means

Subjective Probability (state of knowledge) of Objective Probability

(classical frequency)



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Propagation of Uncertainty

- "Bayesian probability distributions can be propagated through fault trees, event trees, and other logic models. It is difficult or impossible to propagate frequency confidence intervals through fault and event tree models common in PRA to produce corresponding interval estimates on output quantities of interest".

  "Handbook of Parameter Estimation for Probabilistic Risk Assessment", NUREG/CR-6823, SAND2003-3348P
- Part 1 discussed convolution and sampling for combining probability distributions of random variables. Since the Bayesian approach treats parameters and probability as random variables, we can apply these techniques to random variables that are probabilities.

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Major Use of Bayesian Approach for Our Applications

- Estimate uncertainty for logical combinations of basic events that are failure probabilities with uncertainty
  - Uncertainty for basic events in fault trees that are Failure Probabilities
    - Fail on demand
      - Uncertainty in Probability parameter for binomial distribution
    - Fail to operate for prescribed time
      - Uncertainty in Frequency parameter for exponential distribution
    - … Other types of failure
  - Propagate the uncertainty per the logic in the fault tree to provide uncertainty for top event in tree

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Probability of Probability

- **Subjective probability of objective probability (frequency)**
- **Treat objective probability (frequency) as a random variable**
- **Let P(A) be probability of event A: e.g. failure of a component**
  - **P(A) is an objective probability, a frequency**
  - **$P(A) = \lim_{N \to \infty}$ (number of time A occurs / N) for N trials**
- **We are interested in combinations of probabilities of failures in fault trees**
  - **$P(A \cap B)$, probability A AND B fail in a fault tree**
  - **$P(A \cup B)$, probability A OR B fail in a fault tree**

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Probability of Probability

- If A and B mutually exclusive
  - P(A ∩ B) = 0   P(A U B) = P(A) + P(B)
- If A and B independent
  - P(A ∩ B) = P(A)*P(B)   P(A U B) = P(A) + P(B) – P(A)*P(B)
- In general
  - P(A ∩ B) = P(A|B)*P(B) = P(B|A)*P(A)
  - P(A U B) = P(A) + P(B) – P(A|B)*P(B) = P(A) + P(B) – P(B|A)*P(A)
- In fault tree examples in part 1 we assumed P(A) and P(B) were known with no uncertainty
- To address uncertainty in P(A) consider P(A) as a random variable
  - P(A) has a PDF, discrete or continuous. Same for P(B).
- P(A U B) and P(A ∩ B) are functions of the random vector with random variables P(A) and P(B)
  - Lecture 1 discussed function f(z) of Random Vector Z = X x Y where x is Cartesian product
  - Here X and Y are objective probabilities, Z = logical AND, OR
  - We want PDF for f(z)

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Probability for Function of Random Vector (from part 1)

- Probability for Function of a Random Vector Z
  - z = f(r, t), R and T are random variables
  - For discrete case PDF(z) = ∑ PDF(r,t) | f(r, t) = z
  - For continuous case, conceptually
    PDF(z) = ∫PDF(r,t) | f(r, t) = z
  - If R and T are independent random variables
    PDF(r, t) = PDF(r) * PDF(t)

Can be very difficult to solve analytically.
See Meyer text in references.
Can solve by sampling approach.

Now, R and T are probabilities: R is P(A) and T is P(B)

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Probability of Probability

- Two probabilities: the value P(A), P(B)
  - The objective probability
- The probability of the value P(P(A)), P(P(B))
  - The subjective probability of the objective probability
- Probability of a function of probabilities
  - For example P(A U B) is the function
    - The value is P(A) + P(B) - P(A) * P(B|A)
  - If P(A) and P(B) are known the value is known
    - P(A) = p_1 and P(B) = p_2 , A and B independent
    - Value P(A U B) = $p_1 + p_2 – p_1 * p_2$ with no uncertainty
  - If P(A) and P(B) are unknown and treated as random variables
    - The value P(A) + P(B) - P(A) * P(B|A) has a PDF determined by the PDF for the function P(A U B)

Values for events P(A) and P(B)

PDF for values P(P(A)) and P(P(B))

Values for Function P(A U B)

PDF for values for function

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Probability of Probability

- **Will discuss some simple examples for discrete cases tomorrow where we can manually generate the PDF for the function**
- **PDF for the function for continuous case can be difficult or impossible to generate analytically**
  - **For continuous cases we use sampling to generate this PDF**
    - **Examples follow**

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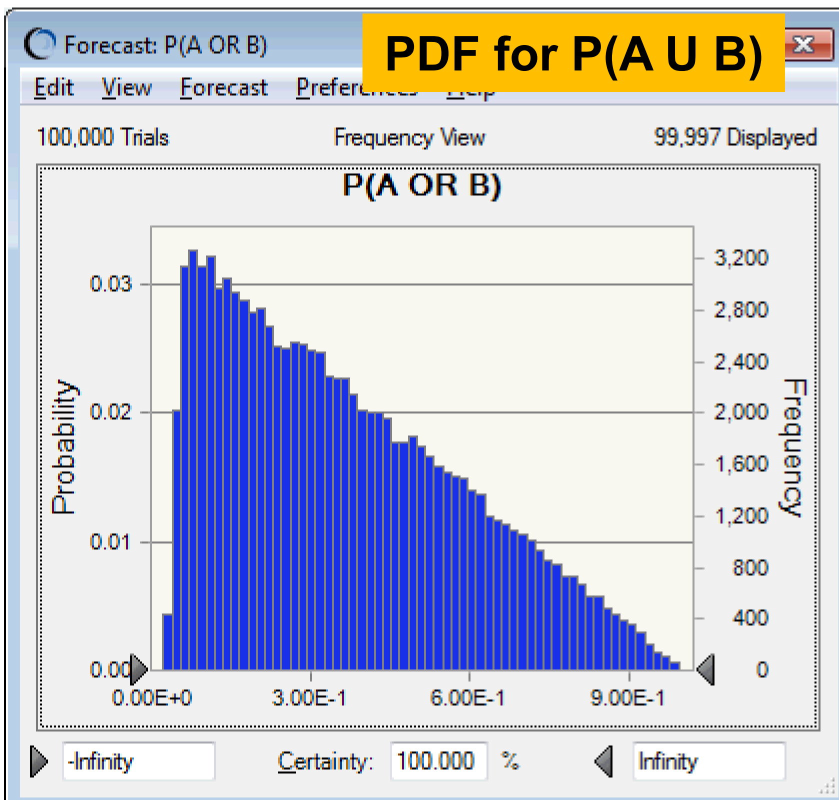
Example
Crystal Ball

- **Estimate the uncertainty for P(A U B) given A and B are independent**
  - **P(A U B) = P(A) + P(B) – P(A) * P(B)**
  - **Treat P(A) and P(B) as random variables with PDFs**
- **Solution using sampling approach in Crystal Ball**



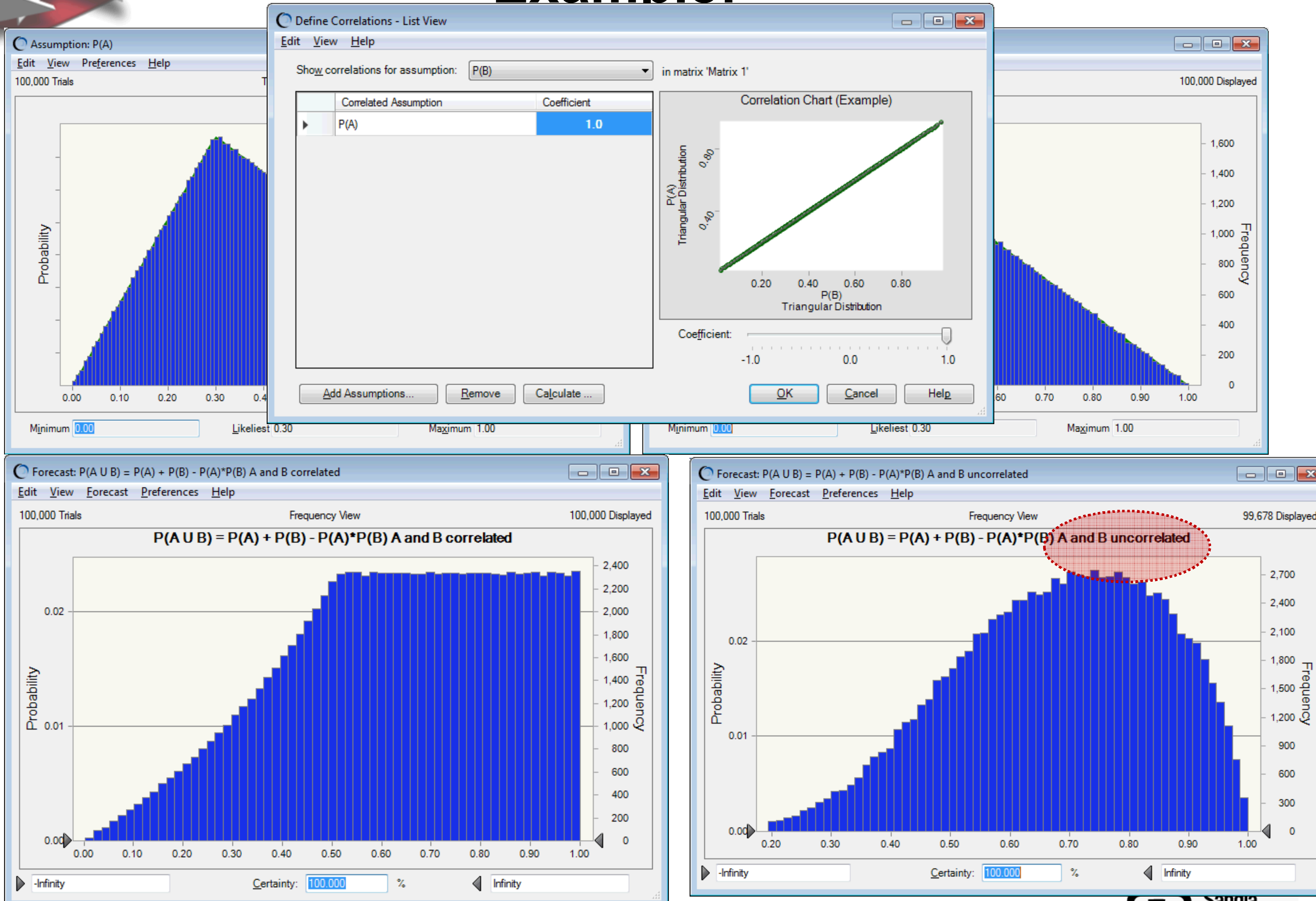
Forecast: P(A OR B)

Edit View Forecast Preferences Help

100,000 Trials | Statistics View | Sheet1!B4

Statistic	Forecast values
Trials	100,000
Base Case	3.57E-1
Mean	3.61E-1
Median	3.22E-1
Mode	---
Standard Deviation	2.26E-1
Variance	5.10E-2
Skewness	0.5665
Kurtosis	2.40
Coeff. of Variation	0.6250
Minimum	2.23E-2
Maximum	9.98E-1
Mean Std. Error	7.14E-4

Example:



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Fail on Demand

- Concern is x (or more) of n items fail on demand
  - Binomial distribution; exactly x of n fail on demand
    - Parameter p: probability an item fails
    - PDF = P(x of n) = $n!/[x!\,(n - x)!]\ p^x\ (1-p)^{(n-x)}$
  - x or more fail
    - P(x or more of n) = 1 – CDF((x - 1)) or, equivalently
    - P(x or more of n) = $\sum_{j=x \text{ to } n}$ PDF(j)
  - Treat p as a random variable, assign it a PDF

Remember: For random variable Z,
CDF(z) is probability Z ≤ z,
CCDF(z) = 1 – CDF(z) is probability Z > z, so
CCDF(z -1) is probability Z ≥ z

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Fail to Operate

- An item fails to operate for time t
- t is time of concern, say 24 hours
  - Exponential distribution
    - Parameter λ: failure rate
    - P(fail within time t) = $1 - e^{-\lambda t}$
  - Treat λ as a random variable, assign it a PDF

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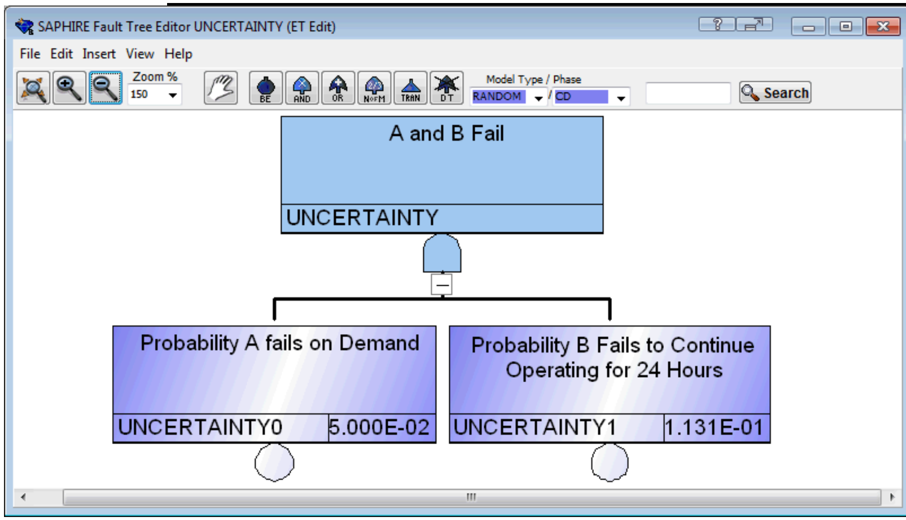
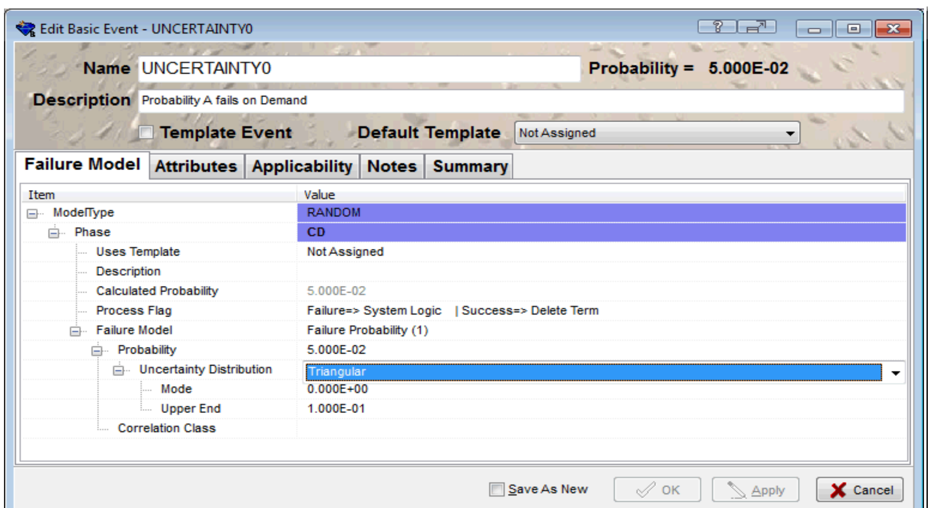
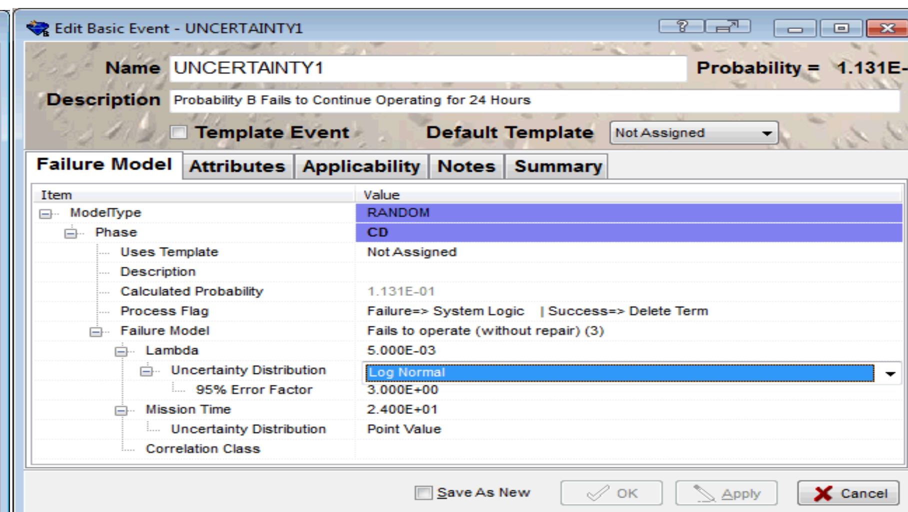
Propagation of Uncertainty: SAPHIRE



...ns to be assigned to basic events in
...robabilities of failure
...vent using sampling process





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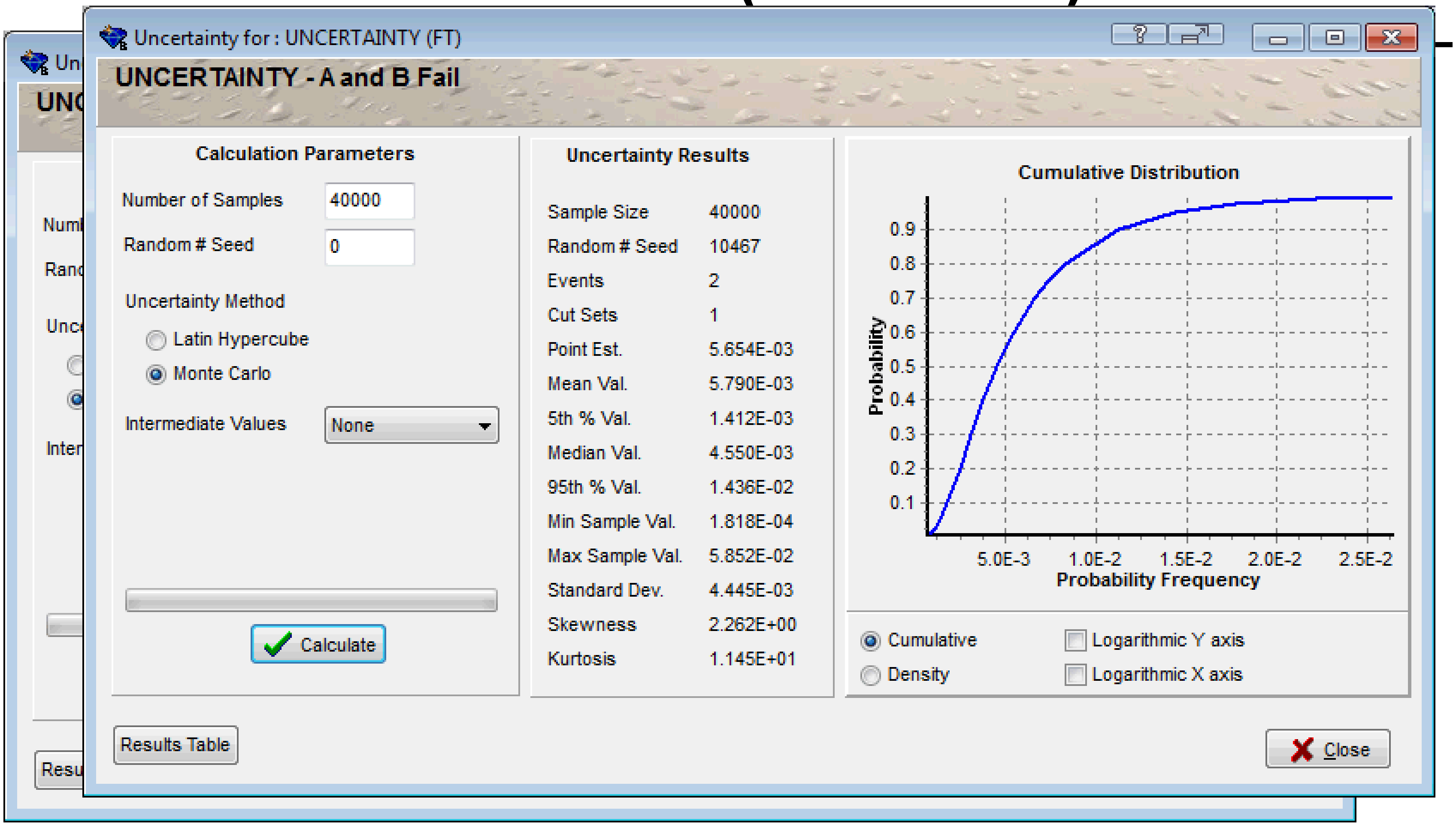
Propagation of Uncertainty: SAPHIRE (continued)



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Propagation of Uncertainty SAPHIRE: Review from Part 1

- **SAPHIRE default treats all events with different names as independent unless correlated**
  - $P(A \cap B) = P(A)*P(B)$
  - $P(A \cup B) = P(A) + P(B) - P(A)*P(B) = 1 – (1 – P(A))*(1 – P(B))$
  - **SAPHIRE default combines cut sets assuming cut sets are independent**
    - **Not exactly accurate if cut sets share basic events**
- **Correlation in SAPHIRE requires P(A) = P(B), uses Same PDFs for P(A) and P(B) in a Bayesian sense, and above become**
  - $P(A \cap B) = P(A)^2$
  - $P(A \cup B) = 2\,P(A) - P(A)^2$

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Propagation of Uncertainty: SAPHIRE N of M Fail

- Concern is any 2 (or more) of 4 identical components fail
- Let each component failure probability be p
- Exact probability 2 (or more) of 4 fail is

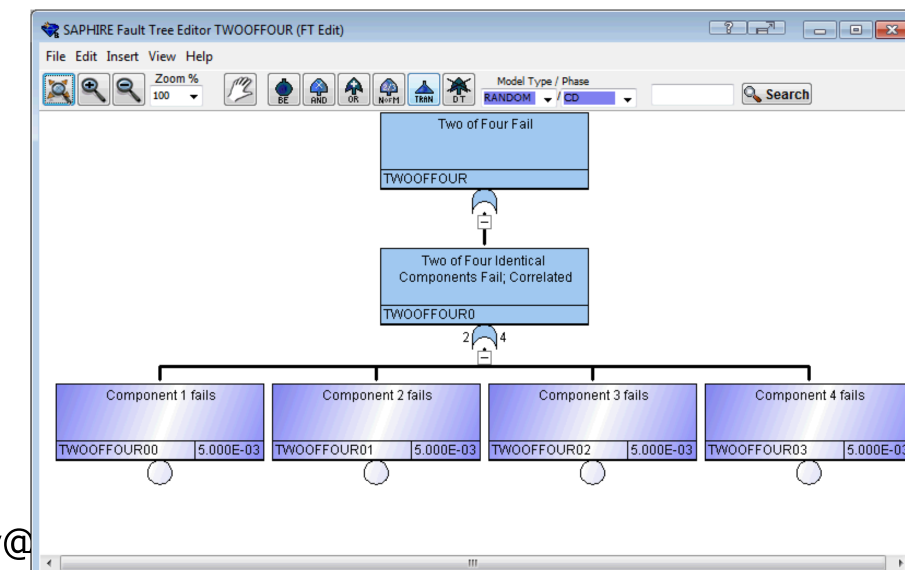
$$\sum_j = \left(\frac{4!}{2!2!}(p)^2(1-p)^2 + \frac{4!}{3!1!}(p)^3(1-p)^1 + \frac{4!}{4!0!}(p)^4(1-p)^0\right)$$

Probability 2, 3, or 4 of 4 components fail out of 4 total where each fails with probability p

- SAPHIRE evaluates as $6*p^2$

6 combinations of 2 of 4 failing

Same result for small p

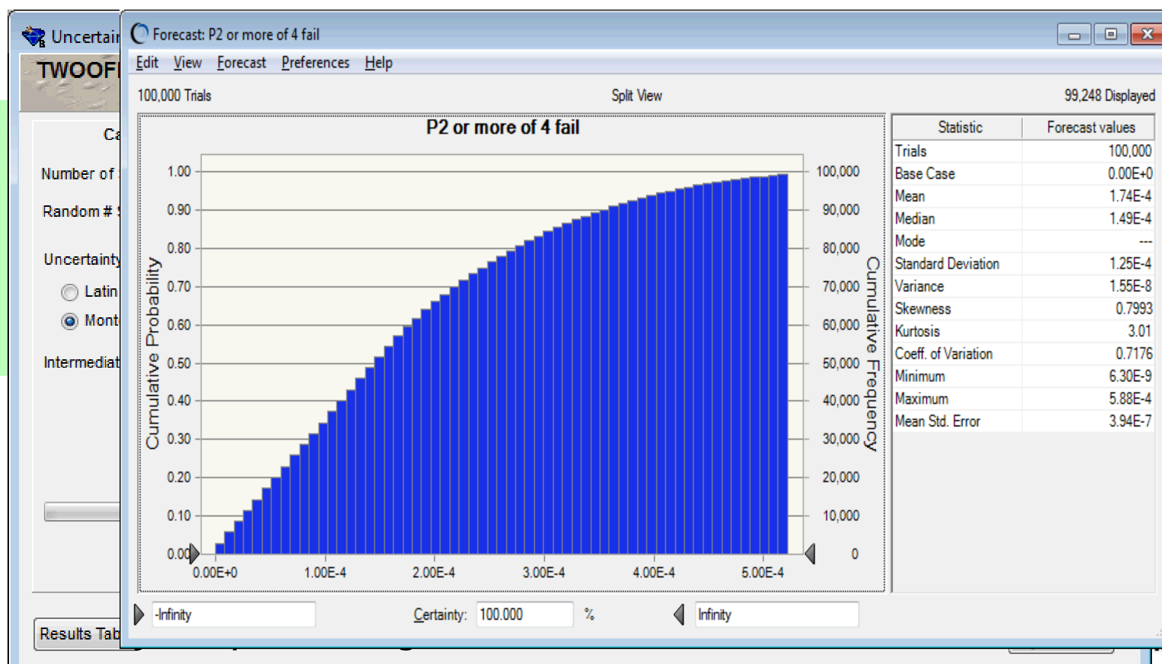


Propagation of Uncertainty: SAPHIRE

- 2 (or more) of 4 fail each with probability uniform [0, 0.001]
  - Exact mean 1.49E-4
- SAPHIRE solution correlated: $6p^2$
  - Point Estimate 1.50E-4
  - CDF

Essentially the same solution from Crystal Ball for exact for small p

SAPHIRE solution for N of M fail is incorrect if M > 2 and probability fail is not small



Propagation of Uncertainty: SAPHIRE N of M Fail

- **Degenerate Case**
  M of 2, Components A and B fail
  - AND: N is 2
    - One cut set A*B
  - OR: N is 1
    - Two cut sets A and B
  - Cut sets are independent
  - SAPHIRE solution same as exact solution

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Bayesian Update: Some Special Cases

- Can use Bayesian update process to generate posterior PDFs for failure probabilities for basic events in fault trees
- Will discuss a few simple cases here
  - See references for more detailed discussion
- f(x) a PDF for a continuous random variable X with parameter θ where we do not know θ
- Treat θ as a specific value of a random variable Θ with PDF g(θ)

What PDF for g(θ)?
What is g(θ|x)?

$$g(\theta|x) = \frac{g(\theta)\, f(x|\theta)}{\int f(x|\theta)\, g(\theta)\, d\theta}$$

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Conjugate Priors

- g(θ) is called a conjugate prior distribution for the parameter Θ if g(θ|x) is a posterior distribution that is a member of the same family of distributions as the prior
  - For X described by the binomial distribution with parameter p, the beta distribution is a conjugate prior for p; range of beta distribution is [0, 1]
  - For T described by the exponential distribution with parameter λ, the gamma distribution is a conjugate prior for λ

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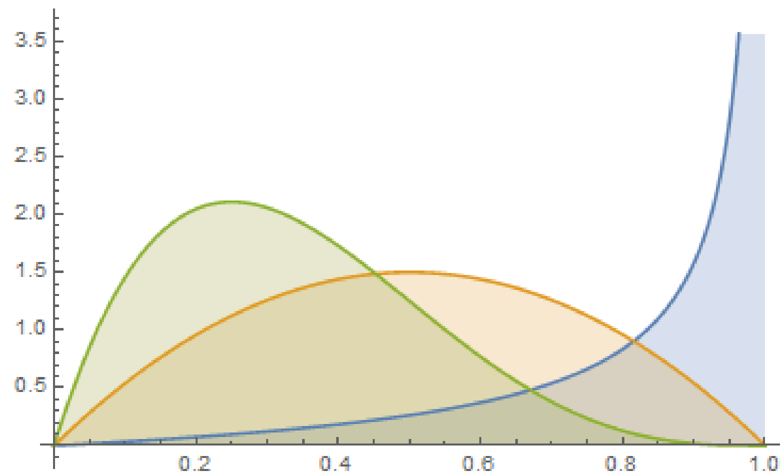
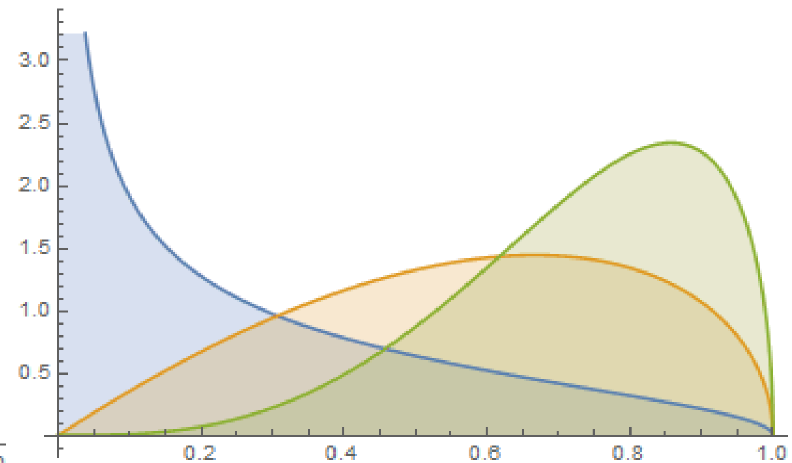
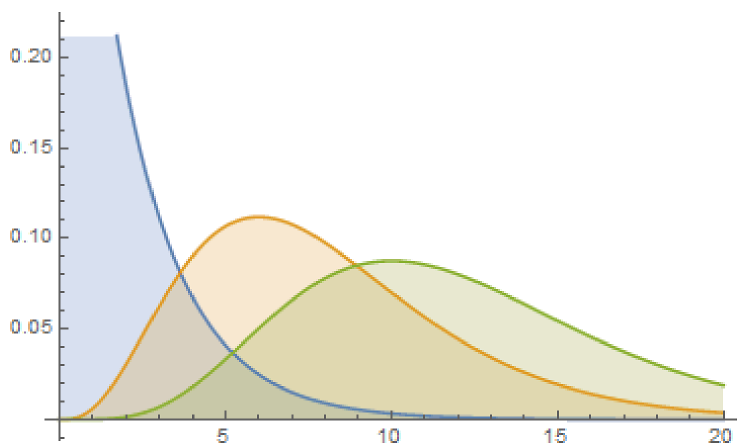
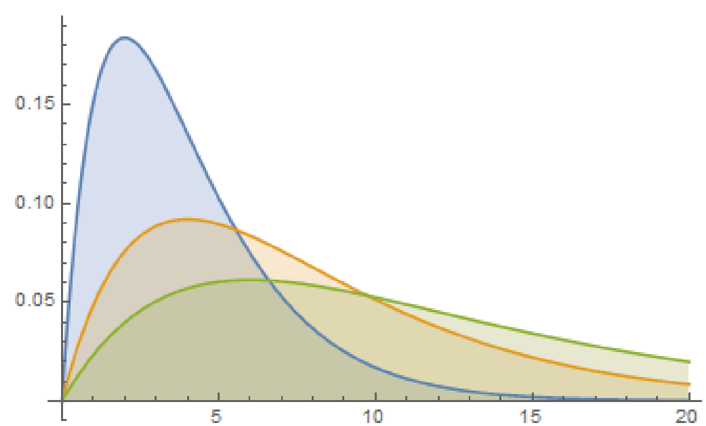
Beta and Gamma Distributions





Beta





Gamma

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Binomial Distribution: Beta Conjugate Priors for Parameter p

- **Two sets of parameters used for beta distribution**
  - **Shape parameters (α, β)** ← **used in this lecture**
    - **Used in Mathematica**
  - **Discrete parameters (x, n) for x failures in n tests**
    - **Used in Martz and Waller reference**
    - **Transformation: $\alpha = x \quad \beta = n - x$**

- **If prior is beta(x_0, $n_0 - x_0$) mean is x_0 / n_0**
- **Observe x failures in n trials**
- **Posterior is beta($x + x_0$, $n - x + n_0 - x_0$) mean is ($x + x_0$) / ($n + n_0$)**

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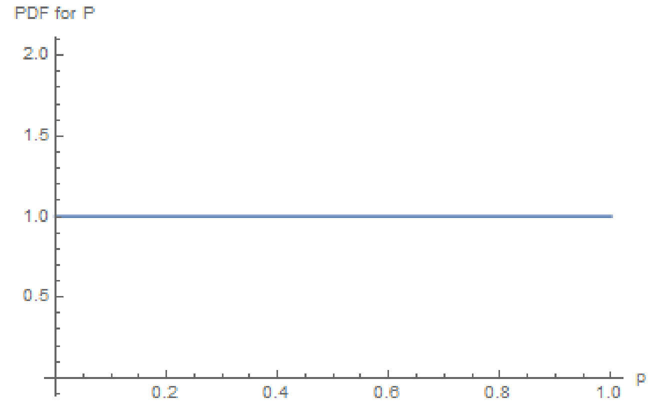
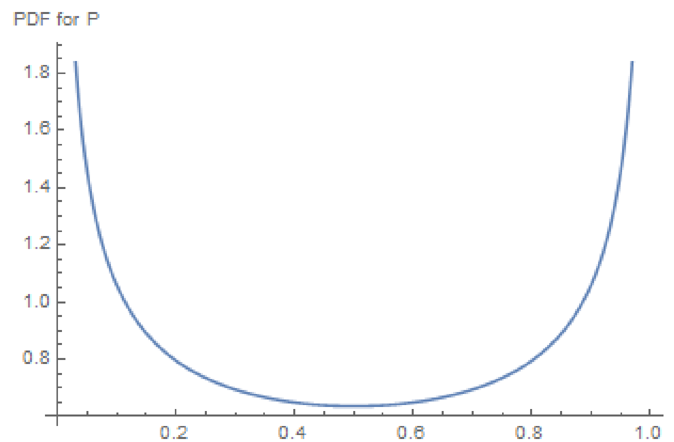
Binomial Distribution: Beta Conjugate Priors With No Information

- With no information
  - Uniform prior: beta(1,1)
    mean ½



Some prefer this: see NUREG/CR-6823

  - Jeffreys noninformative: beta(½, ½) mean ½



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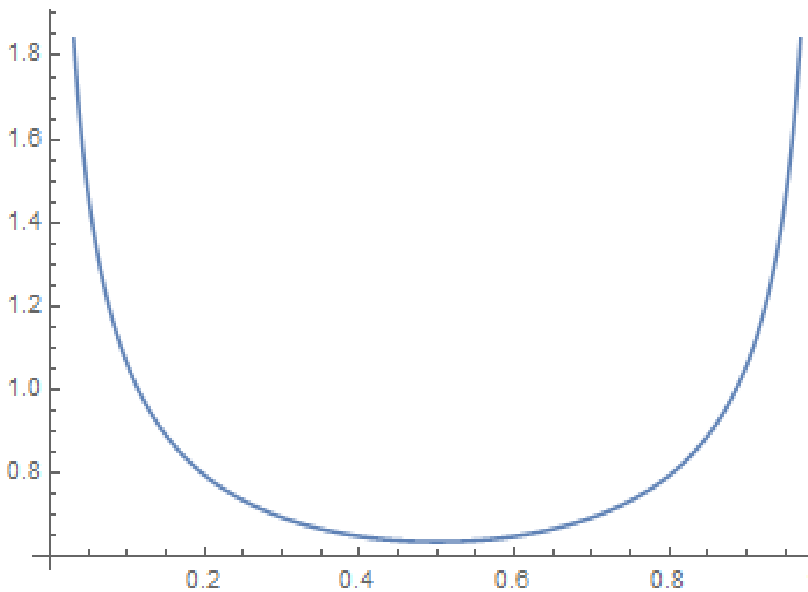
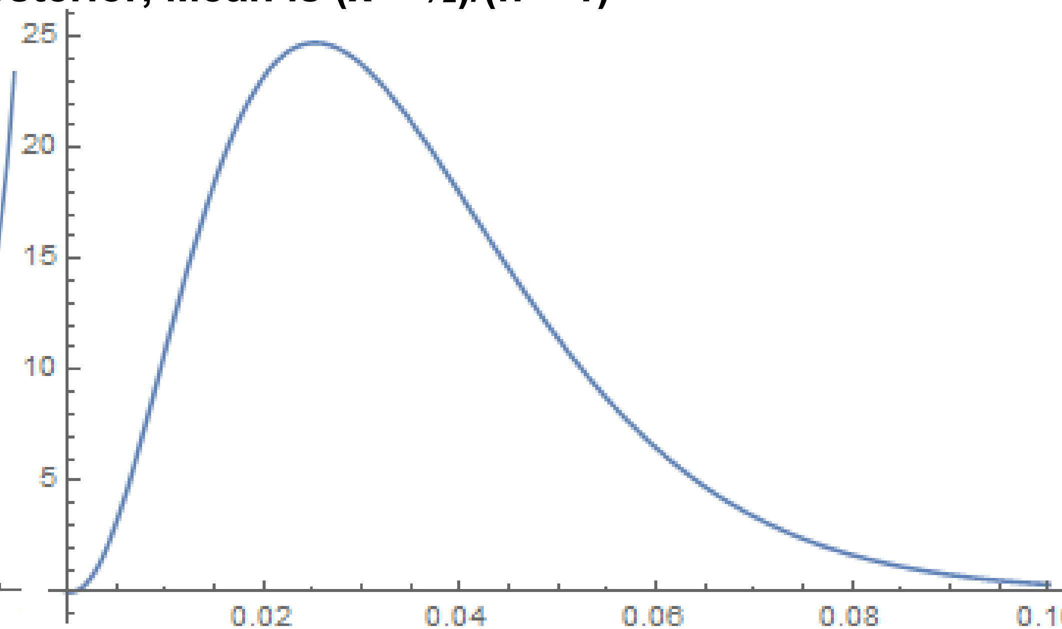
Bayesian Update for p

- No information before doing tests:
  Use Jeffreys noninformative prior
  - beta[½, ½], Mean is 0.5
- Observe x failures in n demands
  - beta[x + ½, n – x + ½] is posterior, Mean is (x + ½)/(n + 1)



PDF for prior: mean 0.5



With n of 100, x of 3 PDF for posterior: mean 0.035

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Situations where Bayesian is Not Useful

- Two examples follow
  - Biased prior
  - Posterior based on little information

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Bayesian: Biased Prior Gives Incorrect Posterior

- Archeologists in a foreign country find an unknown coin from ancient civilization. What is P(Heads)?
- Since Heads and Tails are all the mutually exclusive outcomes of a sample space: P(Tails) = 1 – P(Heads)
- You cannot examine the coin, but based on experience with coins from similar civilizations you assume coin is two-tailed: prior is P(H) = 0 and P(T) = 1
- With these priors, update will always give P(H | tosses) = 0 regardless of the number of tosses and results of the tosses
  - Suppose you later receive the coin, toss it 10,000 times and observe all heads
  - Obviously coin is two-headed
  - But your prior P(H) = 0 forces posterior to always be P(H | tosses) = 0

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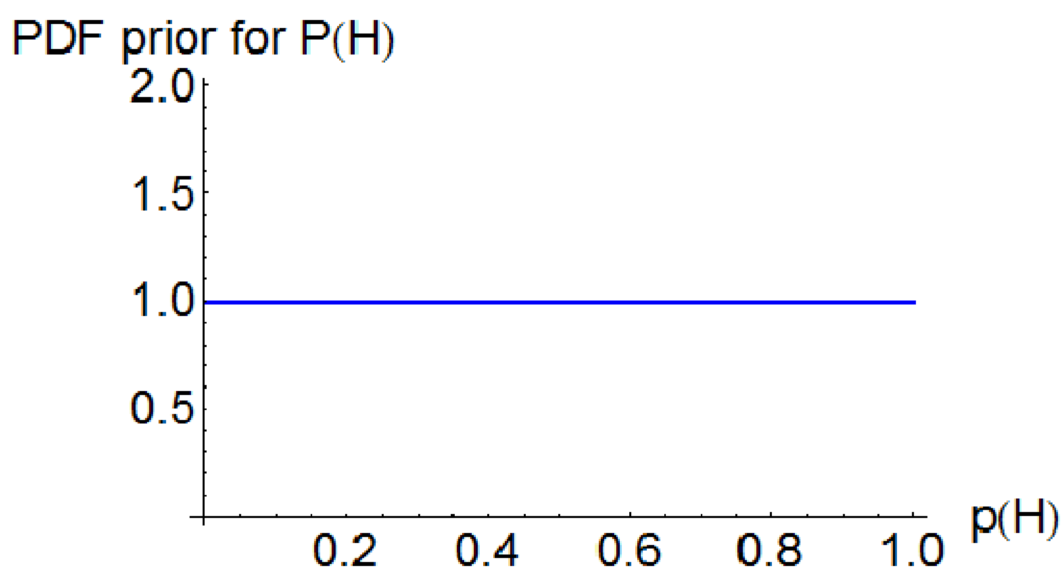
Bayesian: Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- **Unknown coin from ancient civilization.**
- **You assume any value for P(Heads) for prior uniform distribution over [0,1]: beta(1, 1) mean is ½**



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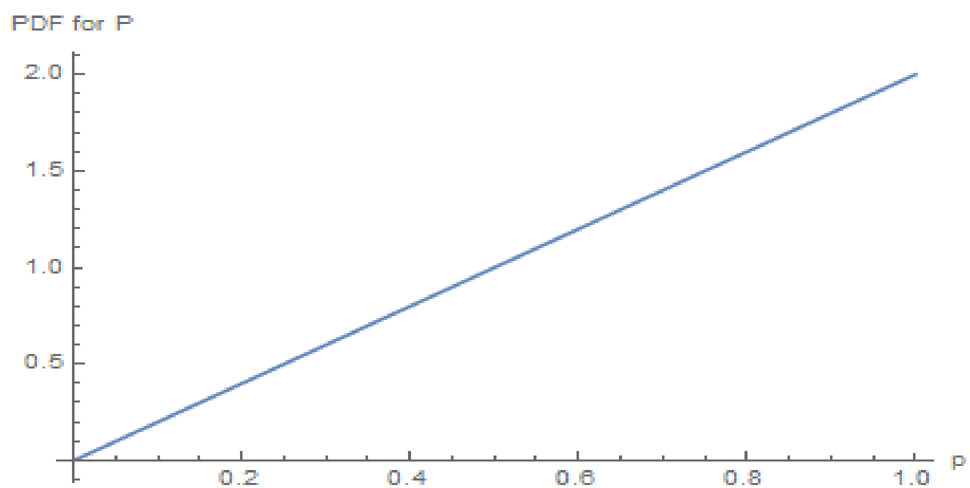
Bayesian:
Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- **Suppose you are told the coin was tossed once and heads was observed; P(Heads) posterior is beta(2, 1) with mean 2/3**



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Bayesian: Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- **Based on this small amount of Information (one toss) your posterior indicates heads is more likely than did prior**
- **If you cannot obtain more information this is the best you can do with Bayesian**
- **But if your prior was not accurate and you have little information your posterior may be misleading**

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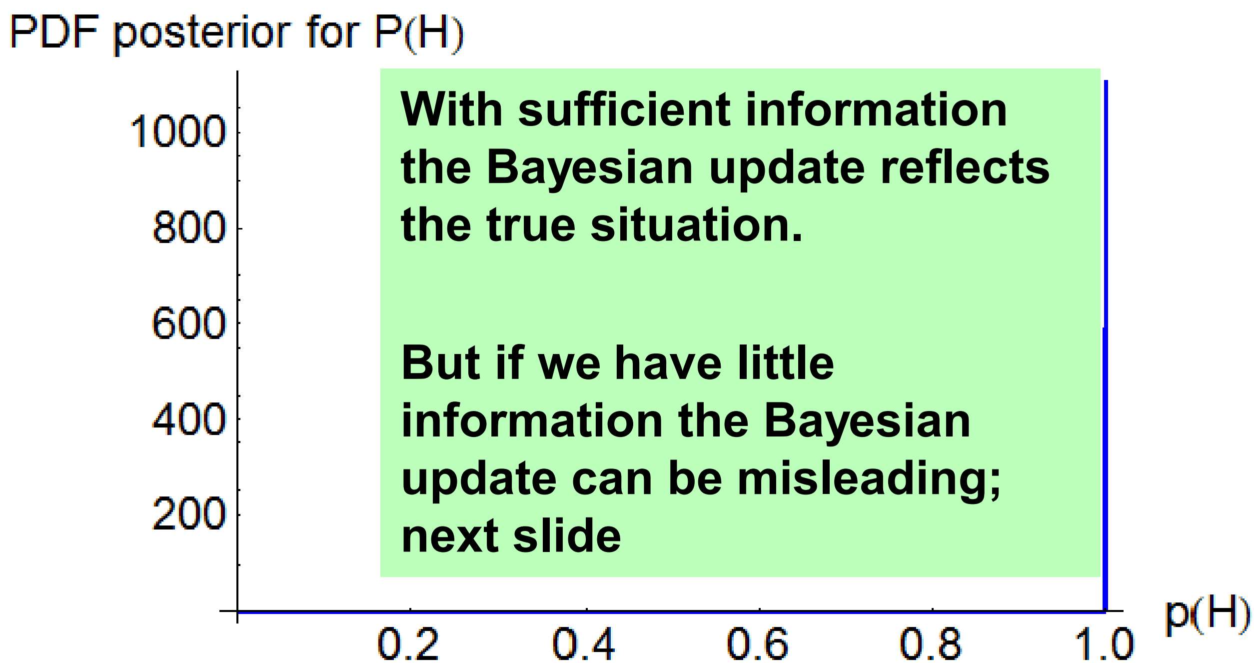
Bayesian: Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- Suppose that you later receive the coin and perform 10,000 tosses and all were heads
- Posterior is beta(10001, 1) with mean 0.9999
- The coin is biased to always be Heads!



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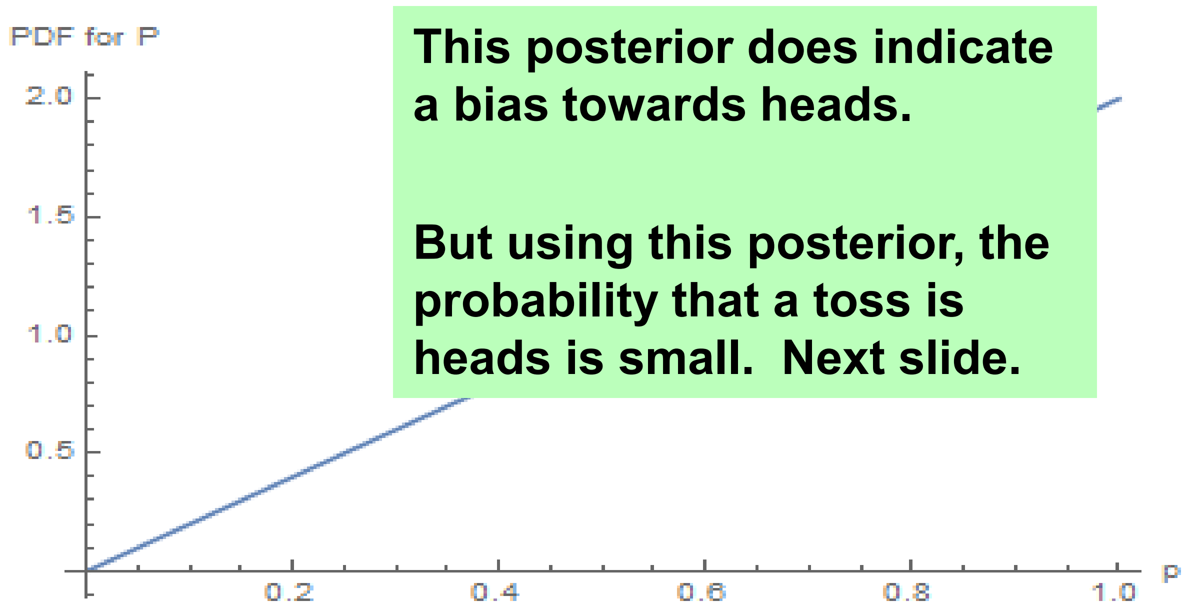
Bayesian: Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- Suppose the coin is two-headed but we only have the information that one toss gave heads. Our posterior for P(H) (from earlier) is beta(2, 1) with mean 2/3.



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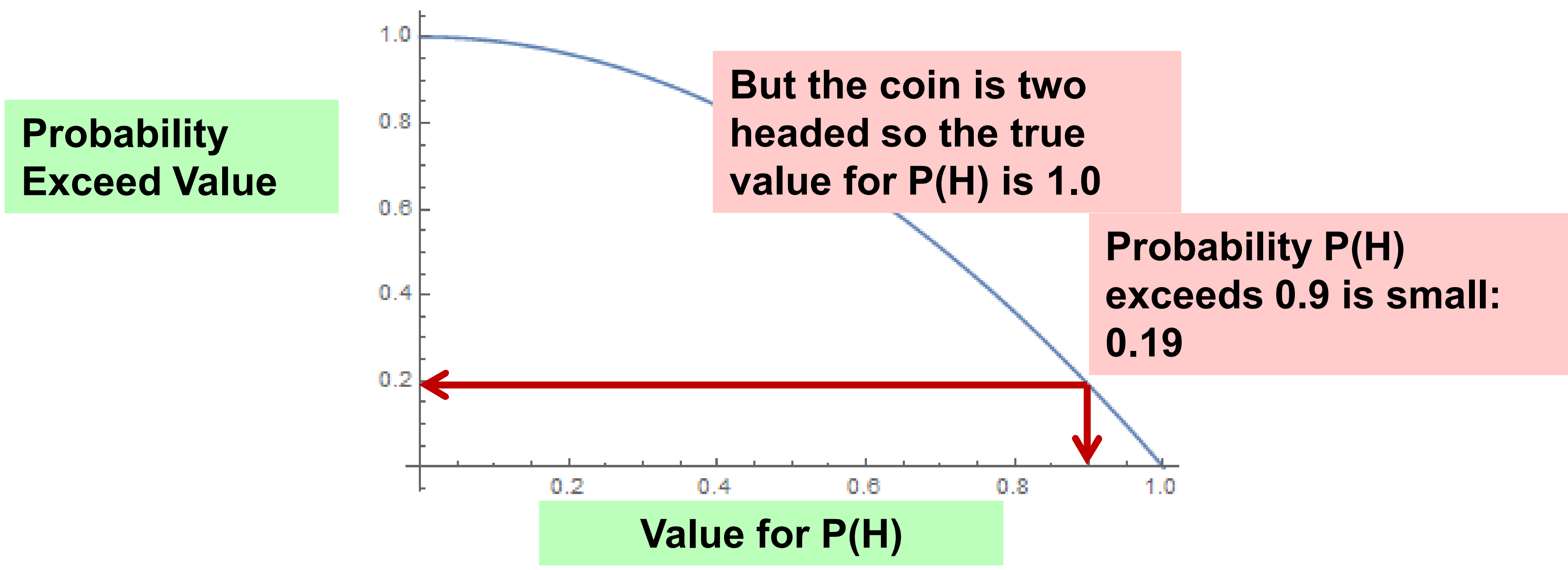
Bayesian:
Example where Will Not Help continued

- **Posterior CCDF**



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Bayesian: Example where Will Not Help continued

- A fault tree can have many cut sets and each cut set can have more than one basic event
  - If our posterior for all basic events in the dominant cut sets is based on little information our answer can be way off
  - For example, assume a cut set has two correlated events each with P(H): value of cut set is $P(H)^2$

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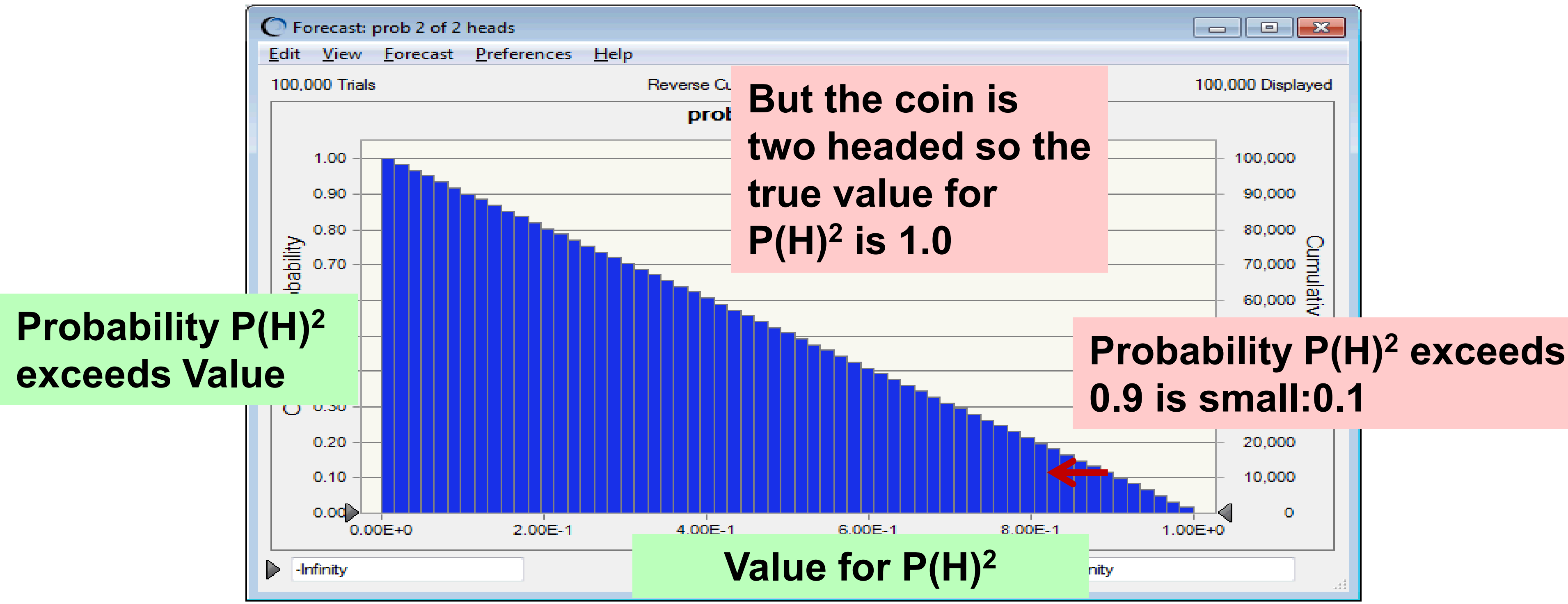
Bayesian: Example where Will Not Help continued

- **Probability 2 of 2 tosses are heads**
  - $P(H)^2$



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Bayesian: Prior With Little Information Gives Posterior that can lead to Misleading Conclusion

- **The prior updated with little information is misleading**

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If Bayesian Cannot Help

- Use techniques that better capture state of knowledge uncertainty
- Belief/Plausibility is such a technique
  - Subject of part 3 of this course
  - We will address this example using belief/plausibility in part 3

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Bayesian Update Insights

- An inaccurate prior *may* be acceptable if extensive information is available for update to posterior

Unless prior totally excludes values that can occur

- A good prior is required if extensive information for update is not available

- A poor prior w[...] can produce [...]

Bayesian approach cannot help you in this situation.

Consider using a measure for uncertainty that is broader than probability, such as belief/plausibility. Belief/plausibility measure of uncertainty is discussed in part 3 of this course.

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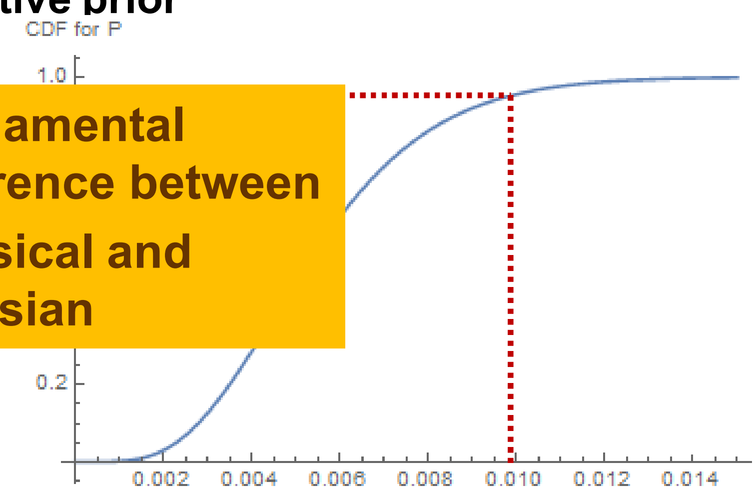
Bayesian and Classical Statistical Inference

- **Both give essentially same answer consistent with the information available**
- **Binomial distribution, large population**
  sample 1000 and observe 5 failures, mean is 0.005
  - **Classical statistical inference 95% (0.05 α) one sided upper confidence level (UCL) for p is 0.010. 95% confidence P in [0, 0.010] (See part 1)**

$$UCL(x)=\frac{(x+1)F_{1-\alpha}(2x+2,2n-2x)}{(n-x)+(x+1)F_{1-\alpha}(2x+2,2n-2x)}$$

UCL is NOT a percentile
p is fixed but unknown

  - **Bayesian with Jeffrey's noninformative prior**
    produces posterior beta(5.5, 995.5)
    mean is 0.0055
    95% percentile is about 0.010



Fundamental difference between Classical and Bayesian

p is a specific value of the random variable P

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Bayesian with Exponential Distribution

- Time to failure T is random variable with PDF

  $\lambda e^{-\lambda t}$ where t is a specific value of T. The parameter λ is the failure rate

  CDF: $Prob(T \le t) = 1 - e^{-\lambda t}$
- Treat λ as a specific value of a random variable Λ
- The gamma distribution is a conjugate prior for Λ
  - Gamma distribution has two parameters $\Gamma(\alpha, \beta)$
    - $PDF(\lambda) = 1/(\beta^{\alpha}\, \Gamma(\alpha)) * \lambda^{\alpha-1} * \exp(-\lambda/\beta)$
      - β has units of 1/time
  - Some developments use $1/\beta$ for the second parameter (NUREG/CR-6823)
    - $PDF(\lambda) = \beta^{\alpha}/\Gamma(\alpha) * \lambda^{\alpha-1} * \exp(-\lambda\beta)$
      - β has units of time
  - We use β with units of 1/time, same as Mathematica

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Exponential Distribution: Gamma Conjugate Priors for Parameter λ

- If prior is gamma(α_0, β_0) mean is $\alpha_0\beta_0$

- Observe total of x failures over a fixed time t
  - Each item that is tested is replaced if it fails before time t
    - See Martz and Waller section 7.1.3 Poisson Sampling

- Posterior is gamma($x + \alpha_0$, $\beta_0 / (\beta_0 t + 1)$) mean is $\beta_0(x + \alpha_0) / (\beta_0 t + 1)$

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Exponential Distribution: Gamma Conjugate Prior With No Information

- **Jeffreys noninformative: gamma(½, 1/0)**

Cannot plot PDF.
An improper distribution: $\beta = \infty$
see NUREG/CR-6823

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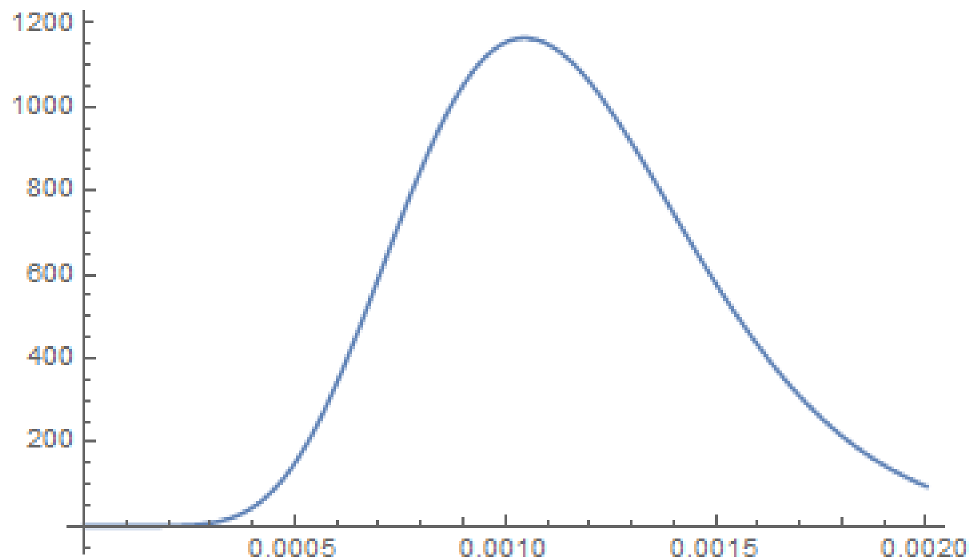
Bayesian Update for λ

- No information before doing tests:
  Use Jeffreys noninformative prior
  - gamma[½, 1/0]
- Observe x failures in time t
  - beta[x + ½, 1/t] is posterior, Mean is (x + ½)/t



With x of 10, t of 9038 PDF for posterior:
mean 0.0011

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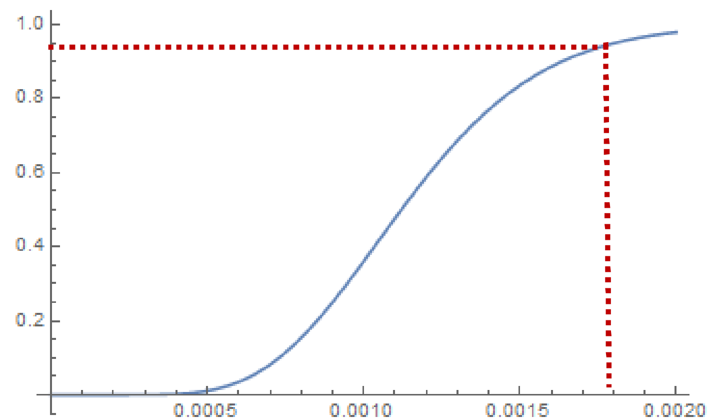
Bayesian and Classical Statistical Inference

- **Both give essentially same answer consistent with the information available**
- **Exponential distribution**
  observe 10 failures over total time 9083, mean is 0.0011
  - **Classical statistical inference 95% (0.05 α) one sided upper confidence level (UCL) for λ is 0.0017 (95% confidence λ in [0, 0.0017] (See part 1)**

$$UCL(\lambda) = \frac{\chi^2_{1-\alpha}(2n)}{2t}$$

UCL is NOT a percentile
λ is fixed but unknown

  - **Bayesian with Jeffrey's noninformative prior**
    produces posterior gamma(10.5, 1/9083)
    mean is 0.0012
    95% percentile is about 0.0018



λ is a specific value of the random variable Λ

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References for Bayesian Approach

- Bayesian Reliability Analysis, H. F. Martz, R. A. Waller, Wiley, 1982
- “Handbook of Parameter Estimation for Probabilistic Risk Assessment”, NUREG/CR-6823, SAND2003-3348P, Sept 2003
- “On the Quantitative Definition of Risk”, S. Kaplan, B. J. Garrick, Risk Analysis, Vol. 1, No. 1, 1981
- “PRA Procedures Guide”, NUREG/CR-2300, US NRC, Jan 1983
- Mathematica Version 10.1, Wolfram Research, Inc.
- SAPHIRE Version 8.1, INL for US NRC
- Crystal Ball software, Version 11.1.2, Oracle.
- “Data Analysis for Scientists and Engineers”, Stuart L. Meyer, Wiley, 1975.

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Part 2 Simple Examples: Day 2

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Simple Example 2-1

"Let's Make a Deal"

Three doors A_1, A_2, A_3 prize behind one door

Monte Hall knows which door has prize

You pick A_1 $P(A_1) = \frac{1}{3}$ $P(A_2) = 1/3$ $P(A_3) = 1/3$ are the priors

If prize in A_1 he opens A_2 or A_3

If prize in A_2 he opens A_3

If prize in A_3 he opens A_2

He opens one of the two doors A_2 and A_3 – that does not have the prize – and asks if you wish to change the door you selected? Should you change?

Suppose he opens A_2

$$P(A_1 \mid \text{open } A_2) = \frac{P(A_1)\,P(\text{open } A_2 \mid A_1)}{P(A_1)\,P(\text{open } A_2 \mid A_1) + P(A_2)\,P(\text{open } A_2 \mid A_2) + P(A_3)\,P(\text{open } A_2 \mid A_3)}$$

$$P(A_2 \mid \text{open } A_2) = \frac{P(A_2)\,P(\text{open } A_2 \mid A_2)}{\sim}$$

$$P(A_3 \mid \text{open } A_2) = \frac{P(A_3)\,P(\text{open } A_2 \mid A_3)}{\sim}$$

$P(\text{open } A_2 \mid A_1) = 0.5$ $P(\text{open } A_2 \mid A_2) = 0$ $P(\text{open } A_2 \mid A_3) = 1.0$

He will not open the door with the prize

$$P(A_1 \mid \text{open } A_2) = \frac{(1/3)(1/2)}{(1/2)(\frac{1}{3}) + (\frac{1}{3})(0) + (\frac{1}{3})(1)} = \frac{1/6}{1/2} = 1/3$$

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Simple Example 2-1 continued

2-1 continued

$$p(A_2 \mid \text{open } A_2) = \frac{(1/3)(0)}{1/2} = 0$$

$$p(A_3 \mid \text{open } A_2) = \frac{(1/3)(1)}{1/2} = 2/3$$

You selected A_1 but $p(A_3 \mid \text{open } A_2) = 2/3$ while $p(A_1 \mid \text{open } A_2) = 1/3$

So you should change your selection to A_3

Suppose he open A_3

$$p(A_1 \mid \text{open } A_3) = \frac{p(A_1)\,p(\text{open } A_3 \mid A_1)}{p(A_1)\,p(\text{open } A_3 \mid A_1) + p(A_2)\,p(\text{open } A_3 \mid A_2) + p(A_3)\,p(\text{open } A_3 \mid A_3)}$$

$$p(A_2 \mid \text{open } A_3) = \frac{p(A_2)\,p(\text{open } A_3 \mid A_2)}{\sim}$$

$$p(A_3 \mid \text{open } A_3) = \frac{p(A_3)\,p(\text{open } A_3 \mid A_3)}{\sim}$$

$p(\text{open } A_3 \mid A_1) = 0.5$ $\quad p(\text{open } A_3 \mid A_2) = 1.0$ $\quad p(\text{open } A_3 \mid A_3) = 0$

He will not open the door with the prize

$$p(A_1 \mid \text{open } A_3) = \frac{(1/3)(\frac{1}{2})}{(1/3)(\frac{1}{2}) + (1/3)(1) + (1/3)(0)} = \frac{1/6}{1/2} = 1/3$$

$$p(A_2 \mid \text{open } A_3) = \frac{(1/3)(1)}{1/2} = 2/3$$

$$p(A_3 \mid \text{open } A_3) = (1/3)(0)/(1/2) = 0$$

You selected A_1 but should change to A_2

You should always change the door you select

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Example 2-2

- Consider OR logic in a fault tree for probability of failure of A or B: P(A U B)
- Assume independence
- P(A U B) = P(A) + P(B) – P(A)*P(B)
- In part 1 we assumed P(A) and P(B) were known
  - If P(A) = 0.4 and P(B) = 0.7
  - P(A U B) = 0.4 + 0.7 – 0.28 = 0.82
- Now we consider uncertainty in P(A) and P(B) using a subjective approach
  - Treat P(A) and P(B) as random variables
    - P(A) and P(B) have PDFs
    - Perhaps obtained with a Bayesian update

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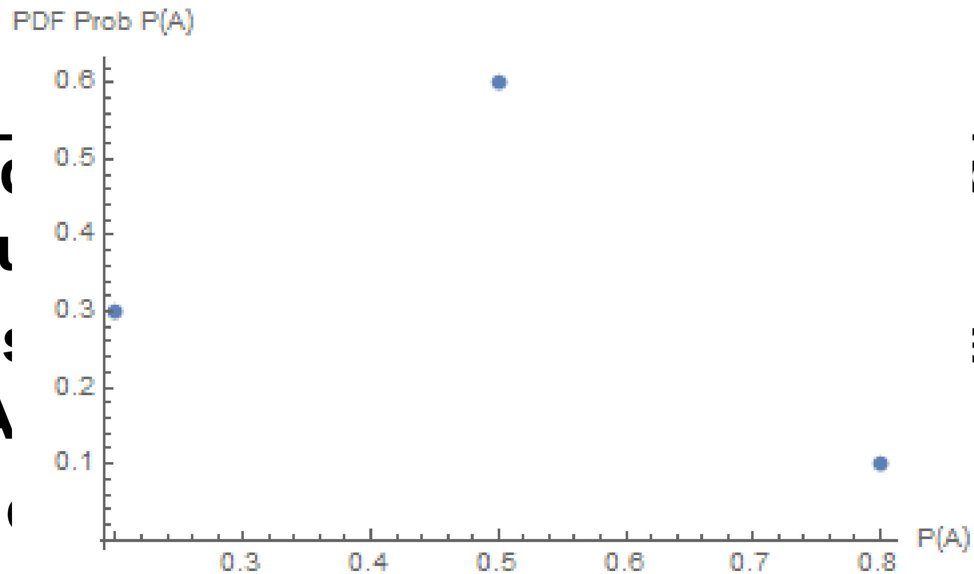
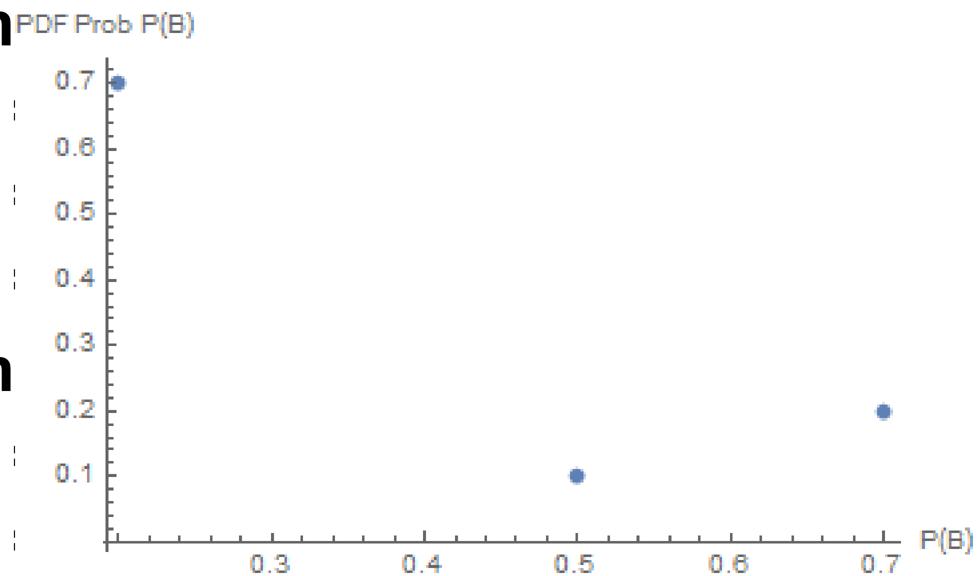
- Yesterd ... or continu
- For this ... ete PDFs for P(A
- So we
- Assum
  - P(A)
  - P(A)
  - P(A)



- Assum
  - P(B)
  - P(B)
  - P(B) = 0.7 with probability 0.2



Example 2-2 continued

P(A) and P(B) are independent

- The **value** of P(A U B) is P(A) + P(B) – P(A)*P(B)
- Each value, call it z, has a probability
  - $P(z) = \sum [P(P(A)) * P(P(B))] \mid P(A) + P(B) - P(A)*P(B) = z$

Values (objective probabilities)			Probabilities (subjective) of Values		
P(A)	P(B)	P(A) + P(B) –P(A)*P(B)	P(P(A))	P(P(B))	P(P(A)) *P(P(B))
0.2	0.2	0.36	0.3	0.7	0.21
0.2	0.5	**0.60**	0.3	0.1	**0.03**
0.2	0.7	0.76	0.3	0.2	0.06
0.5	0.2	**0.60**	0.6	0.7	**0.42**
0.5	0.5	0.75	0.6	0.1	0.06
0.5	0.7	0.85	0.6	0.2	0.12
0.8	0.2	0.84	0.1	0.7	0.07
0.8	0.5	0.9	0.1	0.1	0.01
0.8	0.7	0.94	0.1	0.2	0.02

Value 0.60 has probability 0.03 + 0.42 = 0.45

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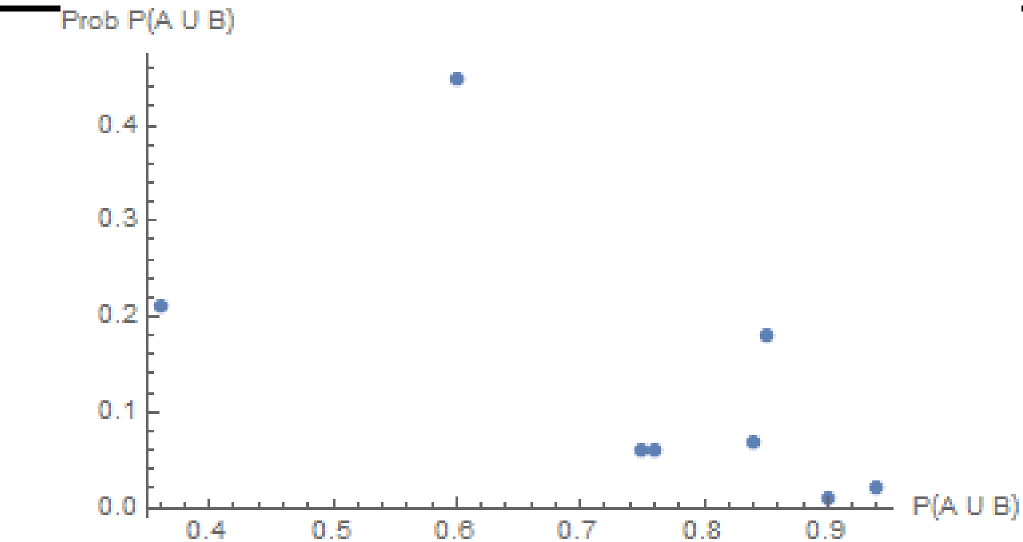
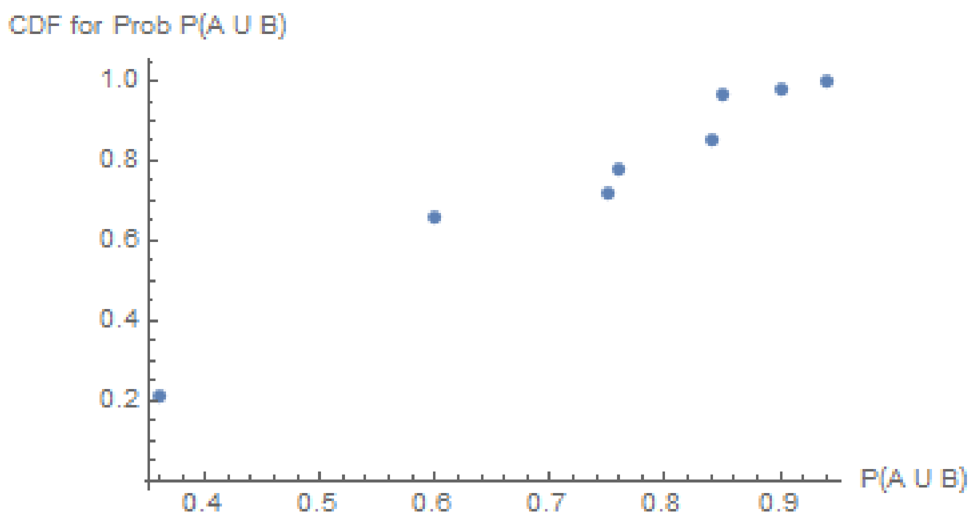
Example 2-2 continued



PDF



CDF

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Example 2-2 continued

P(A) and P(B) are dependent

- Now consider dependence
- The value of P(A U B) is $P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(B|A)*P(A)$
- Each value, call it z, has a probability
  - $P(z) = \sum [P_{joint\ PDF} (P(A), P(B))] \mid P(A) + P(B) - P(B|A)*P(A) = z$

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Example 2-2 continued

- **Example of dependence**
- **P(A U B) = P(A) + P(B) – P(A ∩ B)**
- **As before PDF for P(A) is**
  - **P(A) = 0.2 with probability 0.3**
  - **P(A) = 0.5 with probability 0.6**
  - **P(A) = 0.8 with probability 0.1**
- **As before PDF for P(B) is**
  - **P(B) = 0.2 with probability 0.7**

Dependence

If P(A) is 0.2, P(B) = 0.2 (with probability 1)
If P(A) is 0.5, P(B) = 0.2 (with probability 1)
If P(A) is 0.8, P(B) = 0.5 (with probability 1)

Values (objective probabilities)			Probabilities (subjective) of Values		
P(A)	P(B)	P(A) + P(B) –P(B|A)*P(A)	P(P(A))	P(P(B))	P_{joint}(P(A), P(B))
0.2	0.2	0.36	0.3	0.7	0.3
0.2	0.5	--- (0 joint probability)	0.3	0.1	0
0.2	0.7	--- (0 joint probability)	0.3	0.2	0
0[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.8	0.2	--- (0 joint probability)	0.1	0.7	0
0.8	0.5	0.9	0.1	0.1	0.1
0.8	0.7	--- (0 joint probability)	0.1	0.2	0

P_{joint}(P(A), P(B)) = 0.3 since P(A) of 0.2 has probability 0.3 and P(B) of 0.2 occurs with probability 1 given P(A) is 0.2

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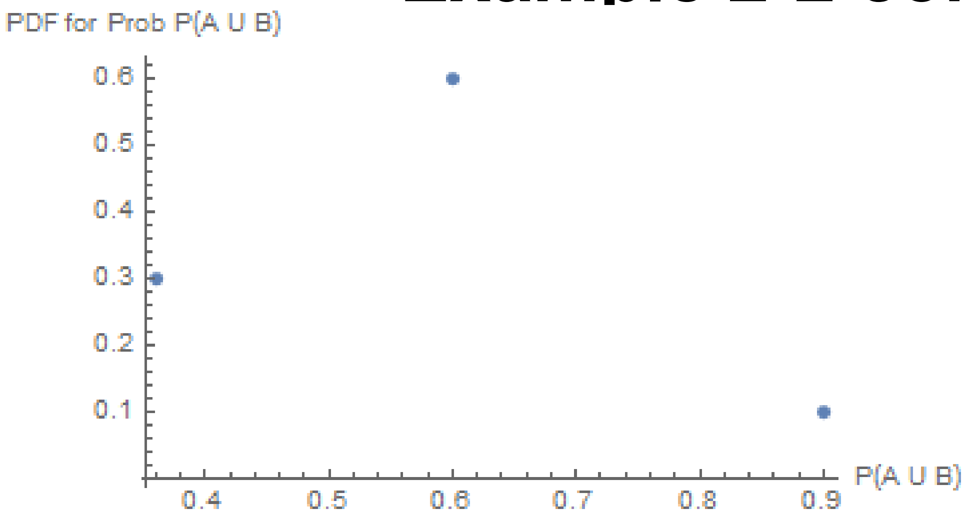
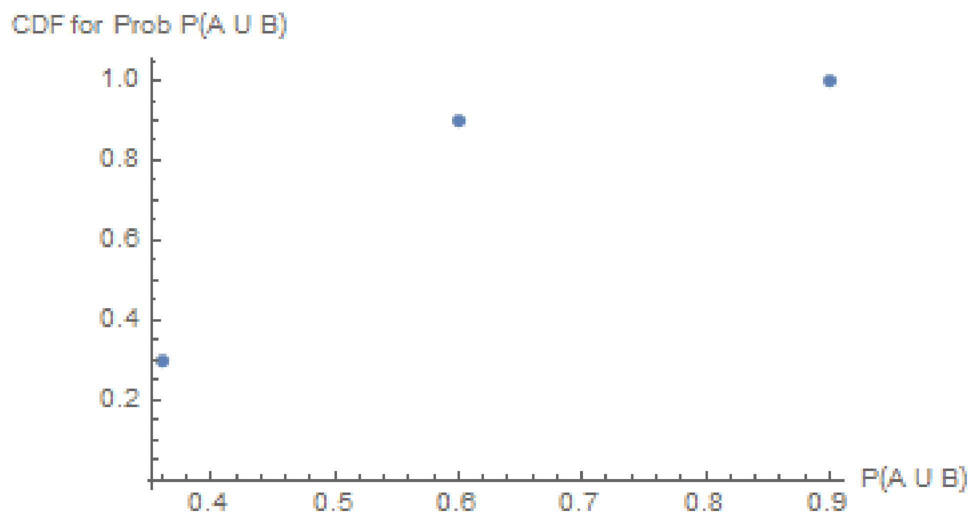
Example 2-2 continued





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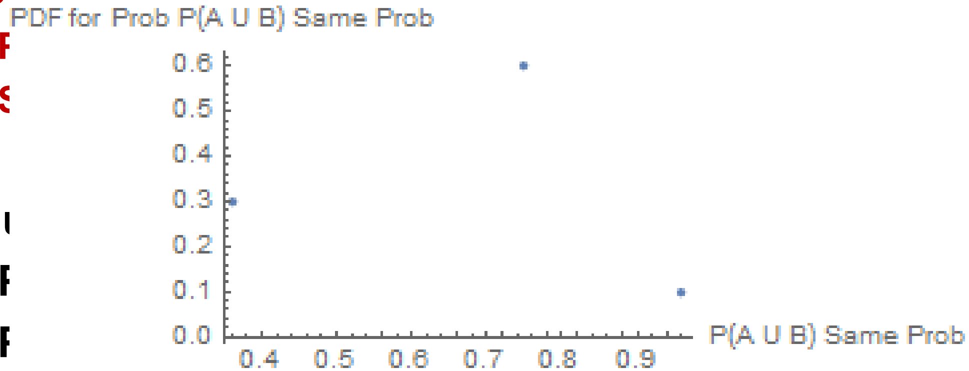
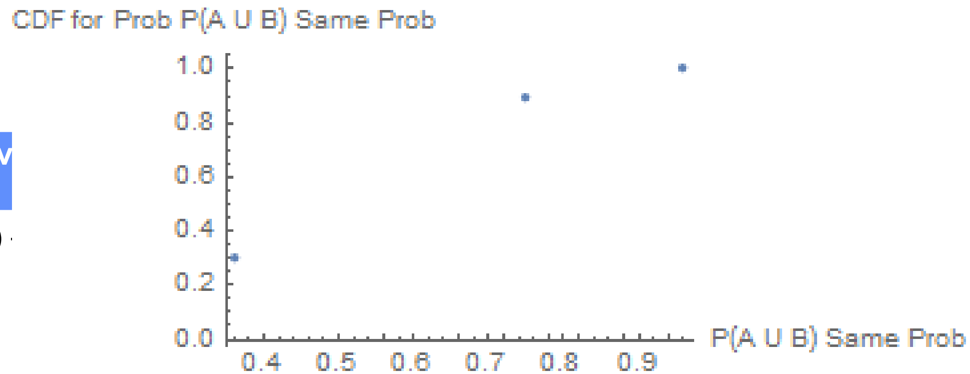
Example 2-2 continued

- Dependence where probability of failure events have same probability
  - F
  - S ... value 1.0; see part 1 lecture



- P(A
- Ass
  - F
  - F
  - P(A) = 0.8 with probability 0.1
- Same PDF for P(B)



Values (objectiv	
P(A)	2*P(A)
0.2	0.36
0.5	0.75
0.8	0.96

0.1 0.1

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Example 2-3

- The correlation we addressed assumed that the dependence is such that the two failure events have the same probability of failure, but that probability is unknown
- Let us also assume we have common cause failure
- Look at the common cause example from lecture 1 considering uncertainty in the probabilities
  - In lecture 1 we assumed the failure probabilities were known: no uncertainty

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Example 2-3 continued
Common Cause from Lecture 1

- Let A and B be failure events for two components
- Frequently, $P(A \cap B) > P(A) * P(B)$ due to dependence from common cause
- Let $A = \{A_i, C\}$ and $B = \{B_i, C\}$ where A_i and B_i are independent failures and C is failure common to A and B. Note that A_i and B_i are independent, A_i and C are mutually exclusive, and B_i and C are mutually exclusive.
- $A \cap B = (A_i \cup C) \cap (B_i \cup C) = (A_i \cap B_i) \cup (A_i \cap C) \cup (B_i \cap C) \cup C = (A_i \cap B_i) \cup C$

Boolean Reduction

mutually exclusive

- $P(A \cap B) = P(A_i \cap B_i) + P(C) - P((A_i \cap B_i) \cap C) = P(A_i) * P(B_i) + P(C)$

independent

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Example 2-3 continued Common Cause from Lecture 1

- Prob A (or B) Fail 10^{-3} with 10% common to A and B
- $P(A_i) = P(B_i) = 9 \times 10^{-4}$
- $P(C) = 10^{-4}$

- From earlier

$$P(A \cap B) = P(A_i) * P(B_i) + P(C) =$$

$$9 \times 10^{-4} * 9 \times 10^{-4} + 10^{-4} = 1.008 \times 10^{-4} \approx 10^{-4}$$

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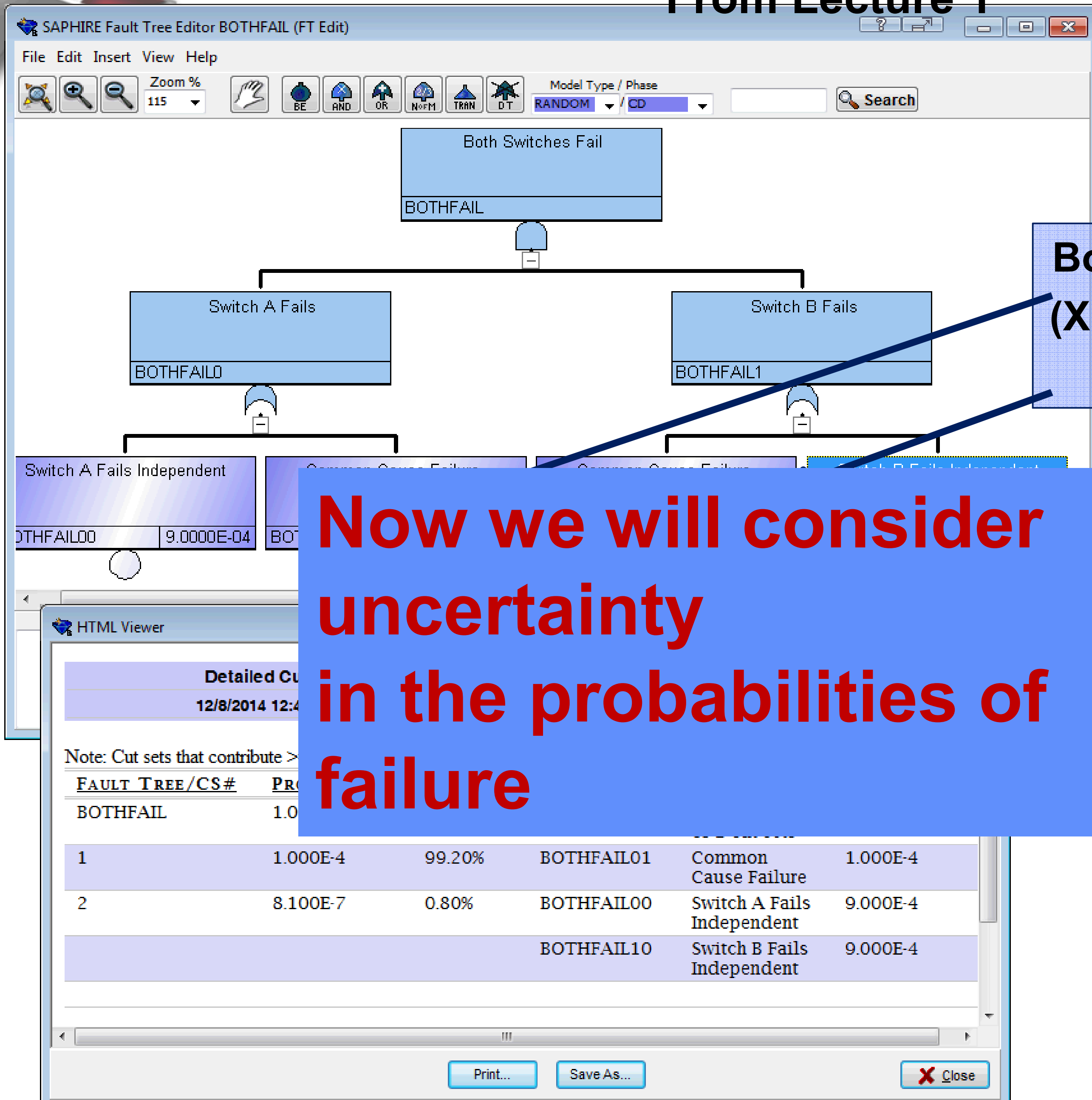
Example 2-3 continued Common Cause Explicitly in Fault Tree From Lecture 1



Boolean Reduction:
$(X \cap X) = X$

Now we will consider uncertainty in the probabilities of failure

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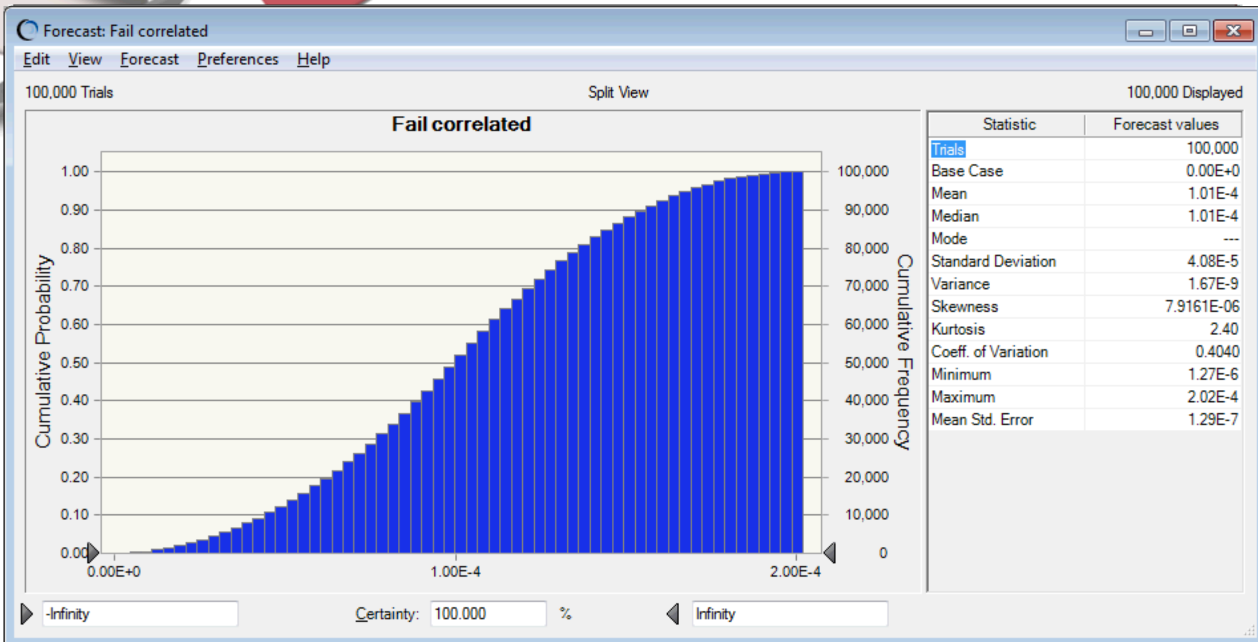
on Cause with

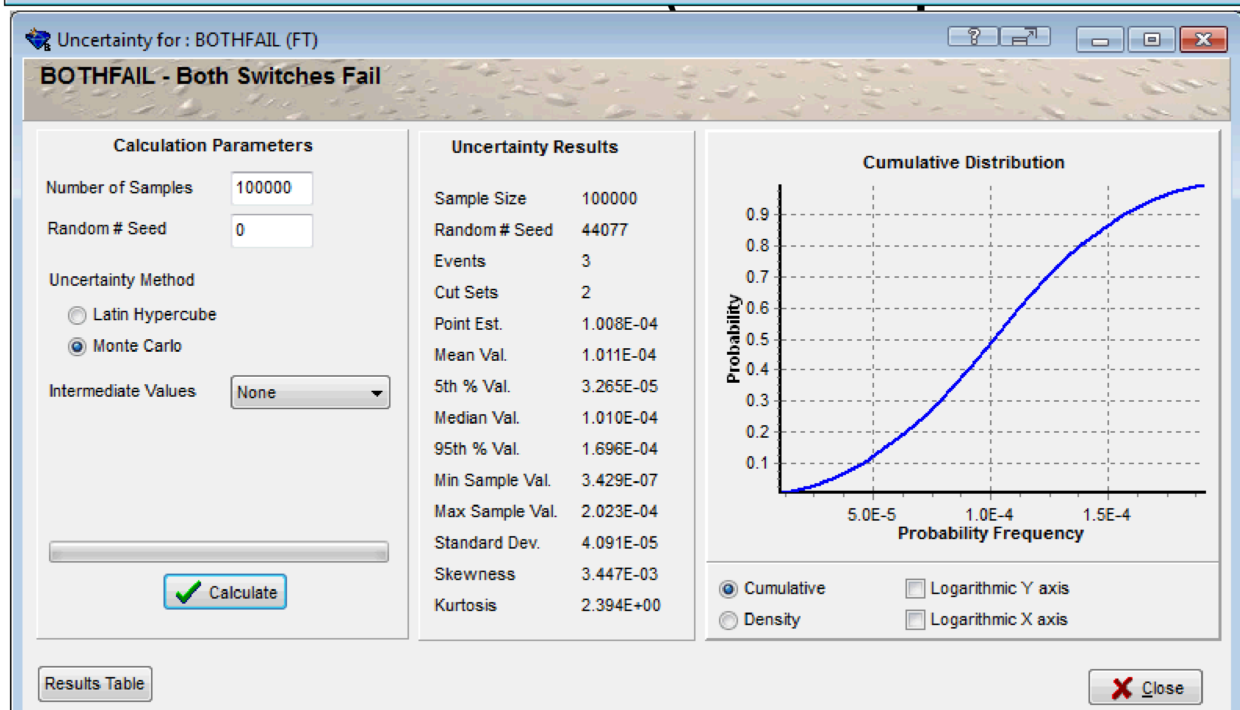
ted

ndependent

e)

e)





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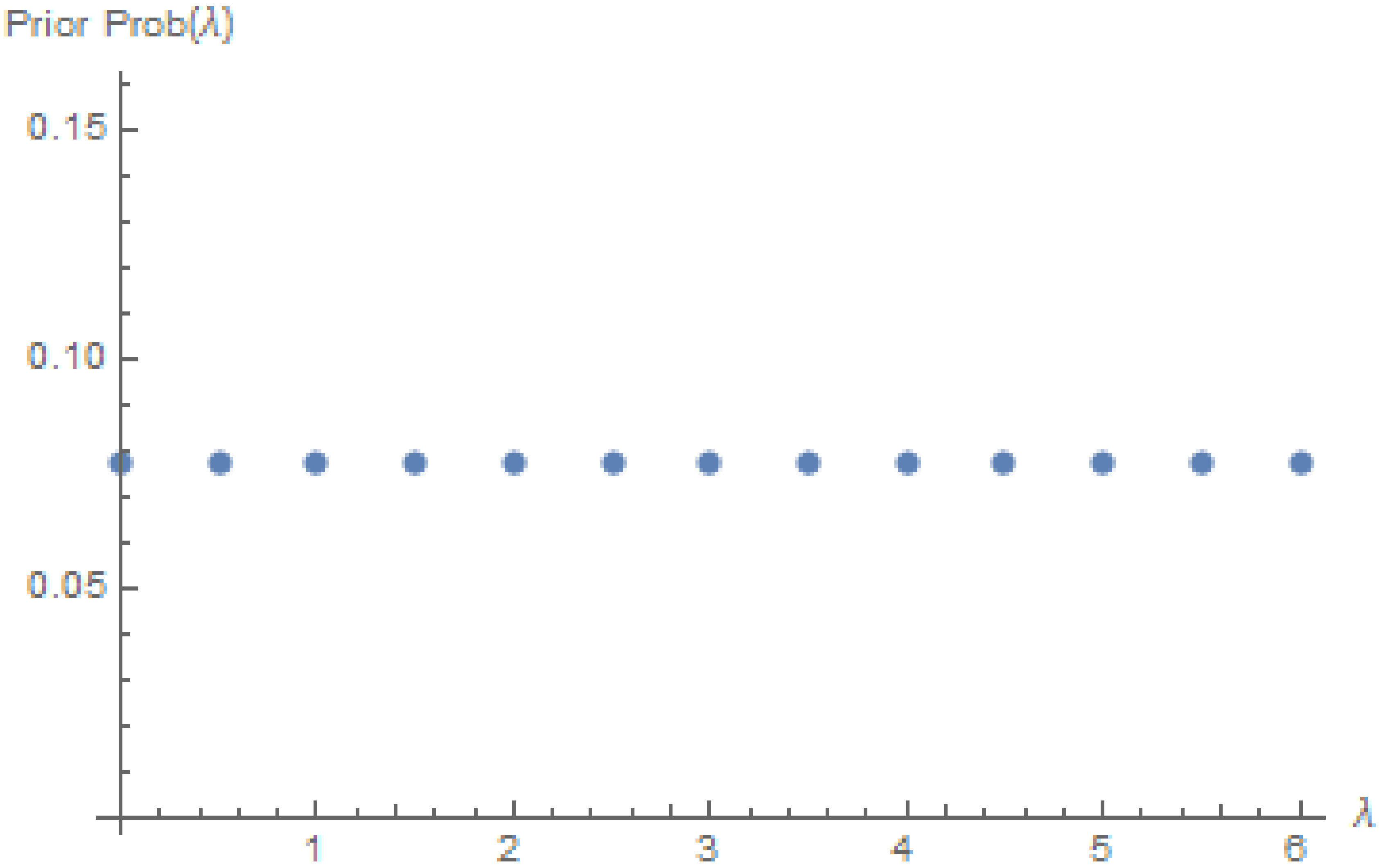
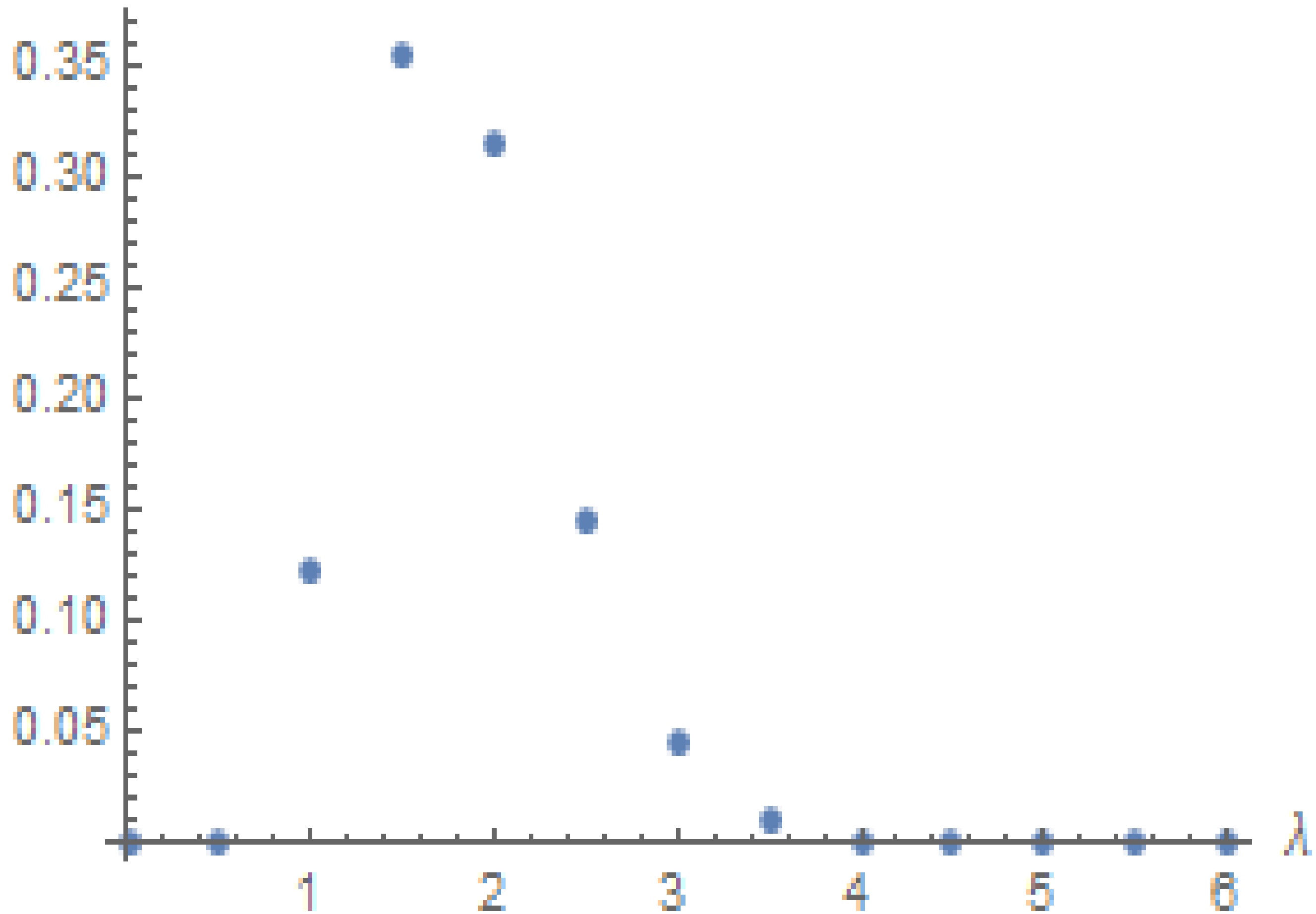
Example 2-4



Posterior PDF for λ given 10 events in 6 yrs



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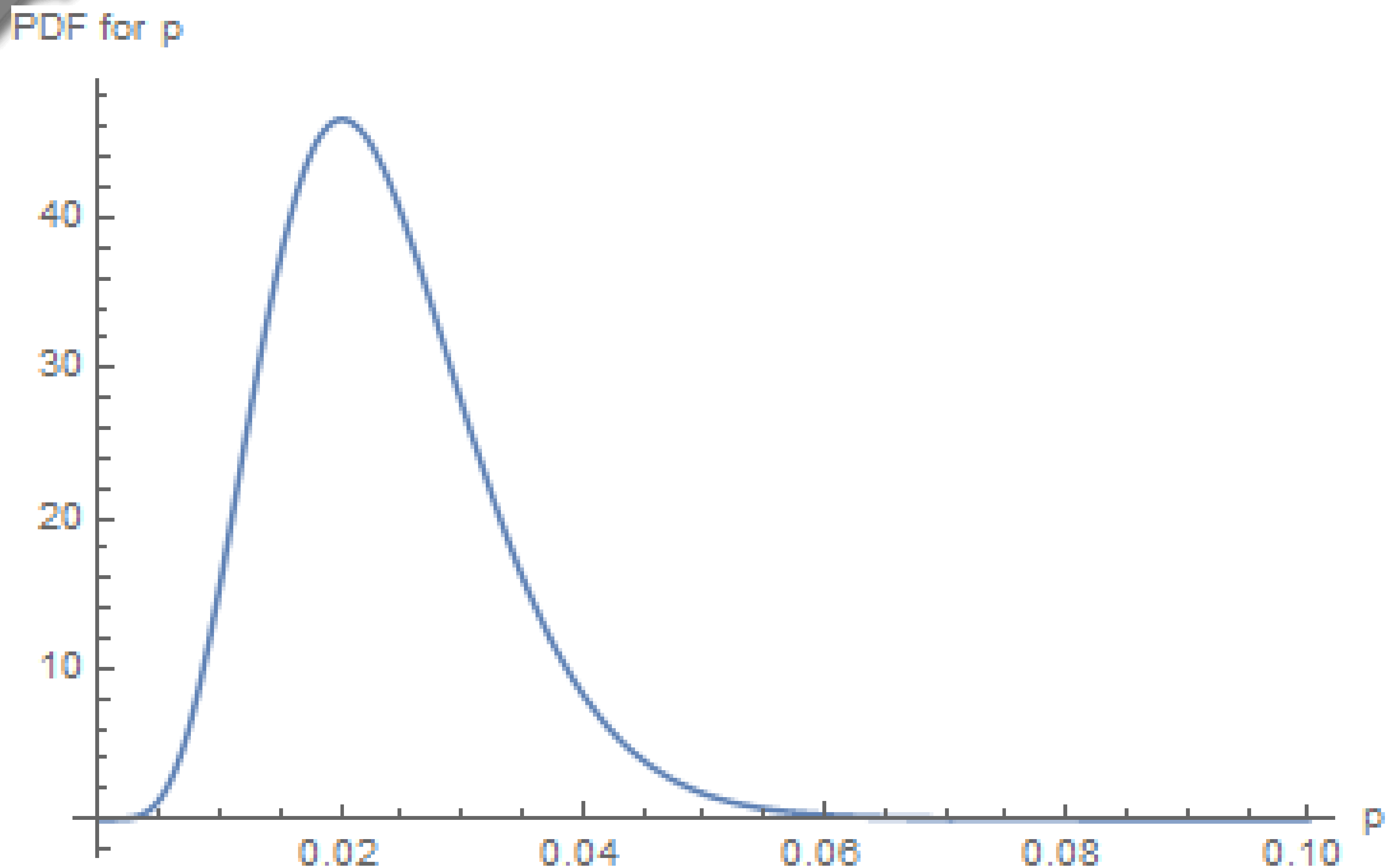

with 6 failures in 275 trials

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Part Two Real World Examples: Day 3

1. W87 new AFA analysis (with uncertainty) (classified)
2. LAC SFI (classified)

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Part 2: Homework

- **See Word Document NST_560_Part2_Problems.docx**
- **Solutions will be handed out at start of part 3**

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