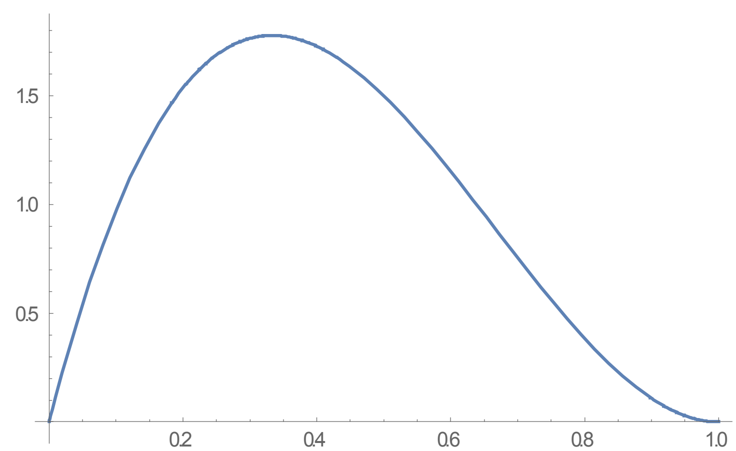
SAND2015-2278PE

Homework Problems for
NST 560:
Surety and Reliability Analysis Techniques that Estimate Uncertainty
Part 2: Bayesian Approach

2-1. The binomial distribution PDF for exactly x of n items failing is $n!/(x!(n-x)!)\ p^x\ (1-p)^{n-x}$. In a Bayesian approach, the parameter p is treated as a specific value of the random variable P. Consider the case where we are concerned with more than one component of the same type failing. Here we are concerned with 3 or more of 5 components failing. Assume the components have the same probability of failure.

(a) Using the binomial distribution, what is the probability that 3 or more of 5 components fail?
(b) If p is very small what is a simple approximation tor the probability that 3 or more of 5 components fail?
(c) If p is very small what is a simple approximation for exactly 2 of 5 components failing?

There is uncertainty in p and we consider it by treating p as the specific value of a random variable P. To be specific, using a Bayesian update process, we specify beta(2,3) as the PDF for P:[1]



Note: p is not small using this PDF.

(d) Since p has uncertainty, describe how you would solve the equation for part (a) with this probability distribution assigned to P?
(e) In your approach for part (d), how do you ensure that you use the same probability of failure for each component?
(f) Are the component failures independent?

[1] Here we use the shape parameters α,β as the parameters for the beta distribution, as discussed in the lecture.

2-2. We are concerned with the failure of two different components; A and B are these failure events. We have uncertainty in the probability of failure of each component and- using a Bayesian approach- we choose to treat the probabilities of failure as random variables. Let P_A denote the random variable for the probability of failure of the first component. Let P_B denote the random variable for the probability of failure of the second component.

Based on our state of knowledge, we assign the following discrete PDFs for the probabilities of failure.

P_A of 0.2 has probability 0.2.
P_A of 0.3 has probability 0.1.
P_A of 0.4 has probability 0.7.

P_B of 0.1 has probability 0.1.
P_B of 0.6 has probability 0.8.
P_B of 0.8 has probability 0.1.

(a) Draw the PDFs for P_A and P_B.
(b) It is possible that failures of A and B are mutually exclusive?

Assume that P_A and P_B are independent.

(c) What is the PDF for failure of either component, P(A U B)?
(d) Draw the PDF for P(A U B).
(e) Draw the CCDF for P(A U B).
(f) What is the expected value (mean) of P(A U B)?
(g) Does it bother you that the mean for P(A U B) is not an allowed value for P(A U B)?
(h) What is the probability that P(A U B) exceeds 0.50 ?

Assume P_B is dependent on P_A such that:
when P_A is 0.2 P_B is 0.6
when P_A is 0.3 P_B is 0.1
when P_A is 0.4 P_B is 0.8
and the PDF for P_A is as before.

(i) What is the PDF for failure of either component, P(A U B)?
(j) Draw the PDF for P(A U B).
(k) Draw the CCDF for P(A U B).
(l) What is the expected value (mean) of P(A U B)?
(m) What is the probability that P(A U B) exceeds 0.50?

2-3. Consider two components. The two components have the same probability of failure P, but P has uncertainty. Using a Bayesian approach we consider P as a random variable with a uniform probability density function (PDF) over [0, 0.5]. Let P_2 denote the probability of the failure of both components, and assume it is the following function of P: $P_2 = P^2$.

In general, for $Y = X^2$, where X is a random variable, it can be shown that the PDF for Y (as a function of X) is:[2]

$$PDF_Y(y) = \frac{1}{2\sqrt{y}}(PDF_X(\sqrt{y} + PDF_X(-\sqrt{y})) \ for\ y > 0$$

(a) Draw the CDF for P.
(b) The PDF for P_2 is infinite at the value p_2 of zero. An associate claims this cannot be the PDF for P_2, since the PDF for P_2 at a specific value is the probability P_2 is equal to that value, and this PDF is infinite at a value zero. Is the associate correct? Hint: review the definition of a PDF for discrete and continuous variables from part 1.
(c) Draw the PDF for P_2.
(d) Draw the CDF for P_2.
(e) Are the component failures probabilities independent?

2-4. Consider a component that must remain operating for a certain time T. Failure of the component can be modeled with the exponential distribution. The probability the component fails to operate during T is $1 - e^{-\lambda T}$ where λ is the failure rate. For λT small, this probability can be approximated as λT. In a Bayesian sense, λ is assumed to be a specific value of a random variable.

In general , for Y = aX + b, where X is a random variable, and a and b are constants, it can be shown that the PDF for Y (as a function of X) is:[2]

$$PDF_Y(y) = \frac{1}{a} PDF_X\left(\frac{y-b}{a}\right)$$

Assume we model the uncertainty in λ with a uniform distribution over [0, 0.001] and are concerned with failure within 24 hours. Use the approximation that the probability of failure, P, is λ*24.

(a) What is the $PDF_P(10^{-5})$?
(b) Draw the CDF for P.

2-5. A colleague proposes to use a subjective approach for the analysis of uncertainty. The proposal is to use a Bayesian approach to produce PDFs for the probability of each outcome in the sample space, and consider the PDFs as independent when combining outcomes for the probabilities of events containing more than one outcome.

[2] For example see the book by Stuart Meyer "Data Analysis for Scientists and Engineers" referenced in the lecture for how to calculate PDF for functions of random variables.

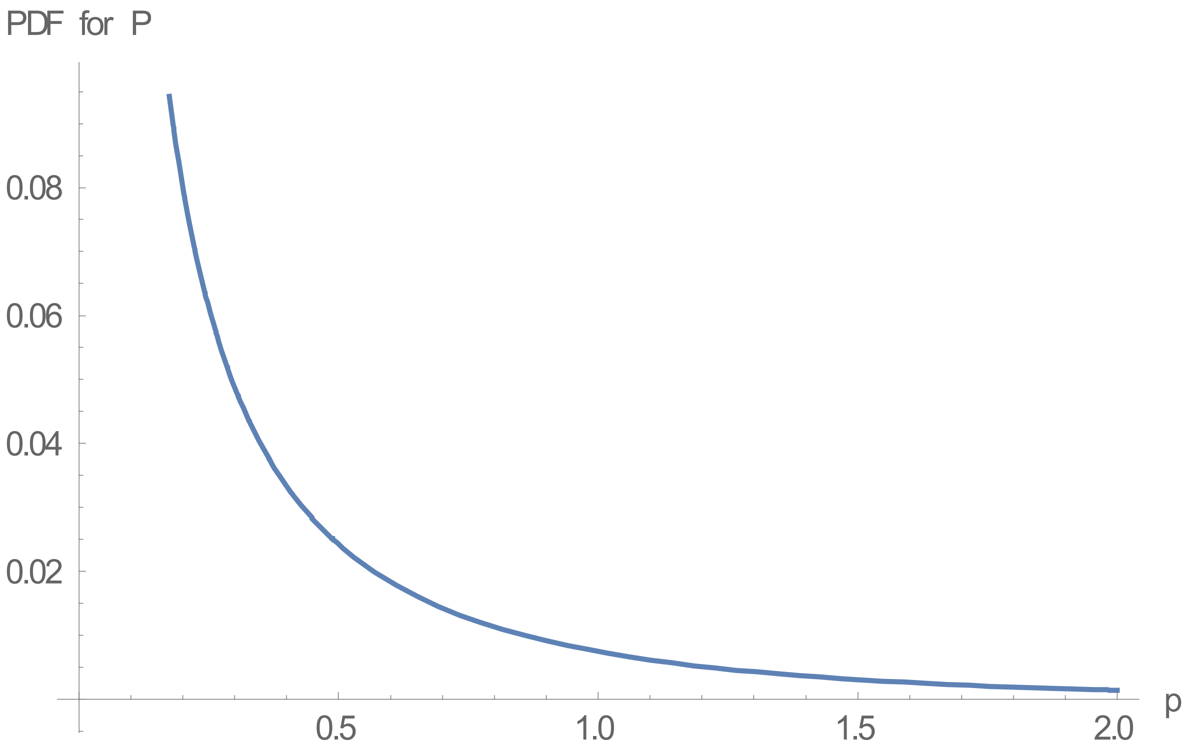
For example, if the sample space is tossing a coin the outcomes are H (heads) and T (tails). P(H) is a random variable for the probability of H, and P(T) is a random variable for the probability of T. Using the approach proposed, one PDF would be produced for P(H) and another independent PDF for P(T).

What is the problem with this approach?

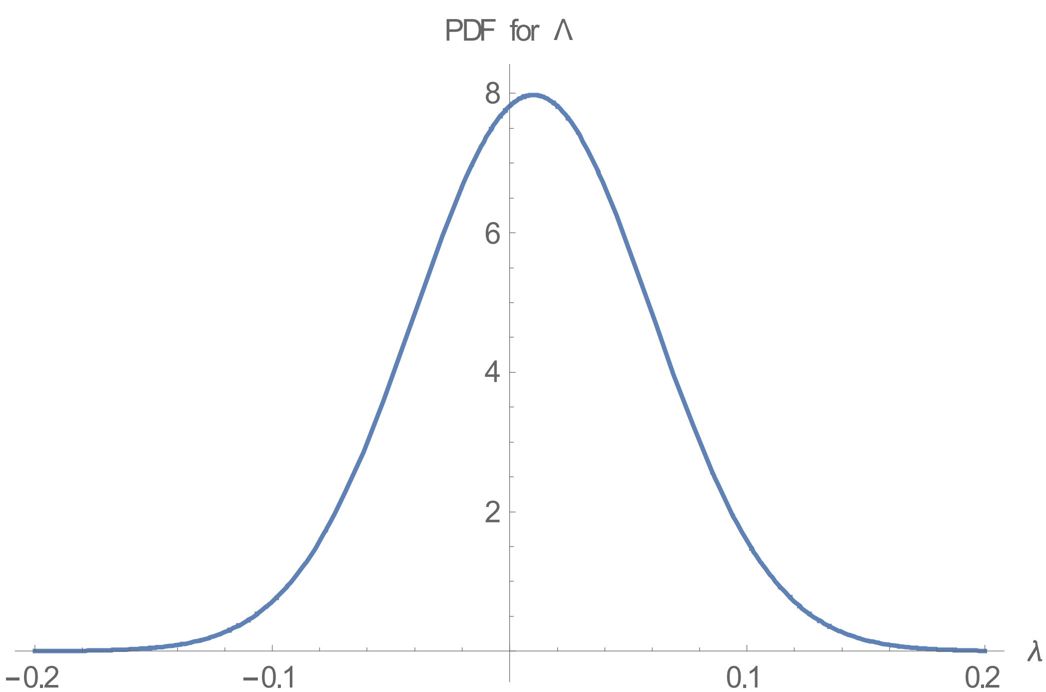
2-6. In a simple example discussed in the lecture we applied a Bayesian approach to selecting one of three doors, one of which has prize behind it. We performed a Bayesian update based on information obtained by opening one door and finding no prize behind that door, for two different cases: (1) you opened the door and (2) Monte Hall opened the door. For the two cases, the results are different for the posterior probabilities that the prize is behind each of the un-opened doors. Why are the results for the posterior probabilities different for the two cases?

2-7. In a simple example discussed in the lecture, we considered the Poisson distribution with the parameter λ, the number of events per unit time. We assumed a prior distribution for λ that was uniform (and discrete) over [0, 6]. We updated the prior given 10 events observed in 6 years. Suppose that instead of observing 10 events in 6 years, we observed 100 events in 6 years. With this information, what do you think of the appropriateness of the prior? If you decide to use a different prior, what might it be?

2-8. The following is proposed as a prior for the probability parameter, p, parameter of the binomial distribution. Why is this prior unacceptable?



The following is proposed as a prior for the failure rate parameter , λ, of the exponential distribution. Why is this prior unacceptable?



2-9. philosophically, how do you feel about the concept of subjective probability and the Bayesian approach?