# Statistical distributions from lens manufacturing data

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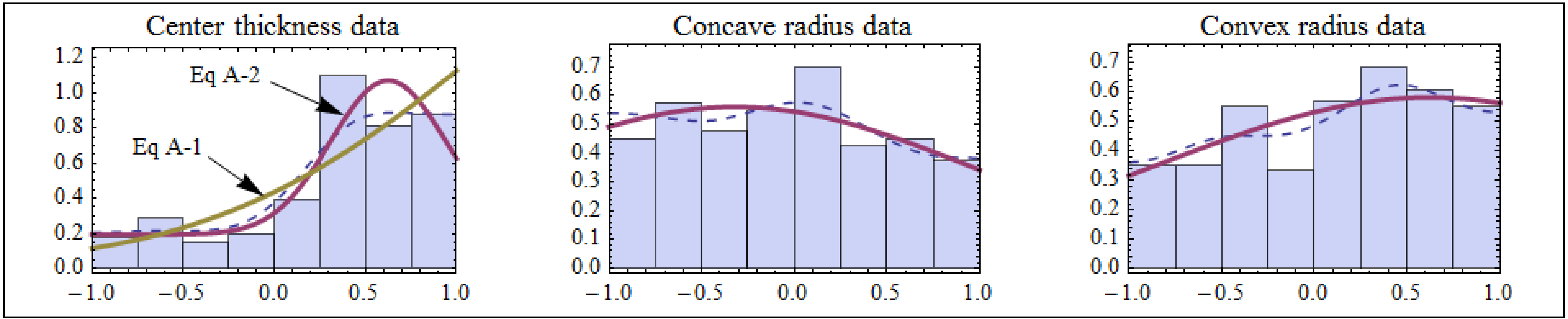
# Agenda

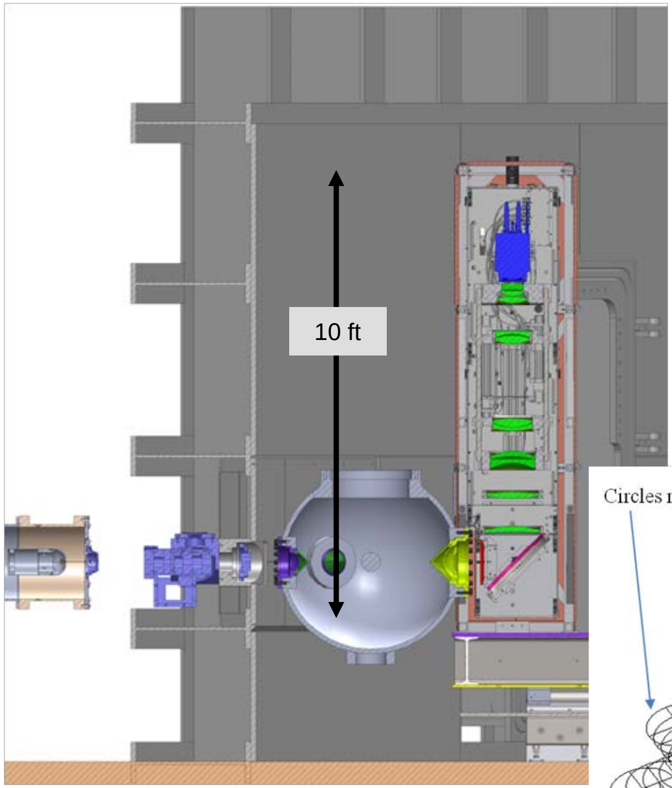
- Motivation for study: Cygnus Zoom Lens
- Statistical tools: Exploratory data analysis, mixture distributions, truncated distributions, etc.
- Data from Optimax
- How to analyze batch data
- How to analyze aggregate data
- **How to use the resultant parameterized aggregate distributions for lens tolerancing**



# Definitions

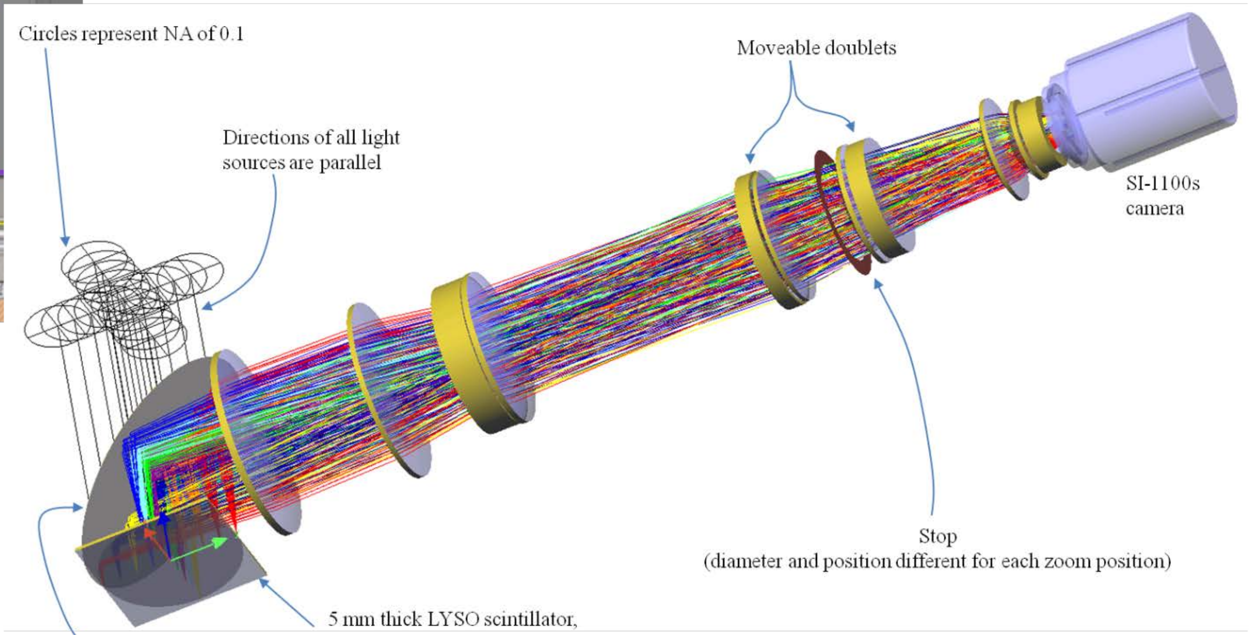
| | |
|---|---|
| Batch | An order of nominally identical lens elements. |
| Aggregate | Distribution related to a class of tolerance of all the lens elements. |
| Mixture | Summation of component distributions (used for bimodal data). |
| Bounded | Distributions only defined on an interval. The interval is defined by the tolerance value T, and then normalized to either (+1, –1) or (0, 1). |
| Truncation | Cut the distribution outside interval and increase uniformly to inside<br>– let f(x) be an un-truncated PDF distribution,<br>– let F(x) be the CDF of f(x), and<br>– let the truncation interval be ($X_{min}$, $X_{max}$).<br><br>The truncated distribution is $f( x )/(F( X_{max}) – F( X_{min}))$ |

# Motivation for this study: Cygnus Zoom Lens




The lenses in this system are quite large. Because of cost and supply chain problems (Japanese earthquake for example), the tolerances were very loose, to be re-optimized later. But systematic deviations between the nominal design value of center thickness and as-built values created problems for us.

Cygnus telecentric zoom lens, position #1

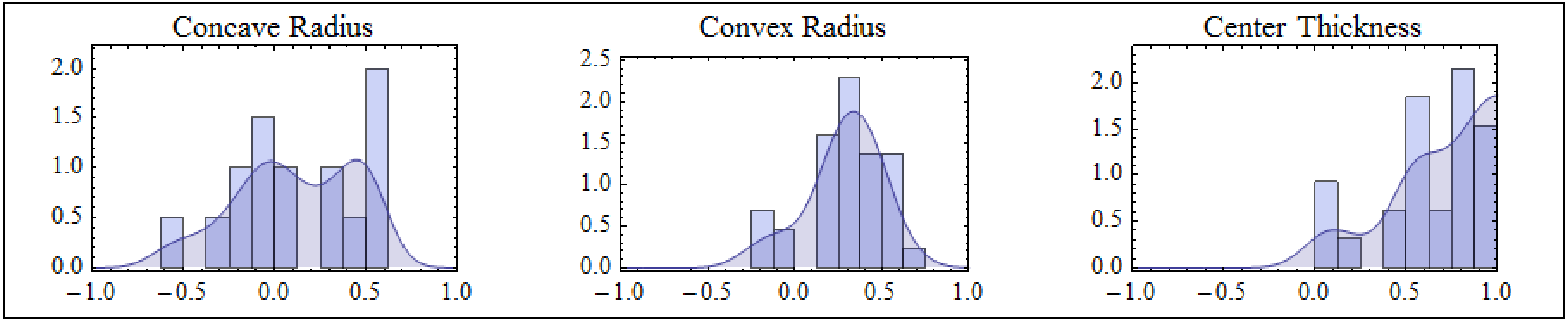


# Data analysis for 3 systems

We made 3 systems. The optics were made by Cosmo. I did some data analysis on the as-built numbers, and noticed some curious trends, particularly with center thickness.

In this case all the lens data is mixed together. The radius error was given in fringes. **All data was normalized from ±T to ±1**, T being the tolerance value. I called Brandon at Optimax; he was quite fascinated and told me that the data seemed fairly typical to him. He explained why center thickness always comes in fat.

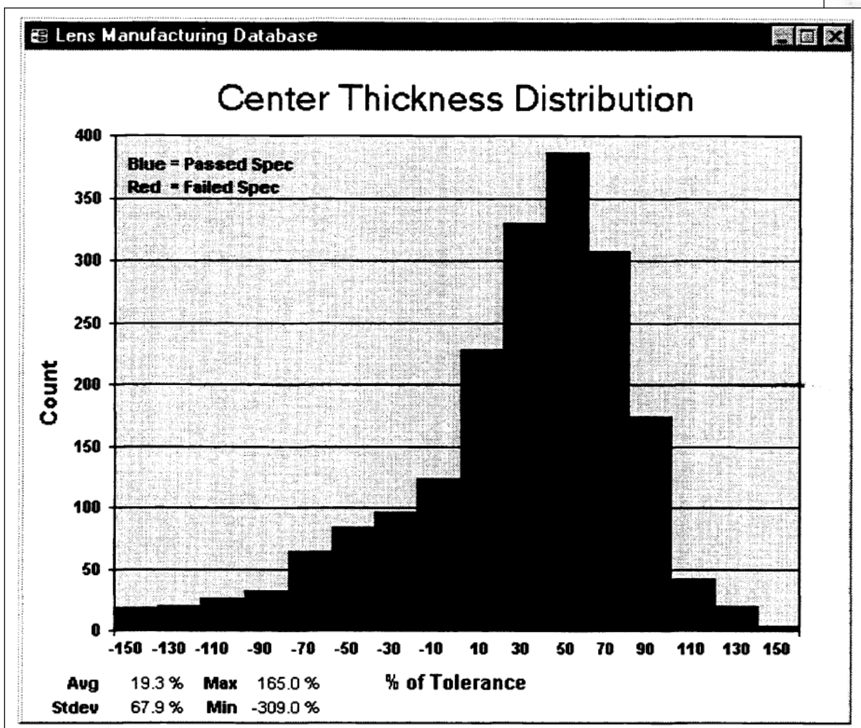
Zoom Lens Data

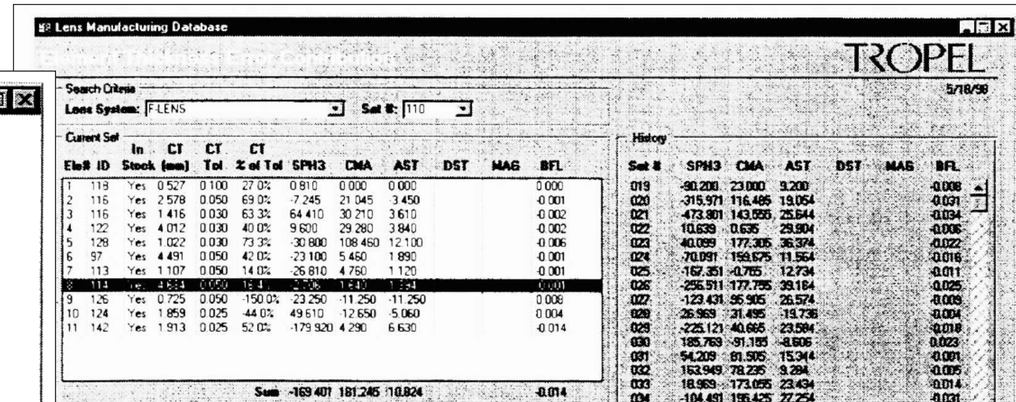



***"Little problems with little lenses turn into big problems with big lenses."***

# Prior work

Companies such as Corning and Raytheon maintain databases of fabrication data, but their data and methods and proprietary. We don't know of any publicly available fabrication statistics.




**Modern lens design using a lens manufacturing database**

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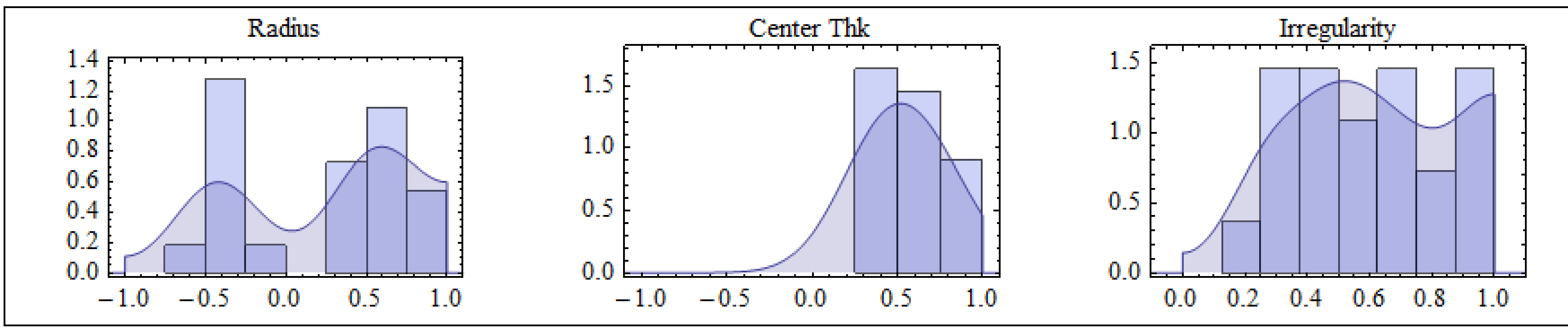
Tropel Corporation

# Example of a lens data set



Optimax made 7 batch data sets available to us for analysis. For the purpose of this discussion, a batch is a lens order consisting of between 10 and 50 lenses, 182 lenses total. Below is an example with 22 lenses. All data are normalized to (+1, -1) for center thickness and radius, (0, +1), for irregularity and wedge.




Shown above are smooth and binned histograms. The "smooth histograms" were made using the Kernel Density Estimate (KDE) method.

How do we deal with bimodality or bounded distributions?

# Nonparametric statistics, exploratory data analysis, smooth histograms and kernel density estimates

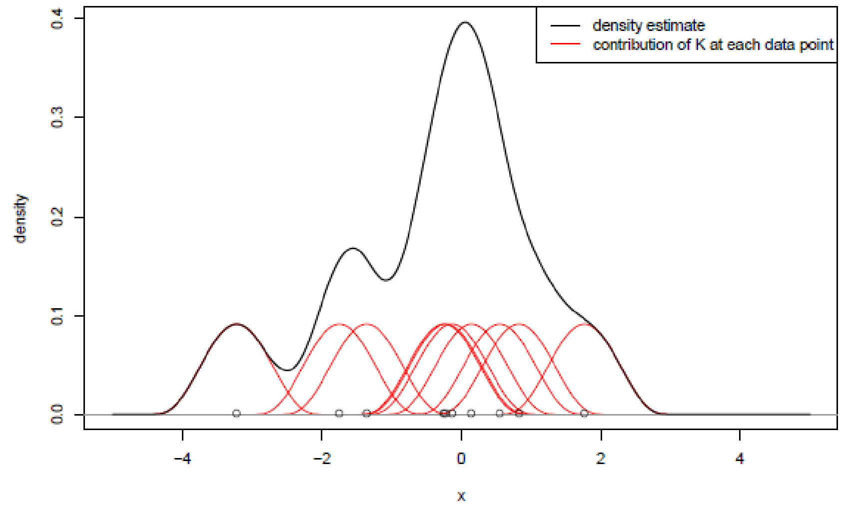



Figure 1.1: Contribution of the kernel function at each data point $X_1, \ldots, X_n$. The sum of the $n$ red curves builds the kernel density estimate (black line).

**Definition 1.2** (Standard kernel density estimator). *Let $n$ be the sample size and $K \equiv K_0$ be a kernel function of support $[-1, 1]$, degree $(0, 2)$ and symmetric around the origin. The standard kernel density estimator based on $X_1, \ldots, X_n$ is*

$$\hat{f}(x) = \int \frac{1}{h} K\left(\frac{x-y}{h}\right) d\mathbb{F}_n(y) = \frac{1}{n \cdot h} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right). \tag{1.2}$$

$$K_{\text{Gauss}}(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \text{ for } t \in \mathbb{R},$$

# Are the kernel density estimates really nonparametric?

Bernard Silverman developed an estimation method for the optimal bandwidth (h). Since then many others have come up with other estimation methods.

**From:** bernard.silverman@stats.ox.ac.uk
**Sent:** Wednesday, May 07, 2014 11:47 AM
**To:** Kaufman, Morris
**Subject:** Lens Study
Great that the work I did all those years ago is still useful.
Thanks for letting me know about it.
Best wishes Bernard

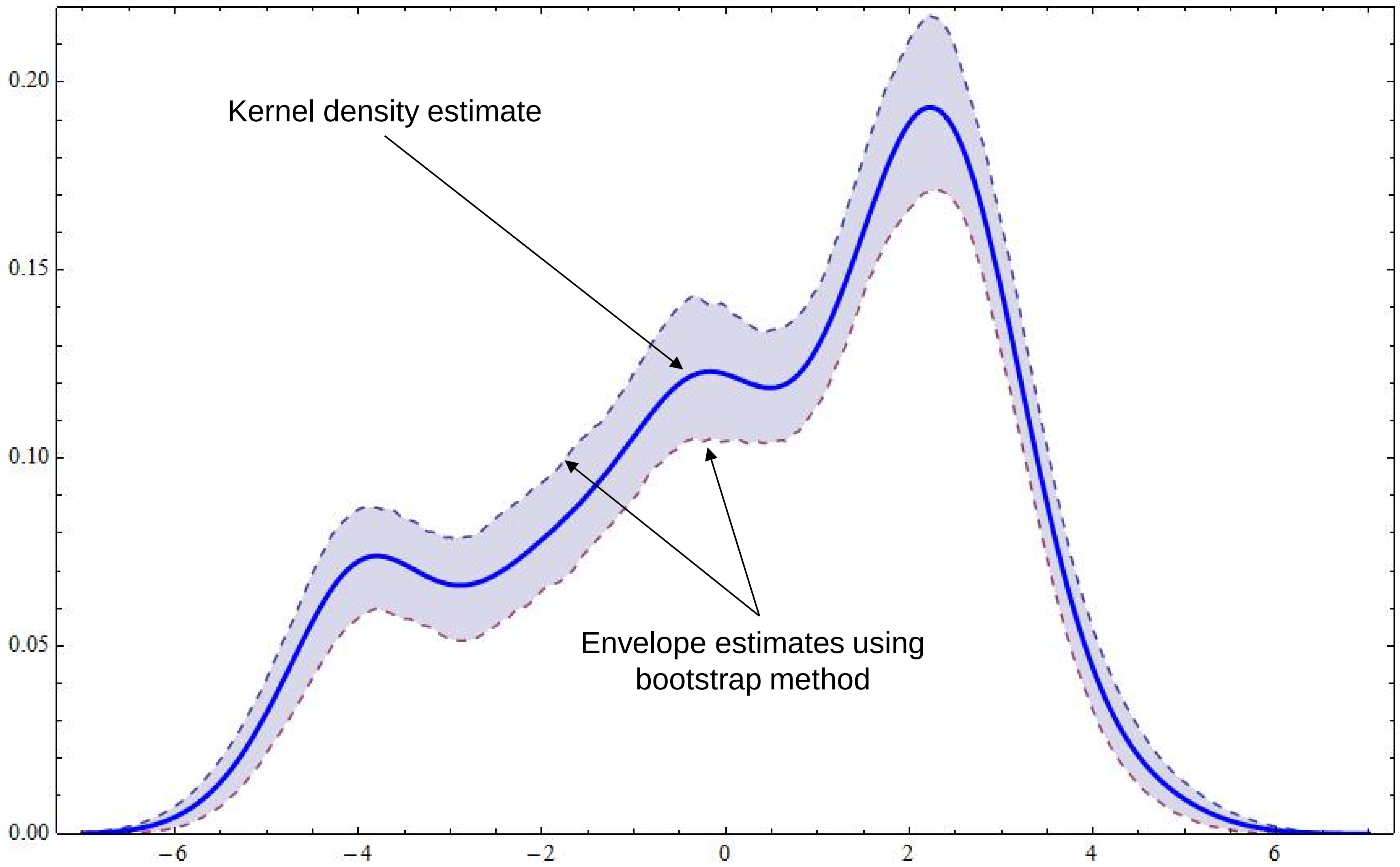
**Practical estimation of the bandwidth** [edit]

If Gaussian basis functions are used to approximate univariate data, and the underlying density being estimated is Gaussian then it can be shown that the optimal choice for *h* is[16]

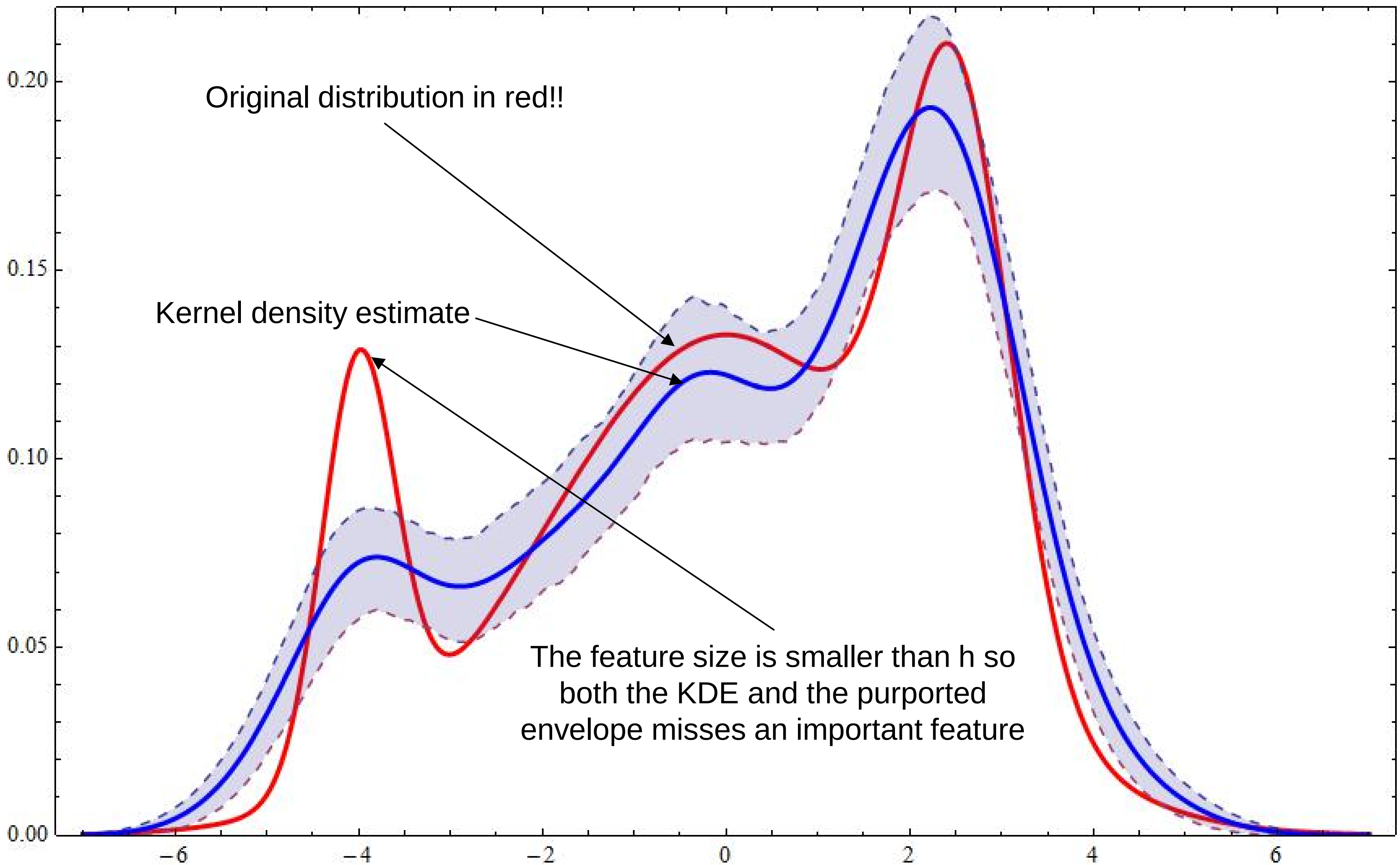
$$h = \left(\frac{4\hat{\sigma}^5}{3n}\right)^{\frac{1}{5}} \approx 1.06\hat{\sigma}n^{-1/5},$$

where $\hat{\sigma}$ is the standard deviation of the samples. This approximation is termed the *normal distribution approximation*, Gaussian approximation, or *Silverman's rule of thumb*.

# The bandwidth h is an important parameter; what can go wrong?

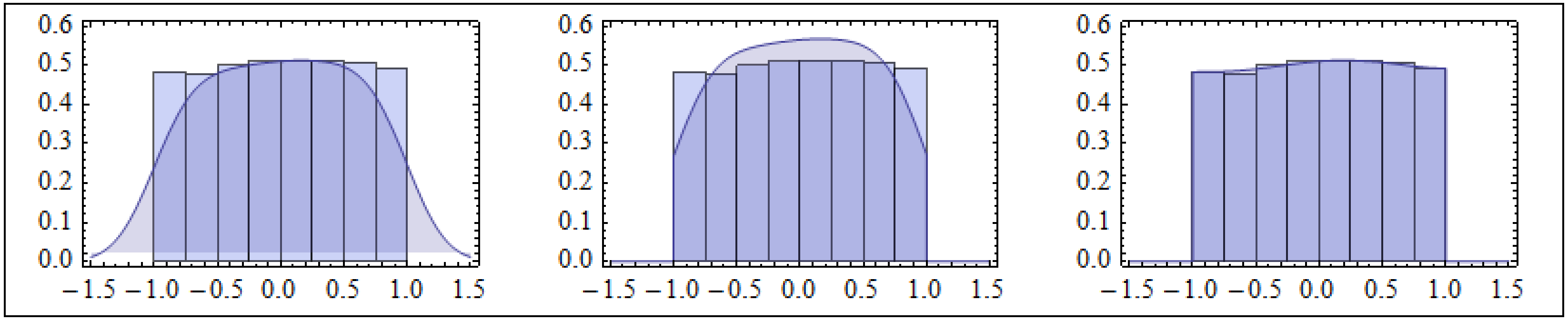


# Features smaller than h will not show up in the KDE



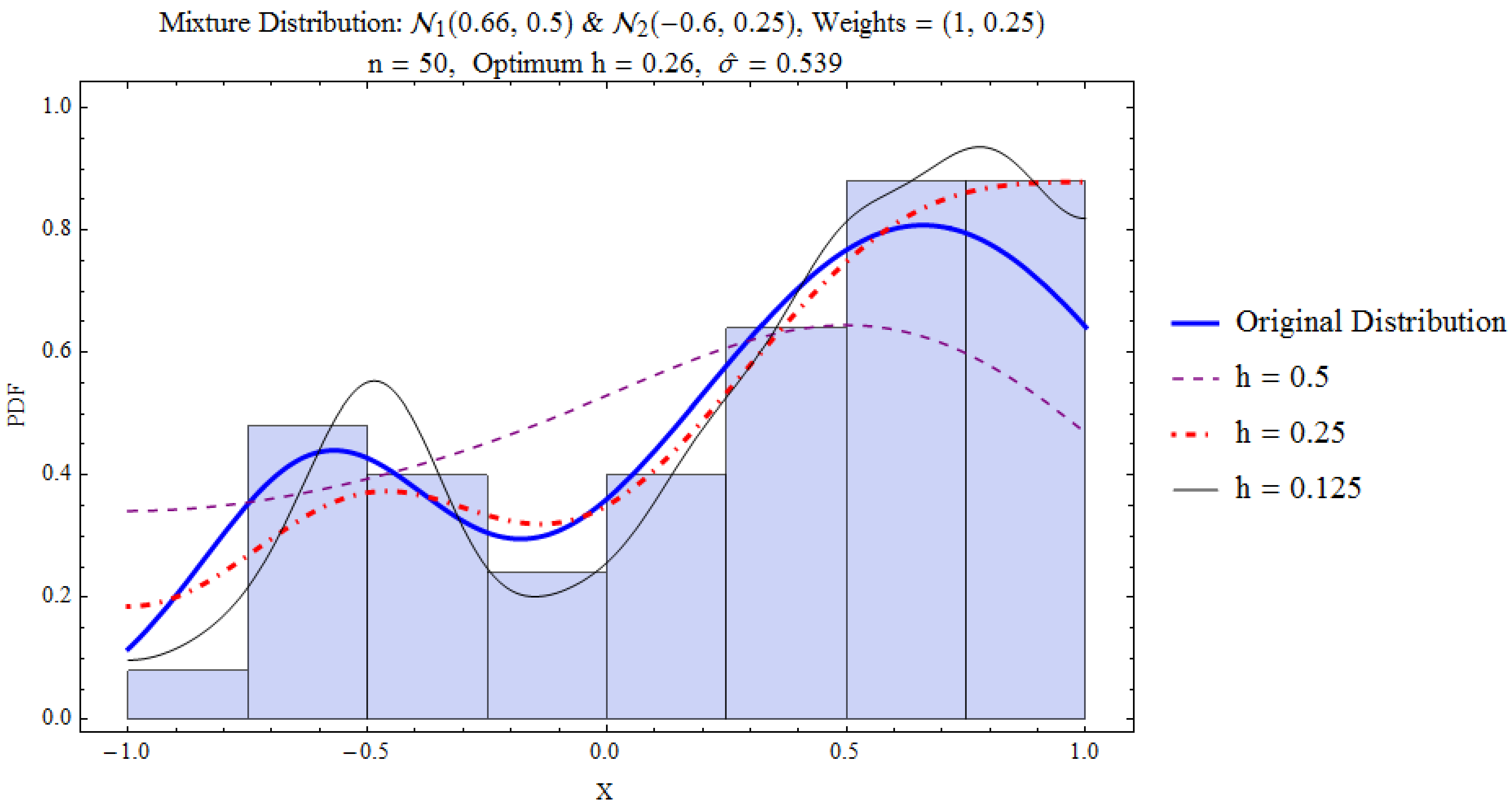
# Bounded distributions pose a tricky problem for modeling

A B C

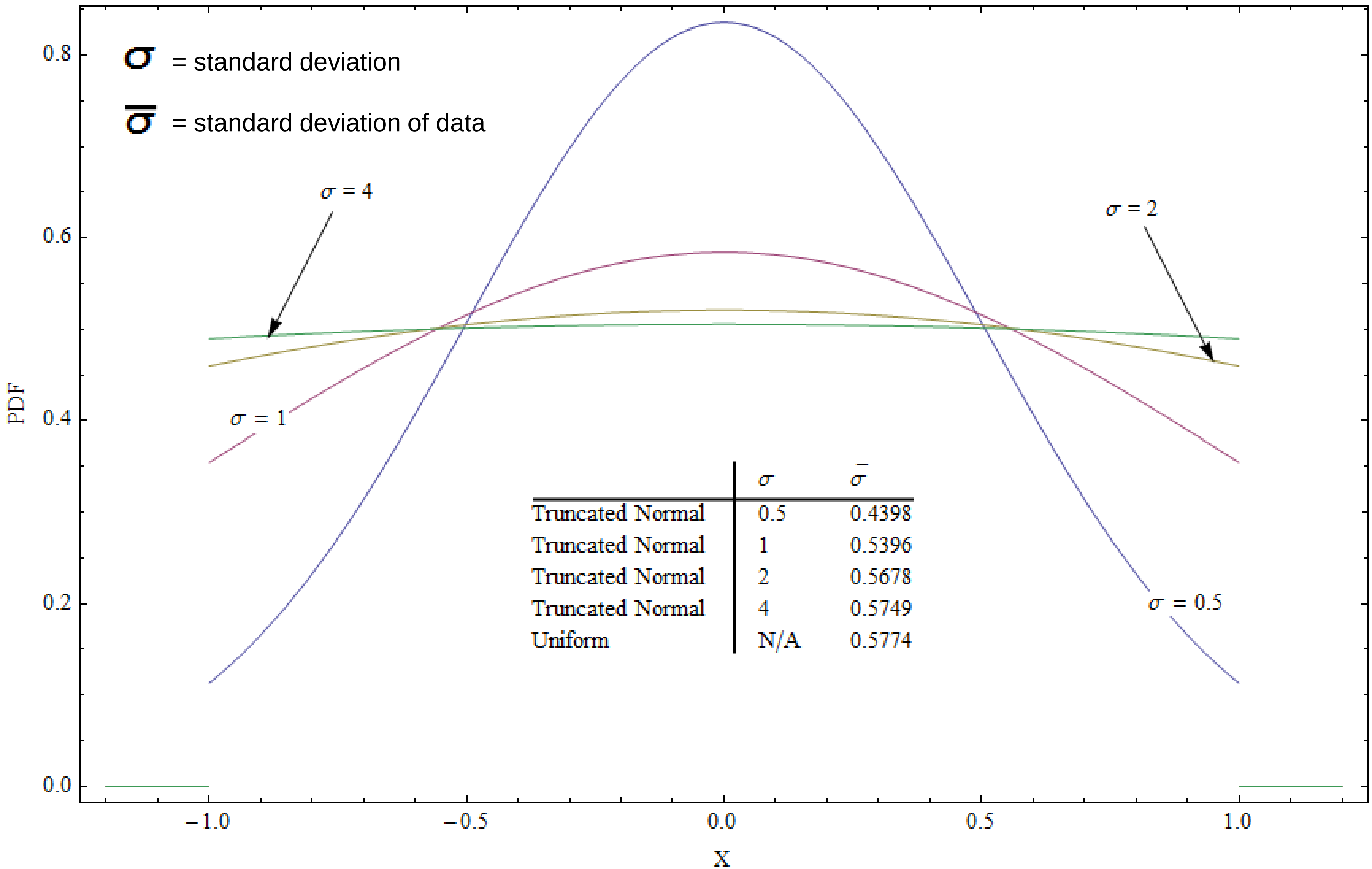



- Random data were generated from a uniform distribution, bounded at (-1, +1).
- An ordinary KDE estimate using a Gaussian kernel demonstrates the “spillover” effect (plot A). The spillover is roughly a 10% effect.
- The truncation algorithm (used in plot B) adds the 10% uniformly to the remainder. The underestimated region on the left and right of plot B demonstrates the “edge bias” effect.
- Plot C was generated using an undocumented feature in Mathematica: SmoothKernelDistribution [data, 0.25, {“Bounded”, {–1, 1}, “Gaussian”}]; in this case, the bandwidth h is set to 0.25

# Simulated bimodal lens data assuming 30 lenses KDE estimates using h-bandwidths of 0.125, 0.25, and 0.5



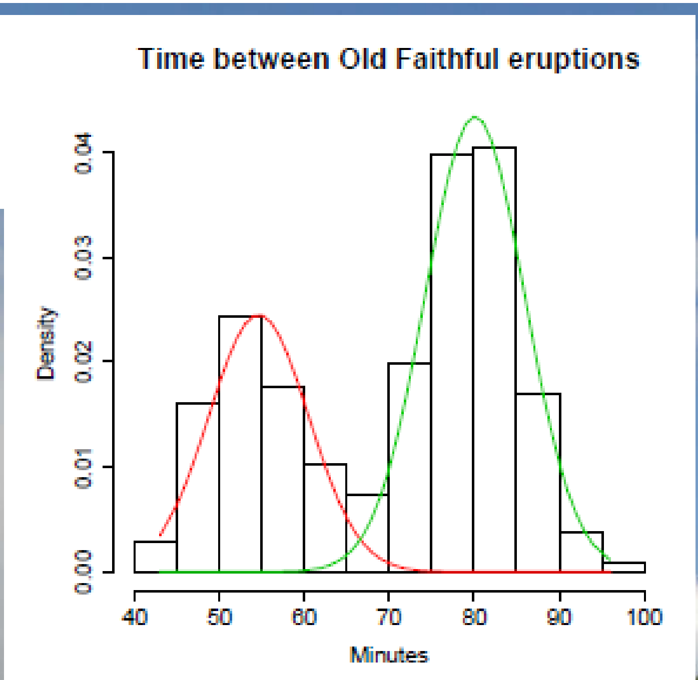
# Truncated distributions behave in a non-intuitive way



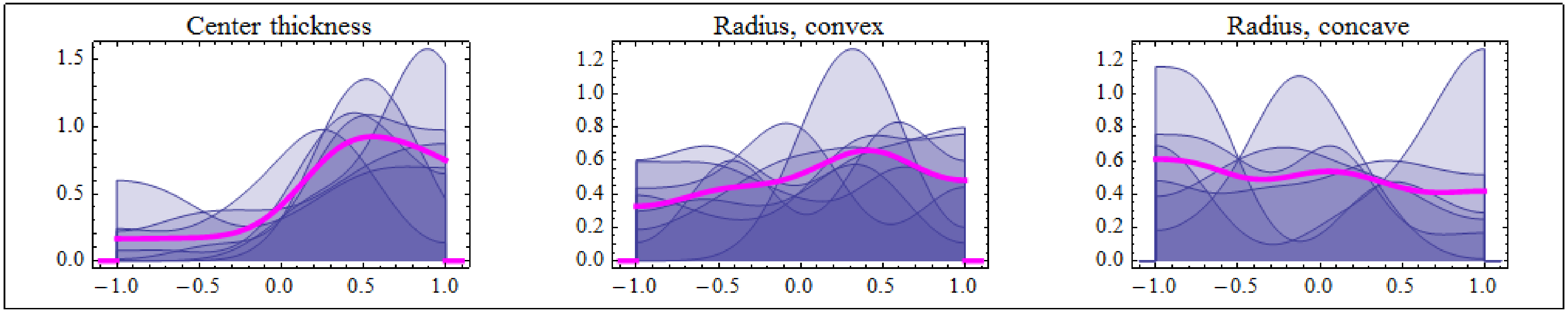

| | $\sigma$ | $\bar{\sigma}$ |
|---|---|---|
| Truncated Normal | 0.5 | 0.4398 |
| Truncated Normal | 1 | 0.5396 |
| Truncated Normal | 2 | 0.5678 |
| Truncated Normal | 4 | 0.5749 |
| Uniform | N/A | 0.5774 |

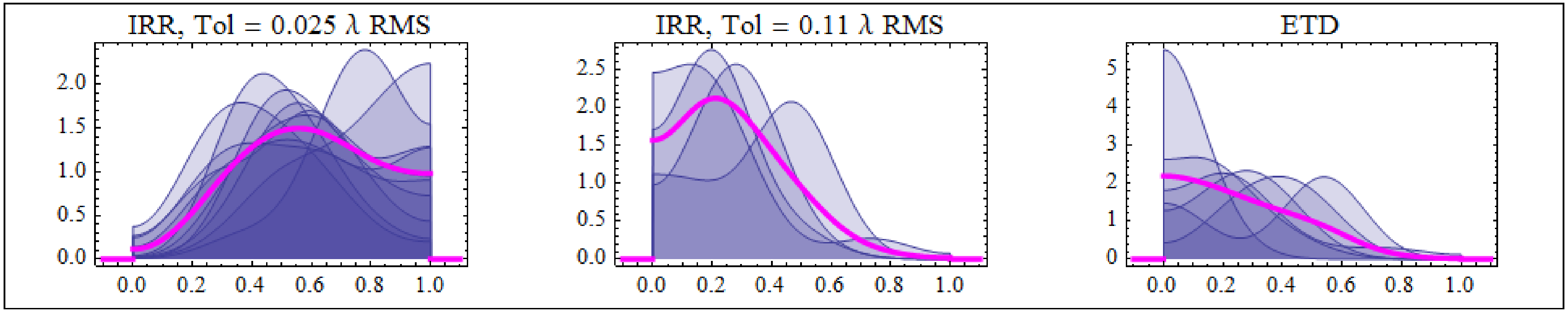
# How should we think about bimodal data? The old faithful geyser is a naturally occurring bimodal random number generator.

Statisticians have an observatory that monitors Old Faithful 24/7. Research of bimodal mixture distributions all reference the Old Faithful geyser.



# Using equal batch weighting, the aggregate mixture distributions are shown with thick lines. The individual lens data are thin lines.

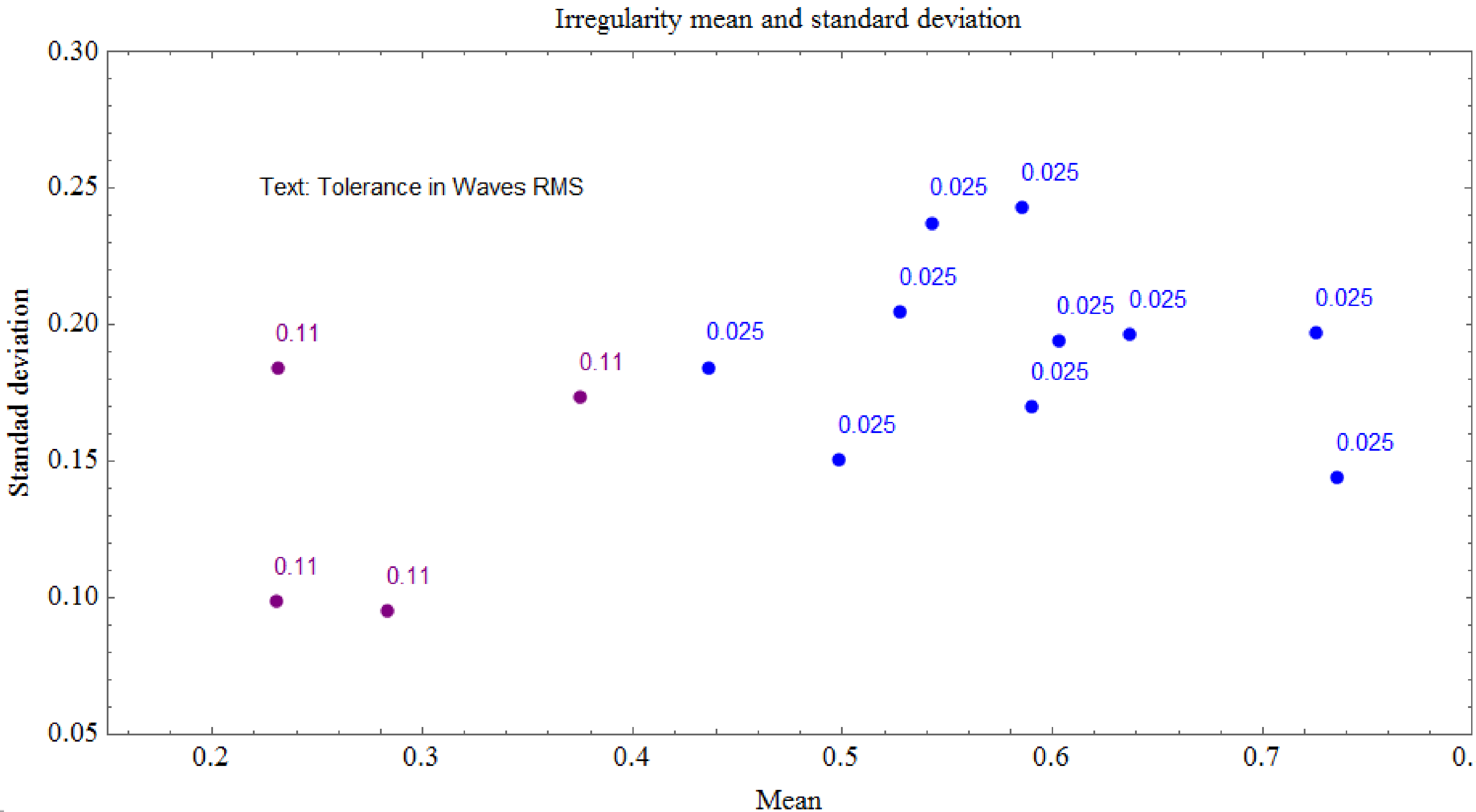


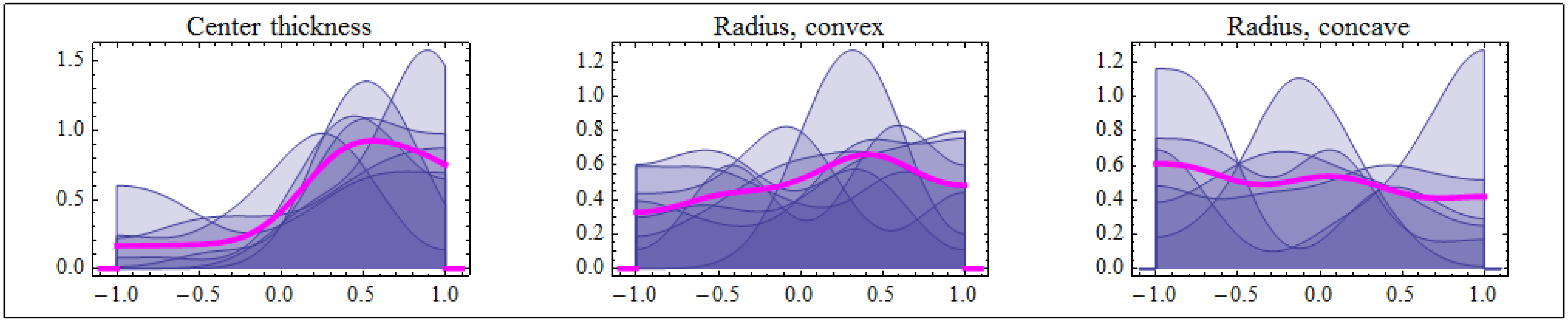

ETD = “edge thickness deviation” = wedge

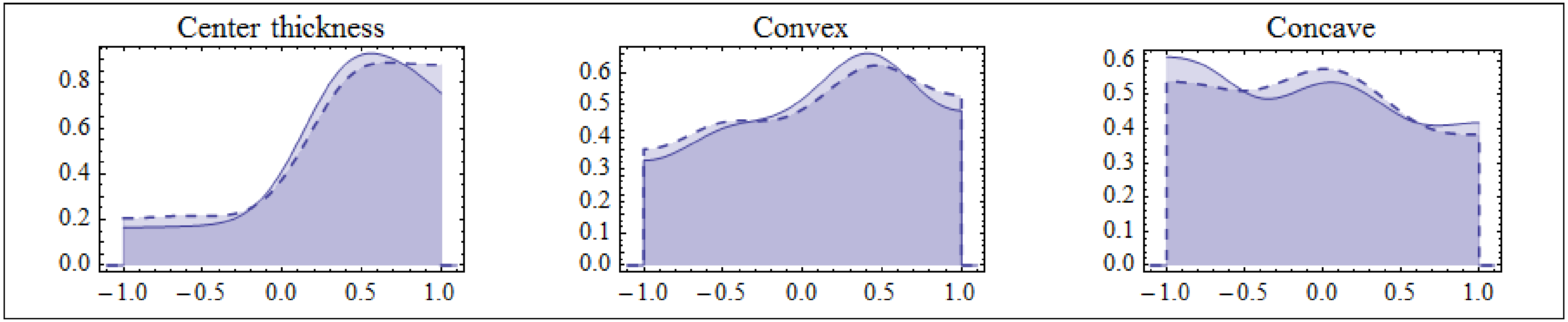
# Irregularity data

Notice that the mean/standard deviation fall into distinct groupings based on precision class. The supplier has to pick from a discrete number of processes. The better precision class will also have a larger mean/standard deviation relative to the tolerance interval.



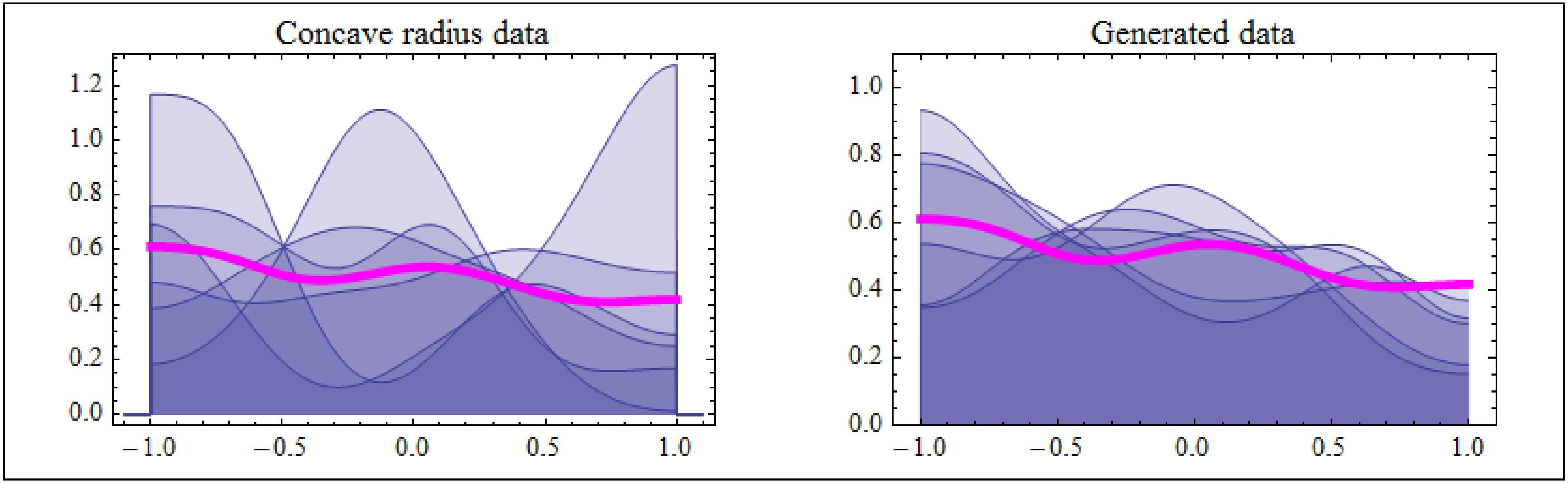
# It turns out that equal lens weighting and equal batch weighting yield similar results




Note: Equal lens weighting is shown with dashed lines, equal batch weighting is shown with solid lines.

# Data from 6 lens batches was used to generate a smooth histogram PDF. That PDF was used to create 6 random data sets.




The data from the actual lenses appear to have much greater variability with respect to the master PDF than one would expect from a random process.

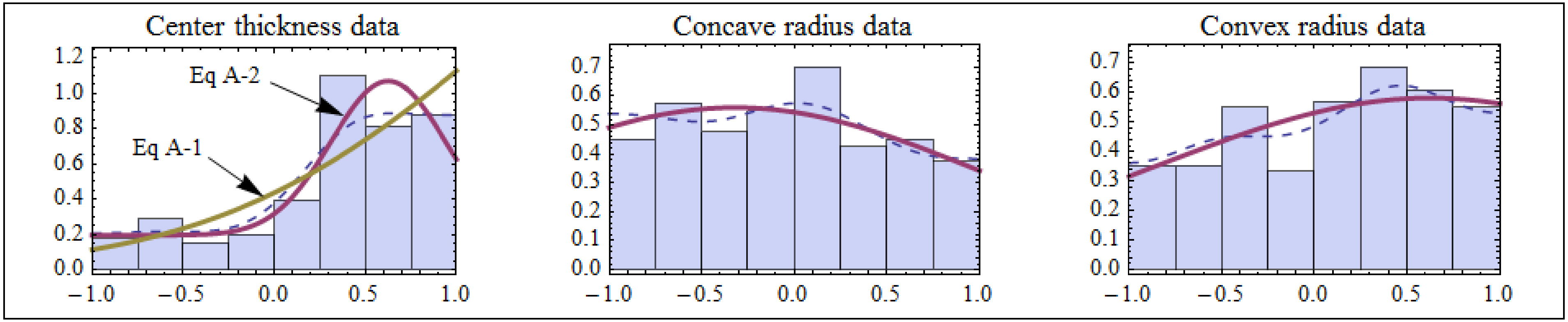
# Tabulated values

Mean and standard deviation data for radii, center thickness, irregularity, and wedge

| | Mean | Standard Deviation | Comments |
|---|---|---|---|
| Convex radius | 0.0898 | 0.5569 | Normalized to (–1, 1) |
| Concave radius | –0.0559 | 0.5559 | Normalized to (–1, 1) |
| Center thickness | 0.3367 | 0.5046 | Normalized to (–1, 1) |
| Irregularity | 0.5748 | 0.2168 | Tol = 0.025 λ RMS, normalized to (0, 1) |
| Irregularity | 0.2722 | 0.1463 | Tol = 0.11 λ RMS, normalized to (0, 1) |
| Wedge | 0.2497 | 0.1818 | Measured w/ ETD, normalized to (0, 1) |

# Parameterized center thickness and radii distributions

## (thick lines, parametric, dotted lines nonparametric)




Concave $N$ [–0.3118, 1.3101]

Convex $N$ [0.5907, 1.4312]

Center Thickness:

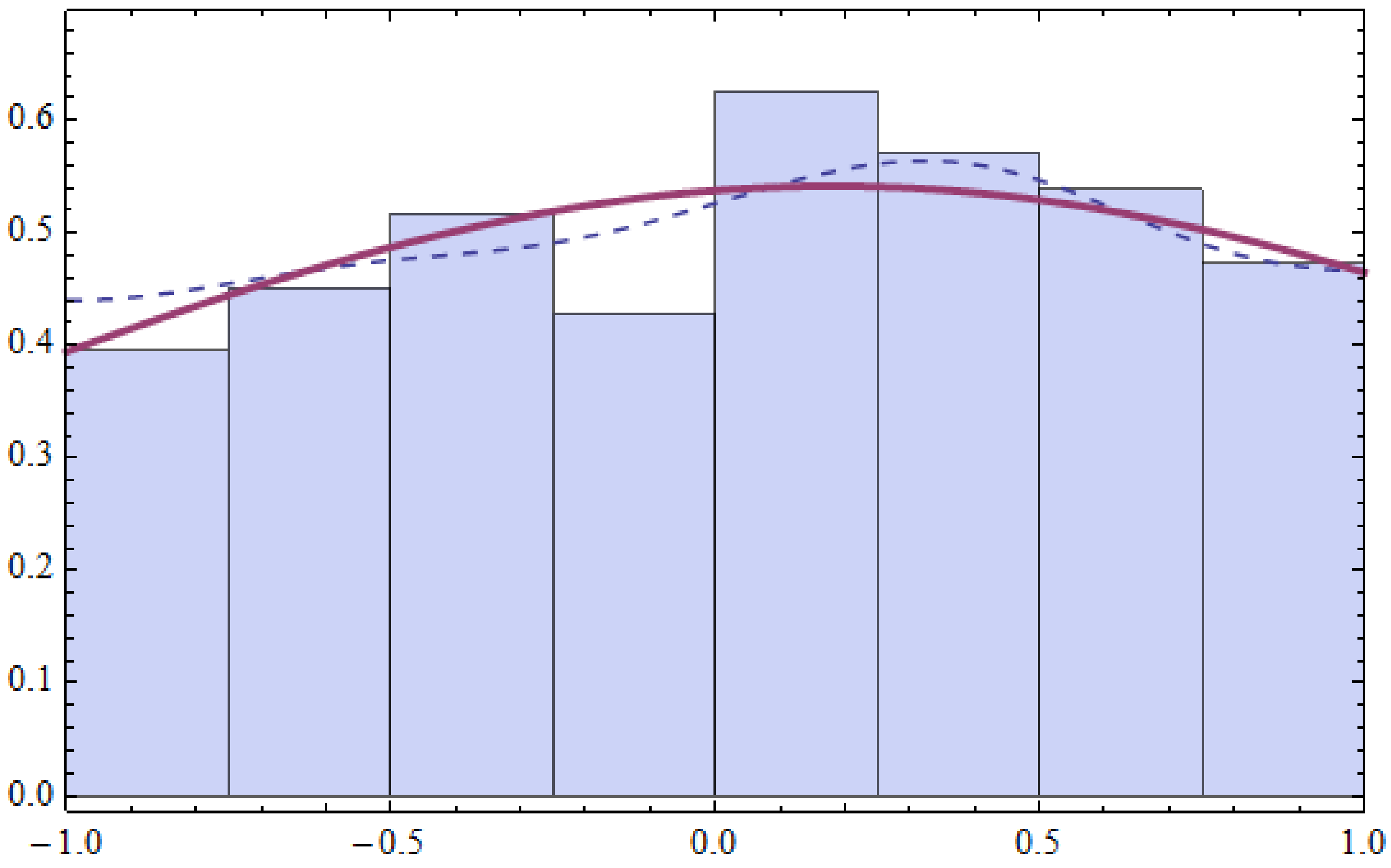
$N$ [2.908, 1.3101] (log-likelihood = –94) (A-1)

or

0.639 × $N$ [0.625, 0.315] + (1 – 0.639) × $U$ [–1, 1] (log-likelihood = –87) (A-2)

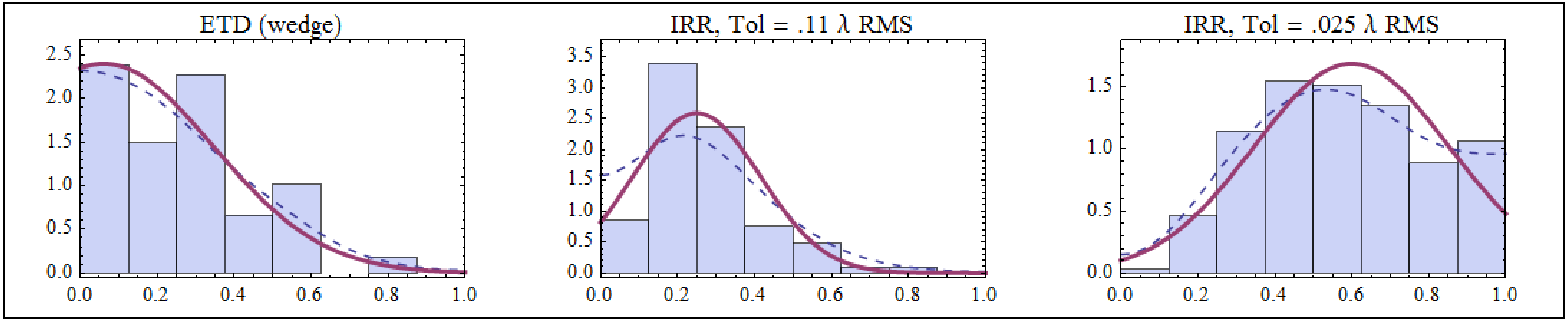
***N* [μ, σ] = Gaussian dist. with mean (μ) and std. deviation (σ) truncated to (–1, 1)**
***U* [–1, 1] = Uniform distribution on the interval (–1, 1)**

# When convex and concave radii are mixed together, the net result is close to a uniform distribution




The mean and standard deviation: 0.026 and 0.56, respectively.
The data was fit to: *N* [0.180, 1.483]

# Parameterized wedge and irregularity distributions
## (thick lines, parametric, dotted lines nonparametric)




ETD (wedge): *N* [0.0607, 0.2843]
Irregularity (tolerance = 0.11 λ RMS); *N* [0.2497, 0.1651]
Irregularity (tolerance = 0.025 λ RMS); *N* [0.5990, 0.2522]

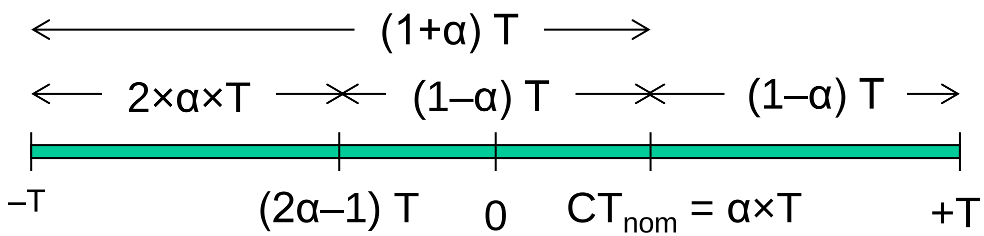
***N* [μ, σ] = Gaussian dist. with mean (μ) and std deviation (σ) truncated to (0, 1)**

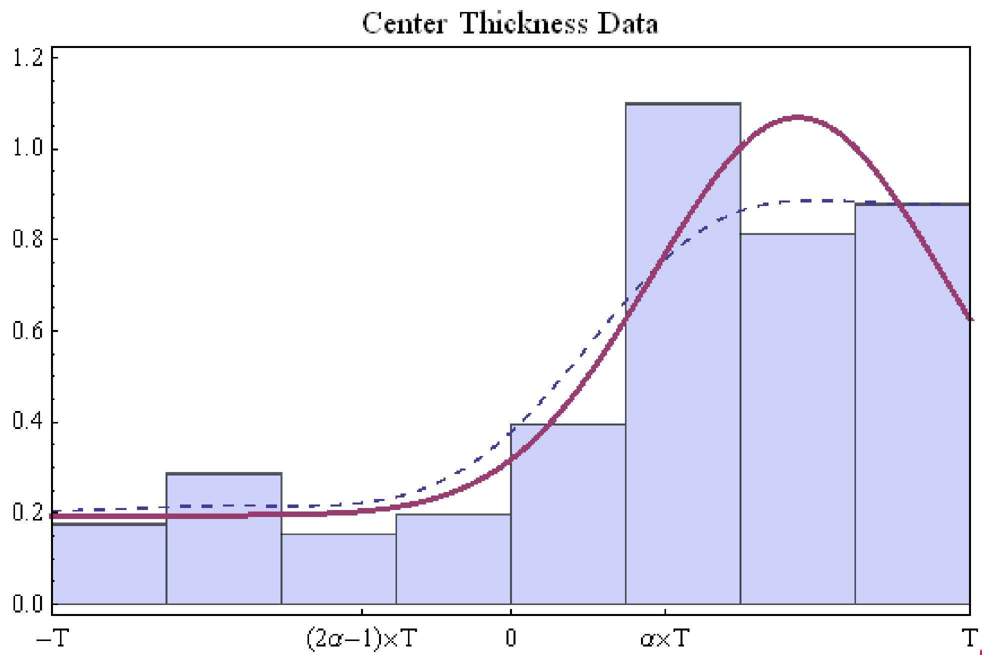
# The structure of an asymmetric tolerance zone

Probabilities (for data under consideration):

– Inside the smaller symmetric zone: Pr $[(2\alpha - 1)\,T < x < T] = 86\%$.

– Outside the symmetric zone: Pr $[-T < x < (2\alpha - 1)\,T] = 14\%$ (the safety bonus).



# A tolerancing scheme using lens statistics with Code V

1. Tolerance system with TOR with $CT_{nom} \pm (1–\alpha)$ T (i.e., symmetric tolerancing zone)
2. Final check with full asymmetrical zone with TOLMONTE using UTOCHNG.seq.

**Notes**

- TOR will not support an asymmetrical tolerance zone
- TOLMONTE with the UTOCHNG.seq would use eq. (A-1) or (A-2).
- Data suggest a reasonable value for α is 0.337.
- One could think of the asymmetrical part (2 × α × T) as a safety bonus for the supplier.
- The final engineering drawings would need to be reformulated for the asymmetrical tolerance zone of $CT_{nom}$ +(1 – α) T and – (1 + α) T.
- The intent is that, knowing the structure of the manufacturing probability distributions, the designer could loosen tolerances a bit and have a nominal optical design that is much closer to the expected as-built values for downstream optomechanical design work.

# Email from ORA

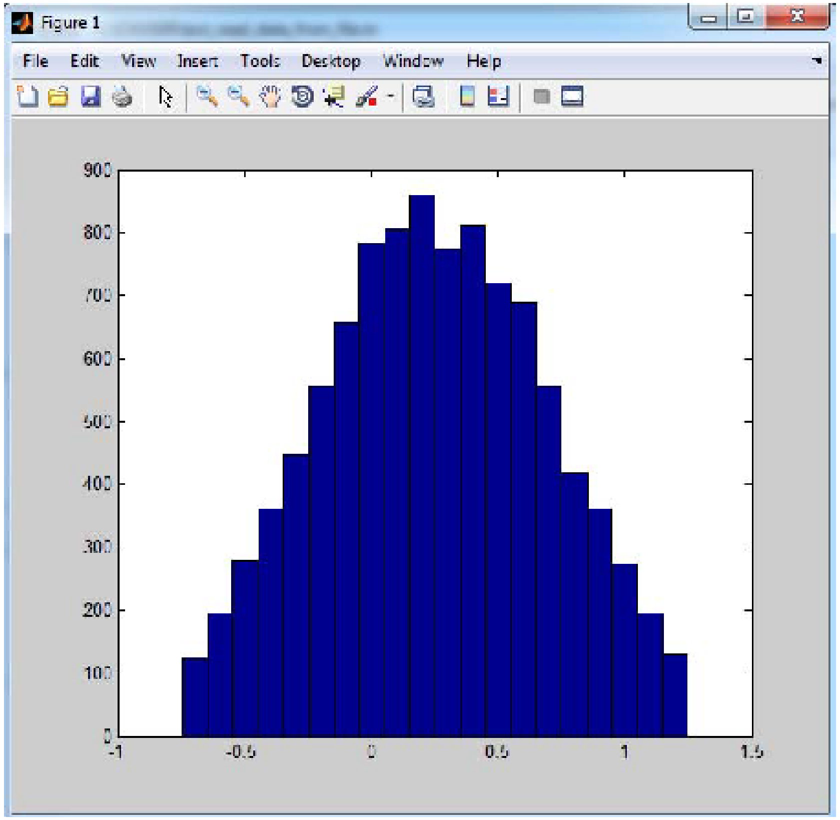
Dear Morris,

There is no way to change the predefined tolerance distributions in TOR option. However, you could apply a user-defined tolerance distribution in TOLMONTE through the UTOCHNG.seq file. Here attached an example of a truncated Gaussian distribution with a sigma = 0.5; mean = 0.25. I have verified the data in MatLab and its histogram is attached below.

Please let us know if you have any other questions.

Best regards,

Weimin Shi



# Questions? Further work?

1. Teasing out the underlying causes: R number, glass softness, batch size?
2. 30% of batches have some thin lenses. Why is extra polishing sometimes needed?
3. Prototype vs. production differences?
4. Why are some of the distributions unimodal, others multimodal?
5. Does it really help to fold the manufacturing data into the design process?

Acknowledgment

The authors thank Daniel K. Frayer of NSTec for his contribution to this work.