

# An Ice Sheet Model Initialization Procedure for Smooth Coupling with Climate Forcing

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# Estimation of ice sheet initial state

(w/ G. Stadler [Courant], and S. Price [LANL] - PISCEES )

## Goal

Find ice sheet initial state that

- matches observations (surface velocity)
- is in <sup>a</sup>equilibrium<sup>o</sup> with climate forcings (SMB)

by inverting for unknown/uncertain parameters

Significantly reduce non physical transients without spin-up.

## Bibliography

- *Arthern, Gudmundsson*, J. Glaciology, 2010
- *Price, Payne, Howat and Smith*, PNAS, 2011
- *Petra, Zhu, Stadler, Hughes, Ghattas*, J. Glaciology, 2012
- *Pollard DeConto*, TCD, 2012
- *W. J. J. Van Pelt et al.*, The Cryosphere, 2013
- *Morlighem et al.* Geophysical Research Letters, 2013
- *Goldberg and Heimbach*, The Cryosphere, 2013
- *Michel et al.*, Computers & Geosciences, 2014

*Perego, Price, Stadler*, **Journal of Geophysical Research**, 2014

# Estimation of ice sheet initial state

(w/ G. Stadler [Courant], and S. Price [LANL] - PISCEES )

**Problem:** what is the initial thermo-mechanical state of the ice sheet?

## Available data/measurements

- *ice extension and surface topography*
- *surface velocity*
- *Surface Mass Balance*
- *ice thickness  $H$  (very noisy)*

## Fields to be estimated

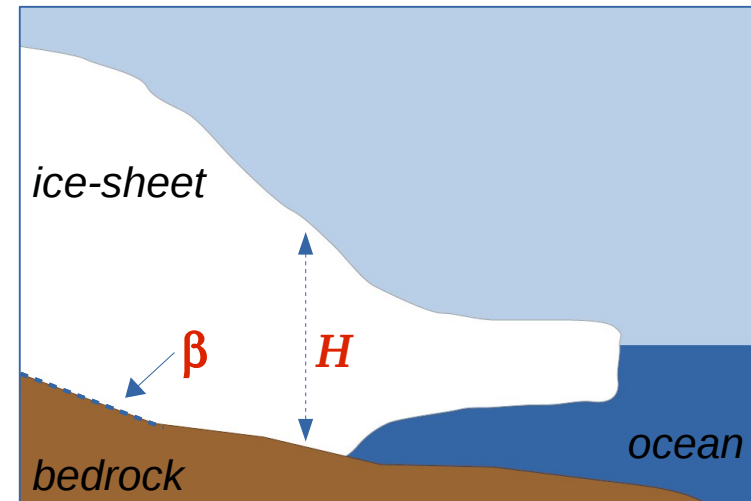
- *ice thickness  $H$*
- *basal friction  $\beta$*

## Additional information

- *ice fulfills **nonlinear Stokes equation** (here we use FO)*
- *ice is almost **at mechanical equilibrium***

## Assumption (for now)

- *given temperature field*



# Estimation of ice sheet initial state

Steady State equations and basal sliding conditions

How to prescribe ice sheet mechanical equilibrium:

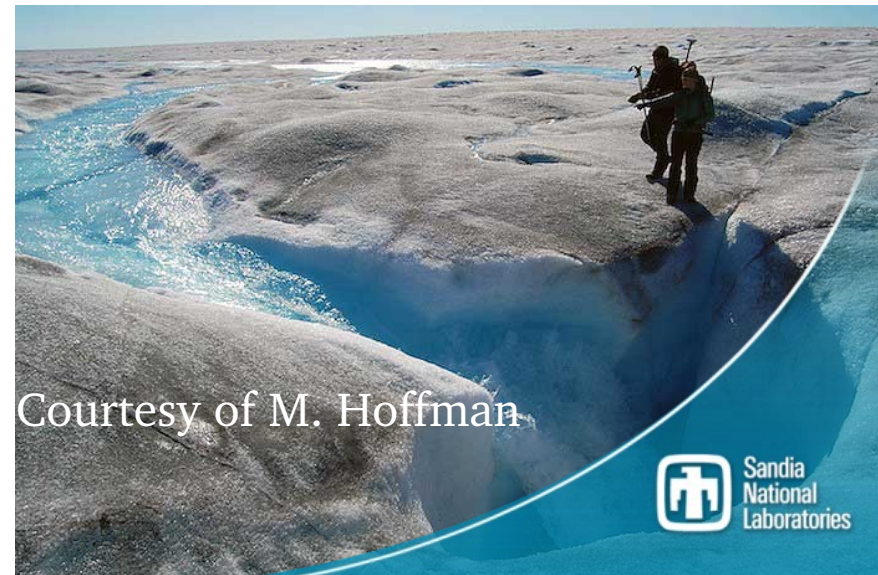
$$\frac{\partial H}{\partial t} = -\text{div}(\mathbf{U}H) + \tau_s, \quad \mathbf{U} = \frac{1}{H} \int_z \mathbf{u} dz.$$

*divergence flux* (pointing to  $\mathbf{U}H$ )  
*Surface Mass Balance* (pointing to  $\tau_s$ )

equilibrium  $\text{div}(\mathbf{U}H) - \tau_s = 0$

Boundary condition at ice-bedrock interface

$$(\sigma \mathbf{n} + \beta \mathbf{u})_{\parallel} = \mathbf{0} \quad \text{on} \quad \Gamma_{\beta}$$



Courtesy of M. Hoffman

# Estimation of ice sheet initial state

PDE-constraint optimization problem: cost functional

**Problem:** find initial conditions such that the ice is almost at thermo-mechanical equilibrium given the geometry and the SMB, and matches available observations.

Optimization Problem:

find  $\beta$  and  $H$  that minimizes the functional  $\mathcal{J}$

$$\begin{aligned}\mathcal{J}(\beta, H) = & \int_{\Sigma} \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 ds && \left. \begin{array}{l} \text{surface velocity} \\ \text{mismatch} \end{array} \right\} \text{Common} \\ & + \int_{\Sigma} \frac{1}{\sigma_{\tau}^2} |\text{div}(\mathbf{U}H) - \tau_s|^2 ds && \left. \begin{array}{l} \text{SMB} \\ \text{mismatch} \end{array} \right\} \text{Proposed} \\ & + \int_{\Sigma} \frac{1}{\sigma_H^2} |H - H^{obs}|^2 ds && \left. \begin{array}{l} \text{thickness} \\ \text{mismatch} \end{array} \right\} \\ & + \mathcal{R}(\beta, H) && \text{regularization terms.}\end{aligned}$$

subject to ice sheet model equations  
(FO or Stokes)

$\mathbf{U}$ : computed depth averaged velocity

$H$ : ice thickness

$\beta$ : basal sliding friction coefficient

$\tau_s$ : SMB

$\mathcal{R}(\beta)$  regularization term



# Estimation of ice sheet initial state

Algorithm and Software tools used

Algorithm	Software Tools
Basal non-uniform triangular mesh	<i>Triangle</i>
Linear Finite Elements on tetrahedra	<i>LifeV</i>
Quasi-Newton optimization (L-BFGS)	<b>Rol</b>
Nonlinear solver (Newton method)	<b>NOX</b>
Krylov Linear Solvers	<b>AztecOO/IfPack</b>



## Details:

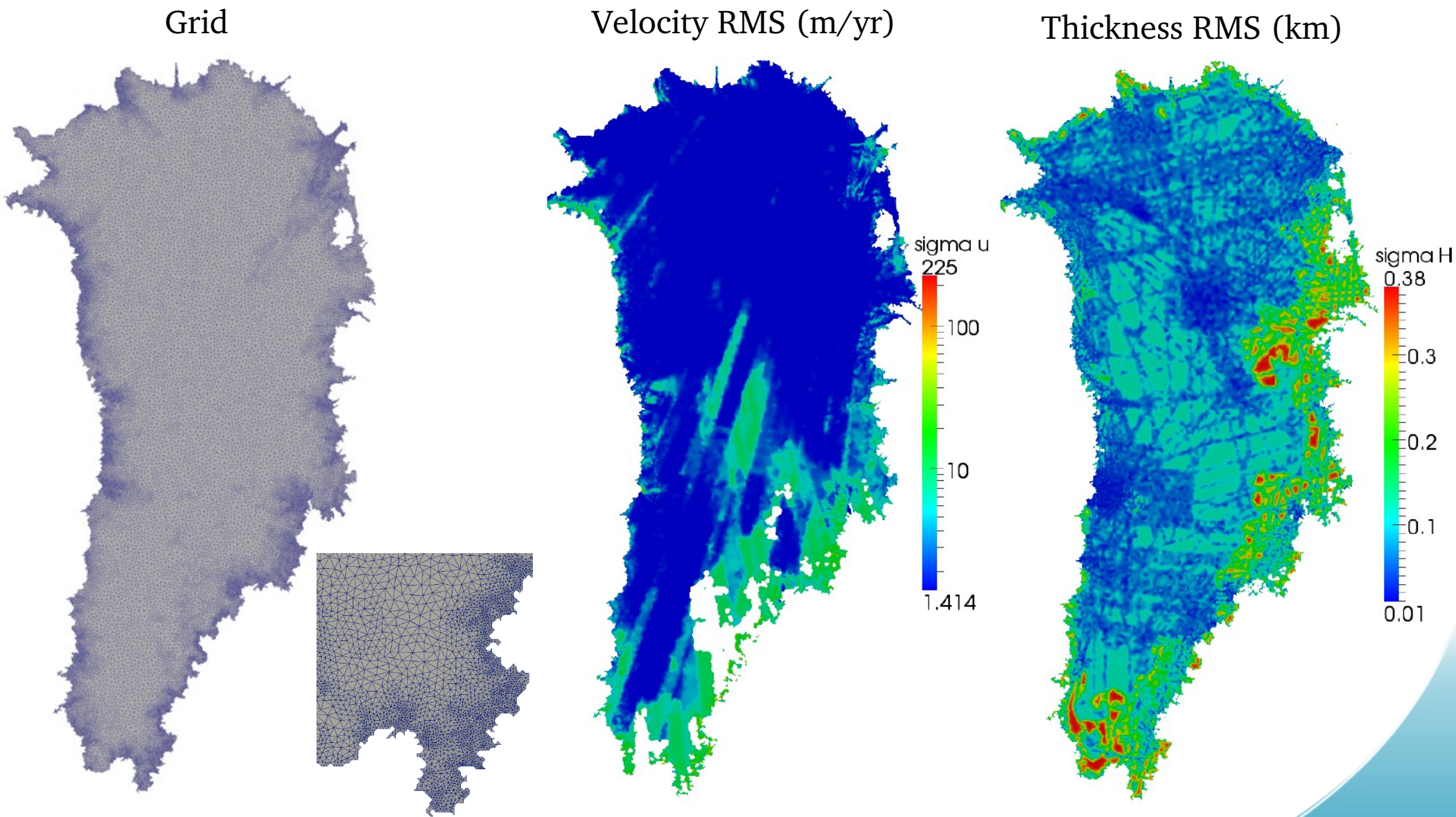
*Regularization terms:* Tikhonov

L-BFGS initialized with Hessian of the regularization terms

$$\left( \frac{1}{2} \beta^T L \beta \rightarrow L \right)$$

# Estimation of ice sheet initial state of Greenland ice sheet

Grid and RMS of velocity and errors associated with velocity and thickness observations



# Estimation of ice sheet initial state of Greenland ice sheet

Inversion results: surface velocities

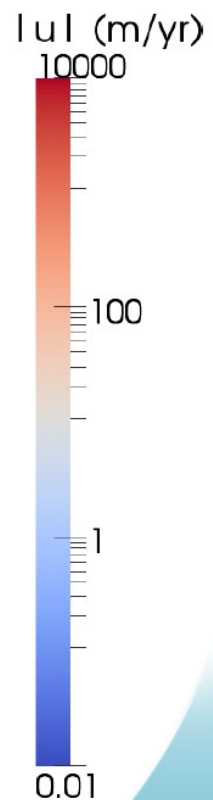
computed surface velocity

common

proposed

observed surface velocity

target

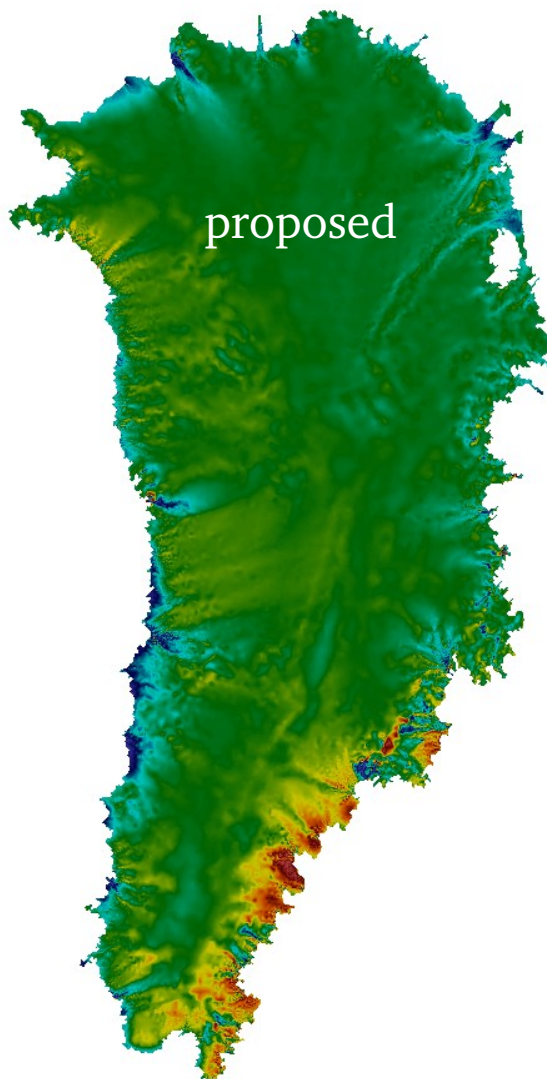
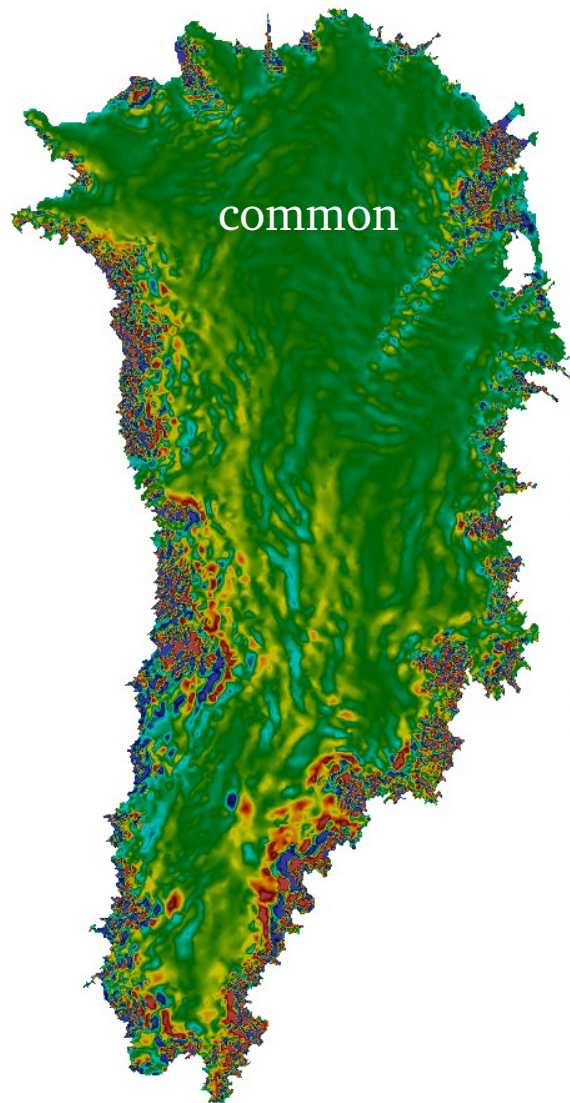




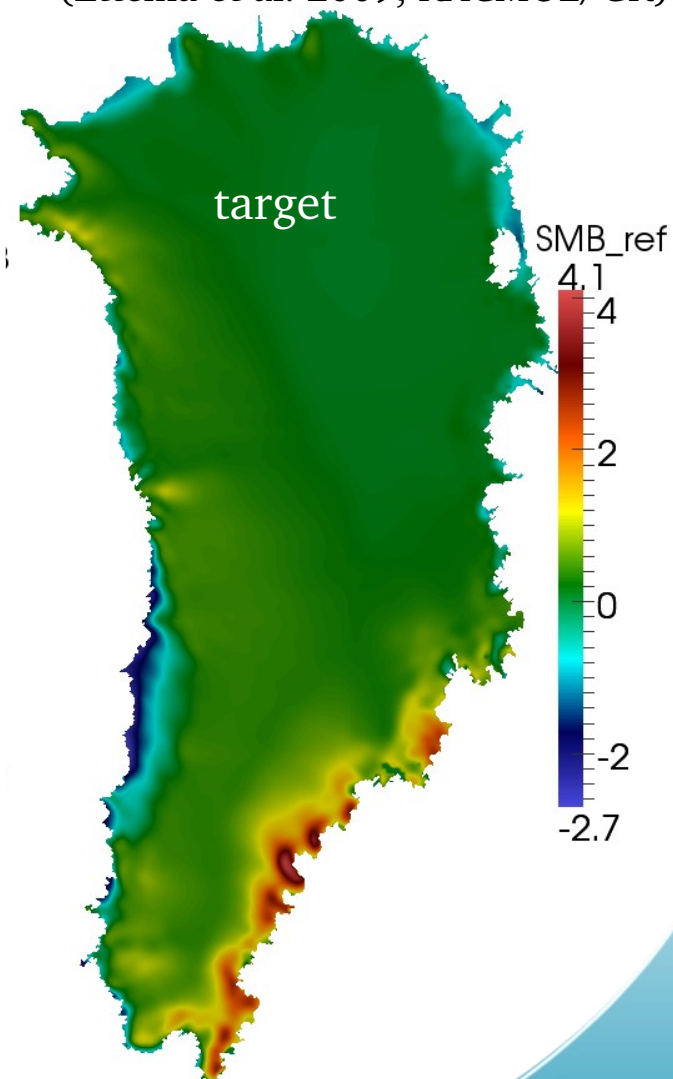
# Estimation of ice sheet initial state of Greenland ice sheet

Inversion results: surface mass balance (SMB)

SMB needed for equilibrium



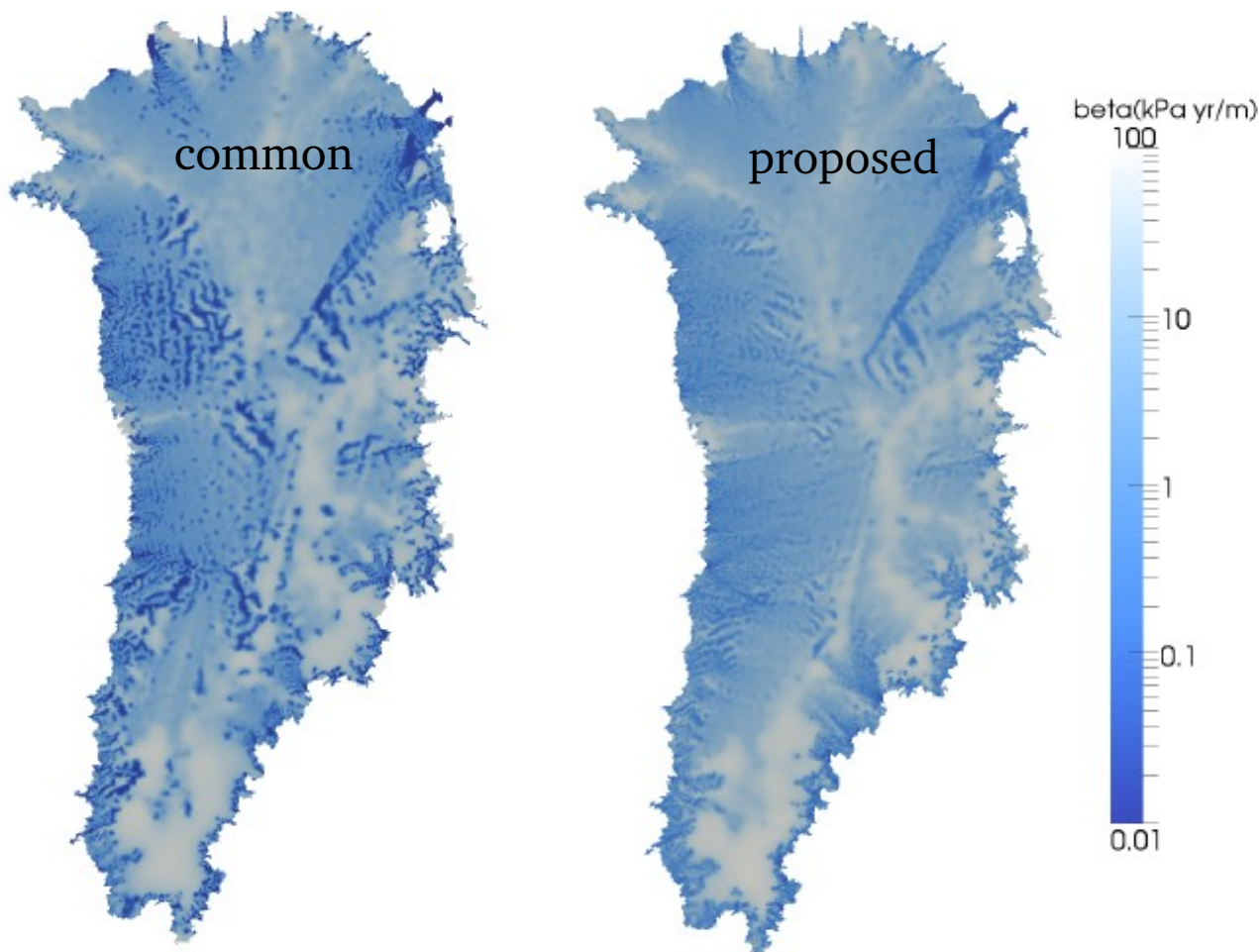
SMB from climate model  
(Ettema et al. 2009, RACMO2/GR)



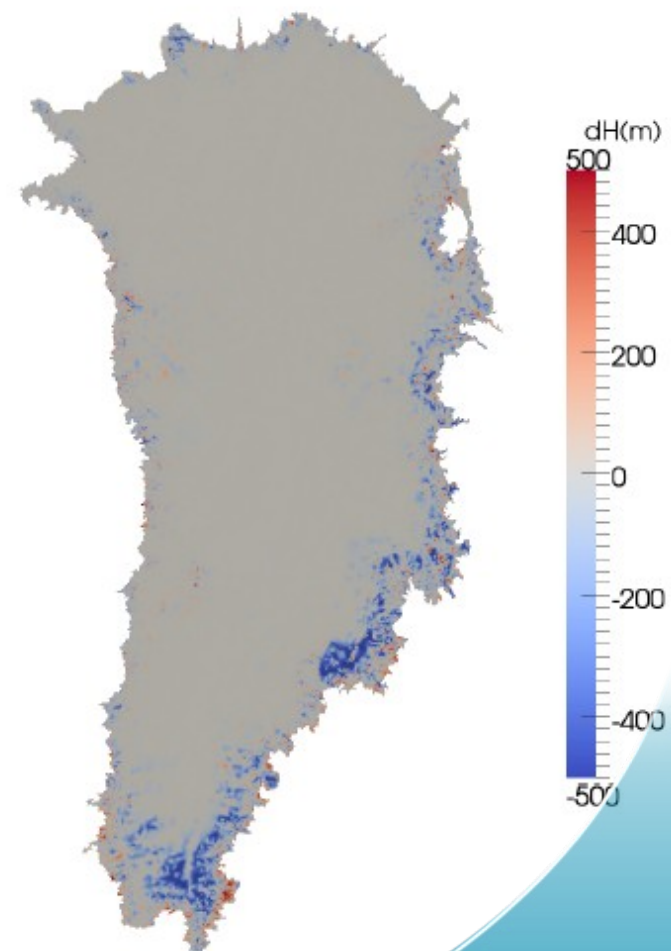
# Estimation of ice sheet initial state of Greenland ice sheet

Estimated beta and change in topography

recovered basal friction



difference between recovered and observed thickness



# Implementation of adjoints capability in newer code Albany-FELIX (w/ E. Phipps, A. Salinger, D. Ridzal and D. Kouri [SNL])

**Albany-Felix:** Albany ice sheet solver (orig. developed by I. Kalashnikova and A. Salinger)

## Why?

- to exploit Automatic Differentiation for computing derivatives
- to exploit Albany/Trilinos ecosystem (e.g. for UQ capabilities using Dakota)
- to use in-house software (better maintainability)

## Features:

- automatic differentiations to compute adjoints and objective functional derivatives
- coupled with ROL (Rapid Optimization Library) package in Trilinos, to perform reduced gradient based optimization
- coupling with Dakota for UQ capabilities

## TODO:

- Implement Hessian to use quasi-Newton methods
- Add shape optimization to be able to invert for bedrock topography
- Improve robustness of inversion and explore different optimization strategies (e.g. SQP)



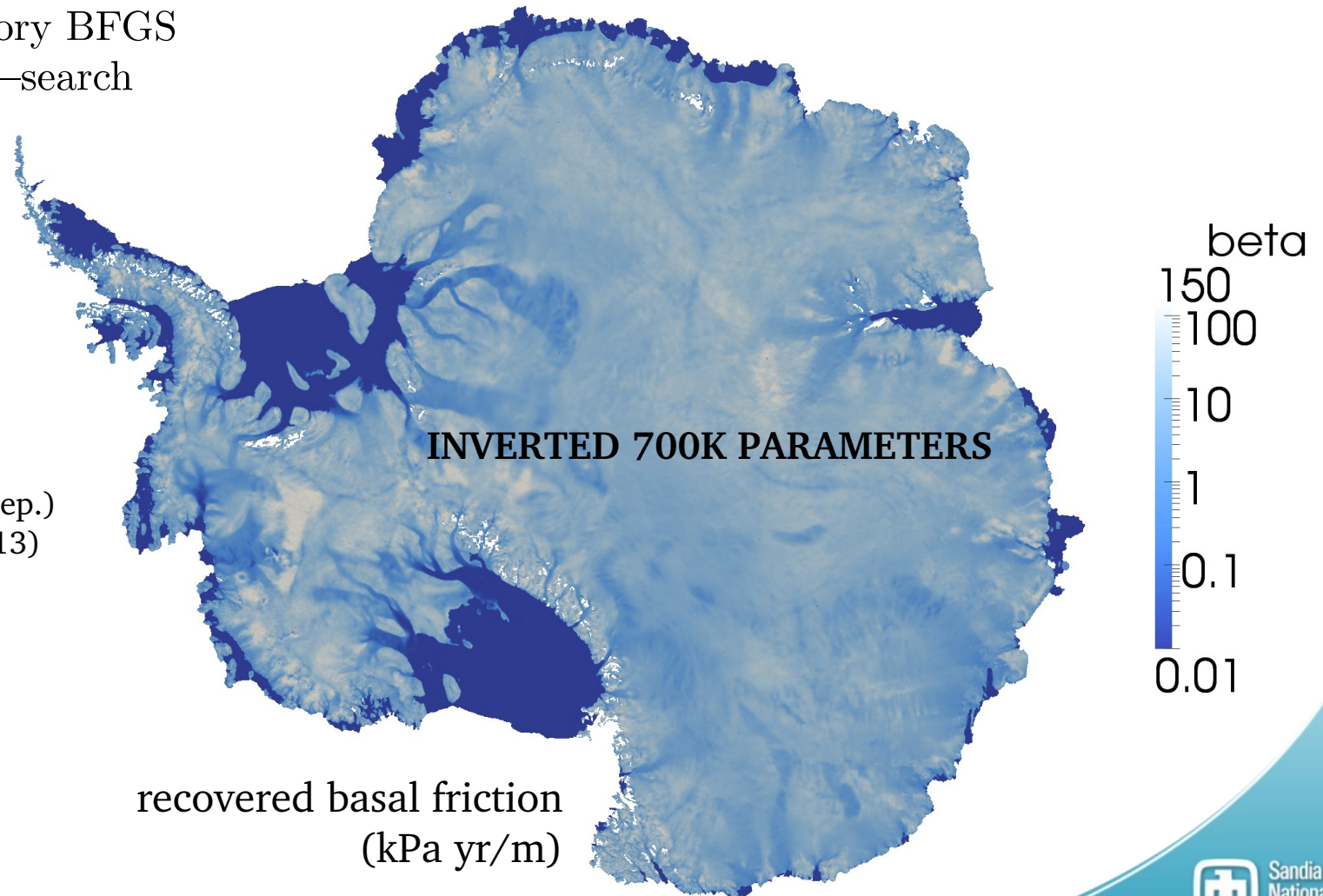
# Antarctica Inversion using Albany-Piro-ROL

Objective functional: 
$$\mathcal{J}(\mathbf{u}(\beta), \beta) = \int_{\Sigma} \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 ds + \alpha \int_{\Sigma} |\nabla \beta|^2 ds$$

ROL algorithm:

- Limited-Memory BFGS
- Backtrack line-search

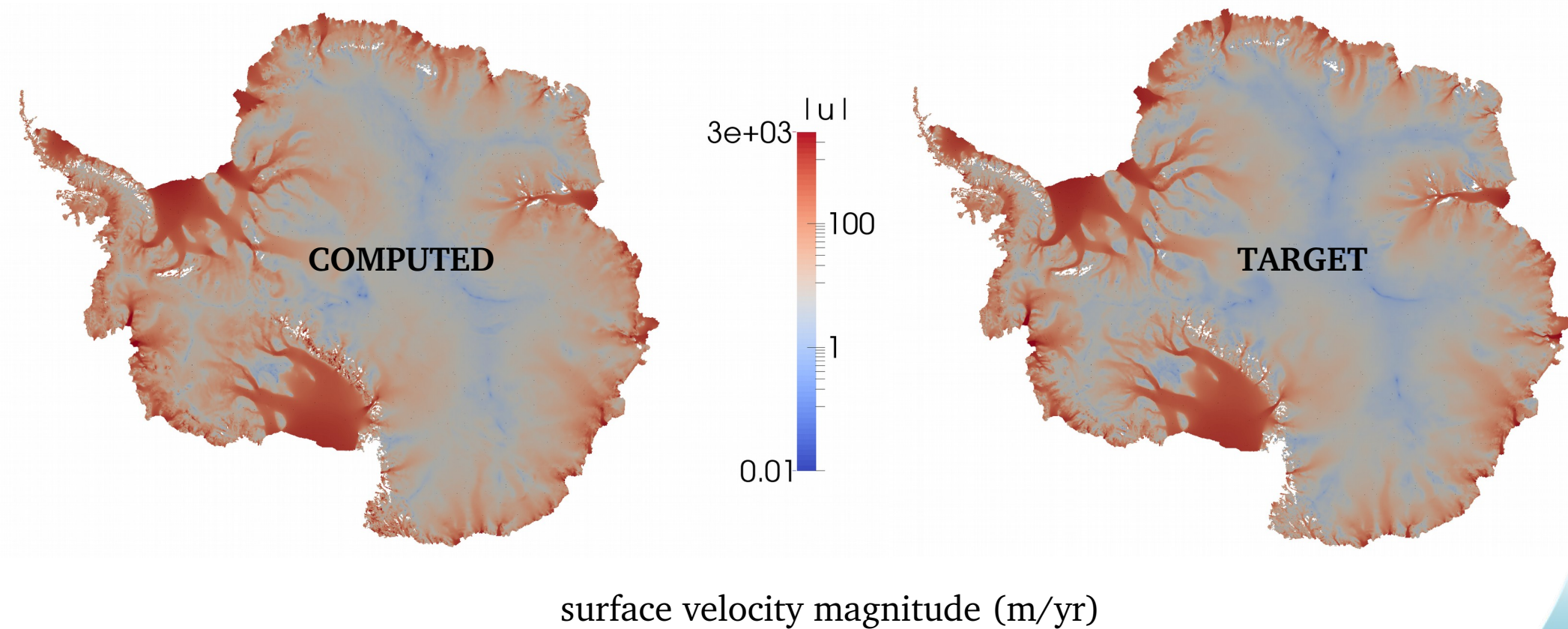
Geometry and fields:  
(Cornford, Martin et al, in prep.)  
Bedmap2 (Fretwell et al., 2013)  
Temperature (Pattyn, 2010)





# Antarctica Inversion using Albany-Piro-ROL

comparison surface velocities, computed vs. target

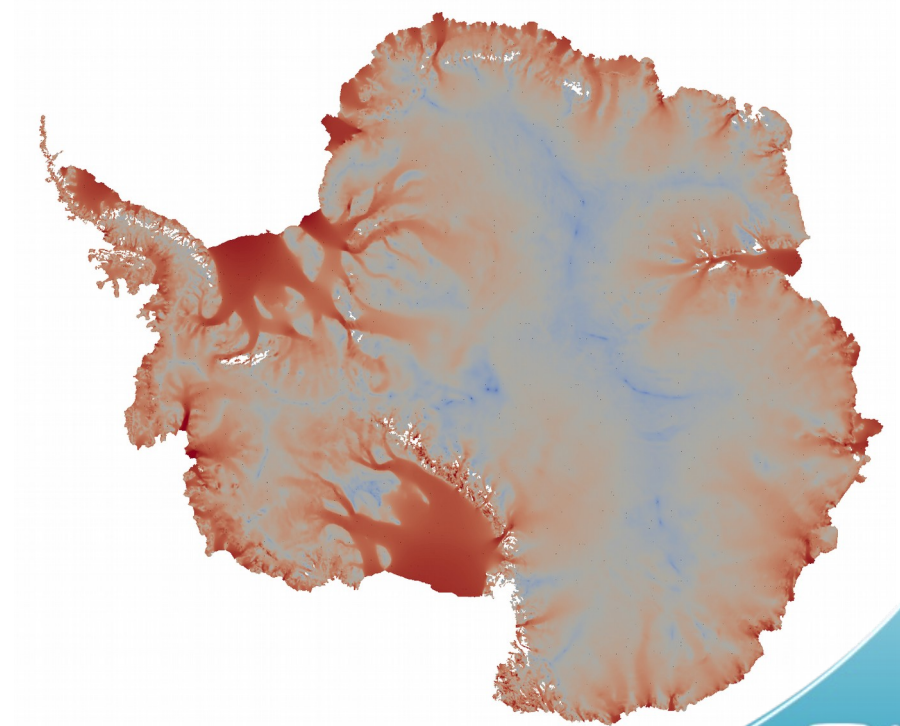
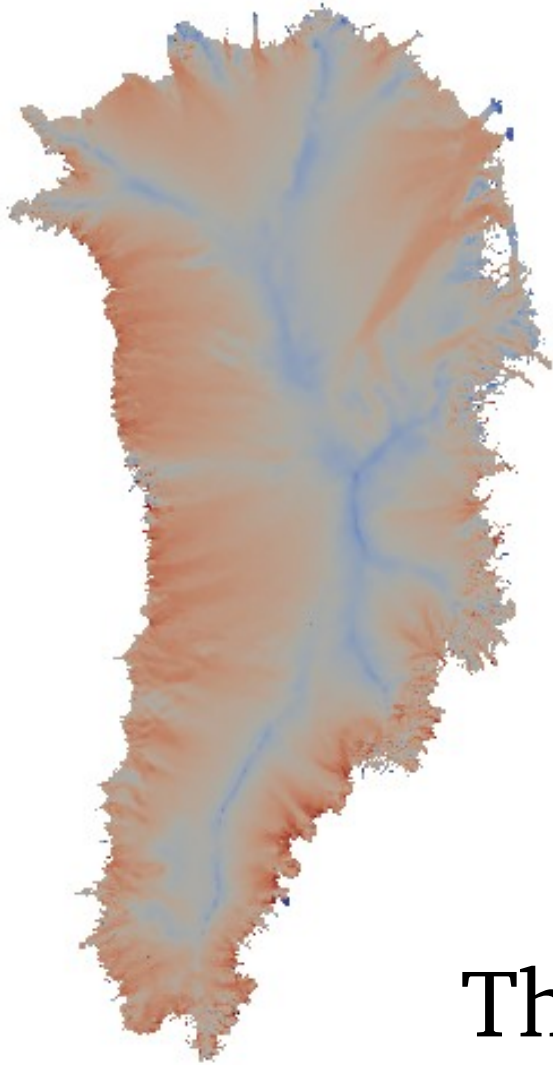


# On-going work

*Bayesian calibration / Uncertainty propagation*

*(w/ M. Eldred, C. Jackson (U. Texas), J. Jakeman, I. Kalashnikova, G. Stadler (Courant) , A. Salinger)*

*Use Hessian of deterministic inversion to estimate Covariance of basal friction distribution (N. Petra, G. Stadler, O. Ghattas)*



Thank you!