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Resiliency of the Power Grid

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Essentially, all models are wrong, but some are useful.

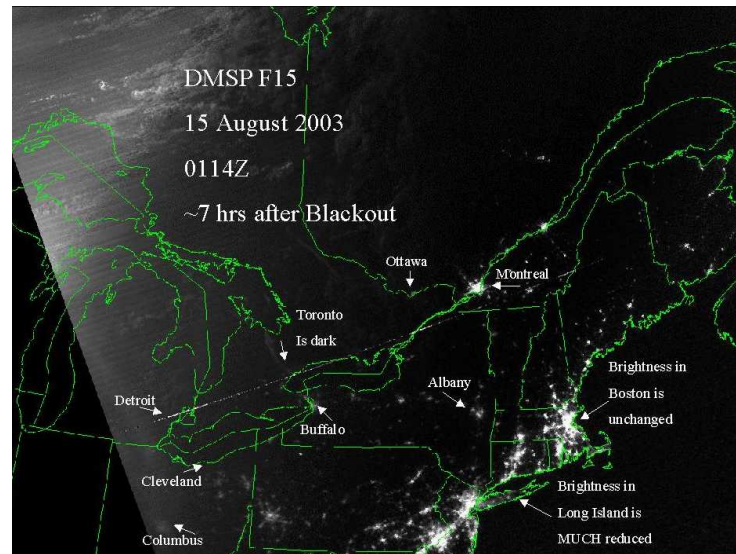
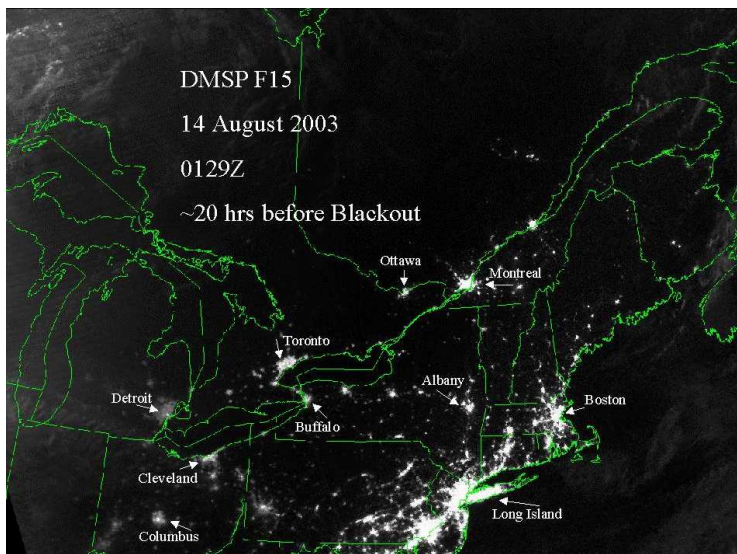
George E. P. Box

Graph models are useful.

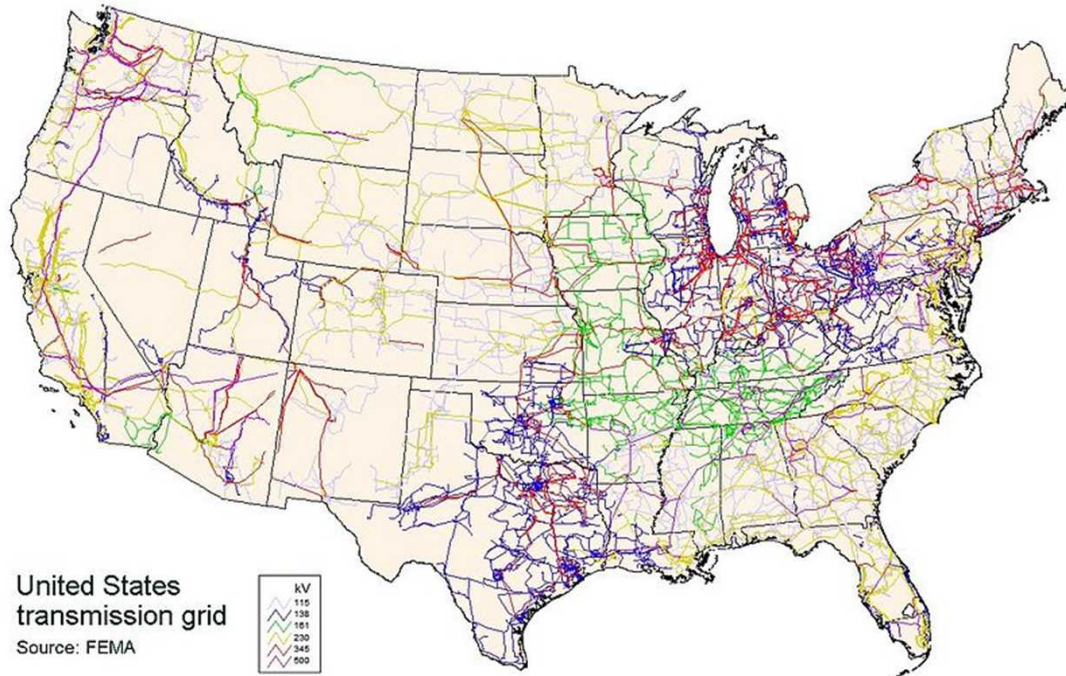
Power blackouts are a global problem



- August 2003 blackout affected 50 million people in New York, Pennsylvania, Ohio, Michigan, Vermont, Massachusetts, Connecticut, New Jersey, Ontario.
- The time to recover from the blackout was as long as 4 days at an estimated cost of \$4-10 B
- Similar occurrences elsewhere: India (2012), Brazil and Paraguay (2009), France-Switzerland-Italy (2003)



The grid's vulnerability increases with its growing complexity



Problem: the current standard requires the system to be resilient to only one failure, because higher standards are not enforceable.

- Uncertainty inherent in many renewable resources and the increasing load on the system force us to operate closer to the feasibility boundary.

Goal:

- detect vulnerabilities of the power network
- Include contingency analysis as a constraint in systems planning

Northeast blackout started with **three** broken lines.

Why is this problem so hard?

Combinatorial complexity

Even the basic approximation lead to NP-hard problems, and heuristics lead to compromise from security.

Multi-level optimization

Defender-attacker-defender games lead to problems where an optimization problems is a constraint for another.

Complexity of power flow

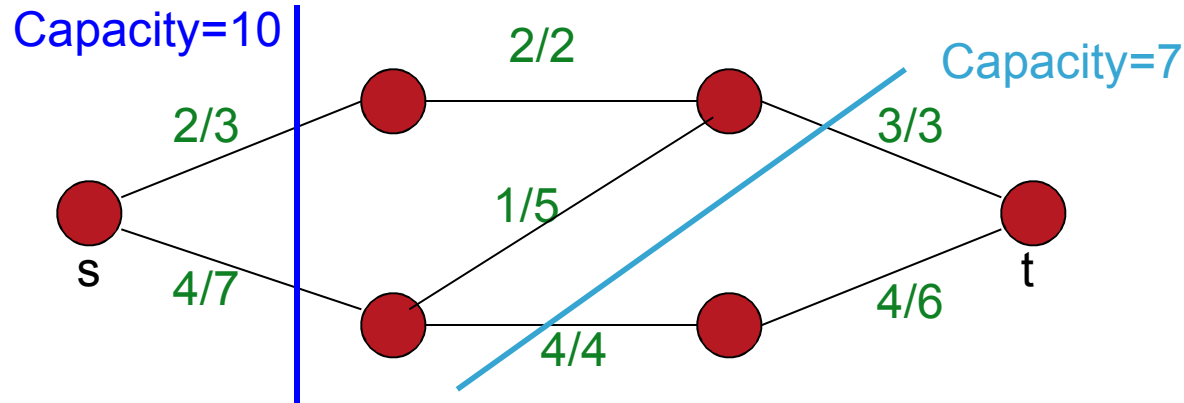
Flow of power can be modeled by differential algebraic equations, which are difficult to solve by themselves; Even steady state is described by nonlinear equations.

Uncertainty

Power system operations inherently involve uncertainties, which is increasing with renewable penetration.

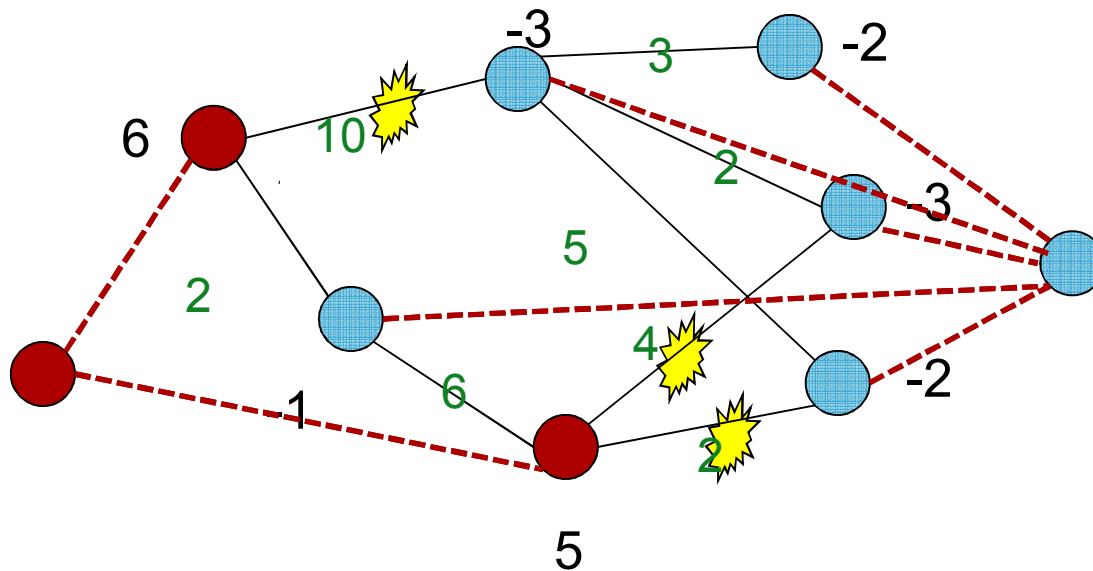
What can we solve?

Maximum-flow and minimum cut



- Given a graph with capacities on edges, a source, s and a terminal, t , push as much flow as possible from s to t .
- Cut is a bipartitioning of the vertices into S and T , so that s in S and t in T .
 - Capacity of a cut is the cumulative capacity of edges between S and T .
 - Min-cut is a cut with minimum capacity.
- Volume of max-flow = capacity of a min-cut.

What can we solve? Network inhibition problem



$k = 0$, max-flow= 11

$k = 1$, max-flow= 7

$k = 2$, max-flow= 5

$k = 3$, max-flow=1

- Cut min. number of lines so that max flow is below a specified bound.
- Shown to be NP-complete (Phillips 1991).
- The classical min-cut problem is a special version of network inhibition, where max-flow is set to zero.
- Can be formulated as MILP with $|V| + |E|$ binary variables.

How useful is this model?

The Good

- We can solve network interdiction problems.

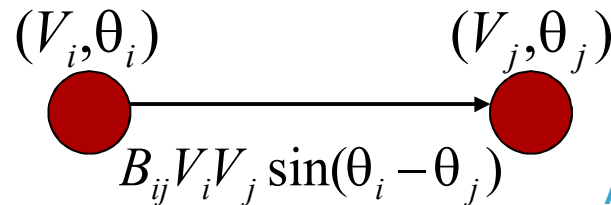
The Bad

- Flow of power is different from the flow model, and thus the results are not applicable.

The Beautiful

- Understanding the combinatorial structure can guide heuristic techniques that model the power flow more accurately.
- Further analysis will show that the gap between the two models is smaller than it looks.

Power flow is guided by potential difference between nodes



Active power

V : voltage

θ : phase angle

B : susceptance

$$B_{ij} V_i V_j \cos(\theta_i - \theta_j)$$

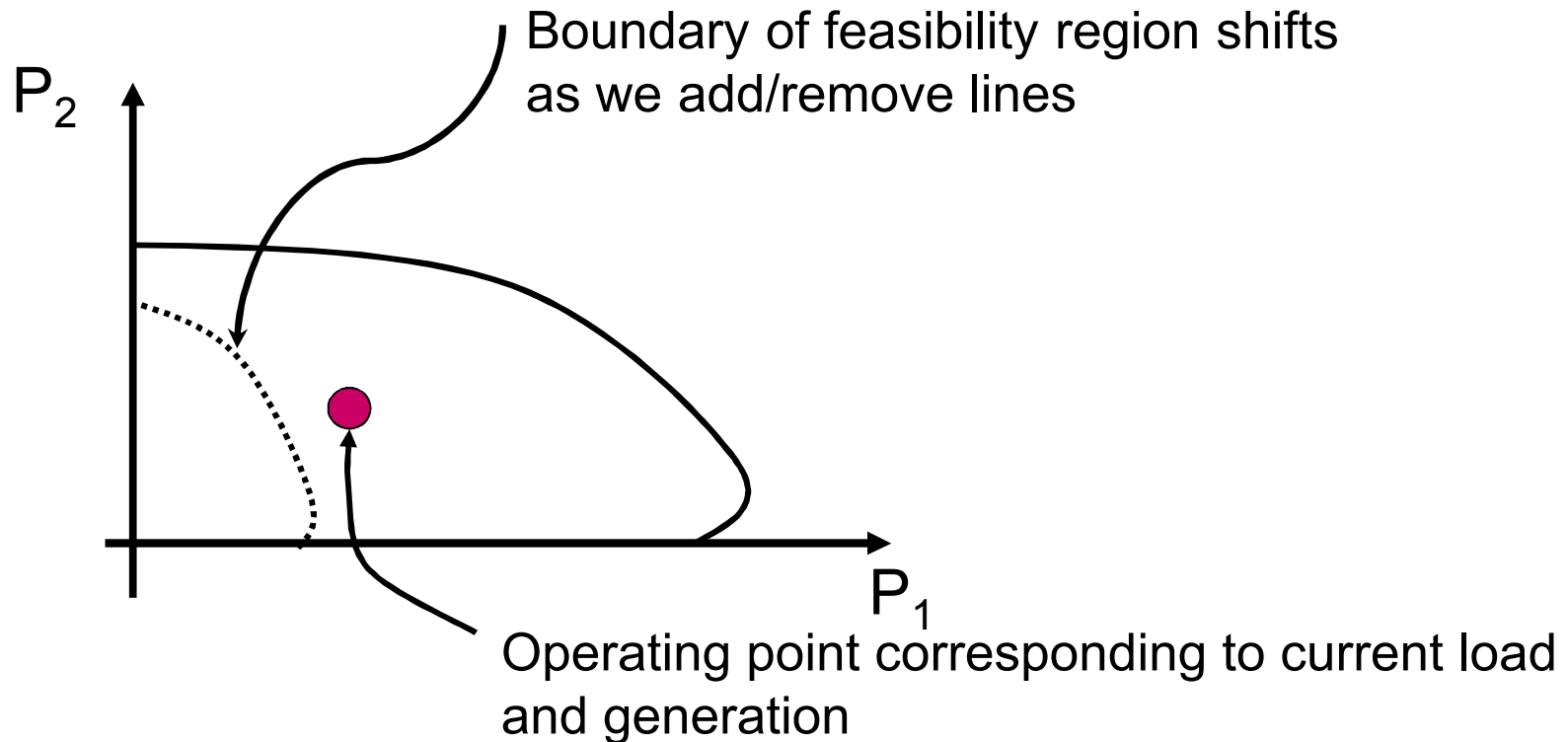
Reactive power

$$-\frac{\pi}{2} \leq \theta_i - \theta_j \leq \frac{\pi}{2}$$

$$V_l \leq V \leq V_u$$

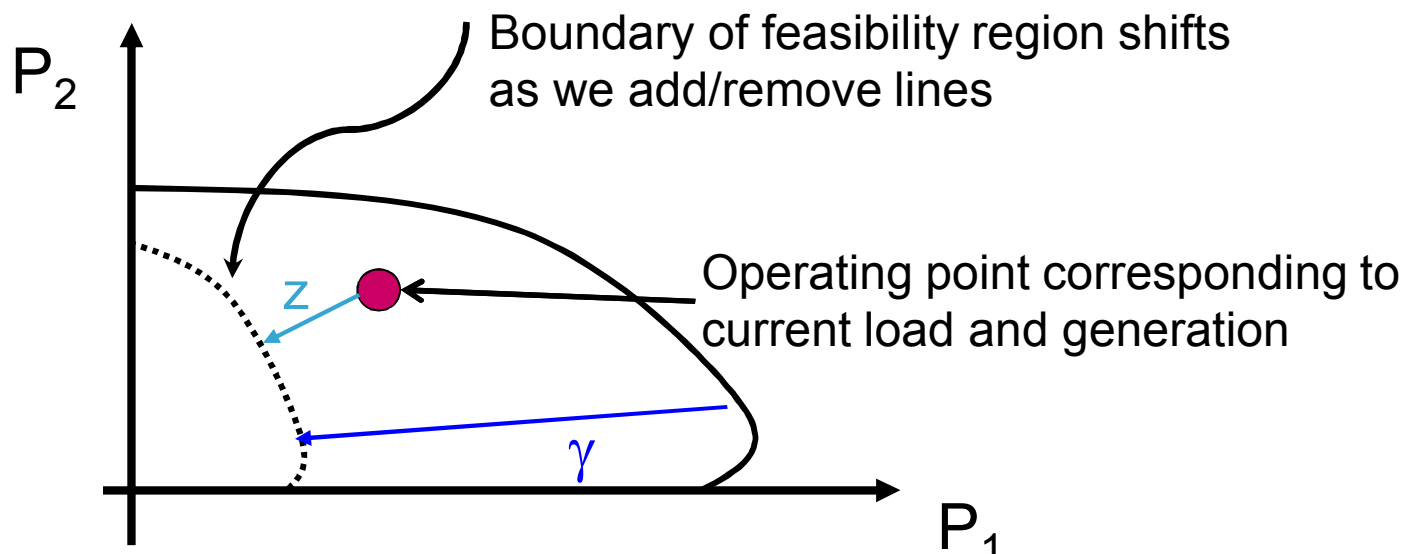
- Simplified model
$$F(A, \theta, p) = A^T B \sin(A\theta) - p = 0$$
 - Fix voltages at 1; work only on active power equations.
- **A** is the node-arc incidence matrix that describes grid topology.
- **p** is the state of the system.
- **Sin(Aθ)** is a diagonal matrix whose entries are the angular difference.

Pictorial representation of a blackout



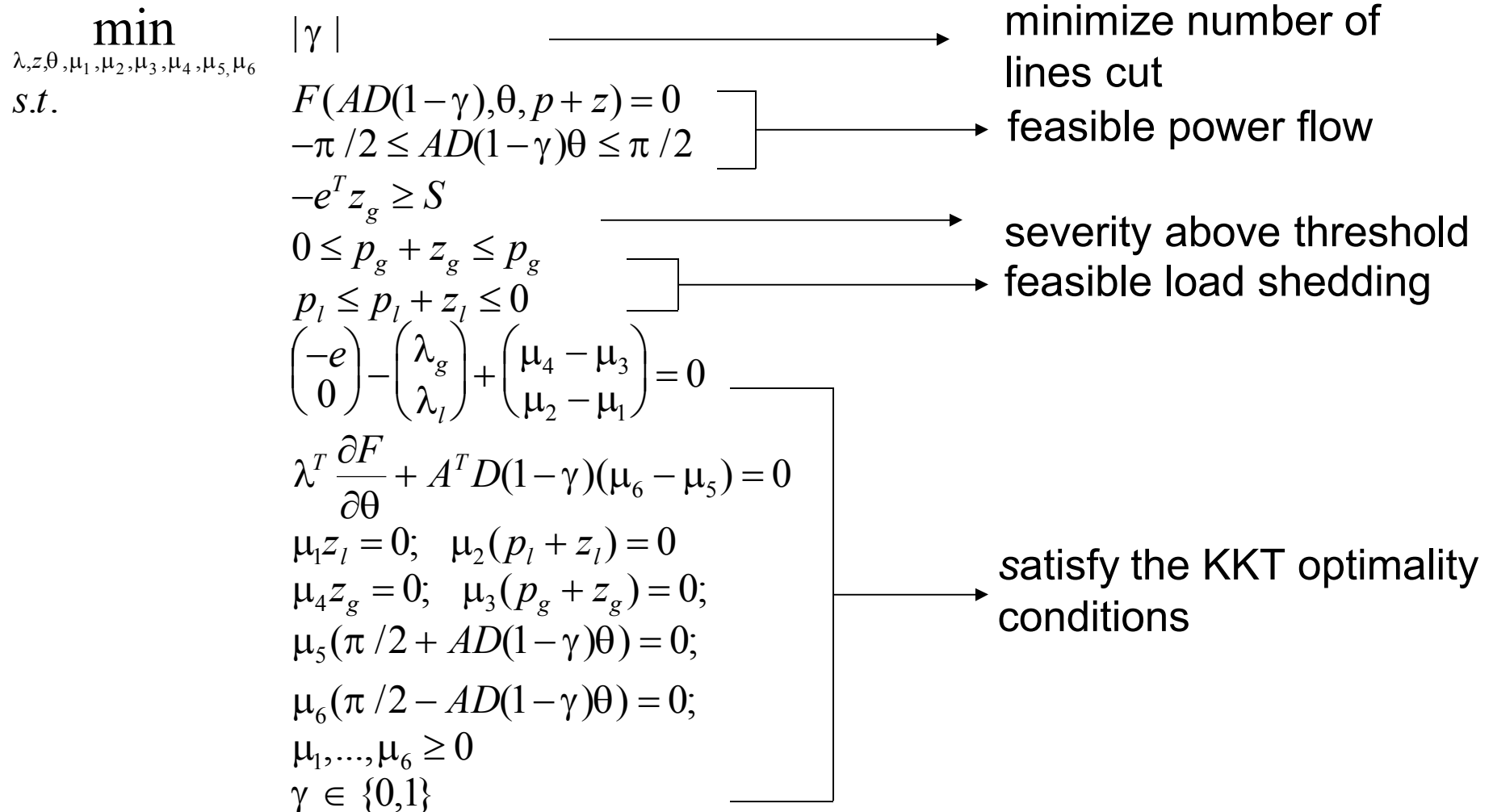
- ❖ Infeasibility of power flow equations lead to a blackout.
- ❖ Cascading is initiated by a significant disturbance to the system.
- ❖ Our focus is detecting initiating events and analyzing the network for vulnerabilities.

Contingency analysis as a bi-level optimization problem

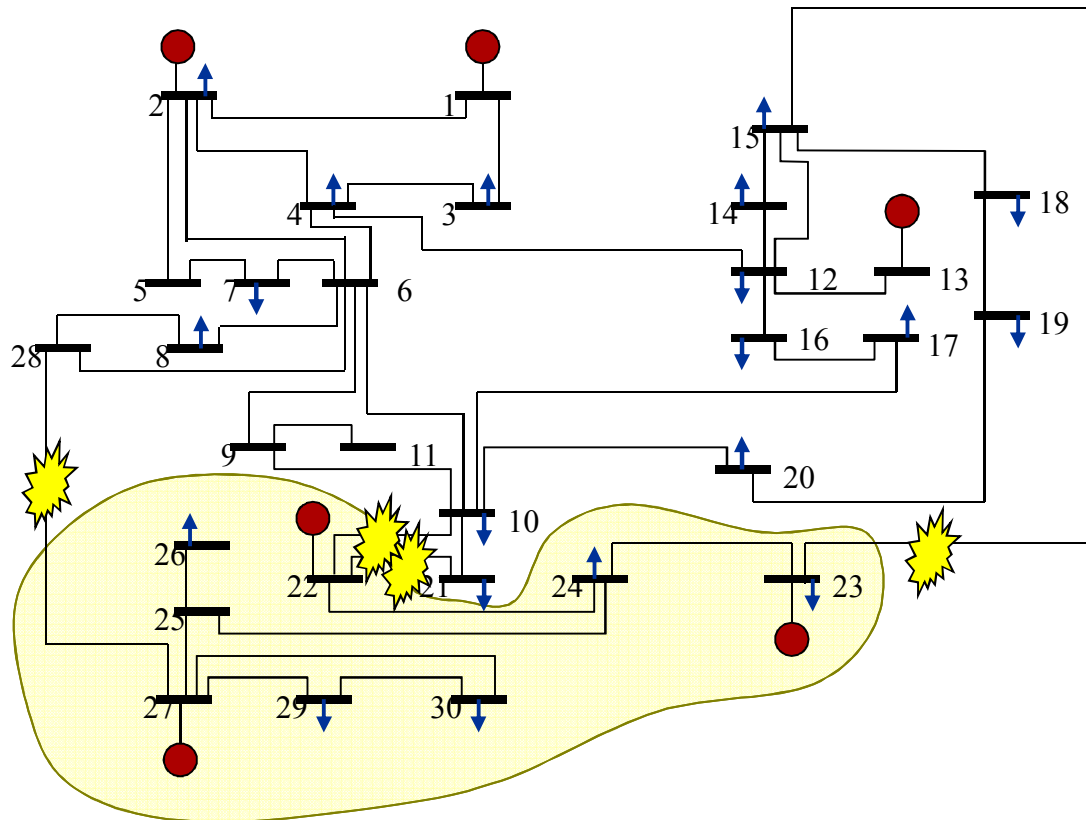


- Add integer (binary) line parameters, γ , to identify broken lines
- Measure the blackout severity as the distance to feasibility boundary
 - The load shedding problem
- Bilevel-MINLP problem
 - cut **minimum** number of lines so that
 - **shortest** distance to feasibility (i.e. severity) is at least as large as a specified target
- Mangasarian Fromowitz constraint qualification conditions are satisfied for a slightly modified system.

This approach leads to a Mixed Integer Nonlinear Program (MINLP)



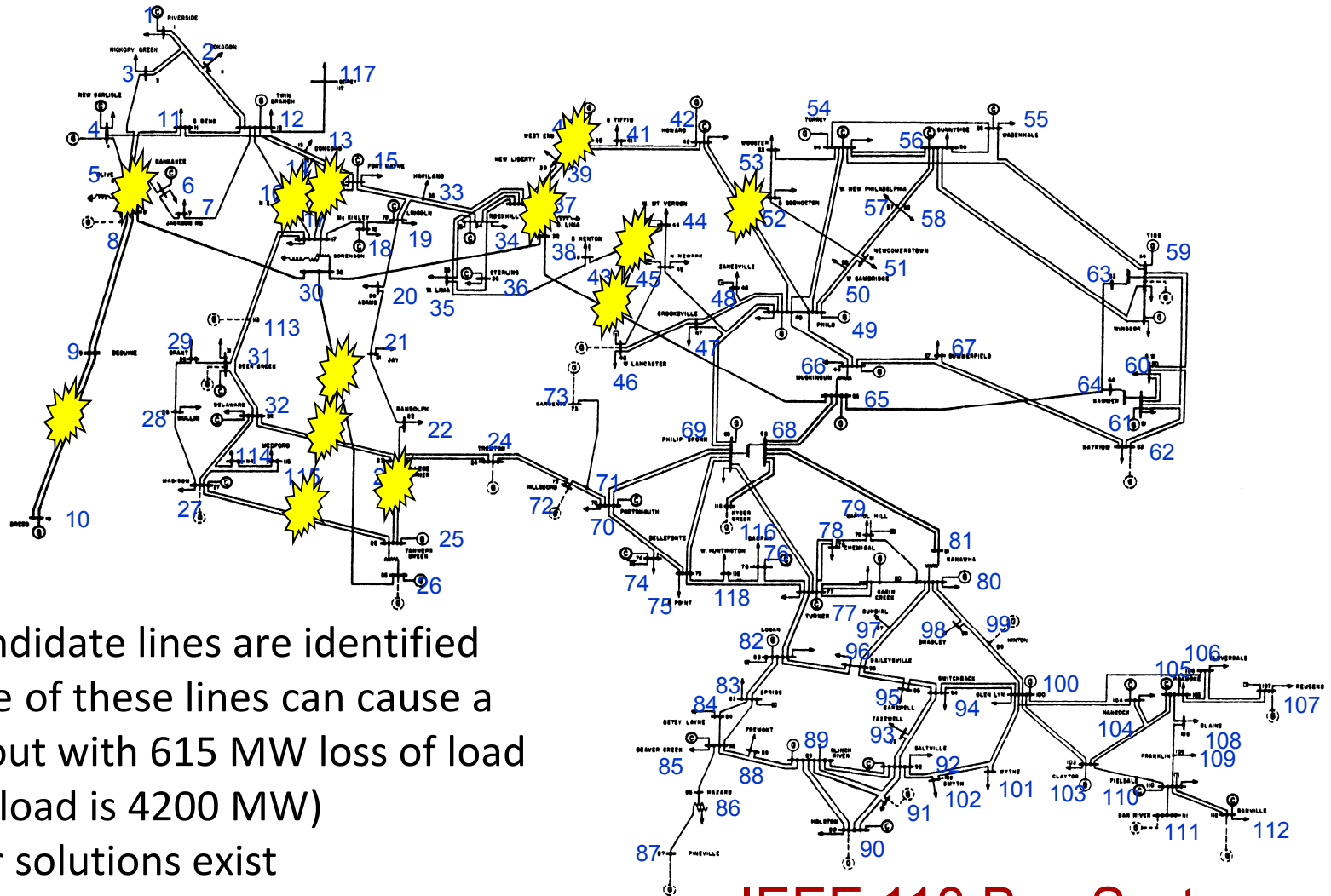
Relaxation works on small problems



IEEE 30-Bus System

- Four candidate lines identified.
- Two are sufficient to cause a blackout.
- Failure of these lines can cause a blackout with 843 MW loss out of a total load of 1655 MW).
- Solutions found using SNOPT.

.... but not on larger problems



- 13 candidate lines are identified
- Failure of these lines can cause a blackout with 615 MW loss of load (total load is 4200 MW)
- Better solutions exist

IEEE 118 Bus System

Power-flow Jacobian corresponds to the graph Laplacian

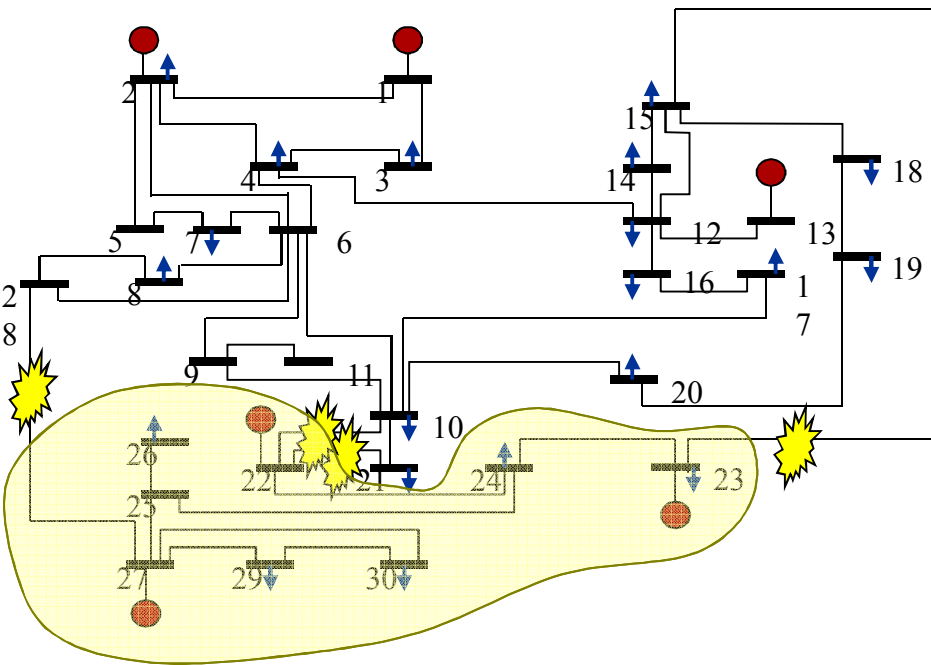
- **Key observation:** *The Jacobian matrix, which characterizes the feasibility boundary, has the same structure as the Laplacian in spectral graph theory.*

$$\frac{\partial F}{\partial \theta} = J = \underbrace{A^T}_{\text{Node-arc incidence matrix}} \underbrace{BD((1-\gamma)\cos(A\theta))}_{\text{Diagonal matrices with non-negative weights}} \underbrace{A}_{\text{Node-arc incidence matrix}}$$

- Feasibility of the optimization problem pushes the Jacobian towards a second zero eigenvalue.
- Second zero eigenvalue corresponds to disconnected regions in the graph.

P. et al., Optimization Strategies for the Vulnerability Analysis of the Power Grid, SIAM J. Optimization, Vol. 20, No. 4, pages 1786-1810, 2010;

Feasible solutions have a combinatorial structure



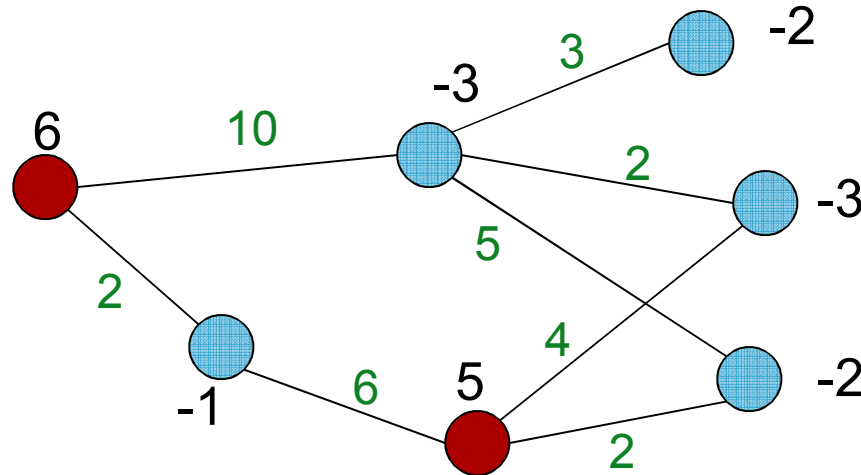
Theoretical analysis of the bilevel MINLP formulation shows:

- System is split into load-rich and generation-rich regions.
- There is at least one saturated line from the generation rich region to the load rich region.
- Blackout size can be approximated by the generation/load mismatch and capacity of edges in between.

Practical application: Exploit the combinatorial structure to find a loosely coupled decomposition with a high generation/load mismatch

Take 2: Vulnerability analysis as a combinatorial problem

Find minimum of number of **power lines**, whose removal decomposes the network into generation-rich and load-rich regions such that the **excess generation of one part minus the capacity of the lines between the two parts** is above a threshold.



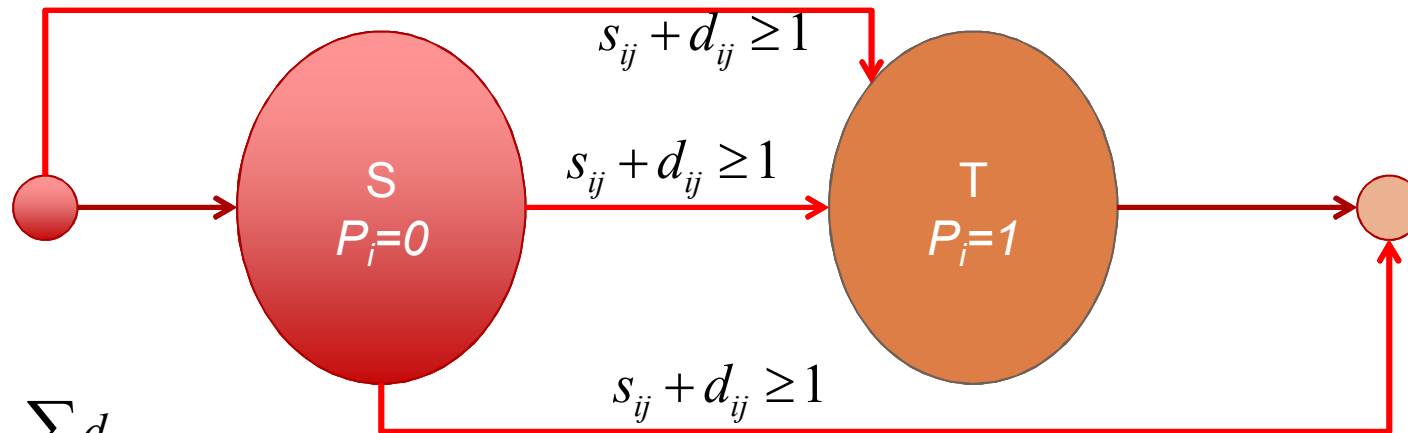
Find minimum of number of **edges**, whose removal **leads to a cut** such that the **capacity of the cut** is below a threshold.

Can we work on a nonlinear model without solving nonlinear equations?



- It is not free lunch, but a good deal.
- Why does it work?
 - We are not proposing a power flow model, we only find why power is not flowing.
 - This is a flow problem.
 - The goal of the load shedding problem is to make this model work.
- What is missing?
 - We can document the underlying assumptions that lead to the graph model.
 - But we cannot quantify the gap between the two models, ... yet.
 - We need a representation of a typical cases.

MILP formulation for network inhibition



$$\min \sum d_{ij}$$

$$s.t. \quad \forall (v_i, v_j) \in E \quad \begin{aligned} p_i - p_j - s_{ij} - d_{ij} &\leq 0 \\ p_i - p_j + s_{ij} + d_{ij} &\geq 0 \end{aligned}$$

$$\sum_{(v_i, v_j) \in E} c_{ij} s_{ij} \leq B$$

$$p_s = 0; \quad p_t = 1$$

$$p_i, d_{ij} \in \{0,1\}; \quad s_{ij} \in [0,1]$$

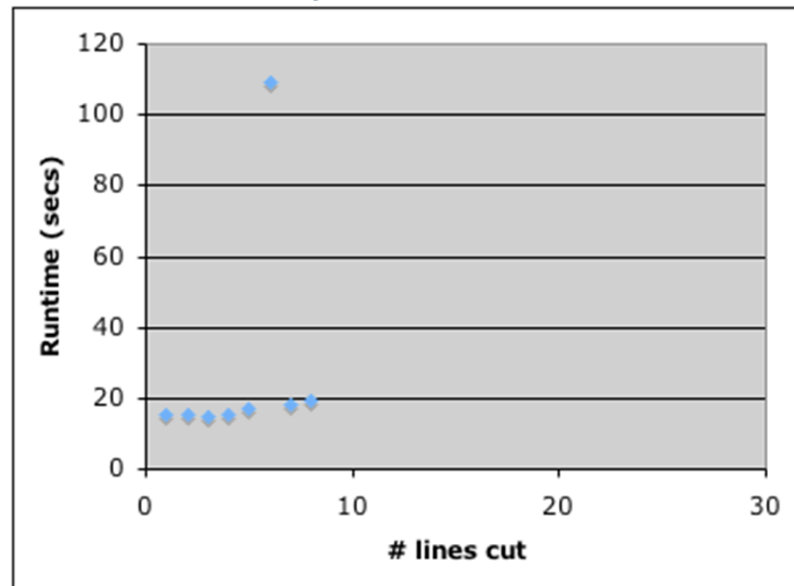
$$p_i = \begin{cases} 0 & v_i \in S \\ 1 & v_i \in T \end{cases} \quad d_{ij} = \begin{cases} 1 & \text{if } e_{ij} \text{ is cut.} \\ 0 & \text{otherwise} \end{cases}$$

$$s_{ij} = \begin{cases} 1 & d_{ij} = 0 \wedge p_i \neq p_j \\ 0 & \text{otherwise} \end{cases}$$

The integrality gap is small, leading to fast solutions.

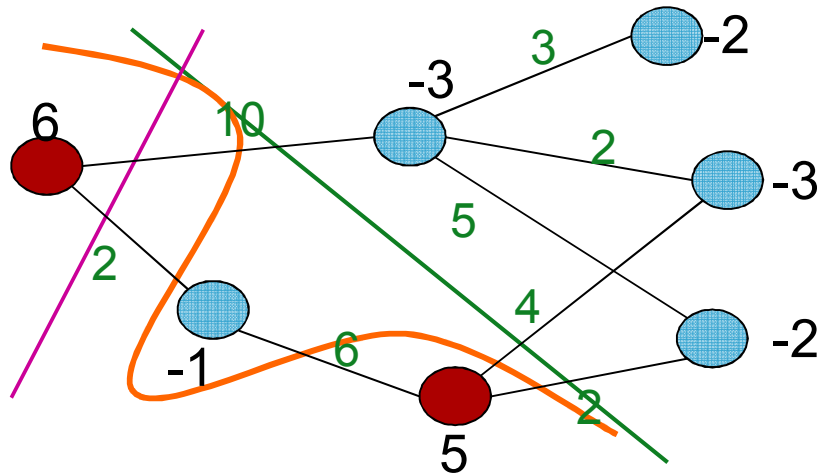
C. Burch, R. Carr, S. Krumke, M. Marathe, C. Phillips and E. Sundberg, A decomposition-based approximation for network inhibition, Network Interdiction and Stochastic Integer Programming, D.L. Woodruff, eds., (2003), pp. 51–66.

Instances of the network inhibition problem can be solved efficiently



- The integrality gap is provably small.
- Only one fractional variable after each solution.
- Experimented on a simplified model for Western states with 13,374 nodes and 16,520 lines, used PICO for solving the MILPs.
- Even the largest instances can be solved in small time, motivating us for more higher objectives.

Take 3: Inhibiting bisection problem



- Divide graph into two parts (bisection) so that
 - load/generation mismatch is maximum.
 - cutsize is minimum.

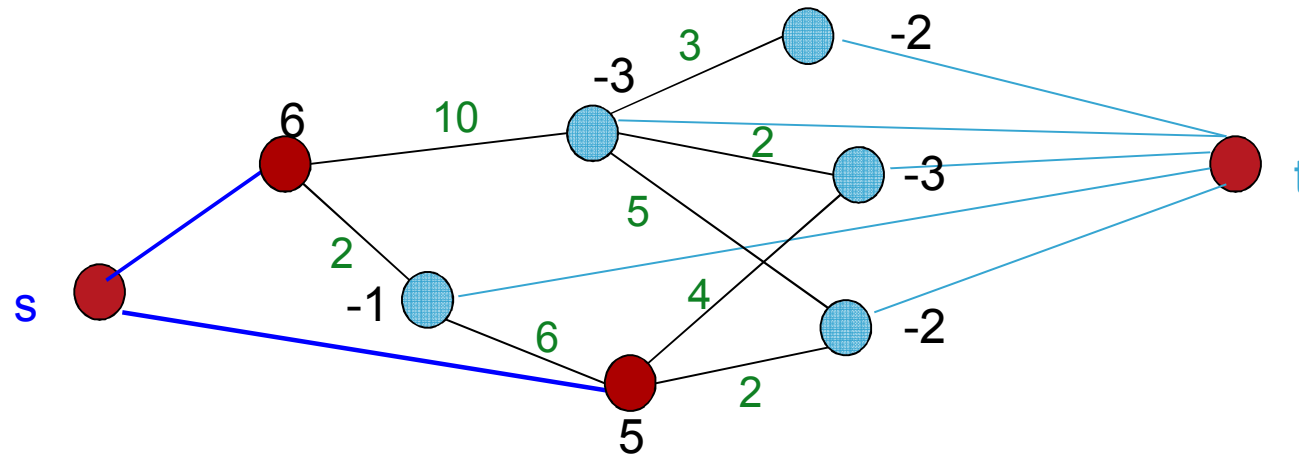
imbalance= 6; cutsize=2

imbalance=10; cutsize=3

imbalance=11; cutsize=5

Lesieutre et al., Power system extreme event analysis using graph partitioning, NAPS 2006.

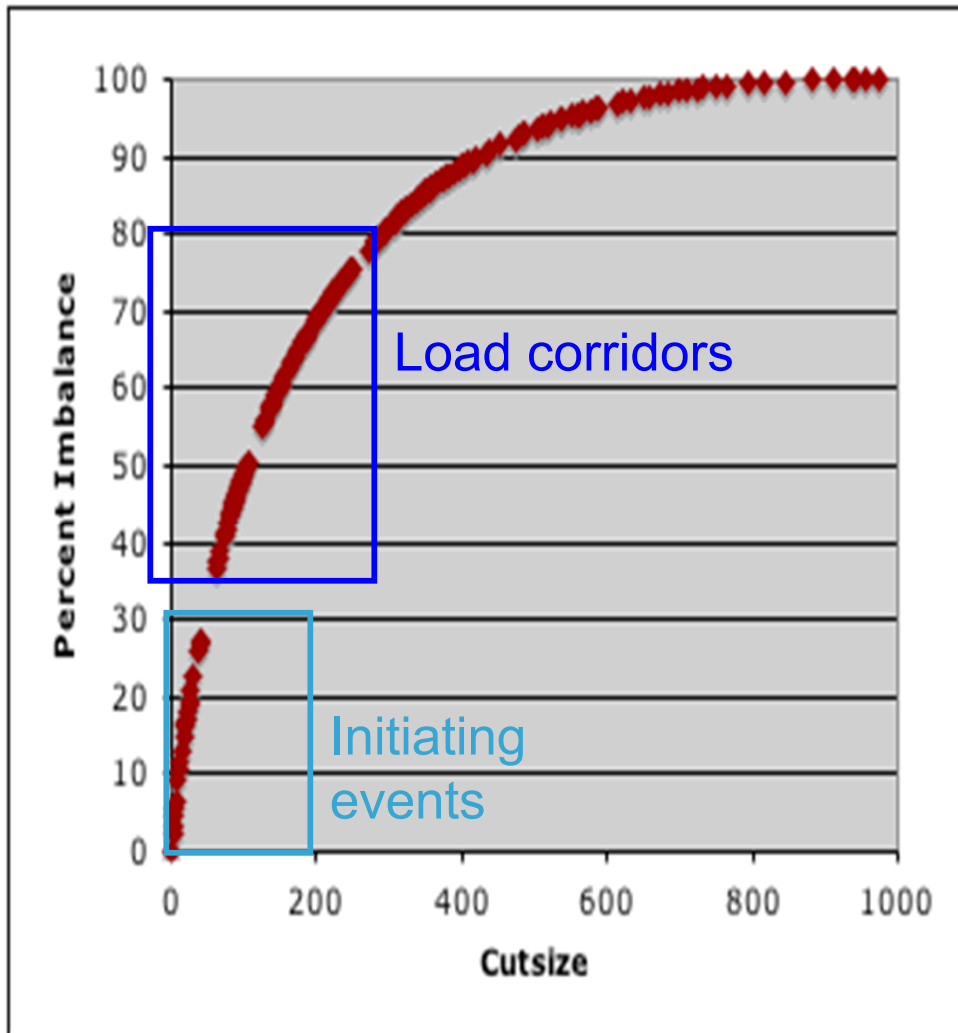
Solving the inhibiting cut problem



- Constrained problem is NP-complete.
- Goal: minimize $\alpha (\text{cutsize}) - (1 - \alpha) \text{ imbalance}$
 - α is the relative importance of cutsize compared to imbalance.
- Solution: use a standard min-cut algorithm.
- Min-cut gives an *optimal* solution to the linearized inhibiting bisection problem.
- Other versions are solvable
 - Minimize cutsize/imbalance
 - Minimize $\text{capacity} * (\text{cutsize} - 1) / \text{cutsize}$

P., Fogel, and Lesieutre, The Inhibiting Bisection Problem, Technical report: LBNL-62142, LBNL.

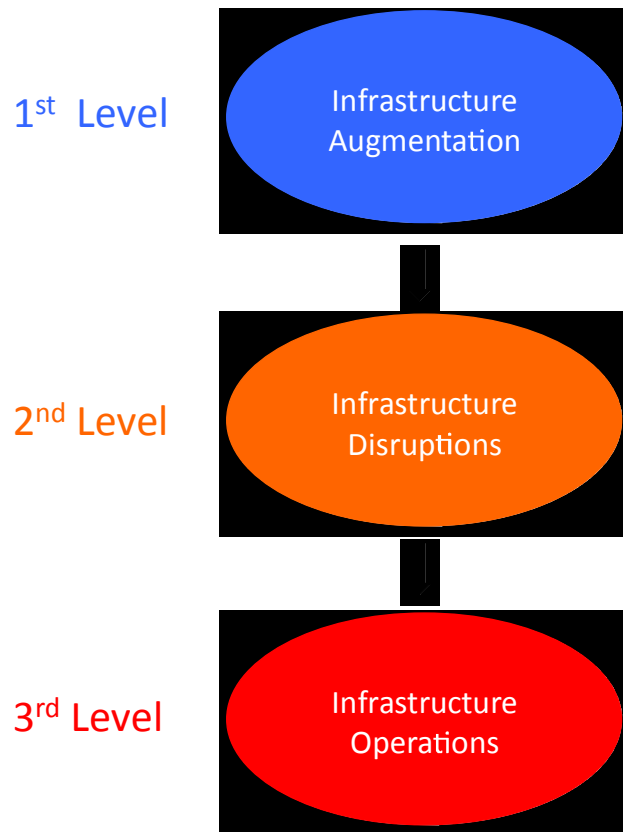
Inhibiting bisection enables fast analysis of large systems



- Experimented on a Simplified model for Western states with 13,374 nodes and 16,520 lines.
- Complete analysis using Goldberg's min-cut solver takes minutes
- Solutions with small cutsizes can be used to detect **initiating events** and groups of vulnerabilities
- Solutions with medium cutsizes reveal **load corridors** that can be used to contain cascading.

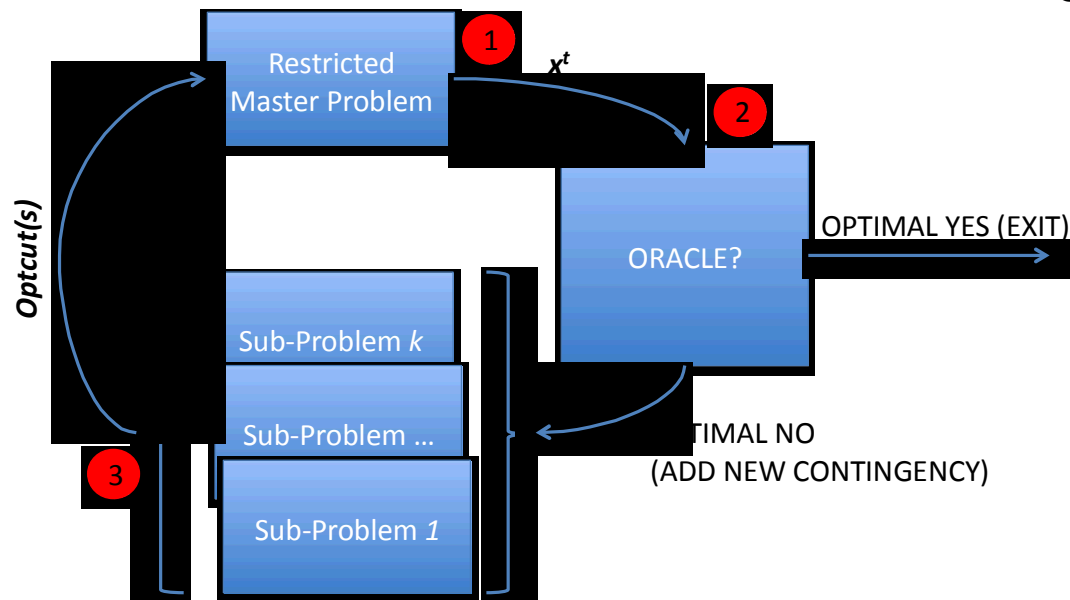
Lesieutre, P., and Roy, Power System Extreme Event Detection: The Vulnerability Frontier, HICSS, 2008.

Finding vulnerabilities is good; fixing them is better: N-k survivable network design problem



- Improve a network efficiently to make it resilient to contingencies
 - **Minimize** the improvement cost such that the **minimum** number of lines for the **maximum** flow to be below a threshold B is above a threshold C.
- Solution approaches:
 - A single problem with a separate set of constraints for each contingency
 - forms a giant problem
 - Bender's decomposition
 - limits the memory requirements
 - the number of subproblems is still very large, prohibitively expensive for large N and k.
 - Proposed Method: Delayed Contingency Generation

Delayed contingency generation



■ Outline of the algorithm

- Solve a restricted master problem to identify candidate lines to add.
- Solve the network inhibition problem
- If we cannot break the network, current solution is optimal
- If not, add a constraint to the master problem for the identified vulnerability.
- New constraints exclude regions of the solution space, as opposed to individual solutions.

- Efficient solution of the interdiction problem is the key enabler.
- The same framework can be applied to planning and operational problems
- We have applied this approach to
 - Unit commitment
 - Transmission and generation expansion

Chen et al. Contingency-Constrained Unit Commitment with Post-Contingency Corrective Recourse, to appear in Annals of Operations Research

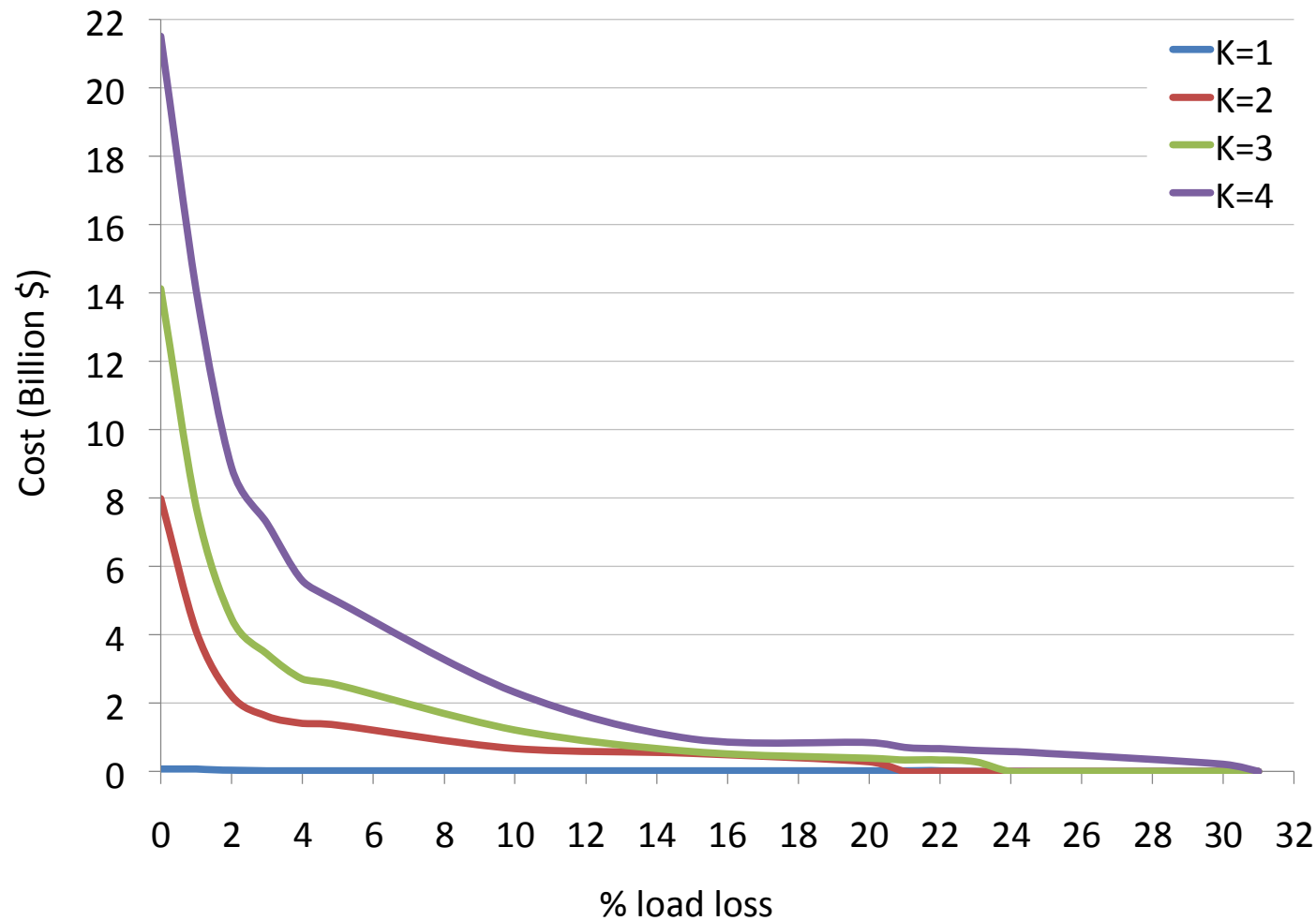
Delayed contingency analysis limits contingencies being considered.

IEEE Test Systems	N	K	No. of contingencies	Total time	MP time	NIP time	SP time	No. of contingencies evaluated
30	82	1	82	0	0	0	0	3
118	358	1	358	4	0	2	1	17
179	444	1	444	19	1	7	10	51
30	123	2	>7K	1	0	1	0	15
118	537	2	>140K	41	3	26	12	58
179	666	2	>200K	174	6	50	118	158
30	164	3	>700K	9	2	5	2	43
118	716	3	>60M	398	25	303	70	128
179	888	3	>116M	653	21	193	439	284
30	205	4	>72M	67	7	23	37	156
118	895	4	>26B	2,708	399	1,698	612	359
179	1110	4	>63B	11,999	4,939	1,822	5,237	899

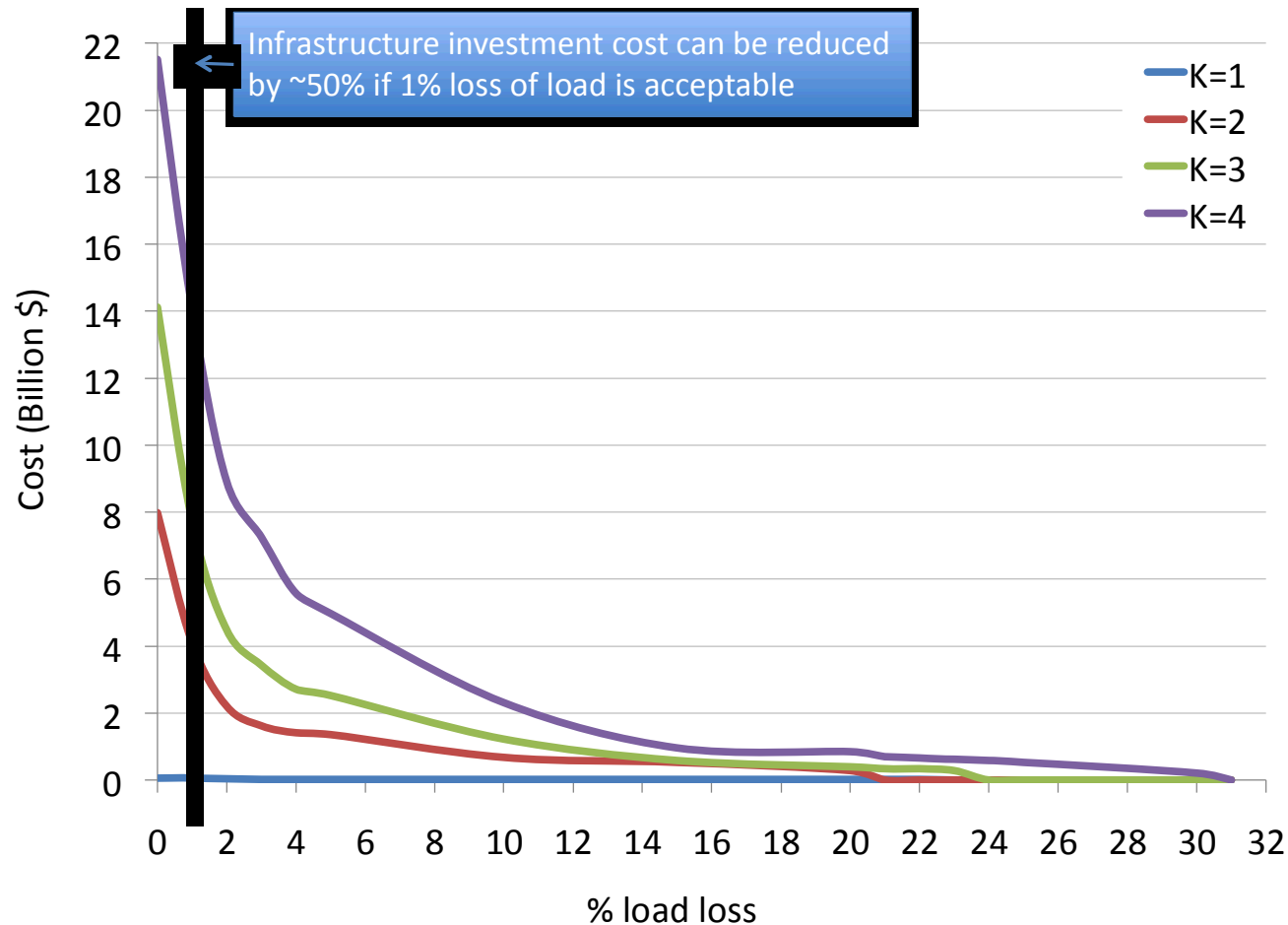
■

Chen et al., Contingency-Risk Informed Power System Design, to appear in IEEE T. Power Systems

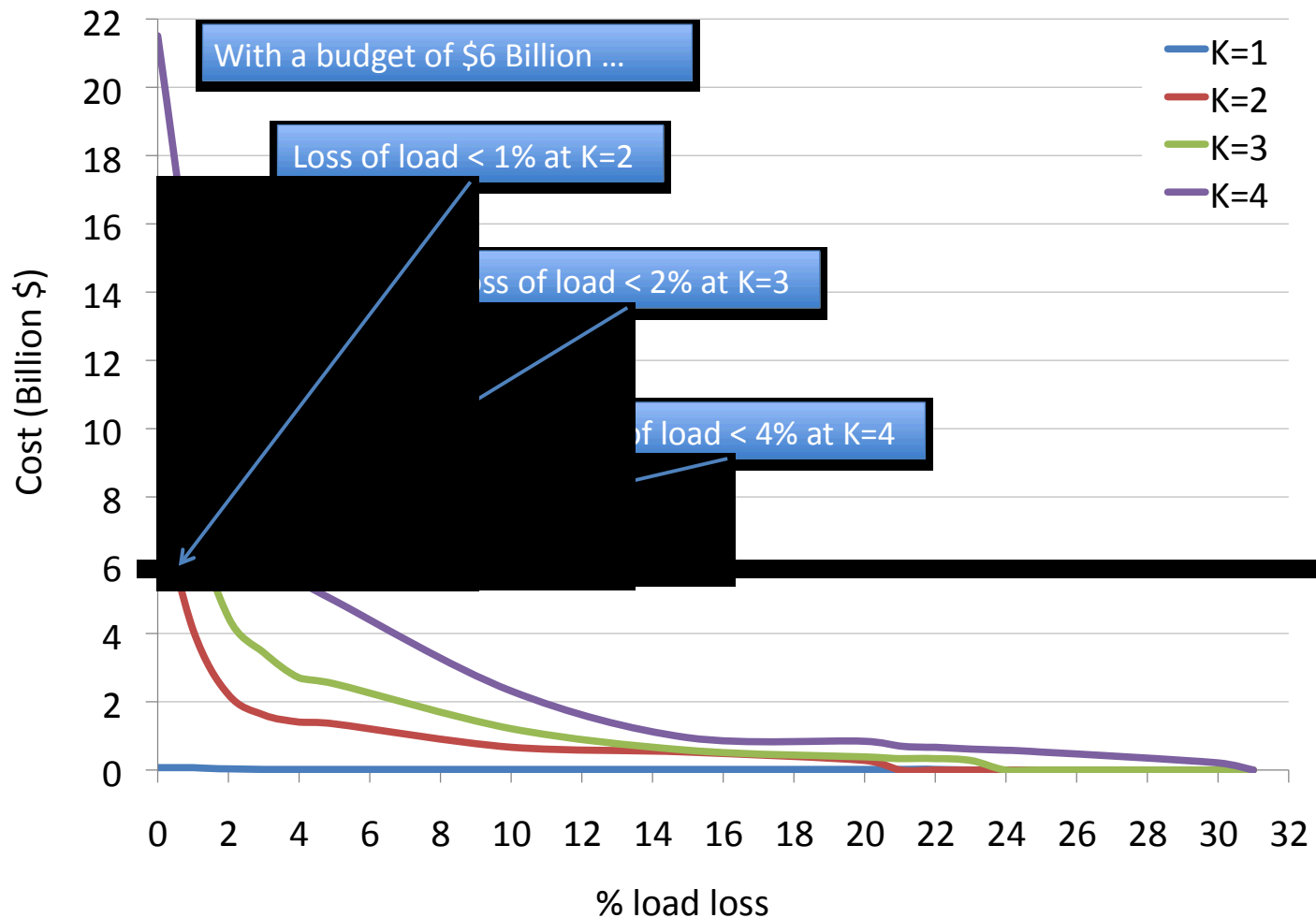
Fast solutions enable analyses of the decision space



Cost of perfectness



Benefits of humbleness



Next Step: Uncertainties of renewables pose a crucial challenge for grid operations



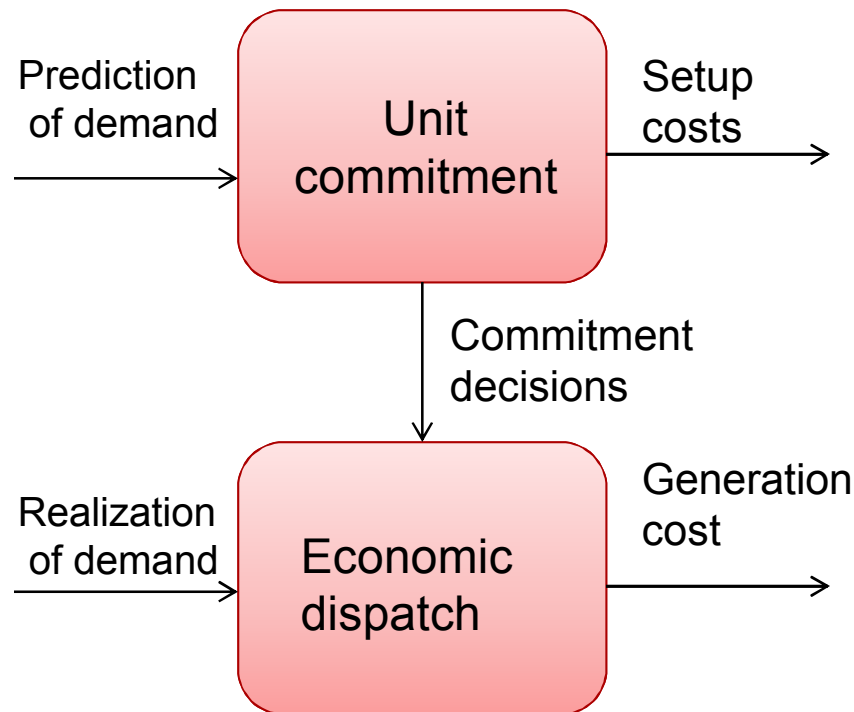
Source: <http://saferenvironment.wordpress.com> [Full link](#)



- Most renewable resources cannot be controlled and involve significant uncertainties.
- High penetration of renewables lead to a significant change in operations due to uncertainty.
- Storage technologies are not adequate enough, yet.
- Operations require decision making under uncertainty.
 - Stochastic optimization is essential.
 - Better models for handling uncertainty are needed.

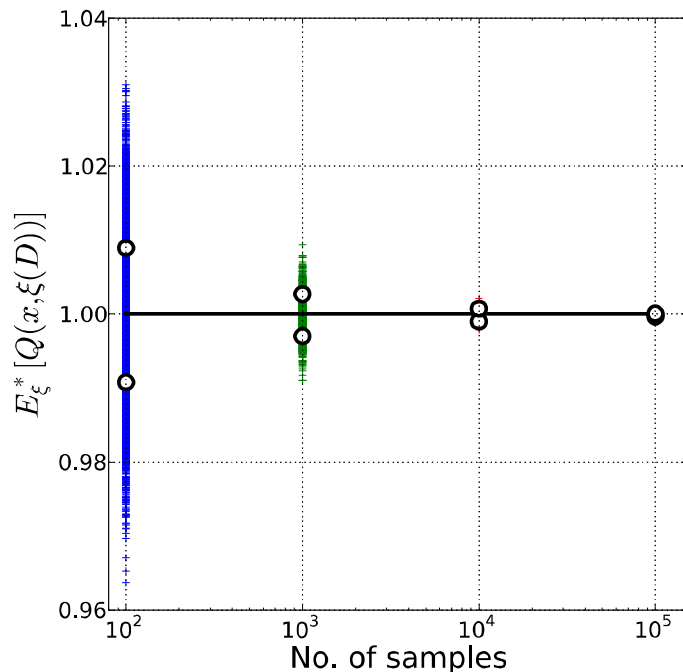
Source: <http://www.thesierraconetelegraph.com/?p=5393>

Operational problems require stochastic optimization



- Sample Problem: Unit Commitment
- Fundamental problem in operations
- Two stage problem
 - Decide on the state of big and slow generators under a prediction of demand/ renewables
 - Operate on a realization of uncertainties to minimize generation costs
- Standard approach: Monte Carlo sampling

Efficient Model for Uncertainty: Polynomial Chaos Expansion



Thiam and DeMarco: "Simply put, when uncertainty is credibly accounted for such methods yield solutions for economic benefit of a transmission expansion in which the "error bars" are often larger than the nominal predicted benefit."

- Error for Monte Carlo
 $\text{Var}(f)/\sqrt{S}$
- Accurate estimations render optimization problems impractical.
- Proposed Solution: Polynomial chaos expansion
 - Commonly used for uncertainty quantification in CSE applications
 - Core idea: preprocess the random variables to build a surrogate that represents random variables compactly
- Promising Initial results:
 - Currently working on adding this to the optimization loop

Safta et al., Toward Using Surrogates to Accelerate Solution of Stochastic Electricity Grid Operations Problems, Proc. NAPS 2014

Concluding remarks

- The grid's vulnerability grows with its growing complexity.
 - We need to do more with less.
 - Renewables bring additional challenges.
- Enforceable security metrics are essential.
 - Problem is extremely complex, thus finding the right abstractions is a challenge.
 - Formal description of the grid, at least some of its properties will boost algorithmic advances.
 - Integration of system dynamics remain as a challenge.
- We have made significant advances in vulnerability analysis.
 - Special structure of a feasible solution to our MINLP formulation can be exploited for a simpler approach for vulnerability detection.
 - Our combinatorial techniques can analyze vulnerabilities of large systems in a short amount of time.
- We can now incorporate vulnerability analysis as a constraint to grid operations and planning problems.
 - Delayed contingency generation approach shows promising results for N-k-e contingency constrained problems.
- We need novel approaches for handling uncertainties.
 - Monte Carlo algorithms cannot provide the desired accuracy in estimations.

Acknowledgements

- Richard Chen, Amy Cohn, Vaibhav Donde, Neng Fan, Yonatan Fogel, Bernard Lesieutre, Vanessa Lopez, Juan Meza, Habib Najm, Adam Reichert, Sandip Roy, Cosmin Safta, Jean-paul Watson, and Chao Yang contributed to this work.

Announcements

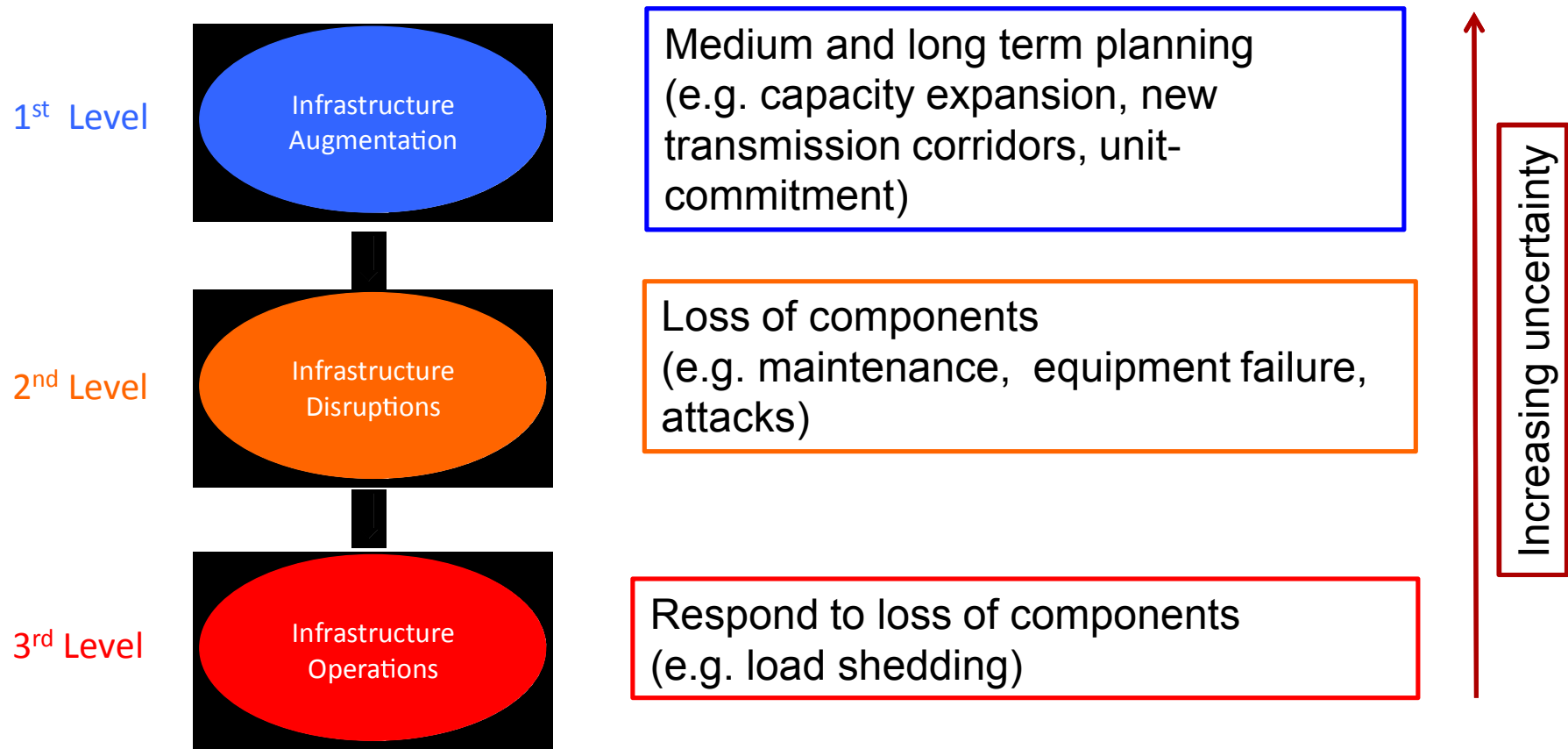
- We always try to create positions for talented researchers.
 - Please contact me
- Network Science under SIAM
 - [3rd SIAM Workshop on Network Science](#), Snowbird, UT, May 17-19, 2015.
 - Co-located with the SIAM C. Dynamical Systems
 - Abstract due by Jan 19th.
 - Mailing list for network science
 - To subscribe, send an email to siam-ns-subscribe@siam.org

Related Publications

- Chen, Cohn, Fan, and P., "Contingency-Risk Informed Power System Design," to appear in IEEE T. Power Systems; preprint available as [arXiv:1305.0780](https://arxiv.org/abs/1305.0780)
- Chen, Fan, P., and Watson, "Contingency-Constrained Unit Commitment with Post-Contingency Corrective Recourse," under revision, preprint available as [arXiv:1404.2964](https://arxiv.org/abs/1404.2964)
- Safta, Chen, Najm, P., Watson, "Toward Using Surrogates to Accelerate Solution of Stochastic Electricity Grid Operations Problems," submitted for conference publication.
- Chen, Cohn, Fan, and P., "N-k-epsilon Survivable Power System Design," in 12th International Conference on Probabilistic Methods Applied to Power Systems (PMAPS12) [arXiv:1201.1530](https://arxiv.org/abs/1201.1530)
- Chen, Cohn, and P., "An Implicit Optimization Approach for Survivable Network Design," Proc. 2011 IEEE 1st International Network Science Workshop (NSW 2011) [arXiv:1109.1801](https://arxiv.org/abs/1109.1801)
- P., Meza, Donde, and Lesieutre, "Optimization Strategies for the Vulnerability Analysis of the Power Grid," *SIAM Optimization*, 20 (4), pp. 1786-1810, 2010.
- Donde, Lopez, Lesieutre, P., Yang, and Meza, "Severe Multiple Contingency Screening in Electric Power Systems," *IEEE T. Power Systems*, 23(2), pp. 406-417, 2008.
- Lesieutre, P., and Roy, "Power System Extreme Event Detection: The Vulnerability Frontier," in *Proc. 41st Hawaii Int. Conf. on System Sciences*, p. 184, HI, 2008.
- P., Reichert, and Lesieutre, "Computing Criticality of Lines in a Power System," *Proc. 2007 IEEE Int. Symp. Circuits and Systems*, pp. 65—68, New Orleans, LA, May 2007.
- Lesieutre, Roy, Donde, and P., "Power system extreme event analysis using graph partitioning," *Proc. the 39th North American Power Symp.*, Carbondale, IL, 2006.
- Donde, Lopez, Lesieutre, P., Yang, Meza, "Identification of severe multiple contingencies in electric power networks," *Proc. the 38th North American Power Symp.*, Ames, IA, October 2005.
- P., Fogel, and Lesieutre, "The Inhibiting Bisection Problem," Technical Report: LBNL-62142, Lawrence Berkeley National Laboratory, Berkeley, CA.

- Questions?

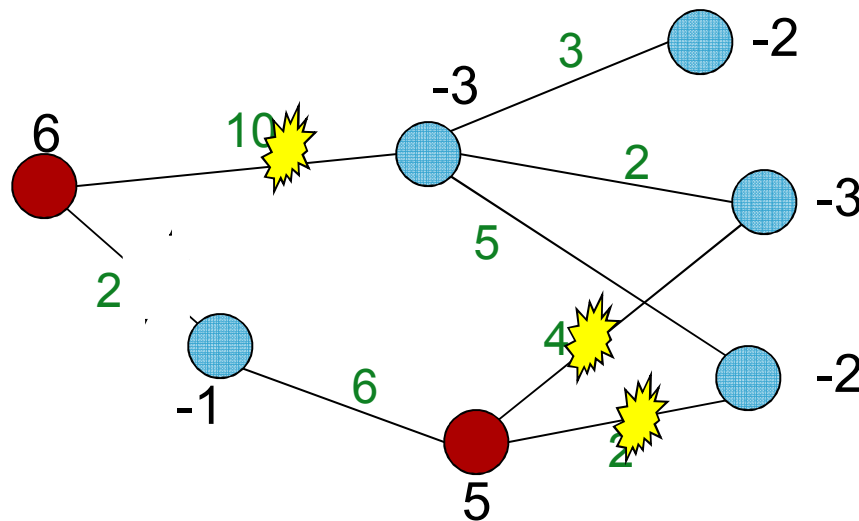
Tri-level optimization problems underlies many challenges



Hierarchy of optimization problems with a modular structure

- Motivation
 - Why is this hard?
 - What can we solve?
 - What is the gap?
-
- Going Back and covering the simplifications
 - Slides 6 7
 - Slides 16 17 transition to the combinatorial problem.
 - Description of uncertainties for robustness
 - Check references
 - Transition into uncertainty

Network inhibition problem



$k = 0$, max-flow = 11

$k = 1$, max-flow = 7

$k = 2$, max-flow = 5

$k = 3$, max-flow = 1

- Cut min. number of lines so that max flow is below a specified bound.
- Shown to be NP-complete (Phillips 1991).
- The classical min-cut problem is a special version of network inhibition, where max-flow is set to zero.
- Can be formulated as MILP with $|V| + |E|$ binary variables.

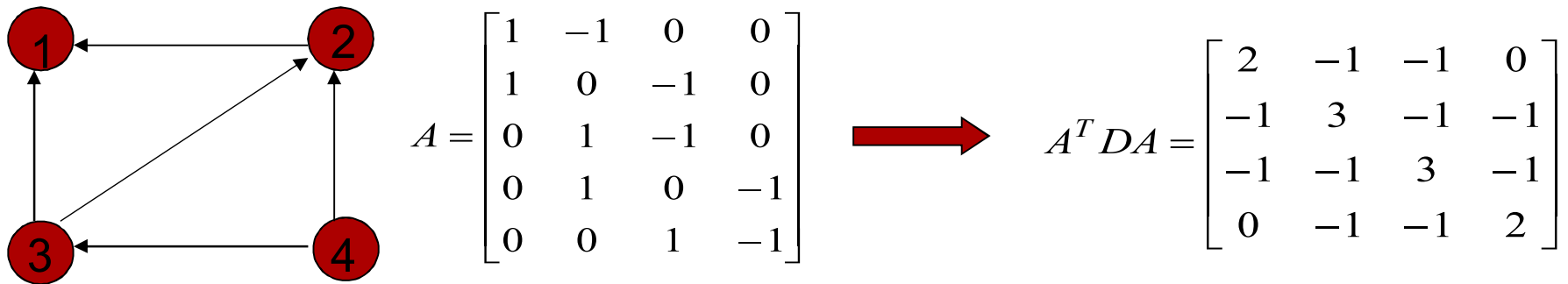
Feasibility boundary and spectral graph theory

On the boundary of feasibility, the power-flow Jacobian will have its second singular vector.

$$\frac{\partial F}{\partial \theta} = J = A^T B D ((1 - \gamma) \cos(A\theta)) A$$

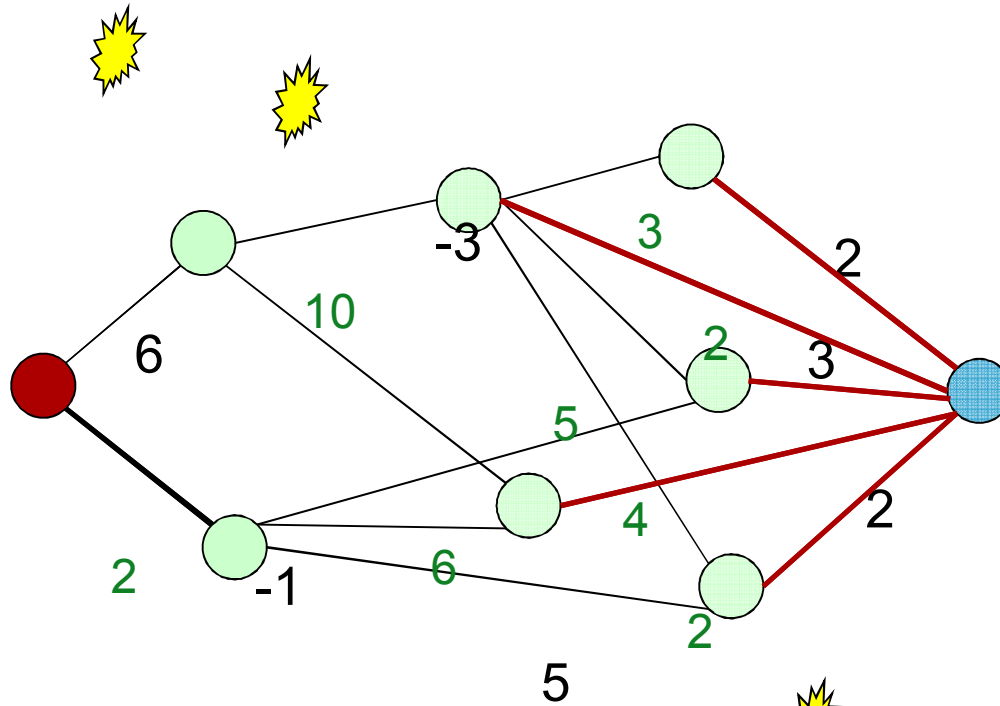
$$Jw = 0; \quad w^T e = 0; w^T w = 1$$

J has the same structure as Laplacian in spectral graph theory.



Uncertainty representation for optimization

What can we solve? Network Inhibition



$k = 1$, max-flow = 7

$k = 0$, max-flow = 11

$k = 2$, max-flow = 5

$k = 3$, max-flow = 1

- Cut k lines to minimize the maximum flow.
- Shown to be NP-complete (Phillips 1991).
- The classical min-cut problem is a special version of network inhibition, where max-flow is set to zero.
- Can be formulated as MILP with $|V| + |E|$ binary variables.