

# Measuring & Influencing Resilience of Adapting Flow Networks

Walt Beyeler, Eric Vugrin, Steve Verzi,  
Geoff Forden, Munaf Aamir,  
Chris Lamb, Sasha Outkin

# Presentation to Sandia National Laboratories' Complex Systems Advisory Panel

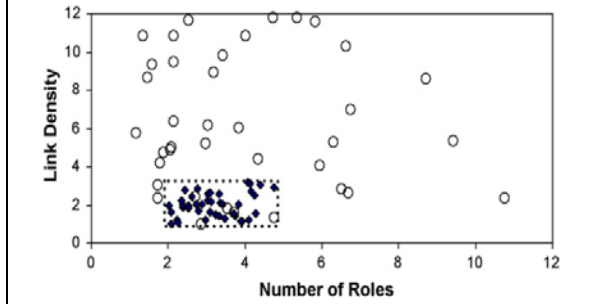
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- This presentation describes a model designed to study the effects of adaptive network growth on the network's resilience, summarizing:
  - Research questions
  - Model formulation
  - Initial results

# Understanding Connections between Network Structure and Performance

that  $c \geq 1$  says simply that the networks being considered are all fully connected. Any value  $c < 1$  would imply that the graph

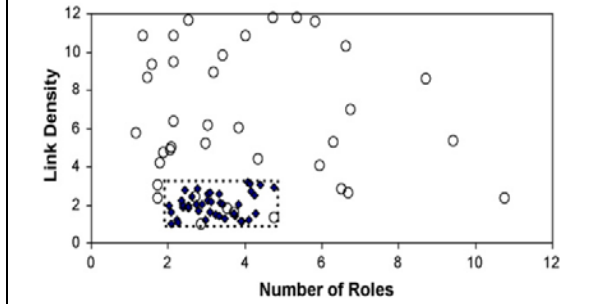


From Ulanowicz et al., Eco. Compl. 6 pp 27-36

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- Maybe this reflects a compromise between being efficient as a system and needing to survive local disruptions
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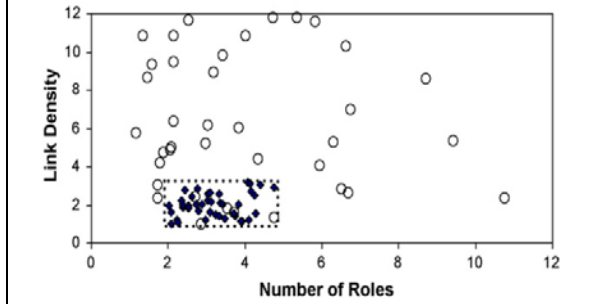
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- Does it leave a signature in the topology?
- Is it a (systemically) good tradeoff?  
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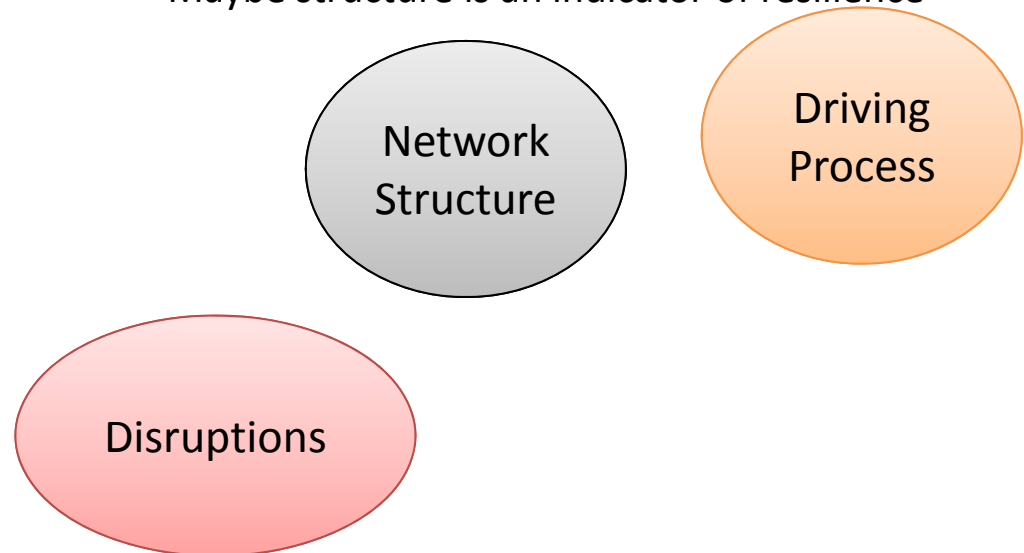


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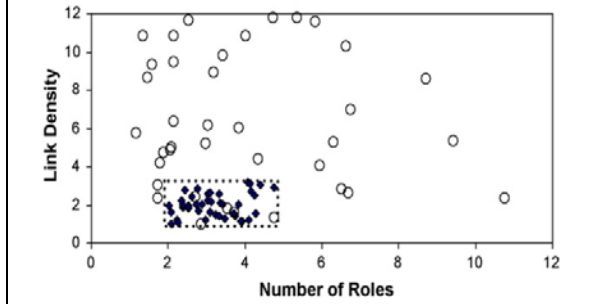
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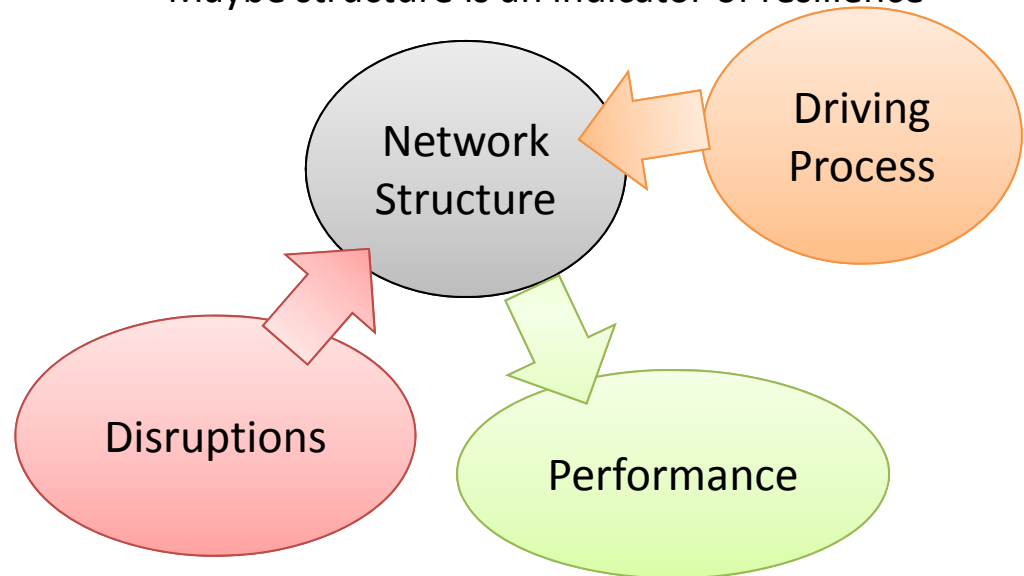


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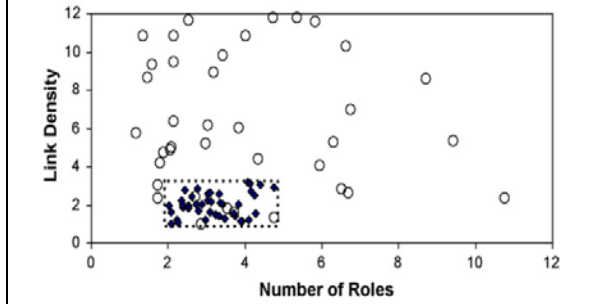
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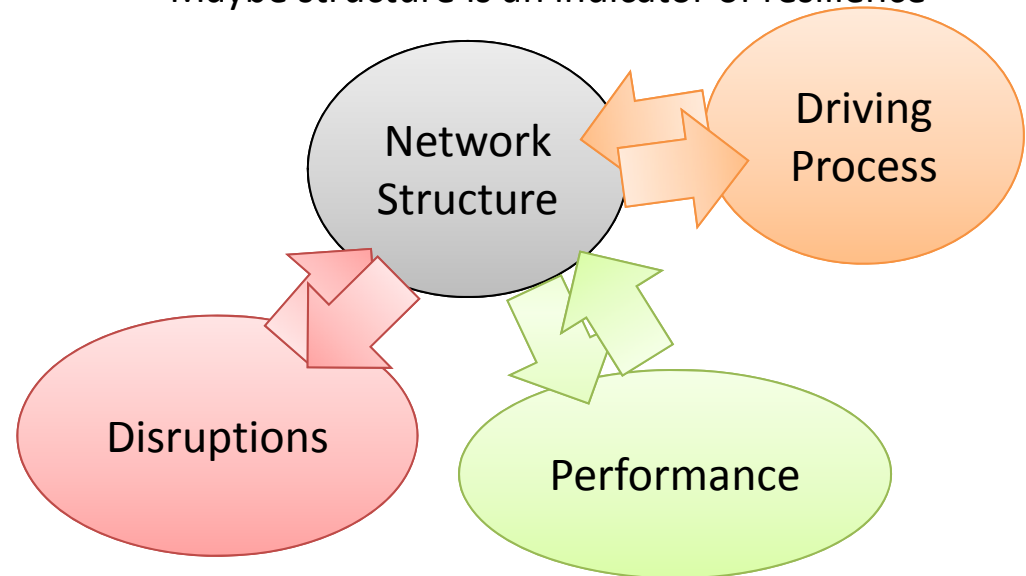


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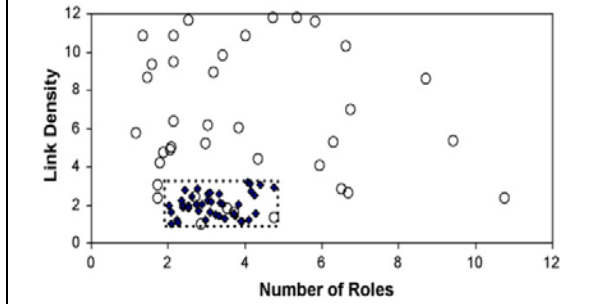
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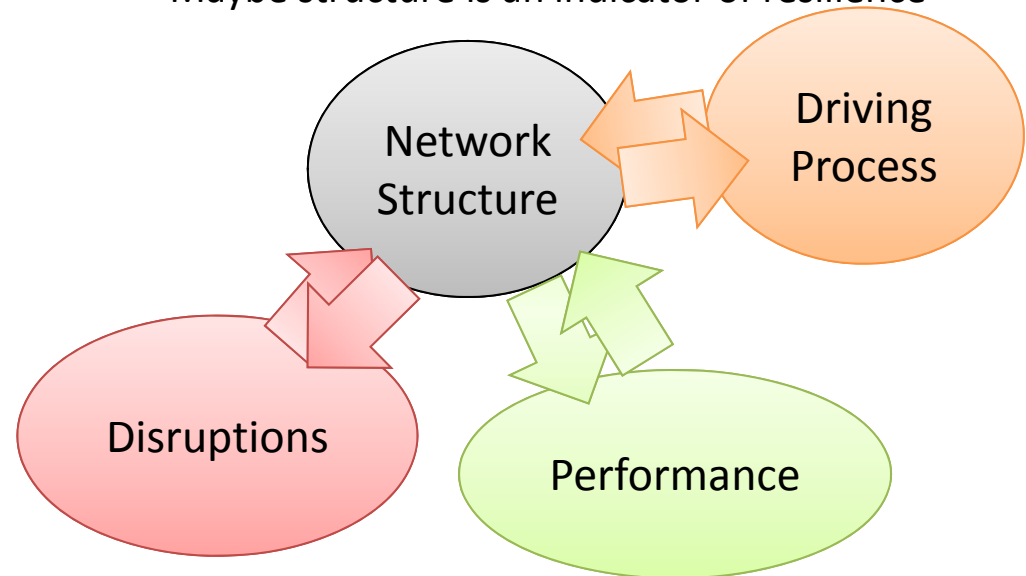


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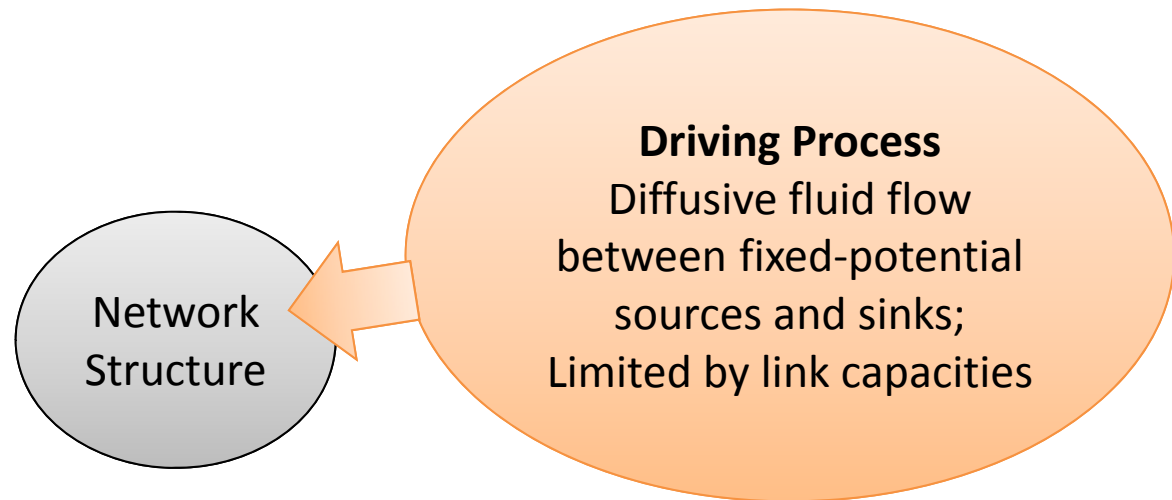
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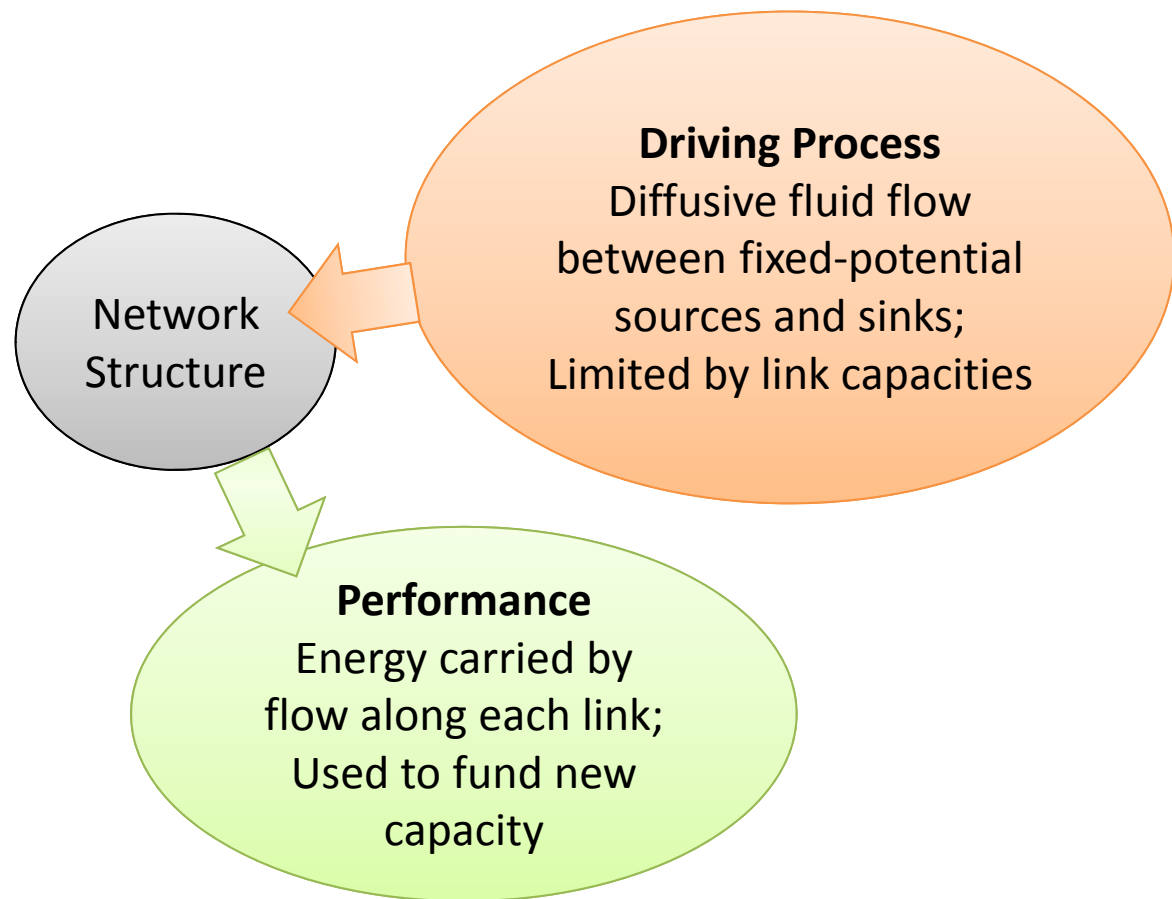
Answers depend on specifics.  
We start with a simple flow model...



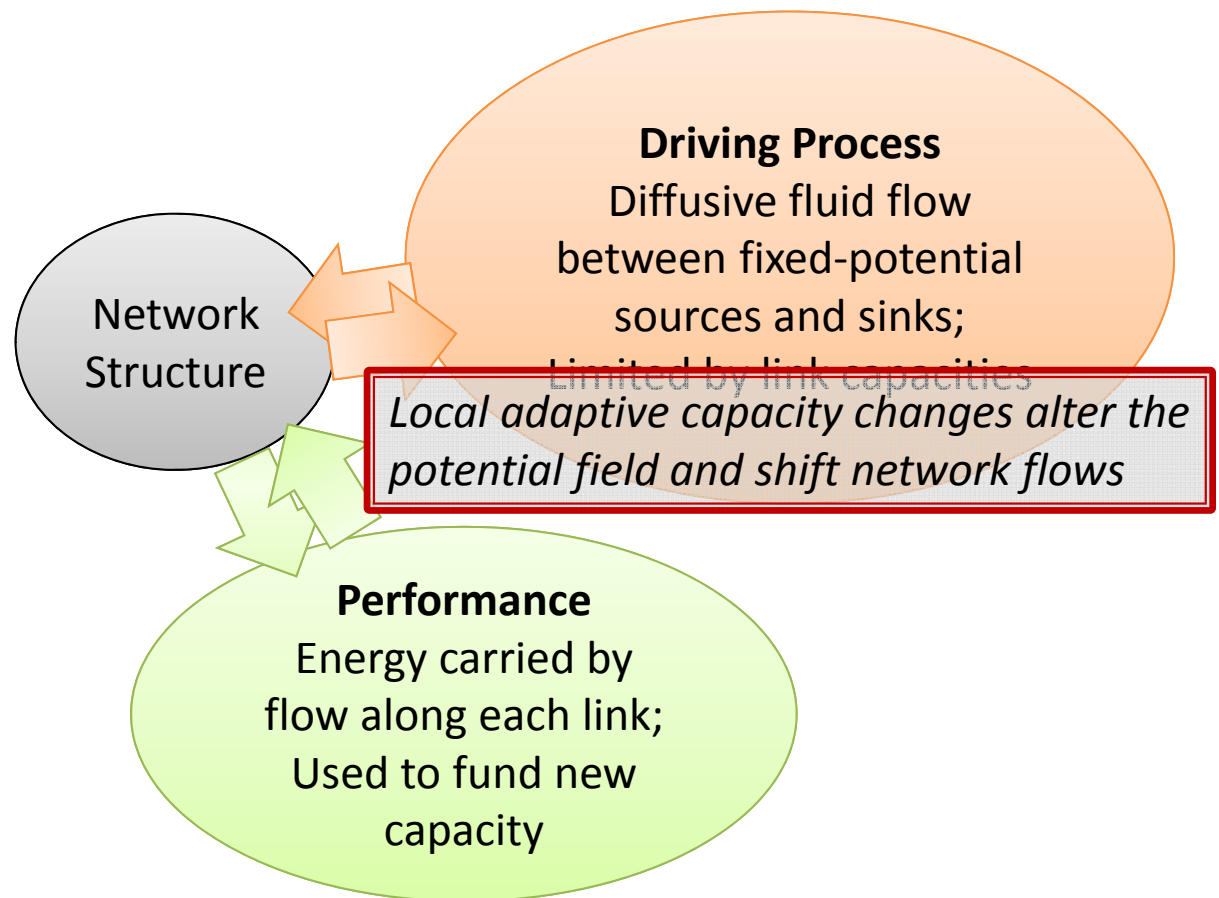
# Model Features



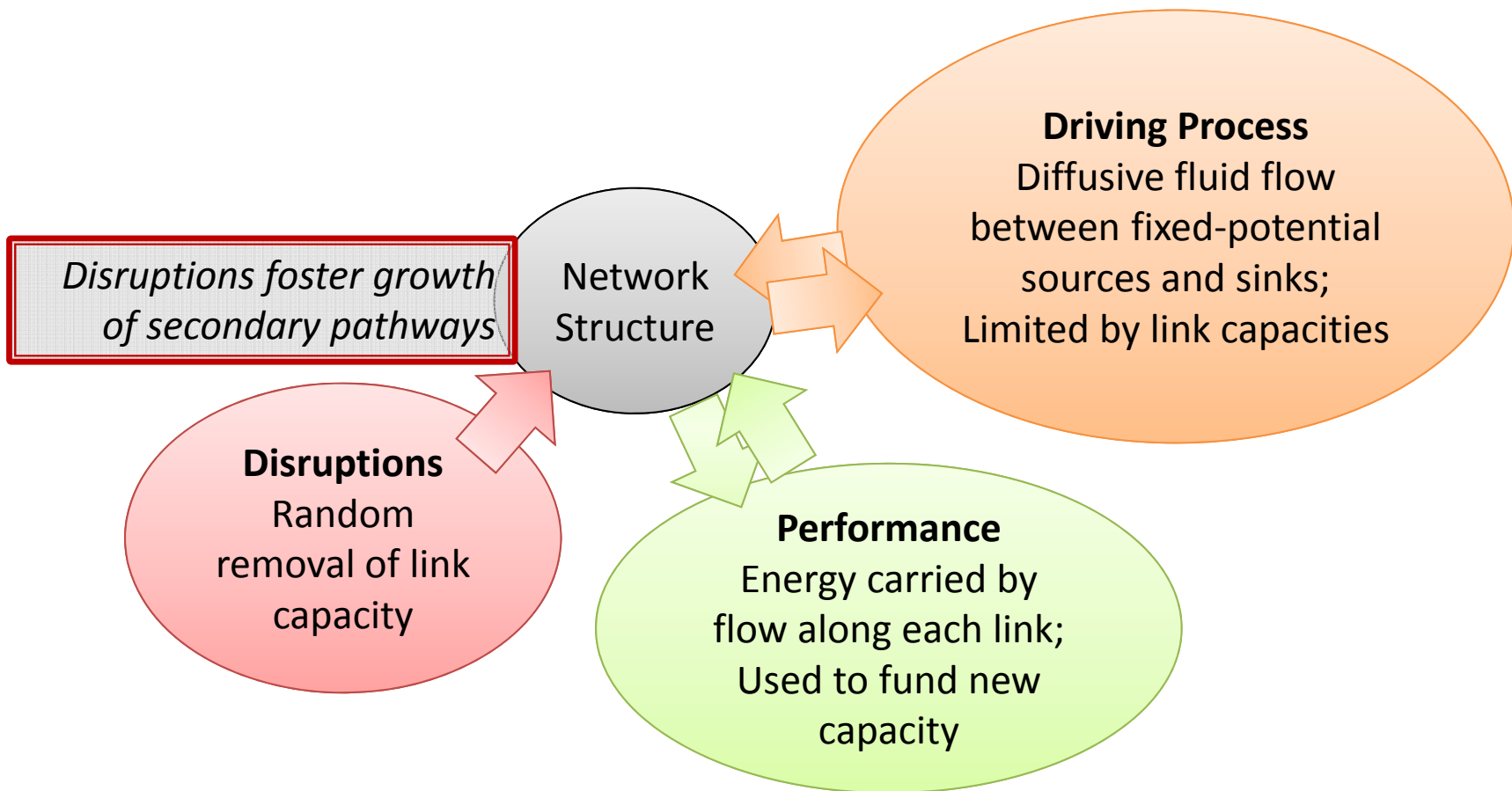
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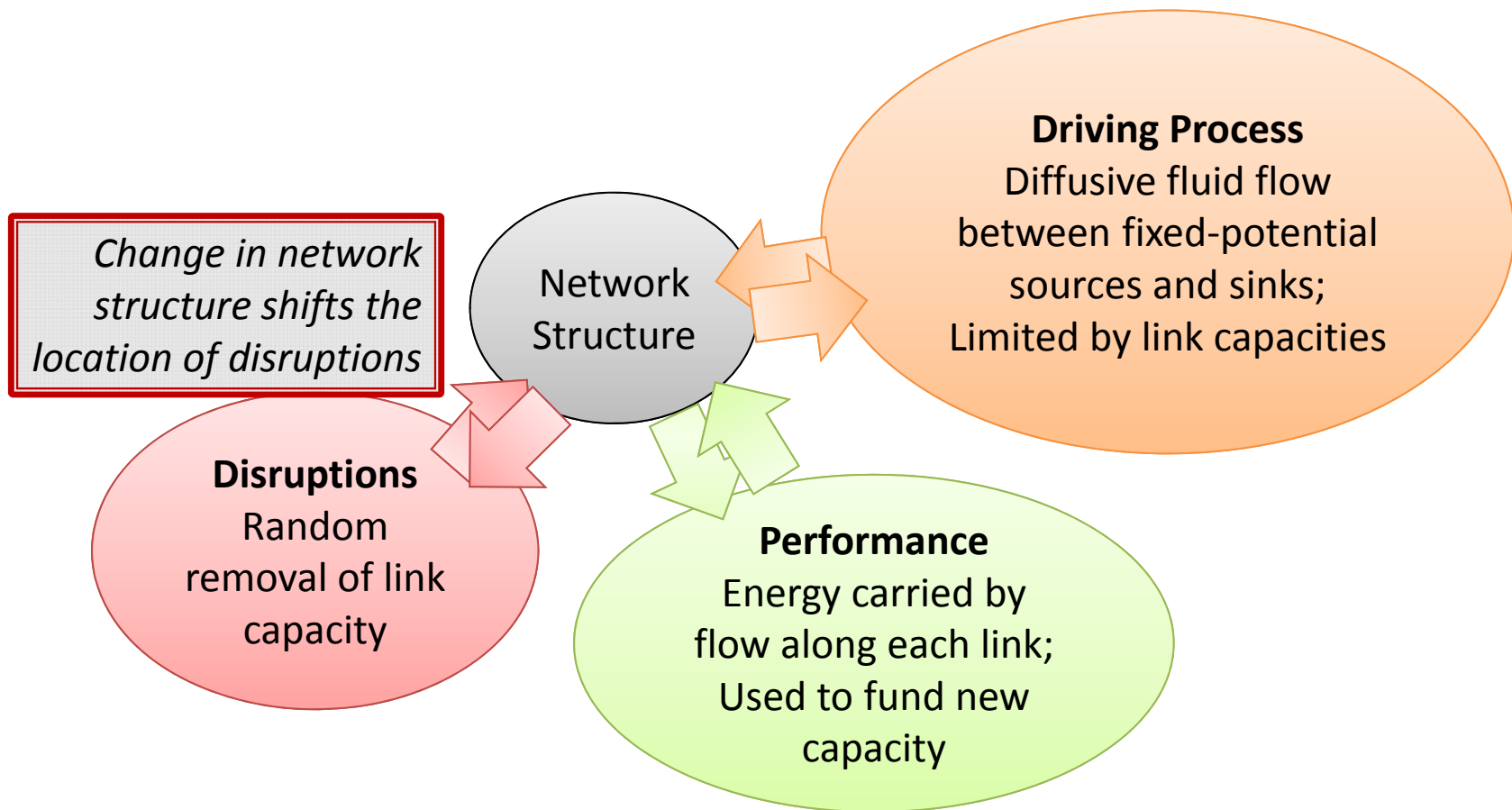
# Model Features



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# Model Features



# Flow and Growth Process Models

Each node  $i$  has a potential  $s_i$

Each link  $ij$  has a capacity  $c_{ij}$

Flow rates are limited by a (directed) capacity associated with each link,  $c_{ij}$ . Assuming  $s_i > s_j$ , the flow from node  $i$  to node  $j$  is given by:

$$q_{ij} = c_{ij} f((s_i - s_j)k_{ij}) \quad (1)$$

In equilibrium, the net flow at each node  $i$  is 0, including any internal sources ( $q_{si}$ ) or sinks ( $d_i$ ):

$$\sum_j q_{ji} + q_{si} - d_i = 0 \quad \forall i \quad (3)$$

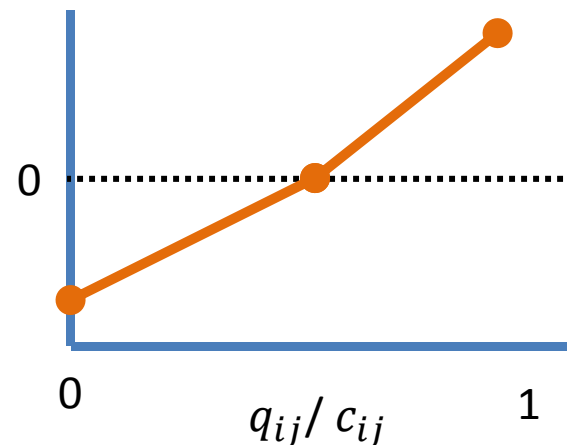
The equilibrium solution  $\{\hat{s}_i\}$  is obtained by solving equations (1-3).

where  $k_{ij}$  is a conductance parameter and the function  $f(x)$  models linear resistance as  $x \rightarrow 0$  and enforces the capacity limit for large  $x$ :

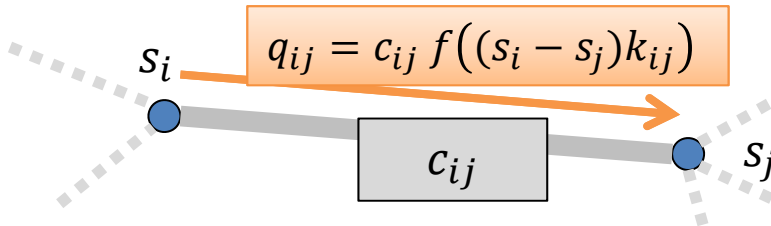
$$f(x) \equiv 1 - e^{-x} \quad (2)$$

Network adapts by changing link capacities in response to utilization:

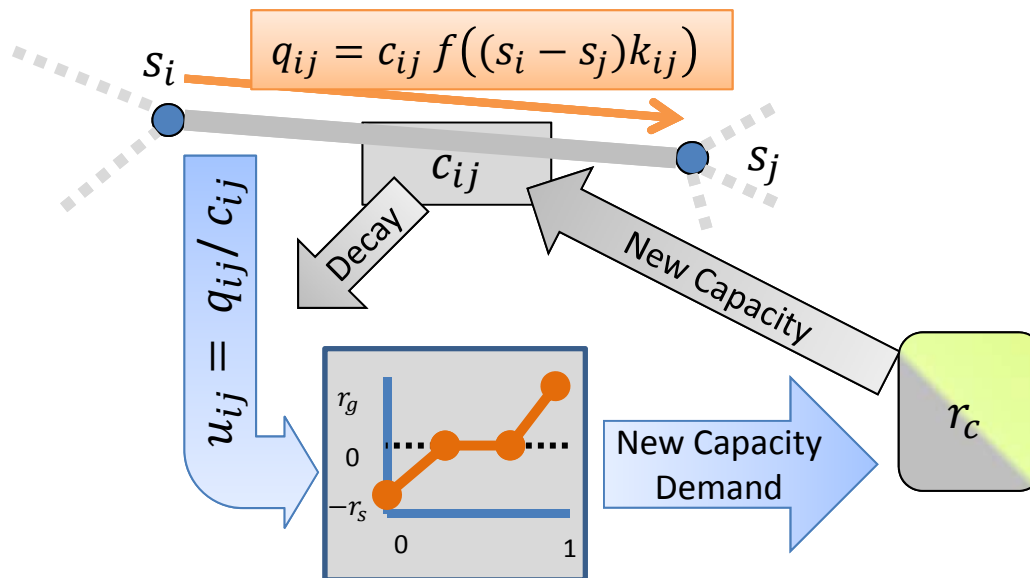
$$\frac{dc_{ij}}{dt}$$



# Link Capacity Dynamics

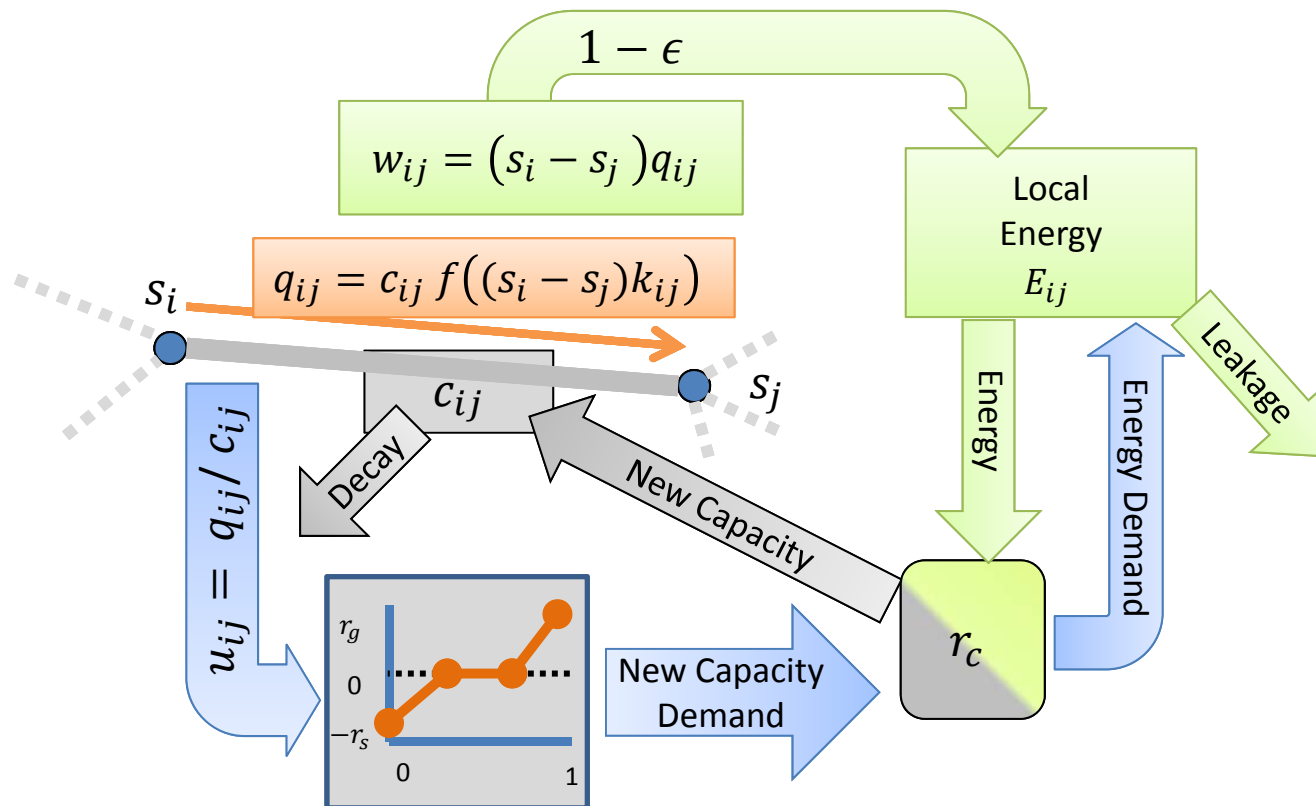


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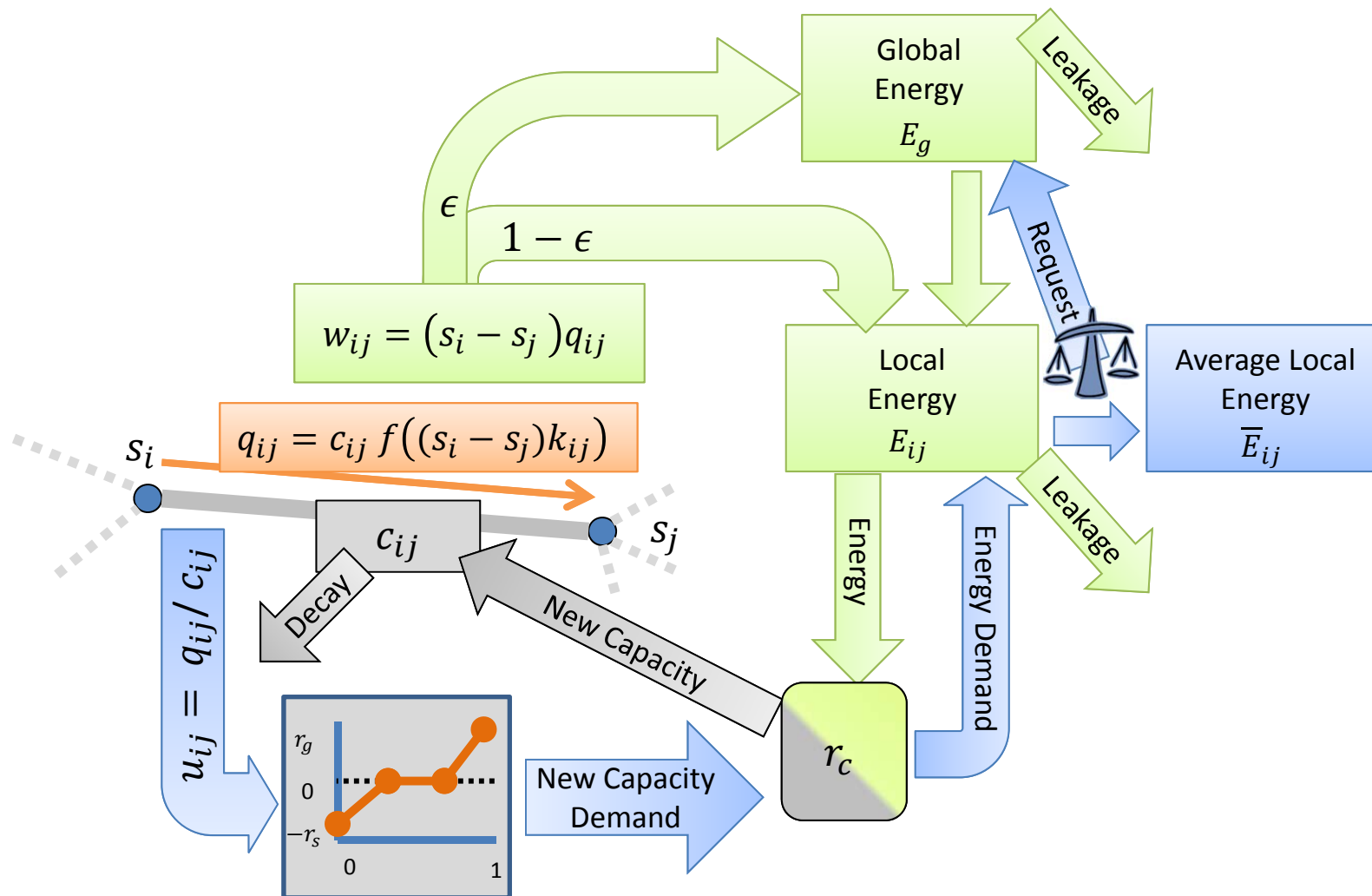




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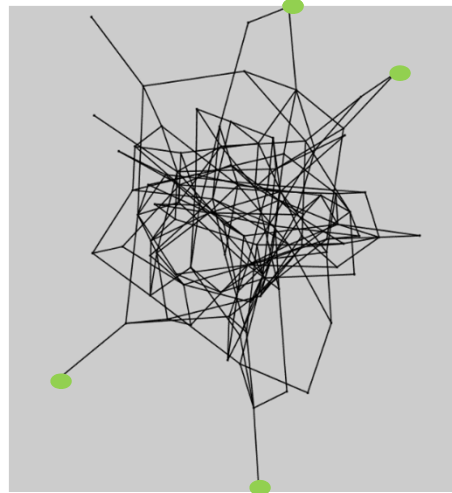


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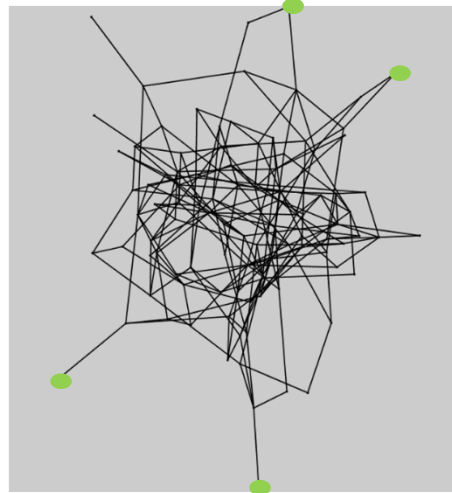
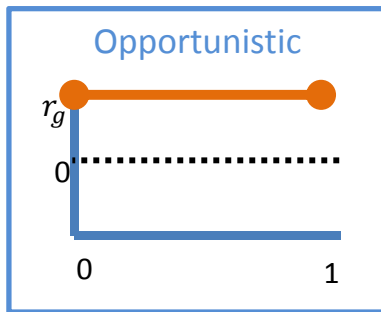
# Adaptation without Disruption

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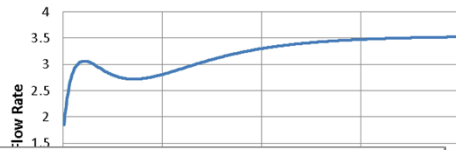


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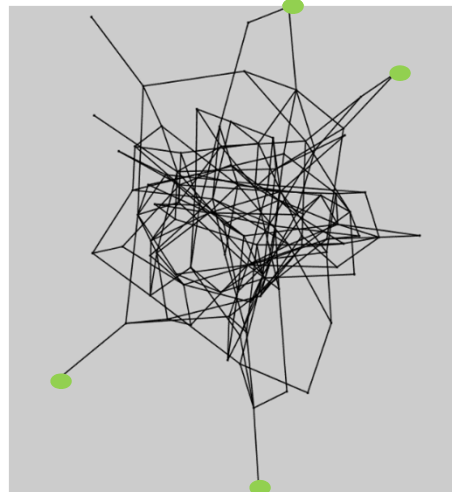
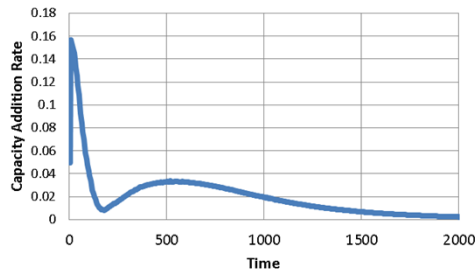
Opportunistic

$r_g$

Network Flow during Adaptation



Capacity Addition during Adaptation

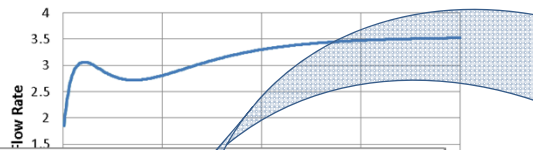


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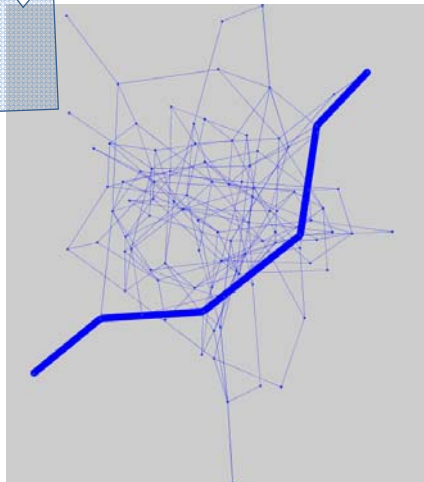
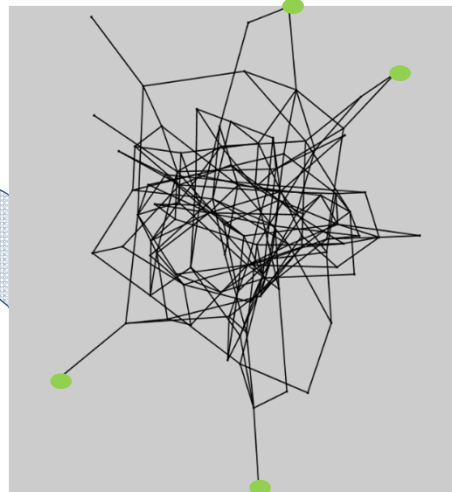
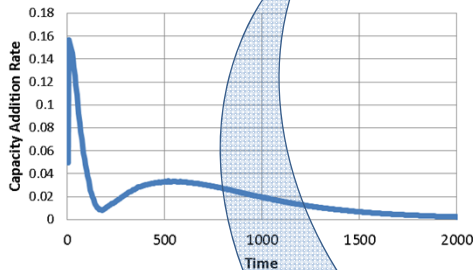
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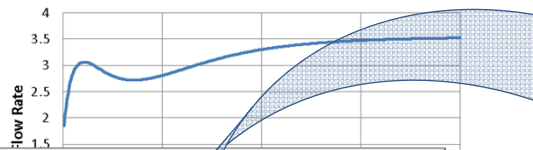
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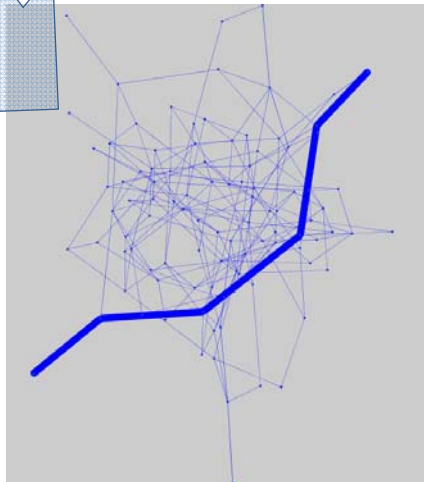
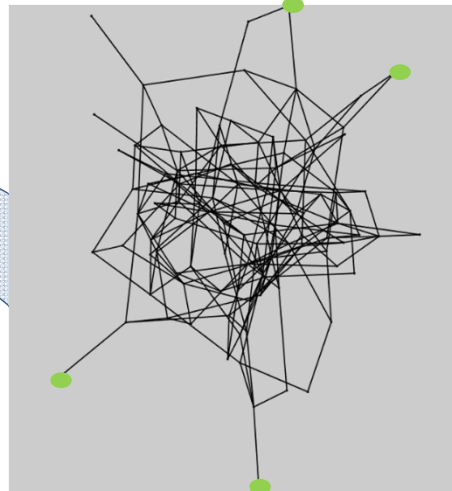
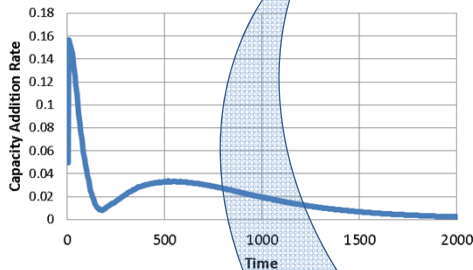
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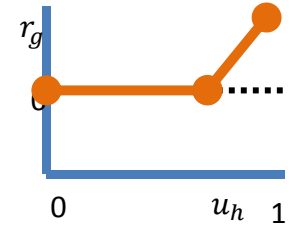
Network Flow during Adaptation



Capacity Addition during Adaptation



Conservative



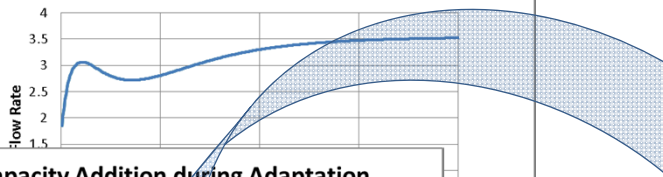
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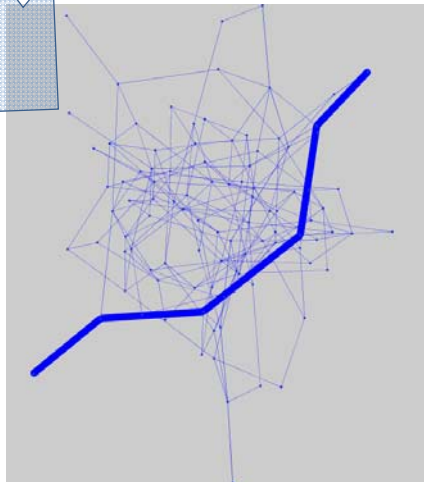
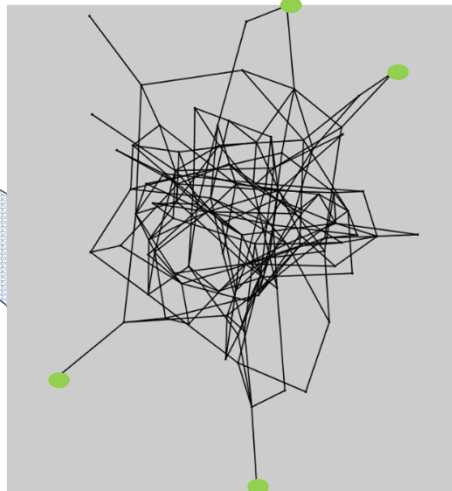
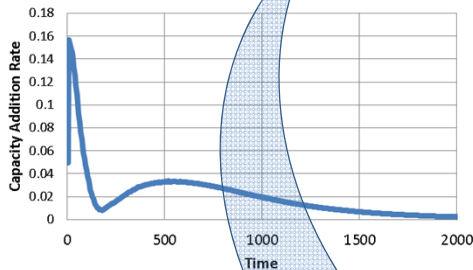
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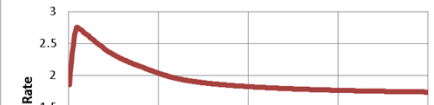
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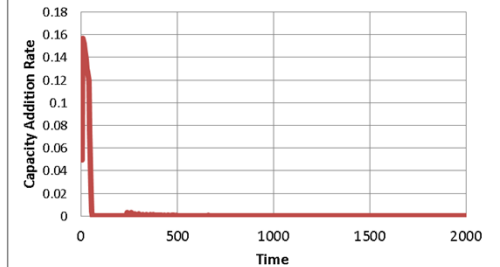
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Network Flow during Adaptation



Capacity Addition during Adaptation





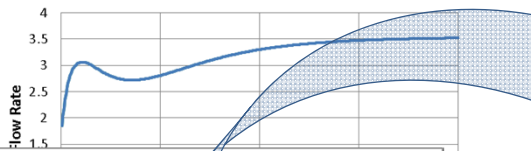
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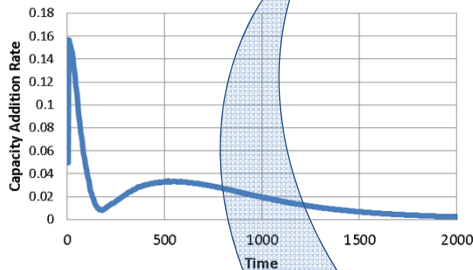
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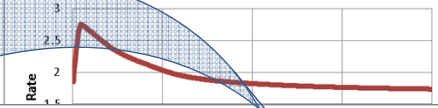
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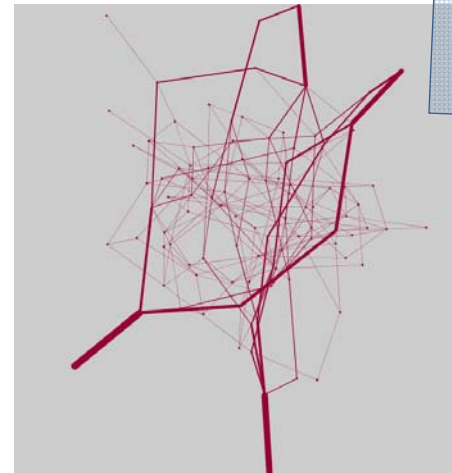
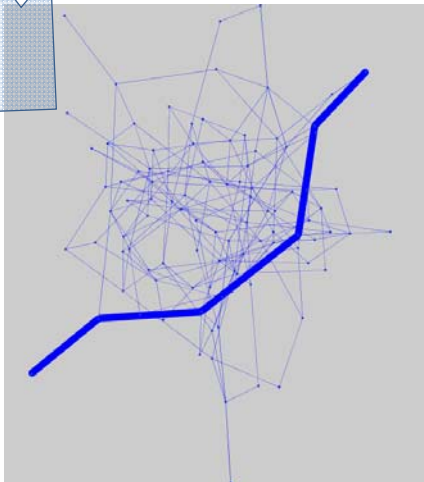
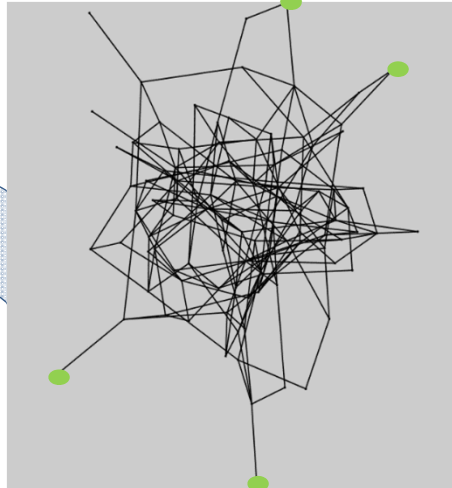
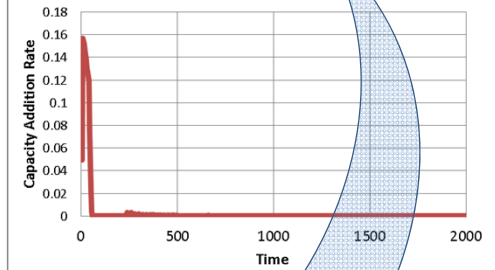
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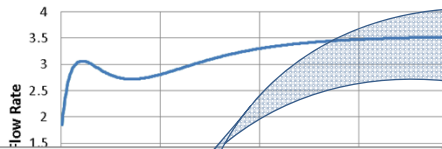
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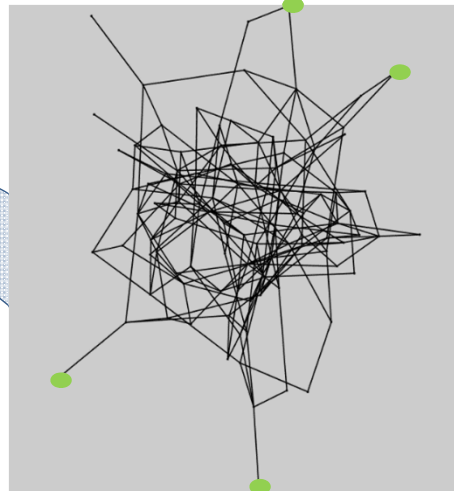
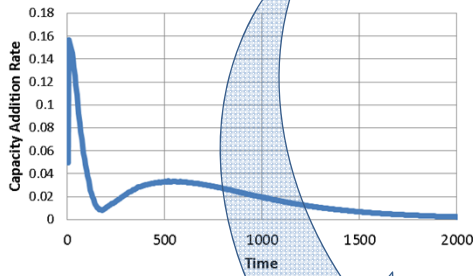
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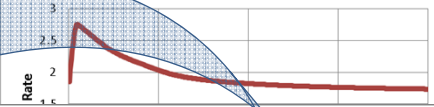
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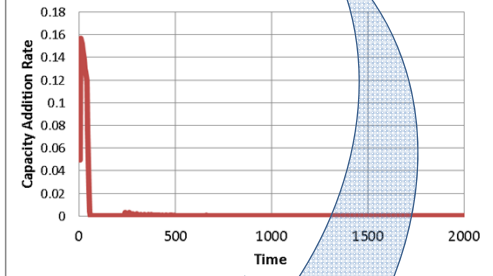
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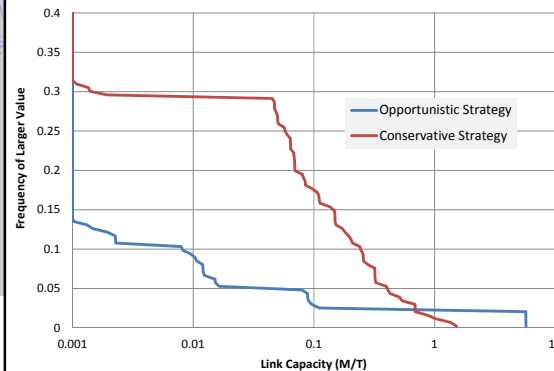
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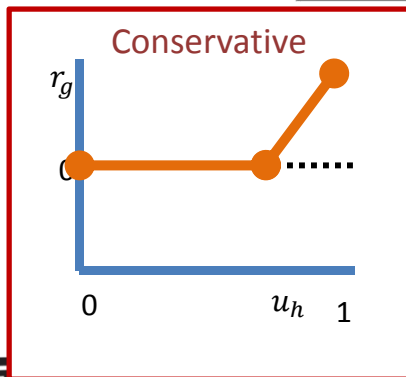
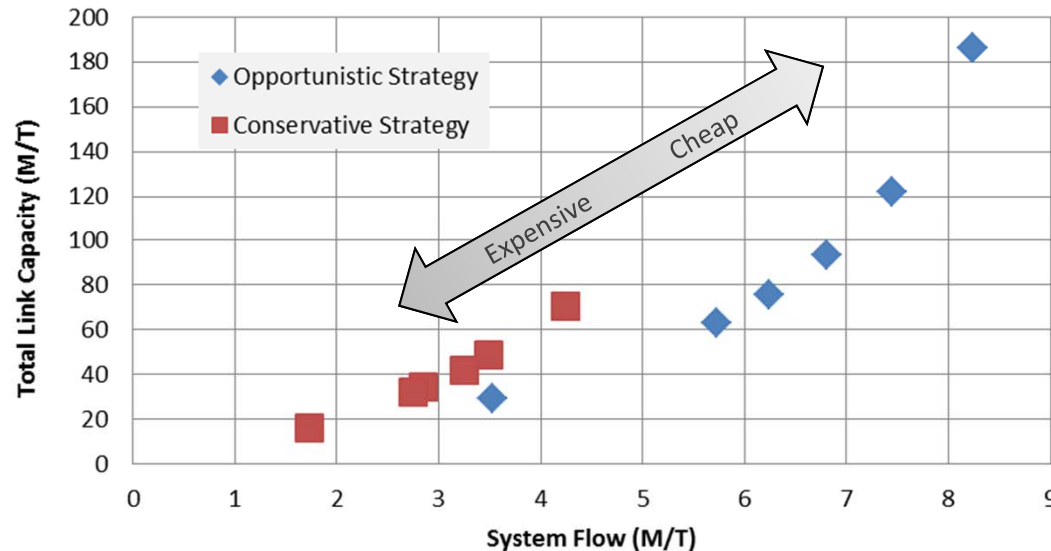


Capacity Distributions in Undisturbed Networks  
Expensive Capacity

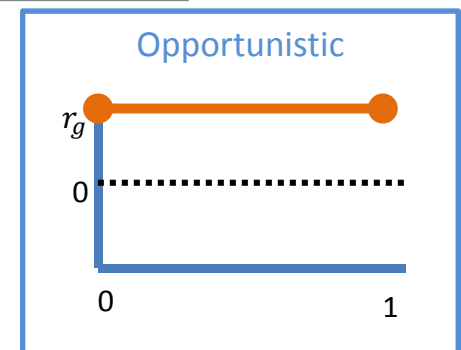


# Local Strategies Shape Configuration and Performance

**Performance of Undisturbed Systems**  
**Effect of Growth Strategies and Capacity Cost**

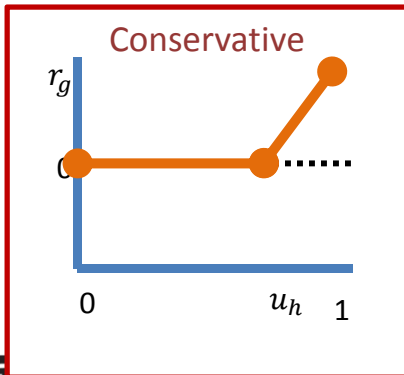
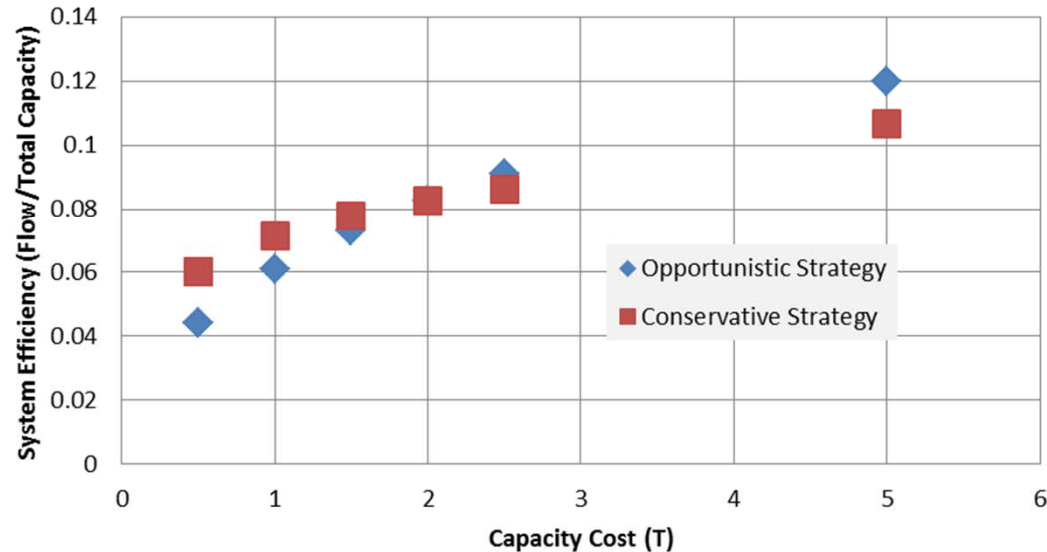


Higher capacity costs induce efficiency

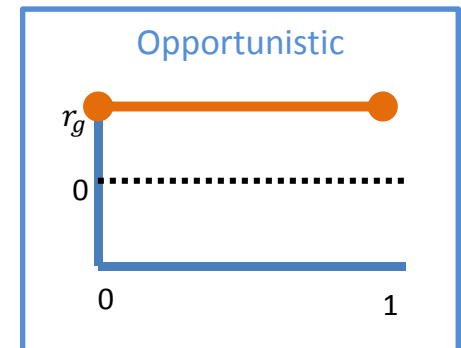


# Capacity Costs Encourage Efficiency

**Performance of Undisturbed Systems**  
**Effect of Growth Strategies and Capacity Cost**



Higher capacity costs induce efficiency

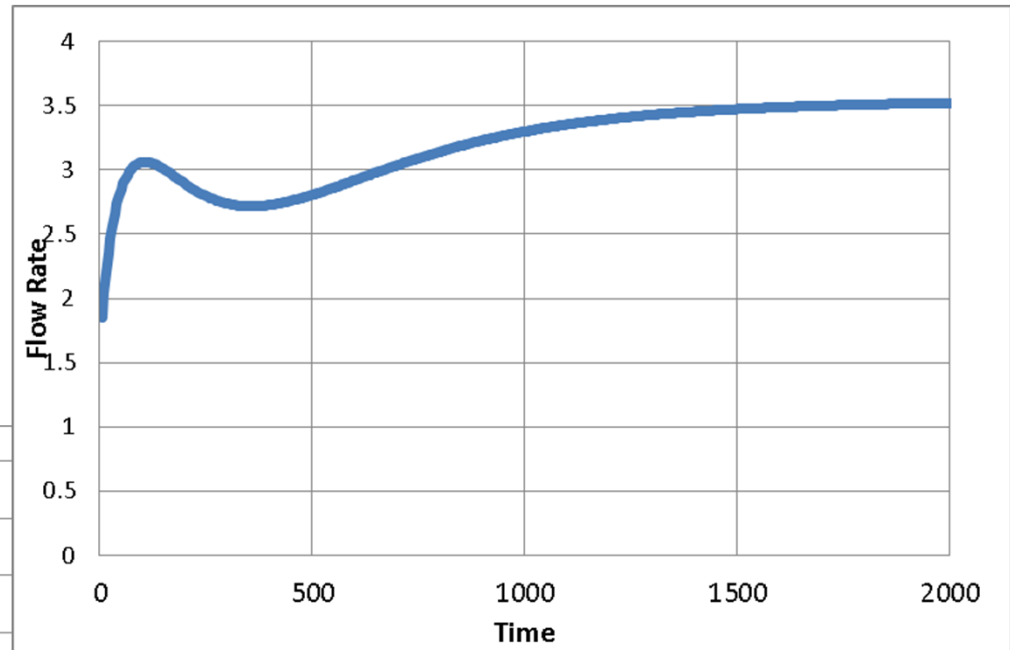
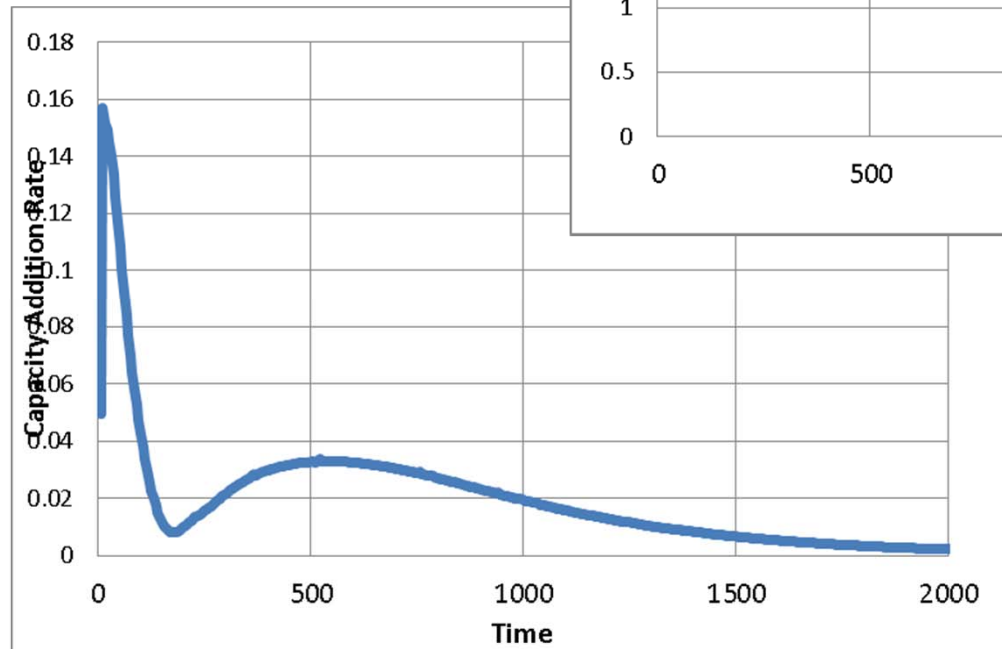


# Influence of Disruption on System Performance

## Opportunistic Strategy with Expensive Capacity

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No Disruption

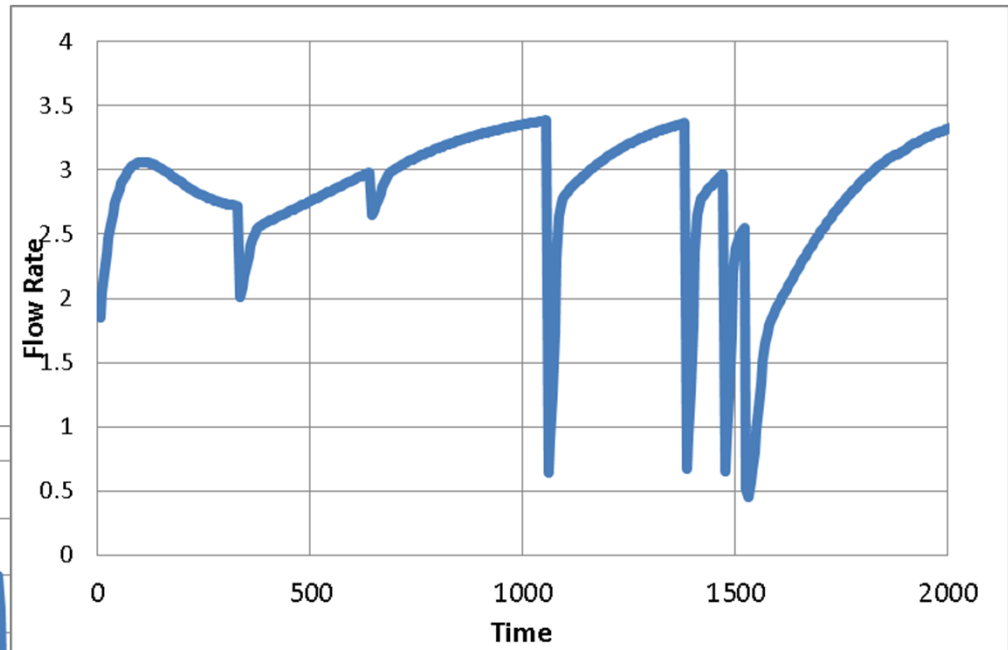
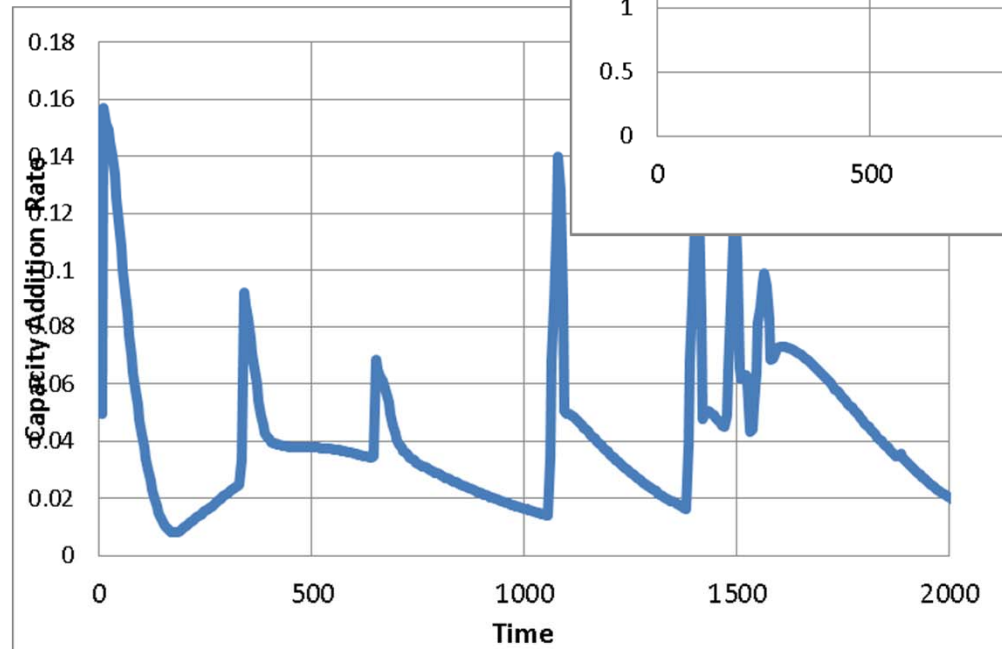


# Influence of Disruption on System Performance

## Opportunistic Strategy with Expensive Capacity

CASOS  
ENGINEERING

Freq = 1/150

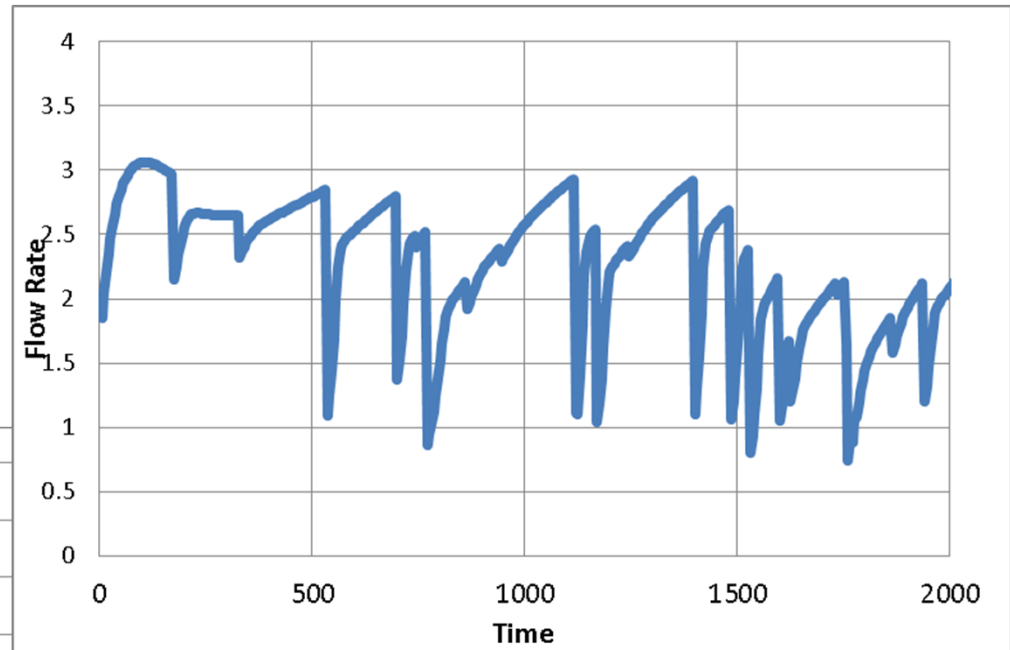
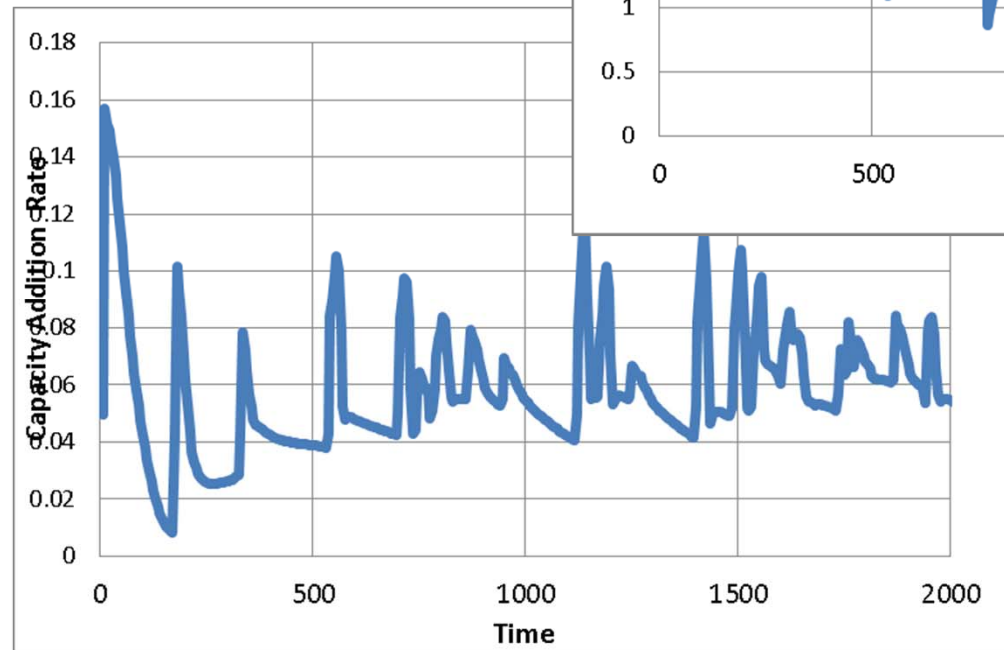


# Influence of Disruption on System Performance

## Opportunistic Strategy with Expensive Capacity

CASOS  
ENGINEERING

Freq = 1/75

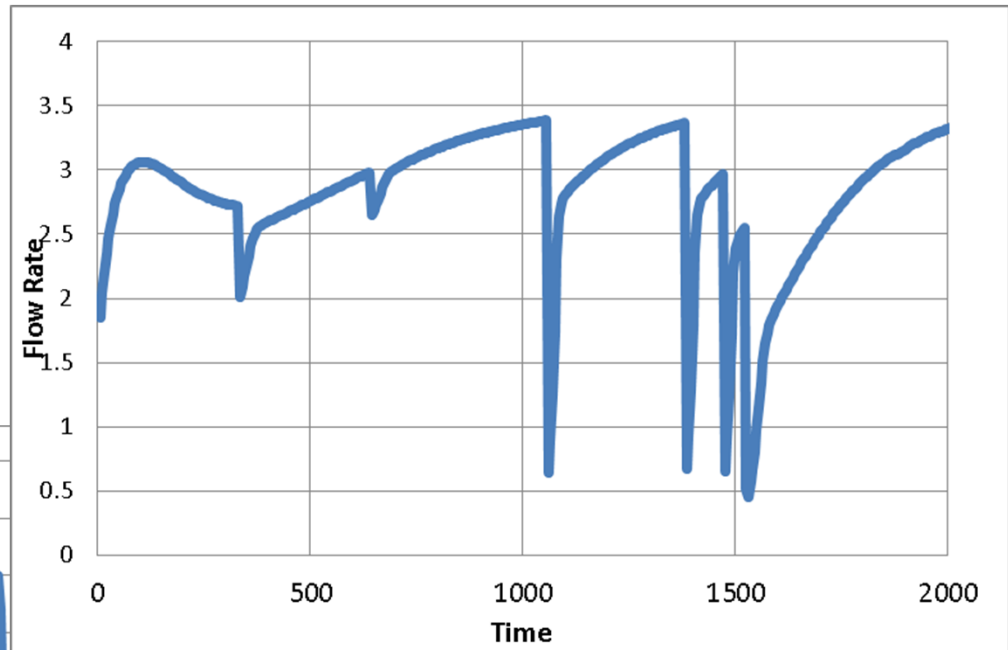
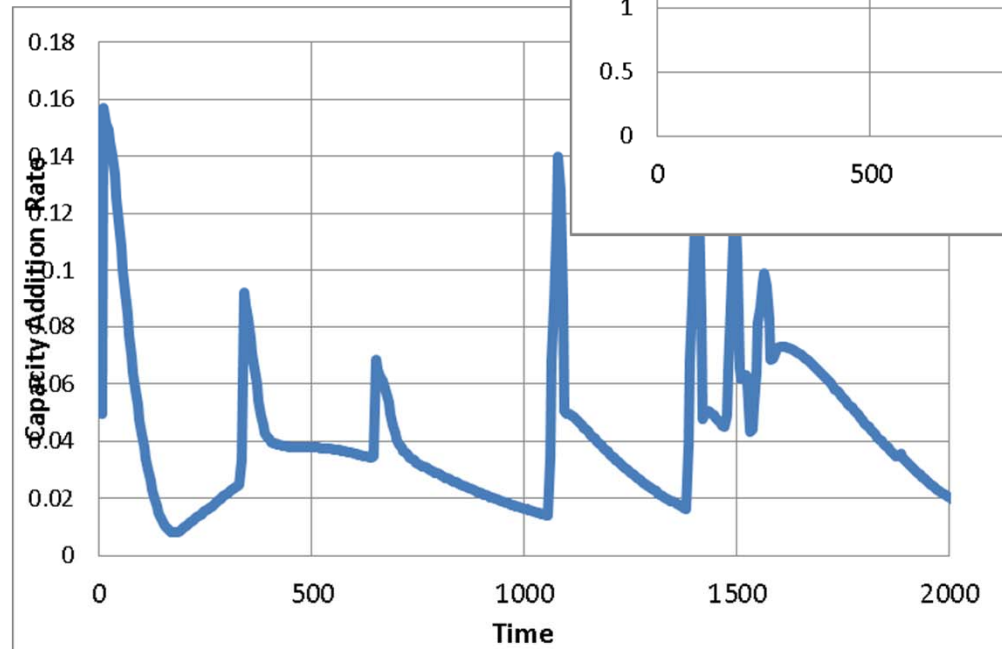


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## Opportunistic Strategy with Expensive Capacity

CASOS  
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Freq = 1/150



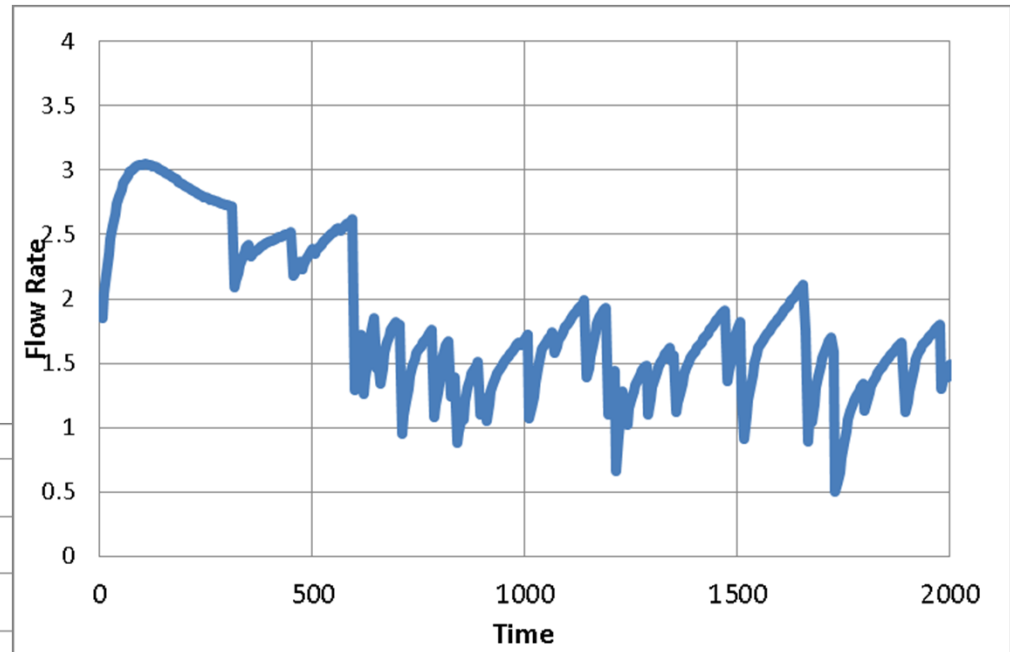
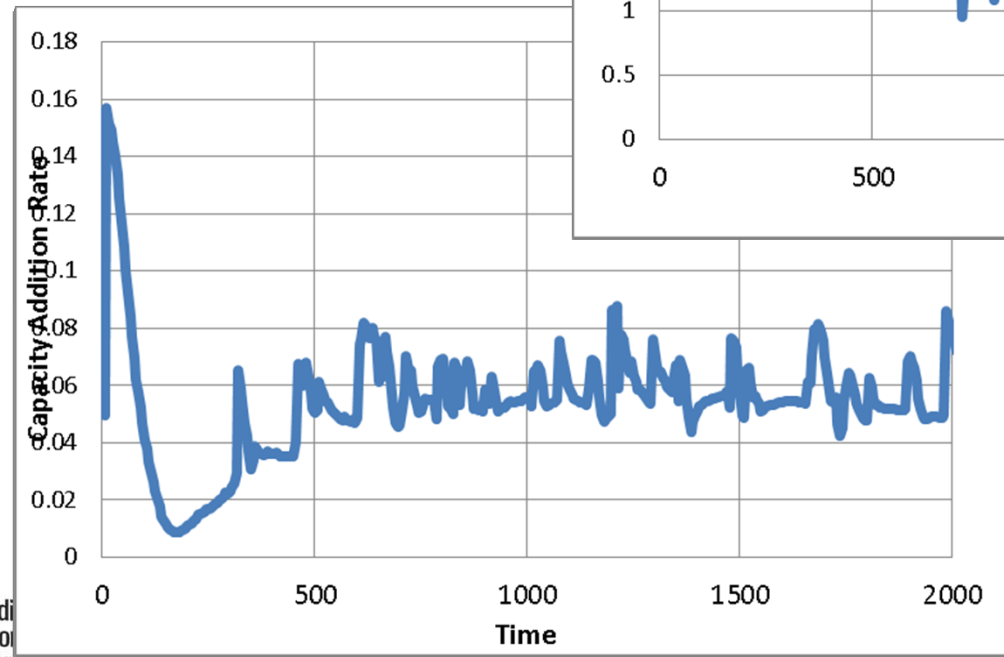


# Influence of Disruption on System Performance

## Opportunistic Strategy with Expensive Capacity

CASOS  
ENGINEERING

Freq = 1/30

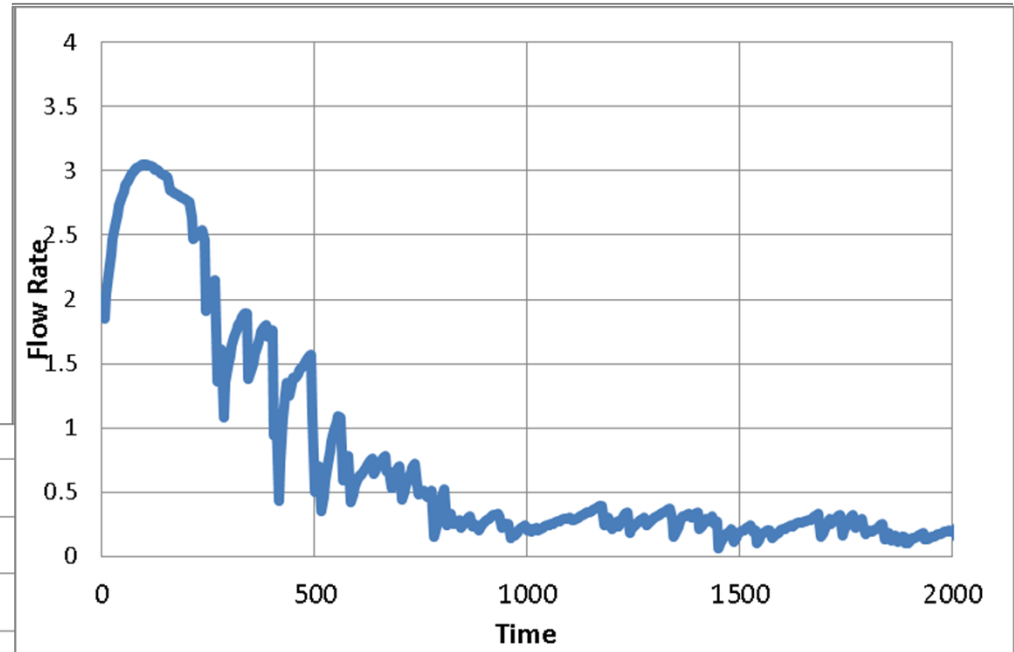
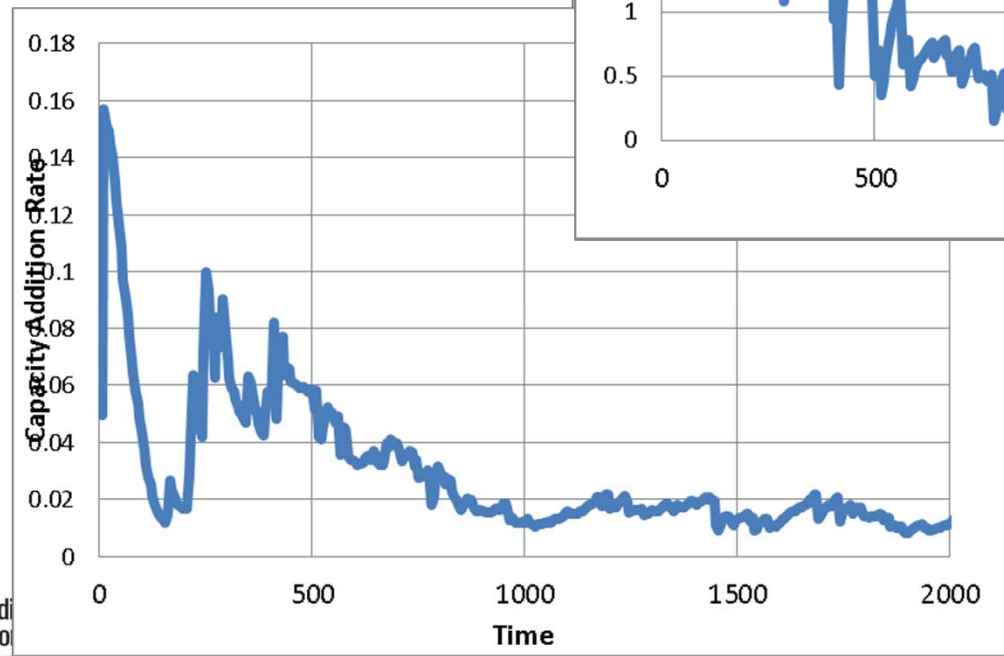


# Influence of Disruption on System Performance

## Opportunistic Strategy with Expensive Capacity

CASOS  
ENGINEERING

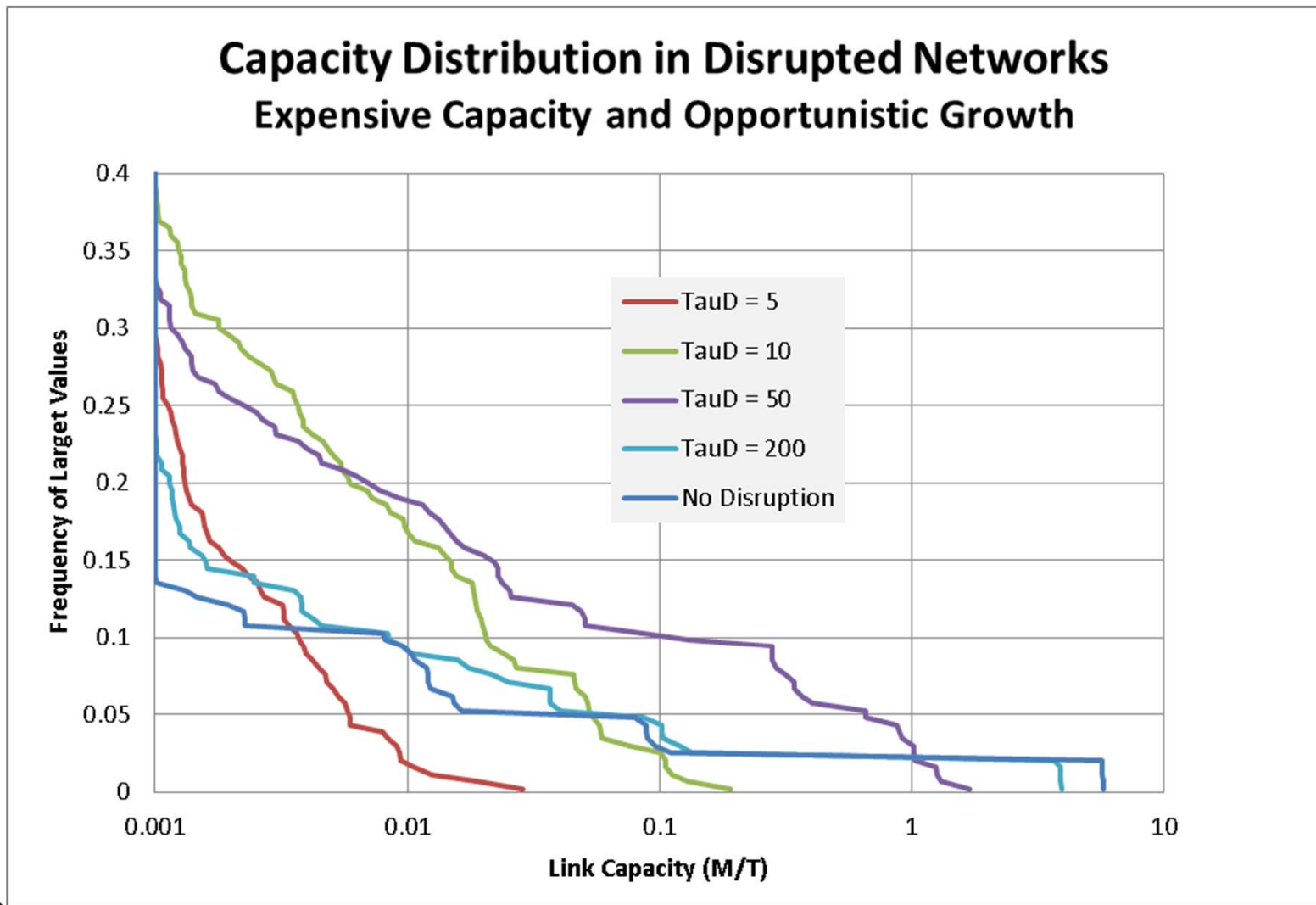
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# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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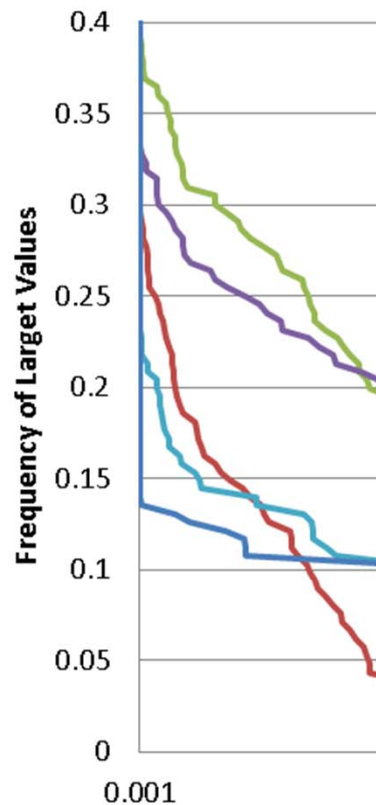
# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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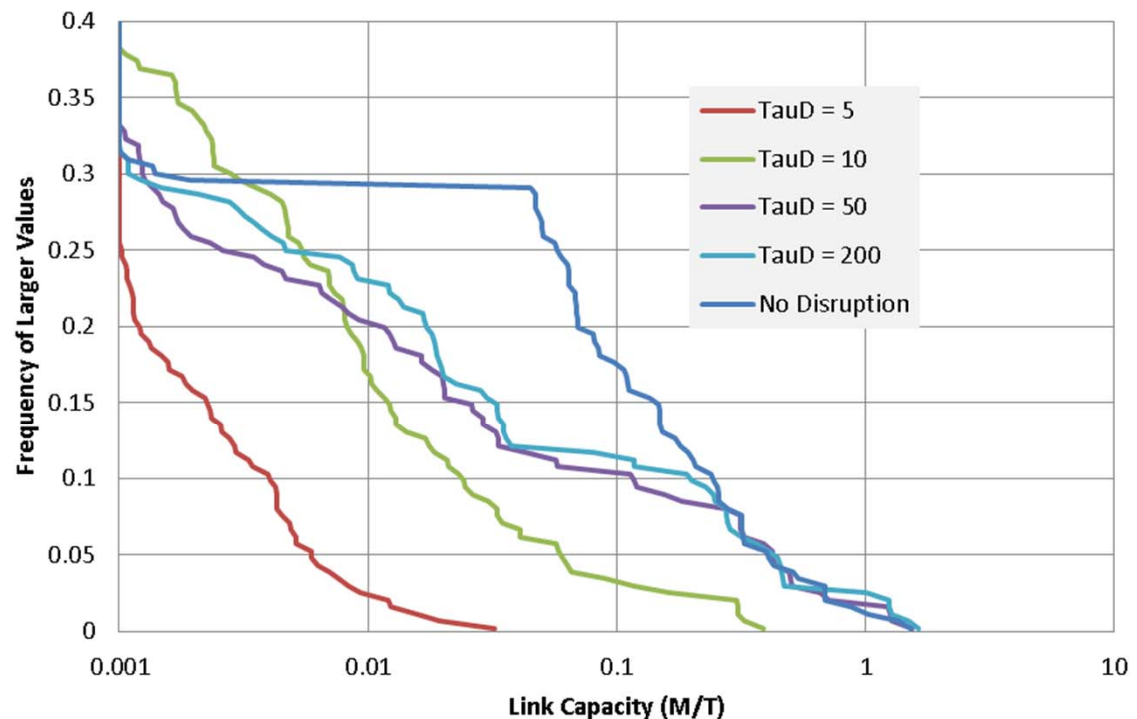
### Capacity Distribution in Disrupted Networks

#### Expensive Capacity and Opportunistic Growth



### Capacity Distribution in Disrupted Networks

#### Expensive Capacity and Conservative Growth

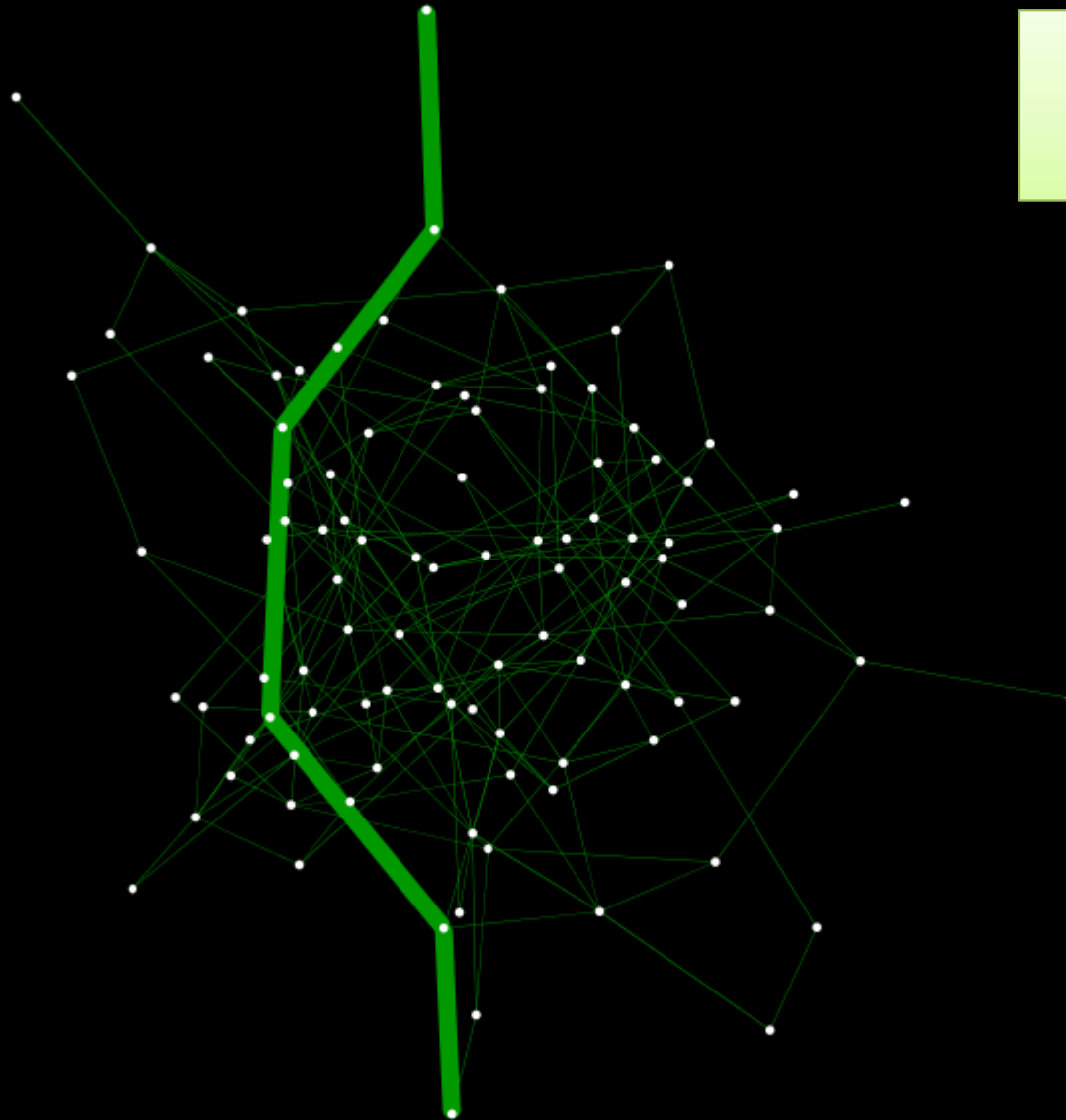


# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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No Disruption



# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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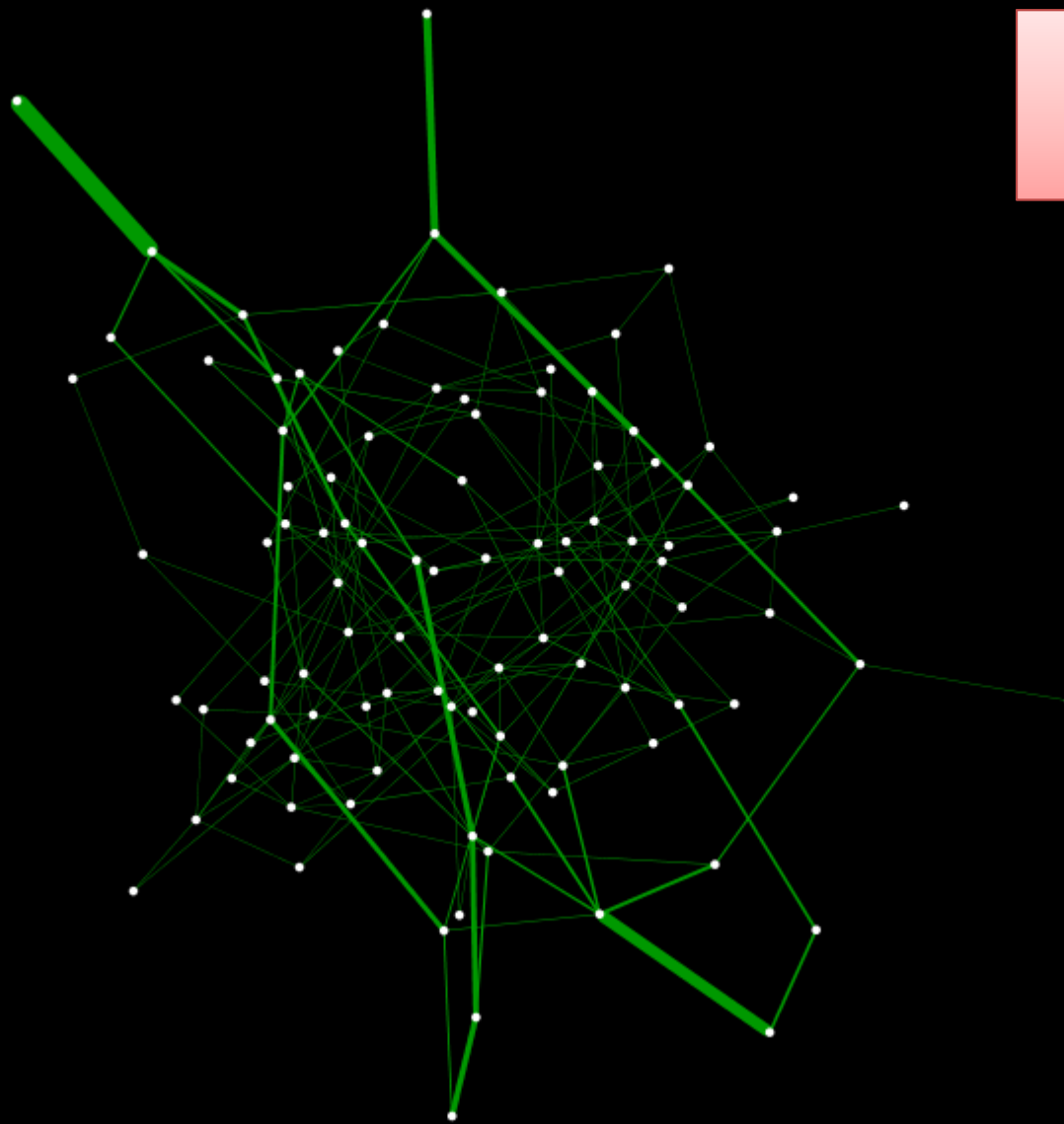


# Influence of Disruption on System Structure

## Opportunistic Strategy with Expensive Capacity

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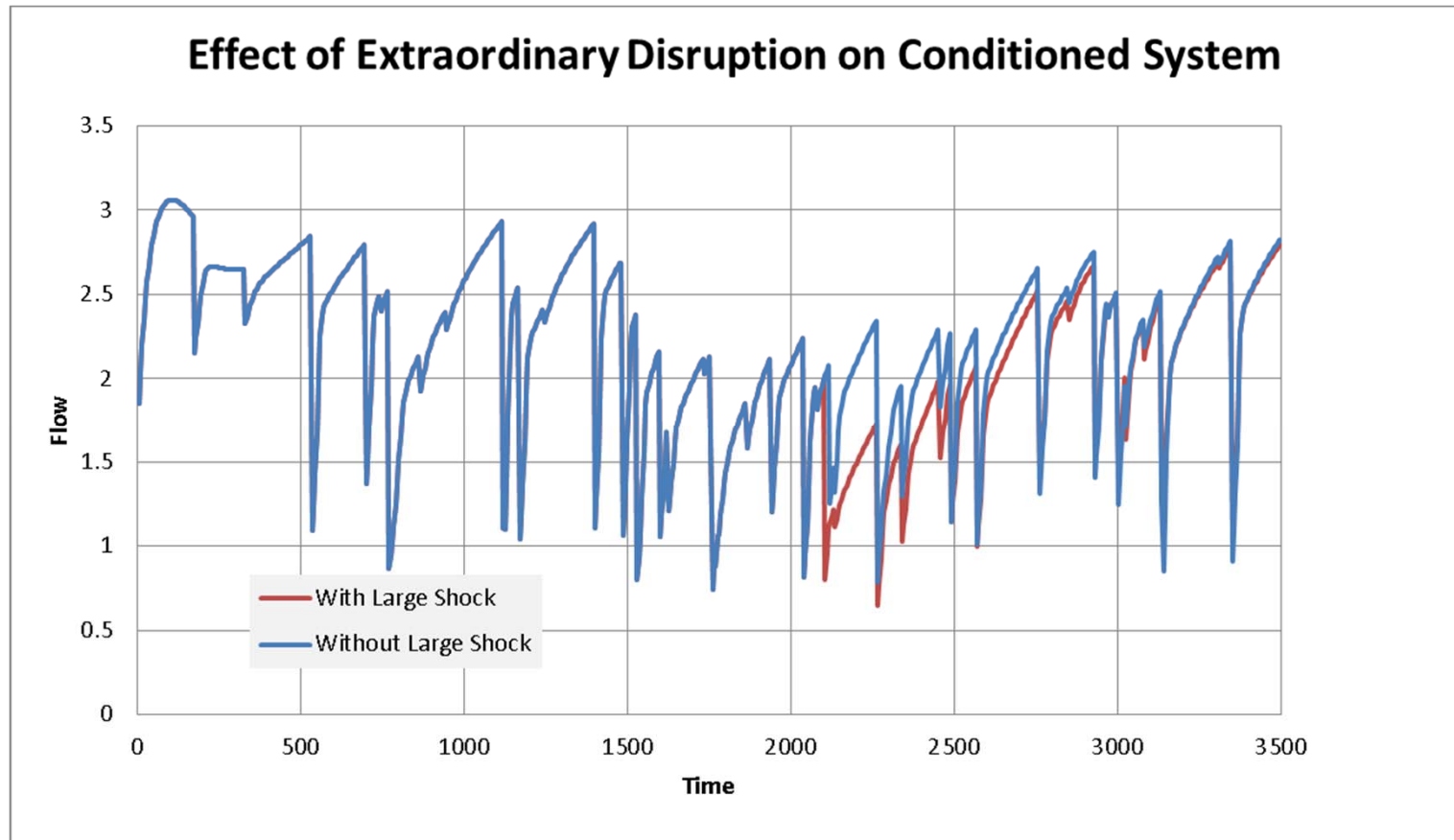
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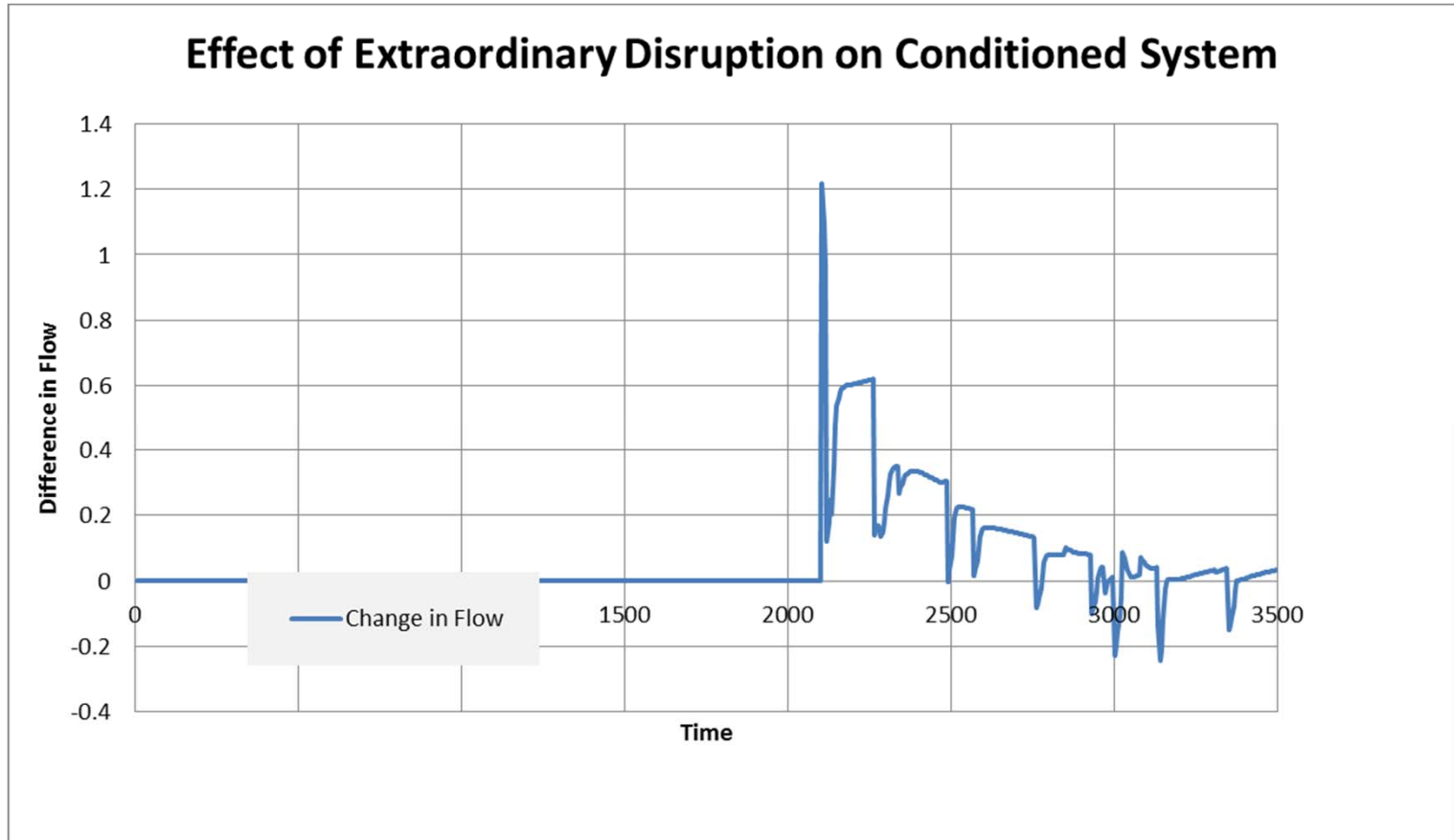


# Assessing Resilience to Extraordinary Disruptions

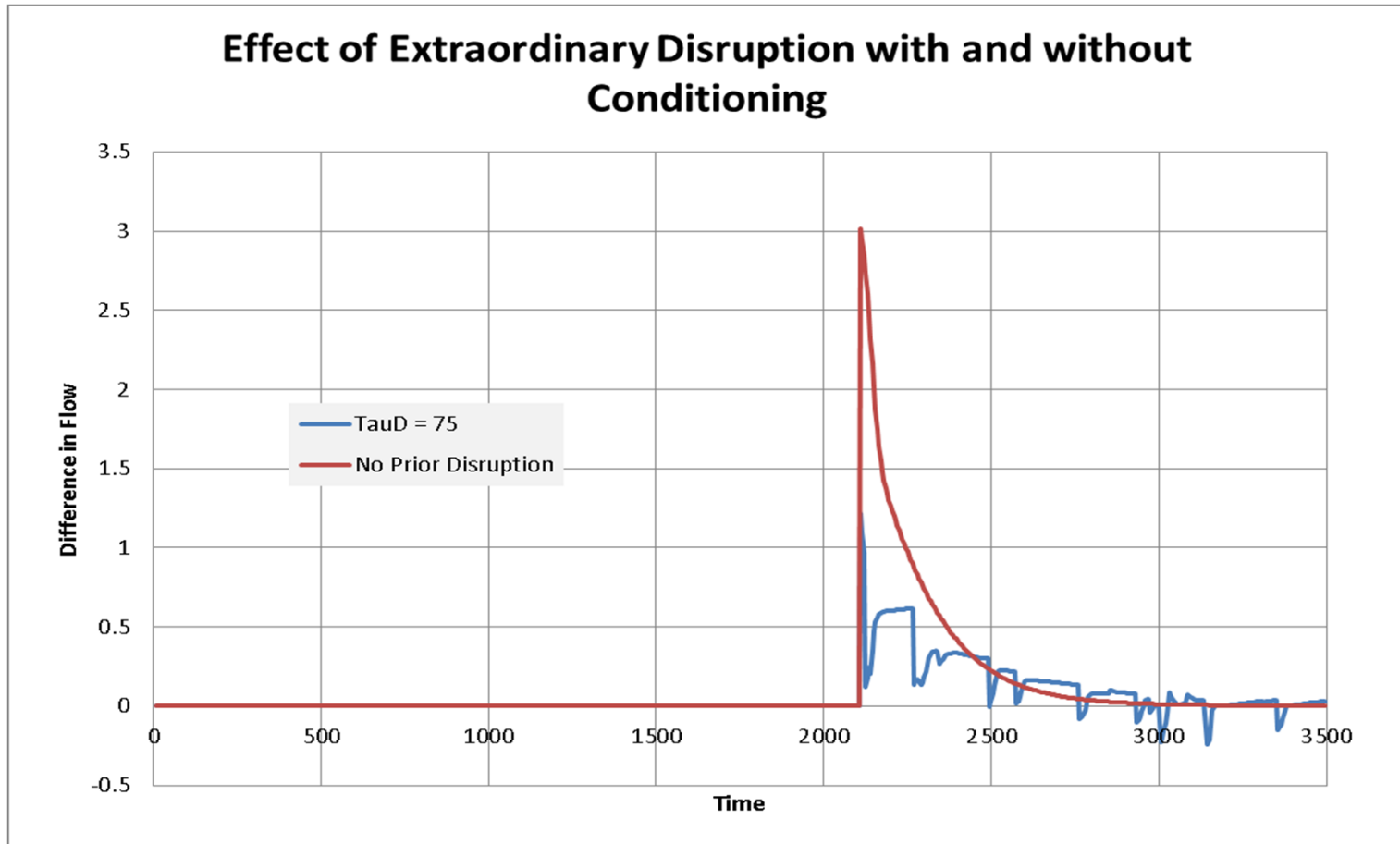
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# Assessing Resilience to Extraordinary Disruptions



# Assessing Resilience to Extraordinary Disruptions



# Summary

- Network structure can reflect an adaptive response, balancing requirements for good nominal performance and resistance to disruption
- Whether this adaptation leads to a resilient system depends on ...
- We are using a simple model of a class of infrastructure systems to understand whether (or under what conditions) adaptation to small disruptions can confer resilience to big ones
- Initial results suggest that
  - The undisturbed system tends toward efficiency
  - Adaptation under disruption can improve performance compared to naïve systems
- Next:
  - Complete and publish an exploration of parameter space for this model
  - Explore application of the approach to other systems of interest having different driving processes and adaptive responses (human networks, communications systems, biological systems)