TORUS: Theory of Reactions for Unstable iSotopes

Final Report from Ohio University

OU PI: Charlotte Elster

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1 Research Report

Mission

The mission of the TORUS Topical Collaboration is to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations, and, by using a new partial-fusion theory, to integrate descriptions of direct and compound-nucleus reactions.

Ohio University concentrates its efforts on the first part of the mission. In this brief report we capture the most important aspects and refer to the full report for details.

Science Highlights

- Separable Potentials for Neutron Scattering off Closed-Shell Nuclei [1].
- Separable Potentials for Proton Scattering off Closed-Shell Nuclei [2].
- Partial Wave Coulomb Wave Functions in Momentum Space [3].
- The Coulomb Problem in Momentum Space without Screening [4].
- Microscopic Optical Potential for Elastic Scattering of ^{6}He [5, 6].

1.1 Separable Representation of Phenomenological Optical Potentials of Woods-Saxon Type

1.1.1 Separable Potentials for Neutron Scattering off Closed-Shell Nuclei

Hlophe and Elster in collaboration with the MSU group

Background and Purpose: One important ingredient for many applications of nuclear physics to astrophysics, nuclear energy, and stockpile stewardship are cross sections for reactions of neutrons with rare isotopes. Since direct measurements are often not feasible, indirect methods, e.g. (d,p) reactions, should be used. Those (d,p) reactions may be viewed as three-body reactions and described with Faddeev techniques. Faddeev equations in momentum space have a long tradition of utilizing separable interactions in order to arrive at sets of coupled integral equations in one variable. While there exist several separable representations for the nucleon-nucleon interaction, the optical potential between a neutron (proton) and a nucleus is not readily available in separable form. For this reason we first embarked in introducing a separable representation for complex phenomenological optical potentials of Woods-Saxon type.

Summary of results: We extended the well-known EST scheme [7] for creating separable representations of two-body transition matrix elements as well as potentials to the realm of complex potentials. Requiring that the separable transition matrix fulfill the reciprocity theorem, we identified a suitable rank-1 separable potential. In analogy to Ref. [7], we generalized this potential to arbitrary rank.

Our calculations were based on the Chapel Hill phenomenological optical potential CH89 [8]. Since the CH89 potential, as nearly all phenomenological optical potentials, is given in coordinate space using Woods-Saxon functions, we first give a semi-analytic Fourier transform of those Woods-Saxon functions in terms of a series expansion. In practice, it turns out that only two terms in the expansion are sufficient for achieving convergence. Note that our approach for deriving the momentum-space optical potential is general and can be applied to any optical potential of Woods-Saxon form. This momentum space CH89 potential is then used in the partial-wave LS integral equation to calculate half-shell t-matrices. These then serve as input to the the generalized scheme for creating separable representations for complex potentials.

We carried out studies of n+^{48}Ca, n+^{132}Sn and n+^{208}Pb, and are able to provide for all cases a systematic classification of support points for partial-wave groups, so that the partial-wave S-matrices are reproduced to at least 4 significant figures compared to the original momentum space solution of the LS equation. We find the low partial waves of the n+^{208}Pb system require a rank-5 separable potential to be well represented in the energy regime between 0 and 50 MeV center-of-mass energy. The support points obtained for this case are well suited to represent all partial waves of the n+^{208}Pb as well as all lighter systems described by the CH89 optical potential.

We found that the rank required for achieving a good representation decreases with increasing angular momentum of the partial wave considered. We developed recommendations for both the rank and the locations of support points to be used when describing medium-mass and heavy systems 0¿generated from the CH89 potential. Our recommendations group together partial waves. We also demonstrated that it is sufficient to determine support points including only the central part of the .¿optical potential; when the spin-orbit interaction is added and the form factors are accordingly modified , the same support points can be expected to yield a good representation.

1.1.2 Separable Potentials for Proton Scattering off Closed-Shell Nuclei

Hlophe, Eremenko, and Elster in collaboration with the MSU group

Background and Purpose: To avoid a screening procedure Mukhamedzhanov derived a three-body theory for (d,p) reactions such that the Faddeev-AGS equations are cast in a momentum-space Coulomb-distorted partial-wave representation, instead of the plane-wave basis [9]. Thus all operators, specifically the interactions in the two-body subsystems must be evaluated in the Coulomb basis, which is a nontrivial task. The formulation also requires the interactions in the subsystems to be of separable form. Proton-proton (*pp*) scattering based on separable interactions was considered some time ago in [10] and [11, 12]. Therein the *pp* interaction was represented in terms of analytic functions, and the parameters in the two lowest partial waves were adjusted to describe the experimentally extracted *pp* phase shifts. While such an approach is viable in the *pp* system, it is not very practical when heavy nuclei are considered, since here many more partial waves are affected by the Coulomb force. Thus our approach for neutron-nucleus scattering must be adjusted to proton-nucleus scattering in order to create the input for (d,p) reaction calculations.

Summary of results: The derivations in the original EST work laid out in [7] set up the scattering problem in a complete plane-wave basis, whereas in this work we need to use a complete Coulomb basis. Consequently, when working in momentum space, we require a solution of the momentum space scattering equation in the Coulomb basis exists. We solve the momentum space Lippmann-Schwinger (LS) equation in the Coulomb basis, following the method introduced in Ref. [13] and successfully applied in proton-nucleus scattering calculations with microscopic optical potentials in Ref. [14].

For deriving a separable representation of the Coulomb-distorted proton-nucleus t-matrix element, we generalize the approach suggested by Ernst, Shakin, and Thaler (EST) [7], to the charged particle case. The basic idea behind the EST construction of a separable representation of a given potential is that the wave functions calculated with this potential and the corresponding separable potential agree at given fixed scattering energies E_i, the EST support points. The formal derivations of [7] use the plane wave basis, which is standard for scattering involving short-range potentials. However, the EST scheme does not depend on the basis and can equally well be carried out in the basis of Coulomb scattering wave functions. In order to generalize the EST approach to charged-particle scattering, one needs to be able to obtain the scattering wave functions or half-shell t-matrices from a given potential in the Coulomb basis.

To demonstrate the feasibility and accuracy of our method, we applied this momentum-space Coulomb EST scheme to proton elastic scattering from ^{12}C, ^{48}Ca, and ^{208}Pb. As example the unpolarized differential cross section for elastic scattering calculated in momentum space and compared with coordinate space values is given in Fig. 1. We found that the same EST support points employed to construct a separable representation of neutron-nucleus optical potentials can be used for the separable representation of the proton-nucleus potential. We showed that the momentum-space S-matrix elements calculated with the separable representation of the Coulomb-distorted proton-nucleus potential as well as the cross sections for elastic scattering agree very well with the corresponding coordinate-space calculation. Since changing from a plane wave to a Coulomb basis preserves the time reversal invariance of the separable potential, the separable Coulomb-distorted proton-nucleus off-shell t-matrix also obeys reciprocity.

We also studied the effects of the short-range Coulomb potential on the proton-nucleus form factor. We found that, with the exception of the lowest partial waves the form factors already vanish at 3.5 fm^{-1}. For the lowest partial waves the short range Coulomb force creates a very slow fall-off for the proton-nucleus form factor at high momenta. The effects of the short-range Coulomb potential quickly decrease as l increases.

In addition, this work demonstrates that when using Coulomb-distorted form factors in A(d,p)B Faddeev reaction calculations carried out in a Coulomb-distorted partial-wave basis, it is mandatory to evaluate neutron and proton-nucleus form factors separately.

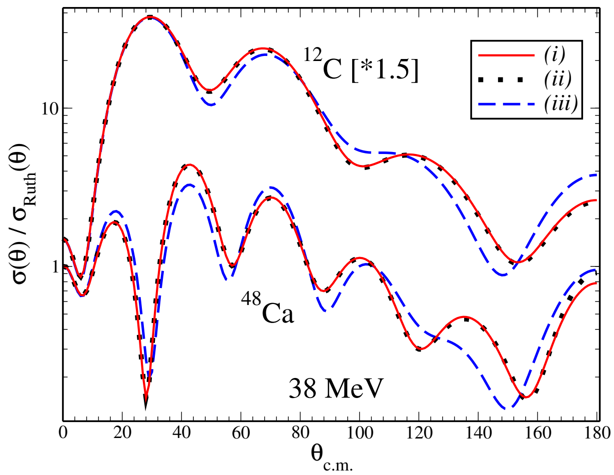

Figure 1: The unpolarized differential cross section for elastic scattering of protons from ^{12}C (upper) and ^{48}Ca (lower) divided by the Rutherford cross section as function of the c.m. angle calculated for a laboratory kinetic energy of 38 MeV. The ^{12}C cross section is scaled by a factor 1.5. The solid lines (*i*) depict the cross section calculated in momentum space based on the rank-4 separable representation of the CH89 [8] phenomenological optical potential, while the dotted lines (*ii*) represent the corresponding coordinate space calculations. The dashed lines (*iii*) show the results in which the short-ranged Coulomb potential is omitted.

1.2 Coulomb in Momentum Space without Screening

1.2.1 Partial Wave Coulomb Wave Functions in Momentum Space

Eremenko, and Elster in collaboration with the MSU group

Background and Purpose: The application of momentum space Faddeev techniques to nuclear reactions has been pioneered in Ref. [15], and successfully applied to (d,p) reactions for light nuclei [16]. However, when extending these calculations to heavier nuclei [17, 18], it becomes apparent that techniques employed for incorporating the Coulomb interaction in Faddeev-type calculations of reactions with light nuclei can not readily be extended to the heaviest nuclei. Therefore, a new method for treating (d,p) reactions with the exact inclusion of the Coulomb force as well as target excitation was formulated in Ref. [9]. This new approach does not rely on screening techniques but rather formulates the Faddeev equations directly in a Coulomb basis. In Ref. [9] generalized Faddeev equations with two charged particles were derived in the AGS form. In order for such an approach to be numerically practical, one needs to have exact expressions for the Coulomb wave function in momentum space as well as reliable techniques to calculate expectation values in this basis.

Summary and Results: The starting point are the Coulomb wave functions, which after a partial wave decomposition can be written as

$$\psi^C_{l,p}(q) = -\frac{2\pi\, e^{\eta\pi/2}}{pq} \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \left[\frac{q^2-(p+i\gamma)^2}{2pq}\right]^{i\eta} (\zeta^2-1)^{-i\frac{\eta}{2}}\, Q_l^{i\eta}(\zeta) \right\} . \tag{1}$$

Here p is the magnitude of a fixed asymptotic momentum and $\zeta = (p^2+q^2)/2pq$. The Sommerfeld parameter is given as $\eta = Z_1 Z_2 e^2 \mu/p$ with $Z_1 Z_2 e^2$ being the total charge and μ the reduced mass of the two-body system under consideration. The spherical function $Q_l^{i\eta}(\zeta)$ in Eq. (1) can be expressed in terms of hyper-geometric functions ${}_2F_1$ as [19]

$$\begin{aligned} Q_l^{i\eta}(\zeta) \;=\; & \frac{e^{-\pi\eta}}{2}\Bigg\{\Gamma(i\eta)\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1;\, 1-i\eta;\, \frac{1-\zeta}{2}\right) \\ & + \Gamma(-i\eta)\,\frac{\Gamma(l+1+i\eta)}{\Gamma(l+1-i\eta)}\left(\frac{\zeta-1}{\zeta+1}\right)^{\frac{i\eta}{2}} {}_2F_1\left(-l, l+1;\, 1+i\eta;\, \frac{1-\zeta}{2}\right)\Bigg\} \end{aligned} \tag{2}$$

under the condition that $|\arg(\zeta \pm 1)| < \pi$ and $|1-\zeta| < 2$, *i.e.*, $-1 < \zeta < 3$. However, care must be taken in its implementation, since there are specific limits of validity of the various expansions of hyper-geometric functions used in its derivation.

A considerable amount of analytical studies and comparisons with the *Mathematica* ® [20] software were carried out by Upadhyay and Nunes, with further details being given in the MSU report. Numerical implementation into robust a computational package and tests against the MSU results were carried out be Eremenko. This suite of codes evaluates the momentum space partial wave Coulomb wave functions for large range of Sommerfeld parameters ($10^{-1} \le \eta \le 10$) with a tested accuracy of about 10^{-6}.

The suite of codes together with a manuscript are published in *Computer Physics Communication* [3], and were already downloaded more than 50 times.

1.2.2 The Coulomb Problem in Momentum Space without Screening

Eremenko, Hlophe, and Elster in collaboration with the MSU group

Background and Purpose: Although the free Coulomb states constitute a basis as well defined as plane waves, the highly complicated nature of their momentum space representation makes it extremely difficult to obtain matrix elements with them. To our knowledge, our work represents the first attempt to obtain such matrix elements with relatively high values of charges. In order to have a chance of numerically realizing the proposed new formulation of Ref. [9] for (d,p) reaction in a Faddeev formulation, it is a mandatory that we carry out 'proof-of-principle' calculations by calculating Coulomb distorted form factors.

Summary and Results: Given the challenge of calculating not only the partial wave Coulomb wave functions $\psi^C_{l,q}(p)$, but also handling their oscillatory singularity when $p = q$ when evaluating integrals of the type

$$u_l^C(p) = \int_0^\infty \frac{dq\, q^2}{2\pi^2}\; u_l(q)\; (\psi^C_{l,p})^\star(q), \tag{3}$$

where $u_l(q)$ is the nuclear form factor. In order to allow for analytical checks, we first used Yamaguchi functions as form factors. Details of this work are given in the MSU report.

After successfully establishing that we correctly implemented the regularization scheme proposed by Gel'fand and Shilov [21], we used the Woods-Saxon type form factors derived in Ref. [1], after adjusting them to p+nucleus scattering.

n Fig. 2 we show in the left panels non-distorted form factors from the separable optical potentials for n+^{12}C, n+^{48}Ca, and n+^{208}Pb. The right panels show the corresponding Coulomb

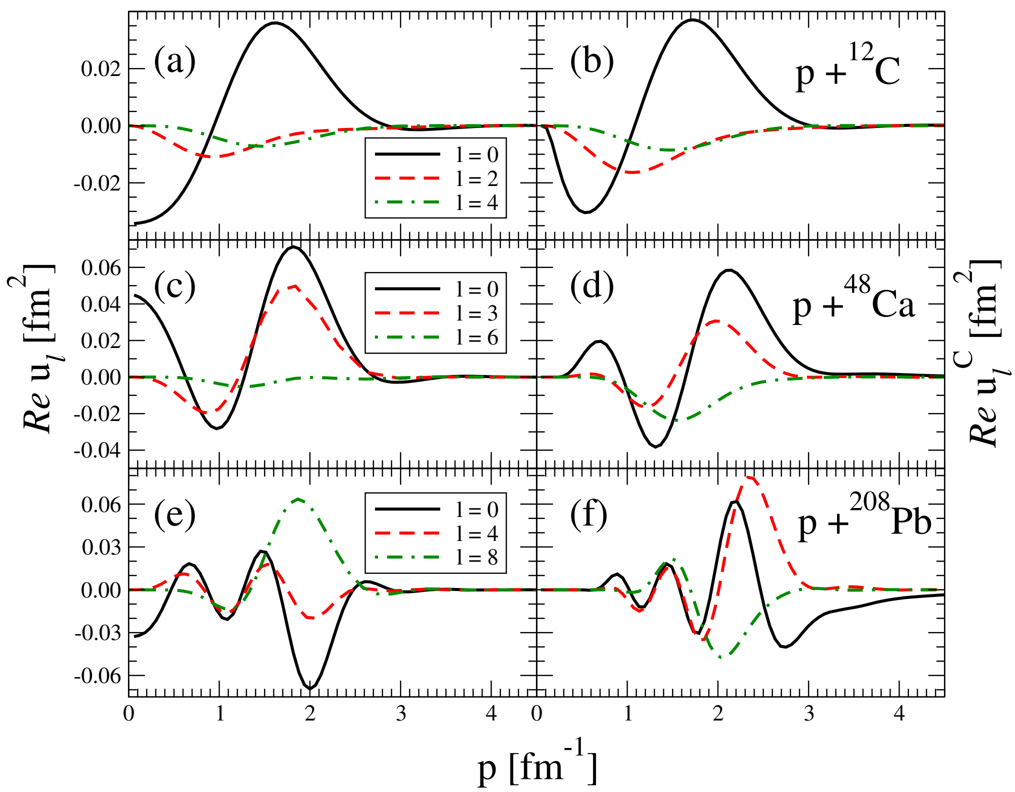

Figure 2: The real parts of the partial wave nuclear form factors $u_l(p)$ (left panels) and the Coulomb distorted nuclear form factors $u_l^C(p)$ (right panels) as function of the the external momentum p for selected angular momenta l: (a) $\Re e\ u_l(p)$ for n+^{12}C; (b) $\Re e\ u_l^C(p)$ for p+^{12}C; (c) $\Re e\ u_l(p)$ for n+^{48}Ca; (d) $\Re e\ u_l^C(p)$ for p+^{48}Ca. (c) $\Re e\ u_l(p)$ for n+^{208}Pb; (d) $\Re e\ u_l^C(p)$ for p+^{208}Pb. The form factors for ^{12}C correspond to the fixed support point $E_{cm} = 30$ MeV, that for ^{48}Ca is at a fixed support point $E_{cm} = 36$ MeV, while the nuclear form factors for ^{208}Pb are at a fixed support point $E_{cm} = 36$ MeV for $l = 0, 4$, and $E_{cm} = 39$ MeV for $l = 8$.

distorted form factors. At zero momentum the nuclear form factors are finite for $l = 0$ while going to zero as p^l for all higher angular momenta as dictated by the partial wave decomposition of the two-body t-matrix they are derived from. In contrast, the Coulomb distorted form factors is also zero for $l = 0$ at $p = 0$. This is associated with the existence of a repulsive barrier at the origin. Comparing the left and right panels of Fig 2 also shows that the Coulomb interaction generally pushes the structure of the form factors from lower momenta to higher momenta. In addition we observe that the heavier the nucleus, the more structure the corresponding form factors exhibit. However, it is interesting to note, that for all nuclei under consideration the form factor goes to zero already at 3 to 4 fm^{-1}, which is a property of the underlying Woods-Saxon ansatz.

In order to carefully study the role of the pole region in the integral of Eq. (3), we perform the integration, but leave out a region of momenta around the pole $p \in [q - \Delta, q + \Delta]$ when

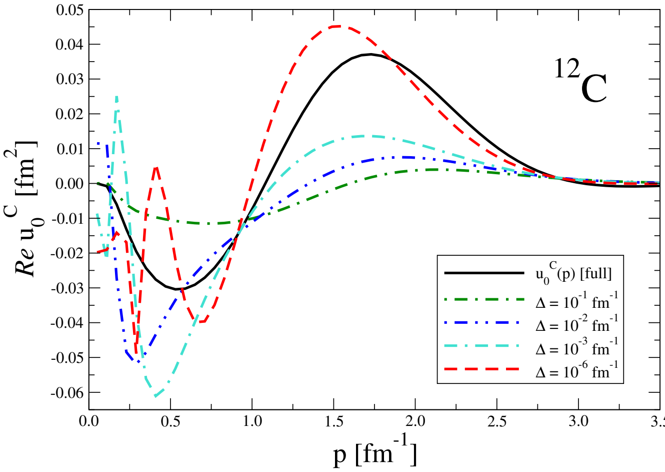

Figure 3: The real part of the $l = 0$ the Coulomb distorted nuclear form factors $u_0^C(p)$ as function of the the external momentum p for ^{12}C at the fixed support point $E_{cm} = 30$ MeV. The solid (black) line shows the full results, while for all other curves an interval of the size Δ has been cut out left and right of the pole p while performing the integration.

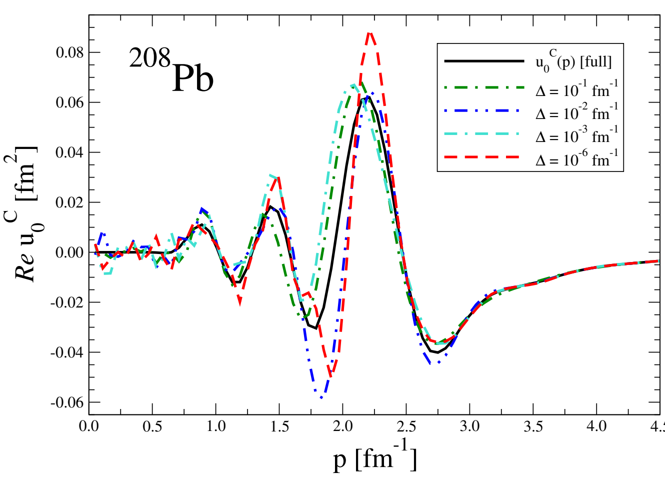

Figure 4: The real part of the $l = 0$ the Coulomb distorted nuclear form factors $u_0^C(p)$ as function of the the external momentum p for ^{208}Pb at the fixed support point $E_{cm} = 36$ MeV. The solid (black) line shows the full results, while for all other curves an interval of the size Δ has been cut out left and right of the pole p while performing the integration.

computing the integral. In Fig. 3 we compare the complete calculation of the real part of the $l = 0$ Coulomb distorted form factor, $u_0^C(p)$, for ^{12}C with calculations of the same integral in which a region Δ around the pole at p was "blended out", i.e. neglected. The complete calculation is the same as shown in Fig. 2. We find that for large Δ (say $\Delta = 0.1$ fm^{-1}) the form factor has little resemblance with the exact one. As Δ becomes smaller, at least in the higher momentum region one can see a continuous build-up towards the exact result. In Fig. 4 we show the identical calculations for the real part of the $l = 0$ form factor for the ^{208}Pb form factor. Here we find that although for all values of Δ considered the form factor computed without the pole region follows the shape of the full form factor, it has quite different values.

To obtain some qualitative insight into this behavior, one has to have the functional form of the Coulomb wave function, $\psi_{l,p}^C(p')$, in mind and consider the dependence on the Sommerfeld parameter η. The smaller η, the more narrowly peaked around the pole p the Coulomb wave function becomes. In case of the ^{208}Pb form factor calculation shown in Fig. 4, η is large and $\psi_{l,p}^C(p')$ has a relatively broad distribution around the pole at p. Consequently, in the integration a relatively large momentum interval of the nuclear form factor $u_l(p')$ contributes. In the case of ^{12}C the Sommerfeld parameter η is already an order of magnitude smaller, and decreases further as function of p, making the momentum distribution of the Coulomb wave function much narrower. In the small p region of Fig. 3 only a relatively small momentum region of the smooth nuclear form factor contributes to the integral. For larger p the value of η becomes smaller and the momentum distribution of the Coulomb wave function even narrower, so that only a very restricted momentum region of the nuclear form factor contributes, leading to the appearance of an almost build-up to the final answer. The Coulomb wave functions contain as one of the leading terms the factor $\exp(-\pi\eta)$, see Eq. (3), thus for large values of η, the contributions in the integrand are smaller. This explains that the variations of the integral for small momenta p are much smaller for ^{208}Pb than for ^{12}C. For example, the value $\eta \sim 1.6$ occurs for ^{12}C at $p \simeq 0.12$ fm^{-1}, while for ^{208}Pb at $p \simeq 1.8$ fm^{-1}. For those momenta both figures show a strong variation of the integral as function of Δ. Once the momenta p become larger, η quickly becomes smaller. In summary, both of these demonstrations show that it is of uttermost importance to carefully treat the pole region in the integral of Eq. (3), since major contributions to this integral come from the region around the pole.

We further implemented Coulomb distorted form factors into the EST formulation for separable optical potentials. In Ref. [22] a rank-1 separable potential was constructed including Coulomb distortions. We implemented a similar approach within the EST scheme. However, since using the sum of Coulomb distorted form factors to construct a p+nucleus optical potential is in general **not** equal to approximating a local Coulomb distorted nuclear potential by a sum of EST form factors, this procedure is not a substitute for dealing with the pinch-singularity which occurs when deriving the expressions for a nuclear potential in Coulomb basis [13]. This results are published in Ref. [4].

1.2.3 Collaboration with MSU

While postdoctoral researcher Dr. Upadhyay was at MSU, Elster and graduate student Hlophe established weekly Skype meetings with the MSU group to discuss and share progress on the projects. After Dr. Upadhyay departed from MSU in the Summer of 2013, funds were directed to OU to hire Dr. Eremenko. Graduate student Hlophe was funded for 9 months by the

TORUS grant. After the funds for the postdoctoral researcher from the TORUS grant ended, Dr. Eremenko was picked up by the DOE nuclear theory grant for one year. During the entire time we continued weekly Skype meeting with MSU to discuss progress, review material, discuss alternative calculations provided by MSU to check the OU results.

2 Other Related Efforts at Ohio University

These efforts related to the TORUS projects were mainly funded through the DOE grant with the nuclear theory group at Ohio University.

Microscopic Cluster Folding Optical Potential for the reaction ^{6}He($\vec{p},\vec{p}$)^{6}He For (d,p) reaction calculations based on a Faddeev formulation, one needs to use as input effective interactions. Our work so far is based on phenomenological optical potentials fitted to a large data set of stable nuclei, from which we derive separable representations. The EST scheme we employ only requires wave functions (or half-shell t-matrices) to construct the separable representation, and thus does not depend on the choice of optical potentials we made. In fact for work with exotic nuclei one should make a more direct contact to microscopic calculations of those effective interactions.

While the typical stable nuclei for which folding models are very successful are mostly spherical, ^{6}He can be understood in few-body models as three-body system consisting of 2 neutrons (n) and a ^{4}He core. Implementing this three-body structure in a cluster model, specifically in a reaction calculation for proton (p) scattering off ^{6}He, was pioneered in Ref. [23].

Elster and collaborator S.P. Weppner developed a microscopic folding optical potential for the reaction ^{6}He(p,p)^{6}He, which takes into account the cluster structure of the ^{6}He nucleus. This work extended the concept of a traditional single-particle folding optical potential in first order in the Watson multiple scattering expansion such that the cluster structure of a halo nucleus is incorporated. In practice, this means that we take into account the internal motion of the valance neutrons with respect to the core. For our calculations we used the density matrix of the three-body cluster orbital shell model approximation (COSMA) introduced in Refs. [24, 25] for the ^{6}He nucleus. We find that the cluster model lowers the cross section for the small angles and brings it closer to the data. Though we do not describe the very small analyzing power at the small angles, we find that the cluster formulation together with a NN t-matrix which takes into account a modification due to the nuclear medium is able to produce a negative analyzing power at larger angles as suggested by the data. Our findings are published in [5].

Microscopic Open-Shell Folding Optical Potential for the reaction ^{6}He($\vec{p},\vec{p}$)^{6}He There is a second issue to be considered when constructing an optical potential for ^{6}He. Despite the ground state being a 0^+, ^{6}He is not a closed shell nucleus. So far, microscopic optical potentials constructed by folding a the NN interaction with the nuclear density matrix assumed closed shell nuclei. In a closed shell nucleus it can be shown analytically that the central and spin-orbit parts of the NN t-matrix, corresponding to the Wolfenstein amplitudes A and C, are the only NN-interaction pieces that enter the optical potential. Elster, Weppner and Ph.D. student A. Orazbayev studied open shell effects in a microscopic optical potential for elastic scattering ^{6}He and ^{8}He with a very simple ansatz, The optical potential needed to describe the reaction is based on a microscopic Watson first-order folding potential, which explicitly takes into account that the two neutrons outside the ^{4}He-core occupy an open p-shell. The folding of the single-particle harmonic oscillator density matrix with the nucleon-nucleon t-matrix leads for this case to new terms not present in traditional folding optical potentials for closed shell nuclei. These findings are published in Physical Review C [6].

Relativistic Formulation of Reaction Theory The Journal of Physics G invited Polyzou and Elster to write an article about relativistic reaction theory [26]. Thus the authors took the opportunity to develop a relativistic formulation of reaction theory for nuclei with a dynamics given by a unitary representation of the Poincaré group. Relativistic dynamics is introduced by starting from a relativistic theory of free particles to which rotationally invariant interactions are added to the invariant mass operator. Poincaré invariance is realized by requiring that simultaneous eigenstates of the mass and spin transform as irreducible representations of the Poincaré group. A relativistic formulation of scattering theory is presented and approximations emphasizing dominant degrees of freedom that preserve unitarity, exact Poincaré invariance and exchange symmetry are discussed. A Poincaré invariant formulation of a (d,p) reaction as a three-body problem is given as an explicit example.

Separable representation of multi-channel nucleus-nucleus optical potentials A three-body ansatz to (d,p) reaction calculations implies that all internal degrees of freedom of the nucleus are integrated out and the nucleus is considered as a single ‘particle’. This approximation may be too severe, since most nuclei are not spherical and can exhibit e.g. rotational excitations. The first work including rotational excitation into a three-body (d,p) reaction calculation was carried out by Deltuva [27, 28], who showed for light nuclei that some reaction cross sections are sensitive to these extra degrees of freedom. Thus, within our path of developing a three-body (d,p) reaction code we need to consider core excitation. Developing the potentials which take those into account is part of the Ph.D. work of graduate student Linda Hlophe, and carried out in collaboration with Filomena Nunes (MSU) and Ian Thompson (LLNL). We assume that the core excitations are of rotational nature. In order to derive a multi-channel separable potential within the previously used EST scheme, we first need to derive and compute the multi-channel LS equations with an optical potential which included rotational excitations. As first test case we chose an example from Nunes et al. [29] with real potentials, ^{11}Be consisting of a ^{10}Be core and a neutron and reproduced the bound state structure given in [29] as well as scattering results in coordinate space (FRESCO) in momentum space with the Fourier transformed potential as well as its separable representation. For real coupling potentials a multi-channel EST scheme had already been derived in Ref. [30]. However for complex potentials we needed to combine this with our previous one-channel work [1] to a multi-channel EST scheme for complex potential. As test case we successfully reproduced the results from Ref. [31], in which the authors suggest a coupled-channel optical potential for neutron scattering from carbon, taking into account the 2^+ and 4^+ excited states of ^{12}C.

Coulomb modified Faddeev-AGS Equations For developing a scheme in which (d,p) reaction can be calculated for heavy nuclei, Mukhamedzhanov [9] derives a three-body formulation which does not include a screening of the Coulomb force. Therein, the Faddeev-AGS equations are cast in the Coulomb-distorted partial-wave representation instead of the plane-wave basis. Up to we have concentrated in practical implementations of the basis and calculating matrix elements of two-body quantities, i.e. form factors, in this basis.

There is however one more issue to deal with, which is ignored in Ref. [9]. For all practical applications the Faddeev-AGS equations are written in Jacobi coordinates, which allows to cleanly separate the center-of-mass momentum and solve the equations in relative coordinates [32]. If two of the three particles are charged, this leads to the well known problem that the repulsive Coulomb potential is in the 'wrong' coordinates in two of the coupled Faddeev-AGS equations [33, 34], since the corresponding Jacobi coordinate of the charged spectator points to the center-of-mass of the pair. In Ref. [9] this issue was not addressed. However, since we are working with Coulomb Green's functions given in Jacobi coordinates, we absolutely must address it and revise the equations of Ref. [9]. Postdoc Vasily Eremenko is currently in the process of reformulating the equations so that they match the formulation we already started to use.

Faddeev-AGS equations in Partial Waves As next step to the numerical realization of the momentum space Faddeev-AGS equations, postdoc Vasily Eremenko derived the general formulation for the iso-spin and spin-angular momentum couplings between for the partial wave equations. Since we work with three distinguishable particles (neutron, proton, and nucleus), we have three coupled Faddeev equations, which need to be represented in a partial wave basis. We work in relative Jacobi coordinates, and have the three different choices of pair and spectator particle, which are all equivalent. One specific basis is picked to carry out the calculation and all other part of the equations need to be expressed in this basis, leading to a set of iso-spin and spin-angular momentum re-couplings. In deriving those we follow the scheme of Balian-Brezin [35, 36], which takes advantage rotational symmetry for specific axes for the explicit evaluation.

Two-neutron halos Nuclei in which the neutron halos involve neutrons in a p-wave relative to the core (e.g. ^{6}He, ^{11}Li) relate to important issues of whether the mechanism of binding in Borromean systems with subsystems involving resonant p-wave interactions is universal. For example, we can ask which features of these systems (e.g. neutron separation energies, radii, E1 dissociation cross sections) are correlated with the energy of the p-wave neutron-core resonance. In order to address this question the Faddeev equations for a three-body system with two (zero-range) resonant p-wave interactions were solved. This was the last part of Chen Ji's Ph.D. work, and was done in collaboration with Phillips and Elster. These results were published in *Physical Review C* [37].

3 Deliverables

Table 1: OU Publications and Presentations for work completed on the TORUS project

	2011	2012	2013	2014	2015	total
Publications		1	2	5	1	9
Proceedings		1	1	1	1	4
Conference Contrib.	3	6	8	2	4	23
Seminars/Colloq.	1	2	4	2		9

3.1 Publications

1. Published paper [5]: Physical Review C**85**, 044617 (2012)
   Elastic Scattering of ^{6}He based on a Cluster Description, S.P. Weppner and Ch. Elster.
2. Published paper [6]: Physical Review C**88**, 034610 (2013)
   Open shell effects in a microscopic optical potential for elastic scattering of $^{6(8)}$He, A. Orazbayev, Ch. Elster, S.P. Weppner.
3. Published paper [1]: Physical Review C**88**, 064608 (2013)
   Separable Representation of Phenomenological Optical Potentials of Woods-Saxon Type, L. Hlophe, Ch. Elster, R.C. Johnson, N.J. Upadhyay, F.M. Nunes, G. Arbanas, V. Eremenko, J.E. Escher, I.J. Thompson.
4. Published paper [38]: Physical Review C**89**, 054605 (2014)
   Reexamining surface-integral formulations for one-nucleon transfers to bound and resonance states, J.E. Escher, I.J. Thompson, G. Arbanas, Ch. Elster, V. Eremenko, L. Hlophe, F.M. Nunes.
5. Published paper [26]: J. Phys. **G41**, 094006 (2014)
   Relativistic Formulation of Reaction Theory, W.N. Polyzou and Ch. Elster.
6. Published paper [4]: Physical Review C**90**, 014615 (2014)
   Coulomb problem in momentum space without screening, N.J. Upadhyay, V. Eremenko, L. Hlophe, F.M. Nunes, Ch. Elster, G. Arbanas, J.E. Escher, I.J. Thompson.
7. Published paper [2]: Physical Review C**90**, 061602(R) (2014)
   Separable Representation of Proton-Nucleus Optical Potentials, L. Hlophe, V. Eremenko, Ch. Elster, F.M. Nunes, G. Arbanas, J.E. Escher, I.J. Thompson.
8. Published paper [37] : Physical Review C**90**, 044004 (2014)
   ^{6}He nucleus in halo effective field theory, C. Ji, Ch. Elster, D.R. Phillips.
9. Published paper [3]: Comp. Phys. Comm. **187**, 195 (2015)
   Coulomb Wave Functions in Momentum Space, V. Eremenko, N. J. Upadhyay, I. J. Thompson, Ch. Elster, F. M. Nunes, G. Arbanas, J. E. Escher, L. Hlophe (TORUS Collaboration).

3.2 Conference Proceedings

1. Proceedings [39]: J. Phys. Conf. Ser. 403, 012025 (2012)
   Nuclear Reactions: A Challenge for Few- and Many-Body Theory, Ch. Elster and L. Hlophe.
2. Proceedings [40]: Few Body Syst. **54**, 1399 (2013)
   Microscopic Optical Potentials for Helium-6 Scattering off Protons, Ch. Elster, A. Orazbayev, S.P. Weppner.
3. Proceedings [41]: Few Body Syst. **55**, 683 (2014)
   Panel Session on the Future of Few-Body Physics, B.L. Bakker, J. Carbonell, Ch. Elster, E. Epelbaum, N. Kalantar-Nayestanaki, J-M. Richard.
4. Proceedings [42]: *arXiv:1410.1227* (in press)
   Separable Optical Potentials for (d,p) Reactions, Ch. Elster, L. Hlophe, V. Eremenko, F.M. Nunes, G. Arbanas, J.E. Escher, I.J. Thompson.

3.3 Presentations at Conferences

1. *"Microscopic Optical Potentials for the Reaction Helium-6 (p,p) Helium-6"*, Inv. Talk, Ch. Elster, Mini-workshop on 'Polarization Phenomena in Proton Elastic Scattering from Unstable Nuclei', December 21, 2011, RIKEN, Tokyo, Japan.
2. *"Application of Three-Body Methods in Nuclear Reactions: ^{6}He(p,p)^{6}"*, Inv. Talk, Ch. Elster, INT workshop on 'Interfaces between Nuclear Reactions and Structure', August 8 - September 11, 2011, Seattle, WA.
3. *"Elastic Scattering of ^{6}He based on a Cluster Description"*, Contributed Talk, S.P. Weppner, Ch. Elster, 2011 Fall Meeting of the APS Division of Nuclear Physics, East Lansing, MI, Bulletin of the American Physical Society,
   Vol. 56, No 12, BAPS.2011.DNP.JE.2
4. *"Spin phenomena is elastic scattering of Helium-6 off Protons"*, Inv. Talk, Ch. Elster, INT workshop on 'Structure of Light Nuclei', October 7-12, 2012, Seattle, WA.
5. *"Towards a Faddeev Description of (d,p) Reactions: Separabilization of Optical Potentials"*, Contributed Talk, Ch. Elster, L. Hlophe, 25th Midwest Nuclear Theory Get-Together, Sept. 7-8, 2012, Argonne, IL.
6. *"Microscopic Optical Potentials for Helium-6 scattering off Protons"*, Contributed Talk, Ch. Elster, A. Orazbayev, S.P. Weppner, The 20th International IUPAP Conference on Few-Body Problems in Physics, August 20-25, 2012, Fukuoka, Japan.
7. *"Nuclear Reactions: A Challenge for Few- and Many Body Theories"*, Inv. Talk, Ch. Elster, Horizons of Innovative Theories, Experiments, and Supercomputing in Nuclear Physics (HITES 2012), June 4-7, 2012, New Orleans, LA.
8. *"Theoretical considerations of internal spin for elastic nucleon-nucleus scattering of ^{6}He"*, S.P. Weppner, A. Orazbayev, and Ch. Elster, 11th Conference on the Intersections of Particle and Nuclear Physics (CIPANP) 2012, May 29-June 3, 2012, St Petersburg, FL.

9. *"Polarization Phenomena in the Reaction $^6He(p,p)^6He$"*, Contributed Talk, A. Orazbayev, S.P. Weppner, Ch. Elster, Annual Meeting of the Ohio Section of APS (OSS12), April 13-14, 2012, Columbus, OH.
10. *"Coulomb distorted nuclear matrix elements in momentum space: I. Formal aspects"*, Contributed Talk, N.J. Upadhyay, V. Eremenko, L. Hlophe, F.M. Nunes, Ch. Elster, Annual Meeting of the Division of Nuclear Physics (DNP), October 24-26, 2013, Newport News, VA.
11. *"Coulomb distorted nuclear matrix elements in momentum space: II. Computational Aspect"*, Contributed Talk, V. Eremenko, N.J. Upadhyay, L. Hlophe, Ch. Elster, F.M. Nunes, Annual Meeting of the Division of Nuclear Physics (DNP), October 24-26, 2013, Newport News, VA.
12. *"Momentum Space Coulomb Distorted Matrix Elements for Heavy Nuclei"*, Contributed Talk, Ch. Elster, V. Eremenko, N.J. Upadhyay, L. Hlophe, F.M. Nunes, G. Arbanas, J.E. Escher, I.J. Thompson, The 22nd European Conference on Few-Body Problems in Physics, September 9-13, 2013, Krakow, Poland.
13. *"Momentum Space Coulomb Distorted Matrix Elements for Heavy Nuclei"*, Contributed Talk, V. Eremenko, N.J. Upadhyay, F.M. Nunes, Ch. Elster, L. Hlophe, G. Arbanas, J.E. Escher, I.J. Thompson (The TORUS Collaboration), 26th Midwest Nuclear Theory Get-Together, Sept. 6-7, 2013, Argonne, IL.
14. *"Towards (d,p) Reactions with Heavy Nuclei in a Faddeev Description"*, Inv. Talk, Talk, Ch. Elster, International Workshop on Nuclear Dynamics with Effective Field Theories, July 1-3. 2013, Bochum, Germany.
15. *"Microscopic Optical Potential for Scattering of ^6He and ^8He off Protons"*, Contributed Talk, Ch. Elster, A. Orazbayev, S.P. Weppner, APS April Meeting 2013, April 13-16, Denver, CO.
16. *"Effect of varying charge and mattter radii on observables in ^6He and ^8He"*, Contributed Talk, A. Orazbayev, Ch. Elster, S.P. Weppner, Spring 2013 Meeting of the APS Ohio-Region Section, March 29-30, Athens, Ohio.
17. *"Separabilization of Optical Potentials in Momentum Space"*, Contributed Talk, L. Hlophe, Ch. Elster, Spring 2013 Meeting of the APS Ohio-Region Section, March 29-30, Athens, Ohio.
18. *"The Coulomb Problem in Momentum Space without Screening"*, Invited Talk, Ch. Elster (for the TORUS Collaboration), Nuclear Theory in the Supercomputing-Era (NTSE-2014), June 23-27, 2014, Khabarovsk, Russia.
19. *"Coulomb distorted T-Matrix Elements in Momentum Space"*, Contributed Talk, V. Eremenko, N.J. Upadhyay, Ch. Elster, F.M. Nunes, I.J. Thompson, G. Arbanas, J.E. Escher, Fourth Joint Meeting of the Nuclear Physics Division (DNP) of the APS and the Physical Society of Japan, October 7-11, 2014, Hawaii.
20. *"About Faddeev-AGS equations for (d,p) reactions on heavy nuclei"*, Contributed Talk, V. Eremenko, L. Hlophe, Ch. Elster, F.M. Nunes, I.J. Thompson, G. Arbanas, J.E. Escher, 21st International Conference on Few-body Problems in Physics, May 18-22, 2015, Chicago, USA.

21. *"Separable forces for (d, p) reactions"*, Poster, L. Hlophe, V. Eremenko, and Ch. Elster, 21st International Conference on Few-body Problems in Physics, May 18-22, 2015, Chicago, USA.
22. *"Towards Faddeev-AGS equations in a Coulomb basis in momentum space"*, V. Eremenko, L. Hlophe, N.J. Upadhyay, Ch. Elster, F.M. Nunes, I.J. Thompson, G. Arbanas, and J.E. Escher, Contributed Talk, INT Workshop on "Reactions and Structure of Exotic Nuclei", March 2-13, 2015, Seattle, WA.
23. *"Microscopic Folding Potentials and Connections to Structure Description"*, Inv. Talk, INT workshop on "Reactions and Structure of Exotic Nuclei", March 2-13, 2015, Seattle, WA.

3.4 Seminars

1. Seminar: Ch. Elster, RIKEN Nishina Center, RIKEN, Japan, December 2011: 'Microscopic Optical Potentials for the Reaction Helium-6 (p,p) Helium-6'
2. Seminar: Ch. Elster, Oak Ridge National Laboratory, November 2012: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'
3. Seminar: Ch. Elster, Livermore National Laboratory, Livermore, CA, December 2012: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'
4. Seminar: Ch. Elster, The Ohio State University, Columbus, OH, February 2013: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'
5. Seminar: Ch. Elster, Notre Dame University, IN, November 2013: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'
6. Seminar: Ch. Elster, NSCL, Michigan State University, November 2013: 'Separable Optical Potentials for (d,p) Reaction Calculations'
7. Seminar: Ch. Elster, Iowa State University, Ames, IA, October 2013: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'
8. Seminar: Ch. Elster, Dept. of Physics and Astronomy, Louisiana State University, Baton Rouge, LA, September 2014, 'Nuclear Reactions: A Challenge for Few- and Many-Body Theory'.
9. Seminar: Ch. Elster, Pacific University, Khabarovsk, Russia, June 2014: 'Spin Phenomena in Elastic Scattering of Helium-6 and Helium-8 off Protons'

4 Student Tracking Information

The following students have received support from this grant during the past twelve months:

Student	Entered OU	Joined NT Group	Projected Graduation	Advisor
Linda Hlophe	Fall 10	Summer 11	Summer 16	Elster

Linda Hlophe receive one quarter (Spring 2012) and one semester (Spring 2013) support from the TORUS trant, while being supported in Summer and Fall 2013 by DOE contract No. DE-FG02-93ER40756 with Ohio University. He was supported in Spring, Summer, Fall 2014, Spring, Summer 2015 by the same grant and will continue to be supported by this grant until his graduation in Summer 2016.

5 TORUS Supported Activities

5.1 Staff

OU Postdoctoral Researcher:
The TORUS project had funding for one postdoctoral researcher for the duration of 4 years. For the initial 3 years of the project the this postdoc was located at MSU. In the 4th year the funds were moved to Ohio University and Dr. Vasily Eremenko was hired. OU received as supplement from the TORUS grant for additional months. Dr. Eremenko's work focused on the numerical implementation of the momentum-space partial-wave Coulomb functions, as well as the analytical and numerical implementation of the Coulomb distorted form factors. In both projects he collaborated with the MSU group. For his second year at Ohio University, Dr. Eremenko was funded by the OU nuclear theory group grant. During this time he developed the partial wave representation of the Faddeev-AGS equations as needed for a (d,p) reaction calculation. During the last months he focused on the derivation of Coulomb modified Faddeev-AGS equations.

Dr. Eremenko will move on to a staff position at Moscow State University in September 2015.

OU Graduate Student:
The TORUS project moved funds to support an OU graduate student for 9 months. From the start of his Ph.D. project at OU, graduate student Linda Hlophe focused on separable representations of optical potentials. First he derived the for neutron-nucleus, then for proton-nucleus scattering. Currently he derives coupled-channel versions to take into account core-excitations in the (d,p) reactions. Linda Hlophe is supported by the OU nuclear theory grant and will graduate in Summer 2016.

5.2 Visitors supported by the grant

The grant contributed to the sabbatical support of Prof. Stephen Weppner, who spent the academic year 2010-11 at Ohio University. After that, Prof. Weppner visited OU for about 2 weeks each in 2012, 2013, and 2014.
The grant supported the visit of Dr. Arnoldas Deltuve in Spring 2015 to OU.

5.3 Travel supported by the grant

The grant supported the travel of the PI, postdoc Eremenko, and graduate student Linda Hlophe to conferences in which TORUS research was presented. Those were the DNP meetings during the grant period, the International Few-Body Conferences in Fukuoka (2012) and Chicago (2015), the European Few-Body Conference in Kracow (2014) and others as shown in Section 3.3. In

addition travel to collaboration meetings at MSU and LLNL by Elster, Eremenko, and Hlophe was supported. The grant also supported travel of Eremenko to the INT and Hlophe to a summer school at ECT* in Trento.

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