

Development of Constitutive Parameters from True Triaxial Tests Performed on Castlegate Sandstone

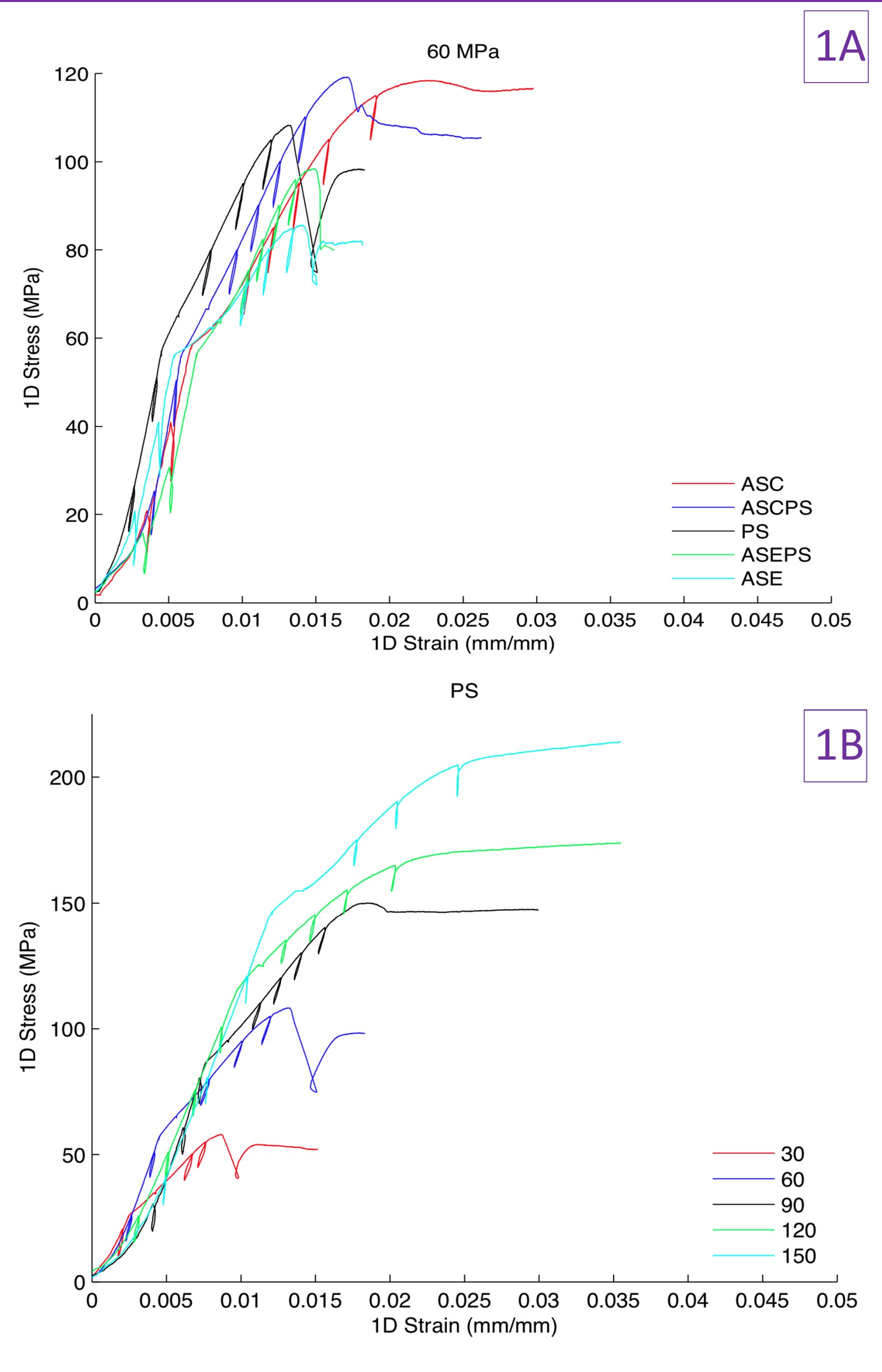
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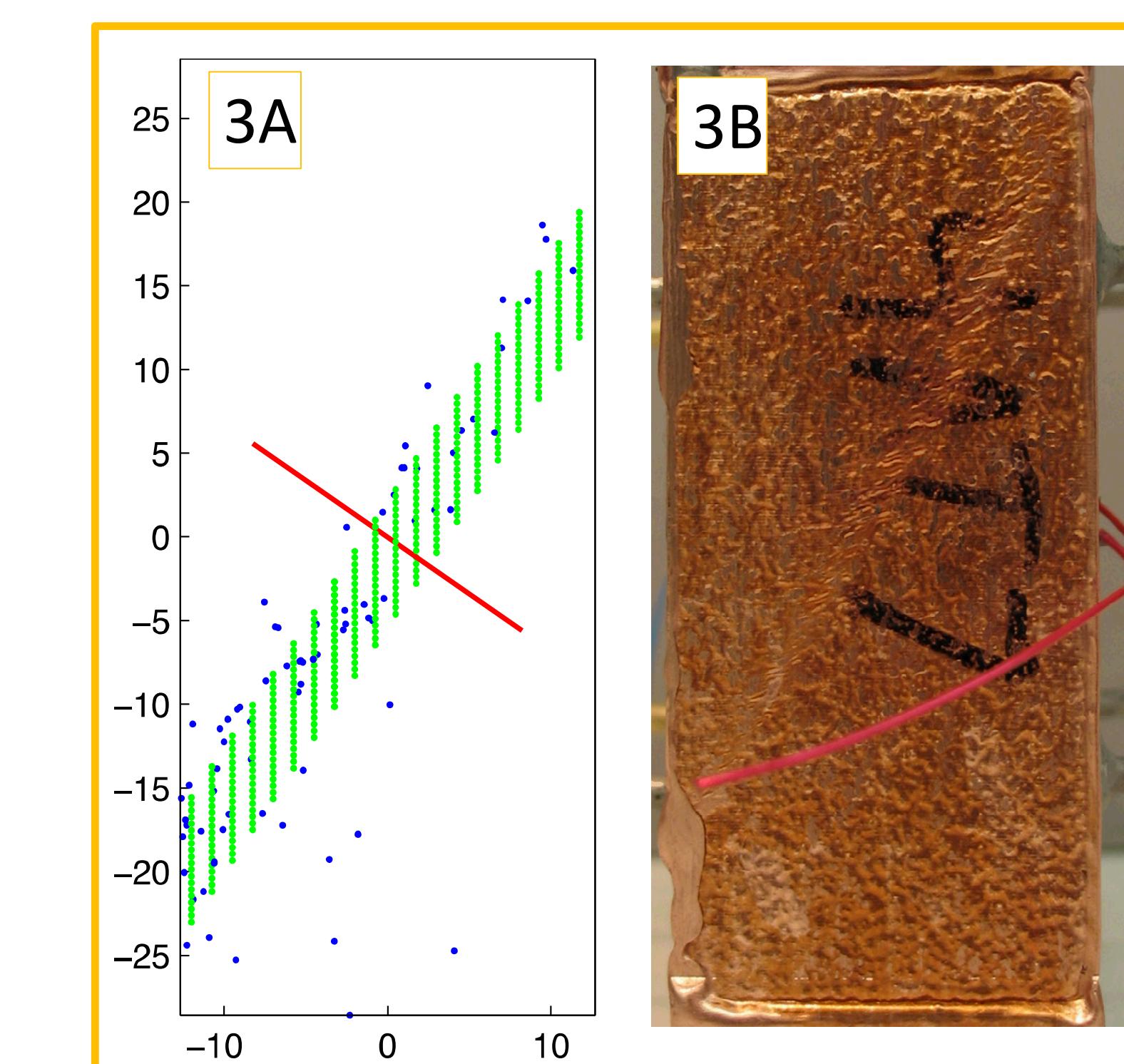
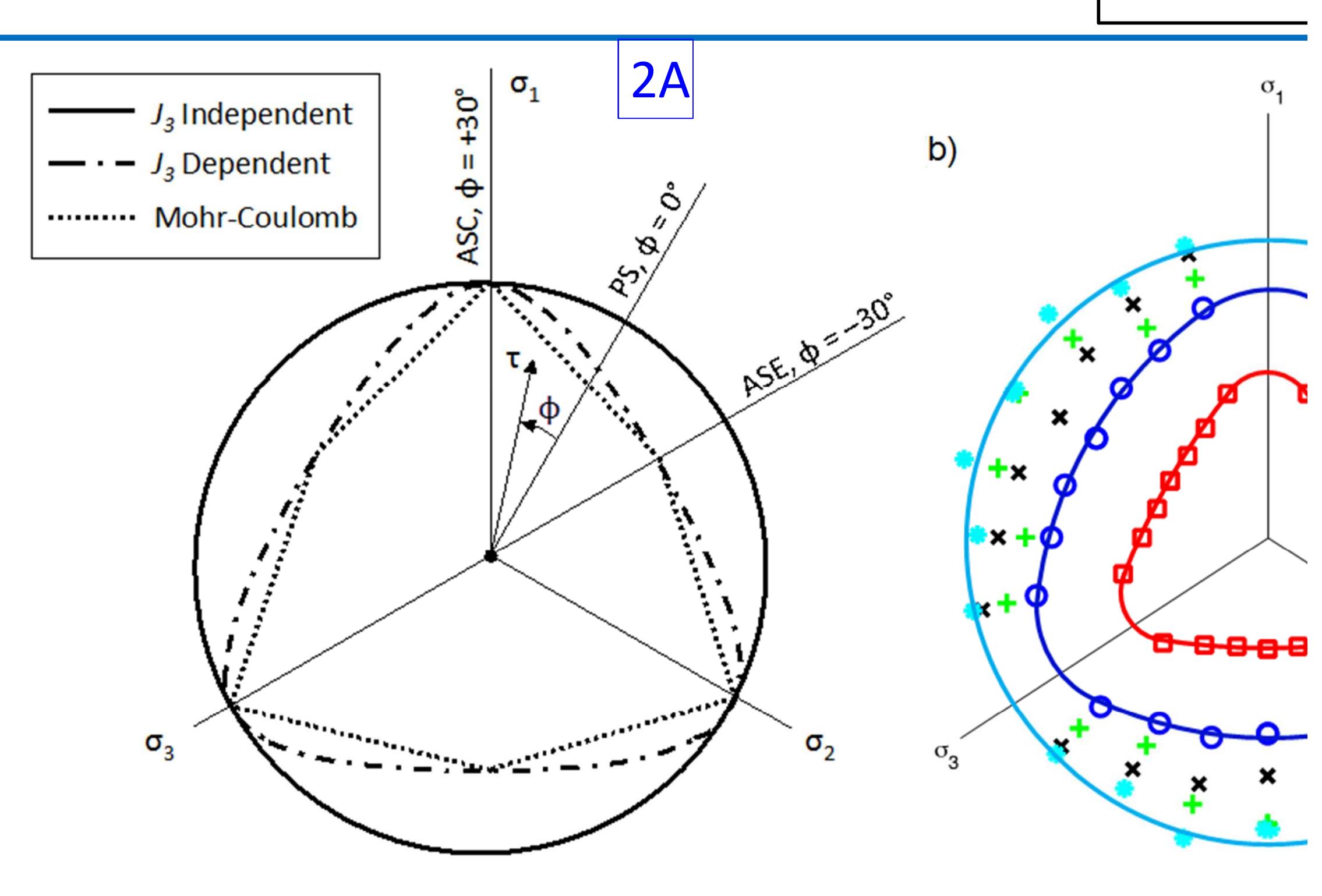
Experimental Methods:

Conducted true triaxial tests to investigate influence of σ_2 on failure

- Used smaller true triaxial apparatus at Sandia (Wawersik et al. 1997)
- Performed tests at constant mean stress, σ , and constant Lode angle, ϕ .
 - Mean stresses: 30, 60, 90, 120, and 150 MPa
 - Assessed J_3 dependence of failure
 - Enabled extraction of bulk and shear moduli from unload loops
 - Lode angles: 30° (ASC), 14.5°, 0°(PS), -14.5°, -30°(ASE)
 - Ranged from axisymmetric compression (ASC) to axisymmetric extension (ASE); determines the stress path in the Pi plane
 - Constant Lode angle, ϕ , is equivalent to constant N_{II}
 - Evaluated strain localization predictions (Rudnicki & Rice 1975)
 - Recorded acoustic emissions (AE); located AE within specimen
 - Plane fit through AE to locate band spatially and temporally (Fig. 3)
 - Onset of localization corresponds to drop in fitting error
 - AE band angle, θ , agrees with angle measured on jacket (Table 1)

Bulk Specimen Response:

- Typical mechanical responses (σ_1 vs. ε_1) are shown in Fig. 1 :
 - 60 MPa constant mean stress tests at five Lode angles (Fig. 1A)
 - 0° (PS) Lode angle tests at five mean stresses (Fig. 1B)
- Mean stress dependence (Fig. 1B):
 - Low mean stress: stress peak and drop; shear band forms
 - High mean stress: stress plateau; compaction band or no band
- Lode angle dependence (Fig 1A): moving from ASC to ASE:
 - Sharper stress peak
 - Lower stress required to initiate failure
- Failure depends on σ_2 (see Fig. 2; octahedral plane)
 - Low mean stress: failure surface is a rounded triangle
 - High mean stress: failure surface is circle
 - Failure depends on third invariant of deviatoric stress, $J_3 = \sigma'_1 \sigma'_2 \sigma'_3$



Ingraham, MD, Issen, KA, Holcomb, DJ, "Response of Castlegate sandstone to true triaxial states of stress" *J. Geophys. Res. Solid Earth*, **118** (2013) p. 536, doi:10.1002/jgrb.50084.

Ingraham, MD, Issen, KA, Holcomb, DJ "Use of acoustic emissions to investigate localization in high-porosity sandstone subjected to true triaxial stresses" *Acta Geotechnica*, **8** (2013) p.645, doi: 10.1007/s11440-013-0275-y.

Localization Conditions:

Predicted band angle, θ , Rudnicki & Rice (1975) (see Eqn. 4) depends on:

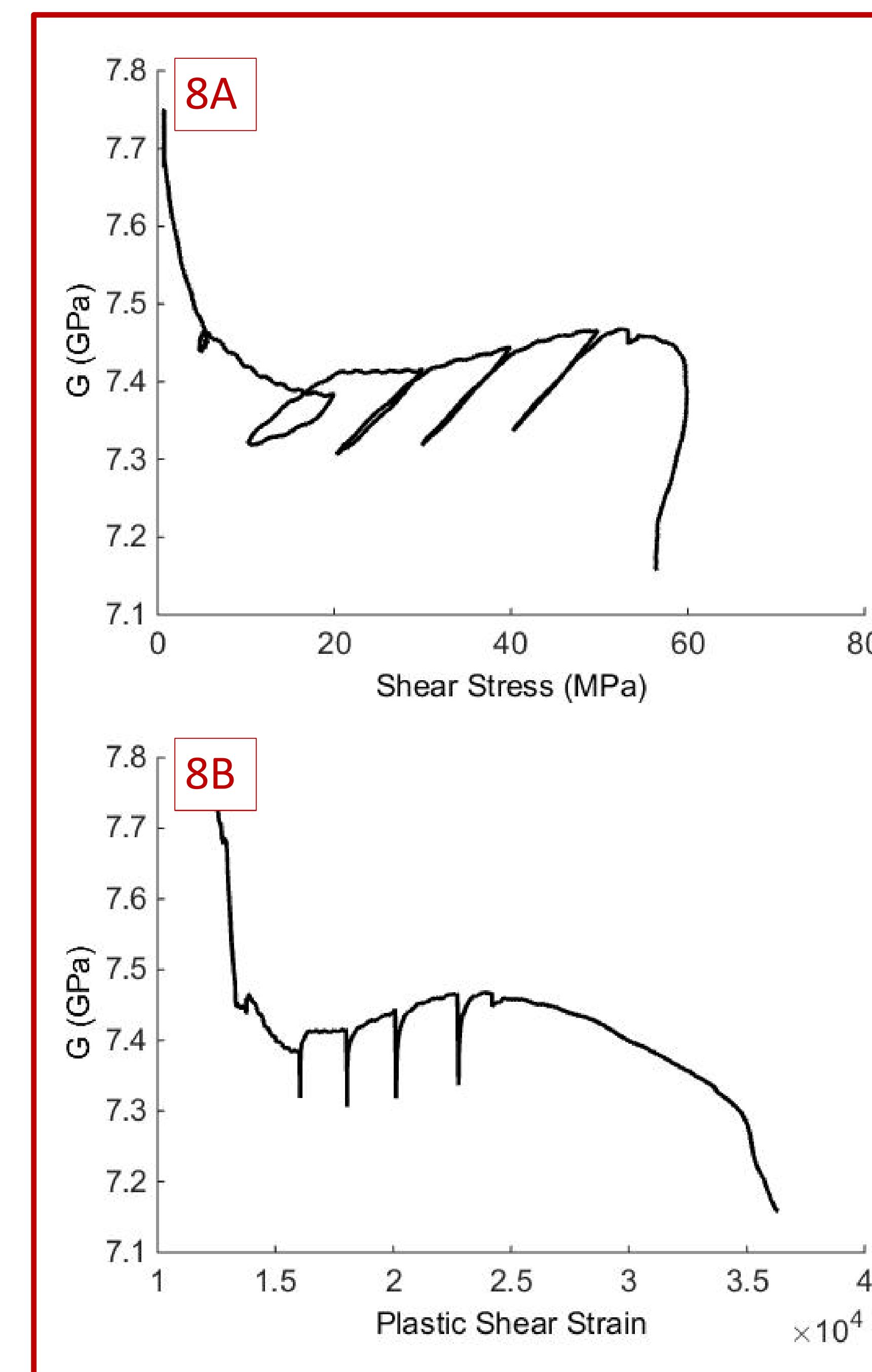
- Lode angle, through parameter N_{II}
- Friction factor, μ , which is yield surface slope
- Dilation coefficient, $\beta = -d^p \varepsilon / d^p \gamma$
- Poisson's ratio, v , for elastic unloading

Strain Separation:

- Elastic bulk and shear moduli, K and G , are
 - Stress dependent (Fig 8A)
 - Plastic strain dependent (Fig. 8B)
- Total strain and strain increment (Eqn. 5)
- Strain separates into four parts (Eqn. 6):
 - A: elastic strain at constant modulus
 - B: strain due to stress dependent moduli
 - C: strain due to plastic strain dependent moduli
 - D: plastic strain
- Plot four strain components (Fig. 7)

Localization Predictions:

- Calculate increment of inelastic strain : C + D
- Plot yield surface; determine friction factor, μ
- Determine β from plot of $d^p \varepsilon$ vs. $d^p \gamma$
- Calculate predicted band angle, θ (Table 1)



4

$$\theta = \frac{\pi}{4} + \frac{1}{2} \arcsin \left[\frac{\frac{2}{3}(1+v)(\beta + \mu) - N_{II}(1-2v)}{\sqrt{4-3N_{II}^2}} \right]$$

$$N_I = \frac{(\sigma - \sigma_3)}{\tau}, N_{II} = \frac{(\sigma - \sigma_2)}{\tau}, N_{III} = \frac{(\sigma - \sigma_1)}{\tau}$$

μ – Friction Factor, β – Dilation Coefficient, v – Poisson's Ratio

5

$$\gamma^t = \gamma^e + \gamma^p$$

$$d\gamma^t = d\left(\frac{\tau}{G(\tau, \gamma^p)}\right) + d\gamma^p$$

$$d\gamma^t = \frac{d\tau}{G} - \frac{\tau}{G^2} \left(\frac{\partial G}{\partial \tau} d\tau + \frac{\partial G}{\partial \gamma^p} d\gamma^p \right) + d\gamma^p$$

6

$$d\gamma_A = \frac{d\tau}{G}$$

$$d\gamma_B = -\frac{\tau}{G^2} \left(\frac{\partial G}{\partial \tau} d\tau \right)$$

$$d\gamma_C = -\frac{\tau}{G^2} \left(\frac{\partial G}{\partial \gamma^p} d\gamma^p \right)$$

$$d\gamma_D = d\gamma^p$$

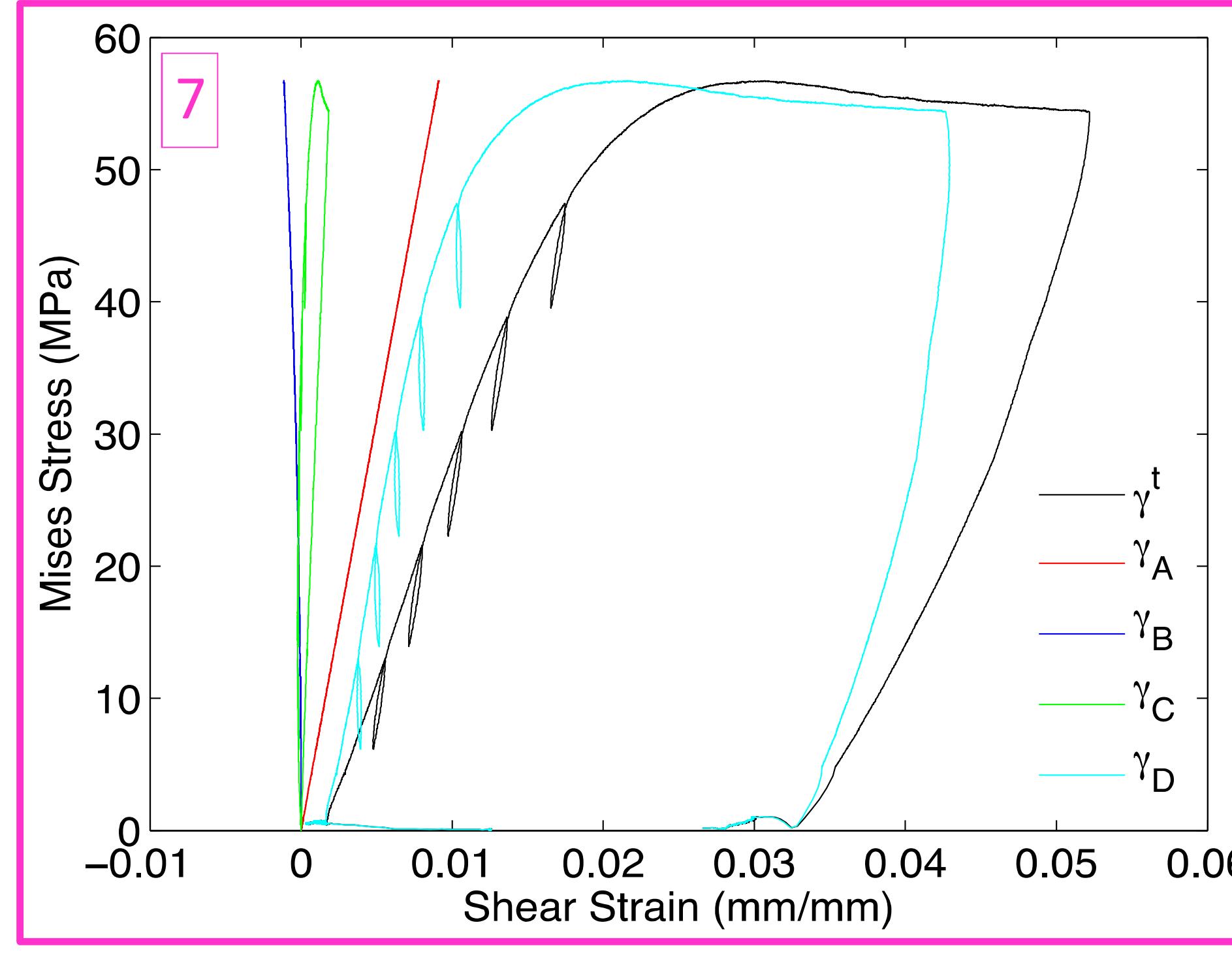


Table 1

Stress State	Mean Stress (MPa)	β	μ	Predicted θ	AE θ	Measured θ	Response Type
ASC	30	0.76	0.56	59	Conj. Bands	55-60	Shear
ASC	60	0.23	0.31	48		23	Shear
ASC	90	0.01	0.09	42	10-23	NA	CL
ASC	120	-0.29	0.2-0.3	37:33	5-15	NA	CL
ASC	150	-0.66	-1.1:-3	3:0	NL	NA	CL
PS	30	0.09	0.94	57	58	61-80	Shear
PS	60	0.55	0.80	62	63	64	Shear
PS	90	0.08	0.67	54	54	58	Shear
PS	120	-0.23	0.2-0.7	42:33	NL	NA	NL
PS	150	-0.75	-1.5:-4.4	15:0	16-25	NA	CL
ASE	30	0.76	0.85	80	51	65	Shear
ASE	60	0.65	0.49	68	NA*	70	Shear
ASE	90	0.04	0.13	54	41	46	Shear
ASE	120	-0.17	0.2-1.9	50:23	Conj. Bands	45	Shear
ASE	150	-0.21	-1.8:-6	25:0	10-25	NA	CL

Conclusions:

- Mechanical response and failure (band orientation and failure surface) depend on σ_2
- Elastic moduli evolve with stress and plastic strain
- Strain separation was used to determine onset of yield and yield surfaces
- Reasonable agreement between localization predictions using strain separation and experimental results for Castlegate sandstone