

# Development of Constitutive Parameters from True Triaxial Tests Performed on Castlegate Sandstone

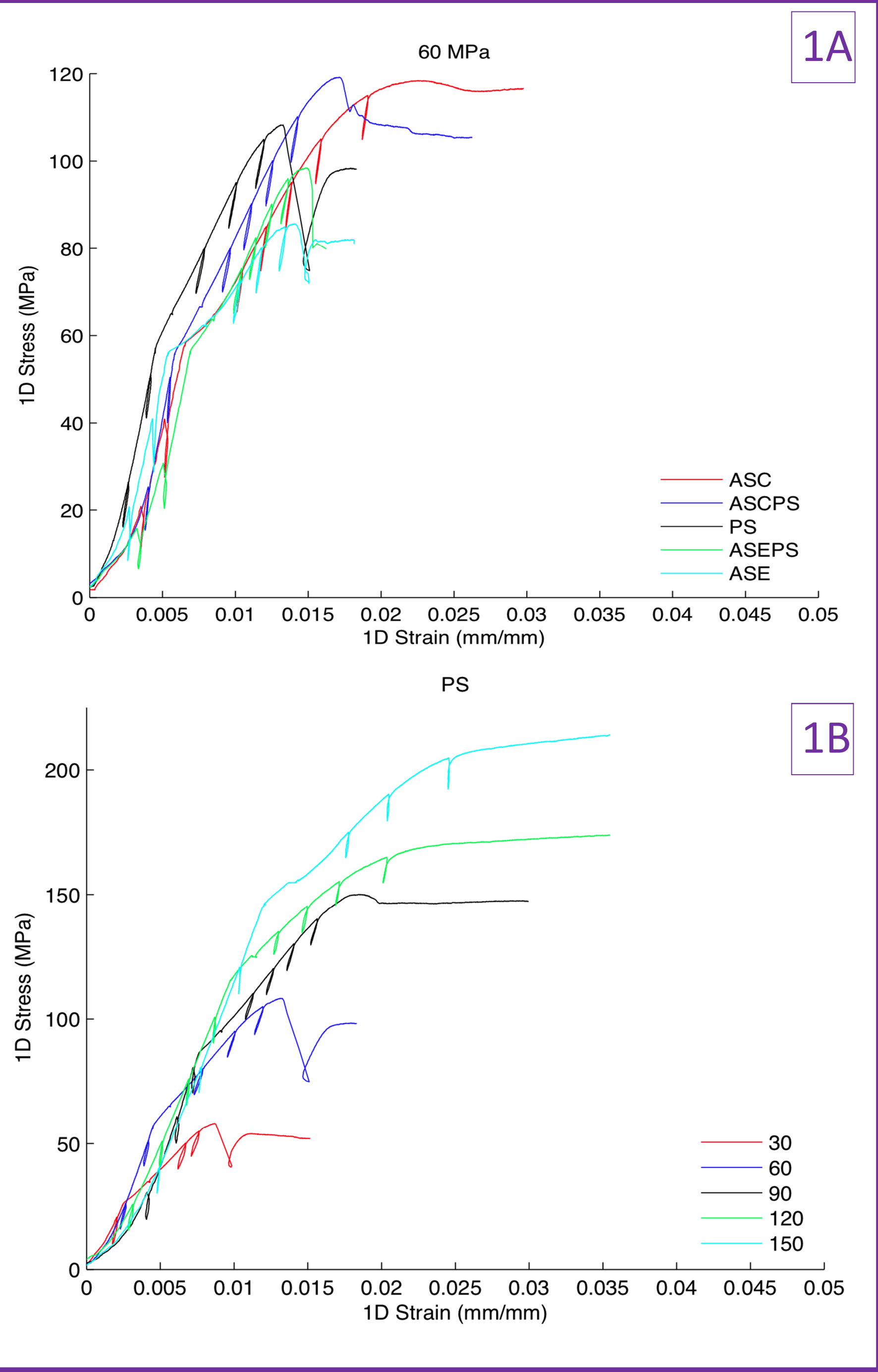
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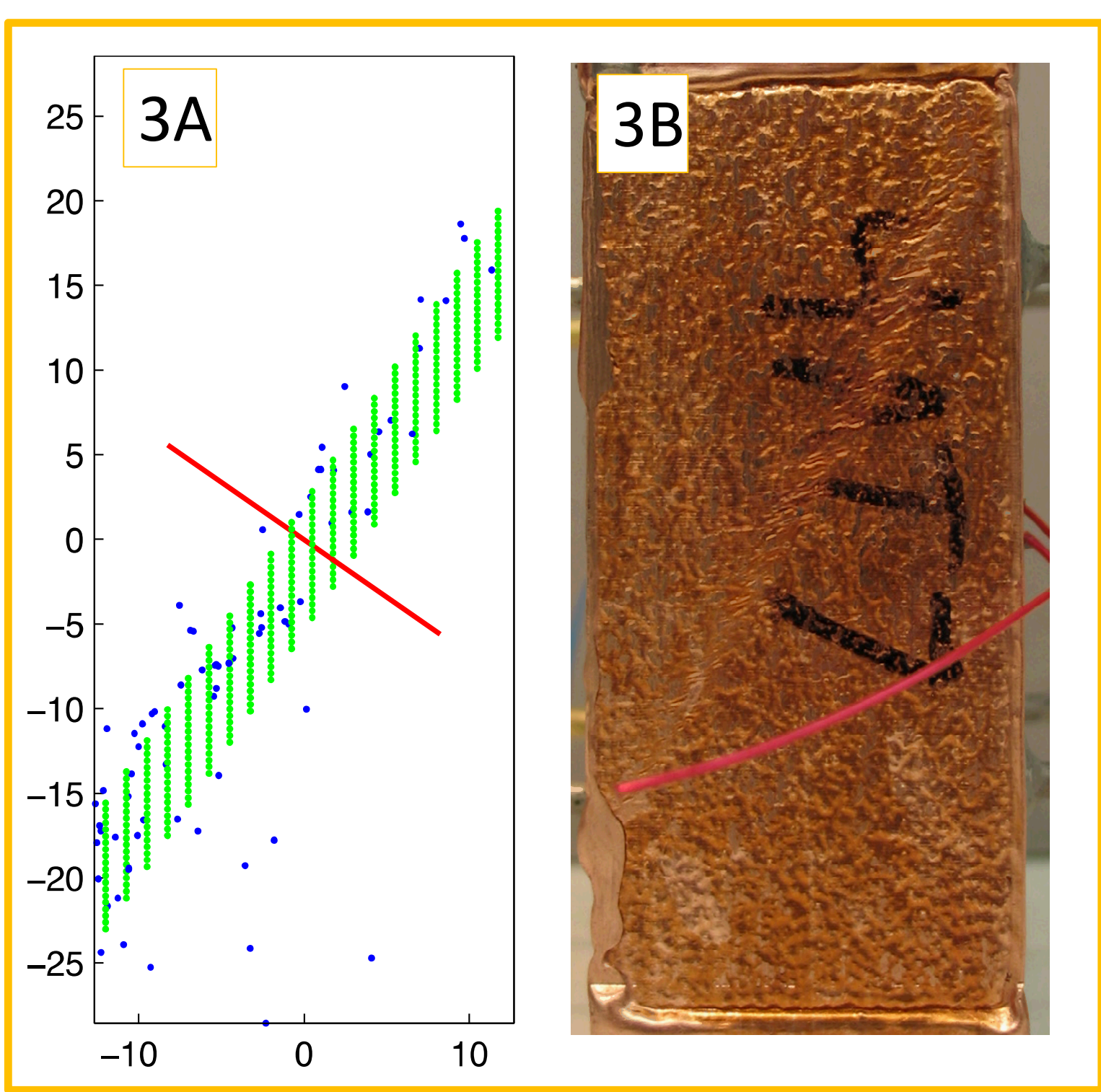
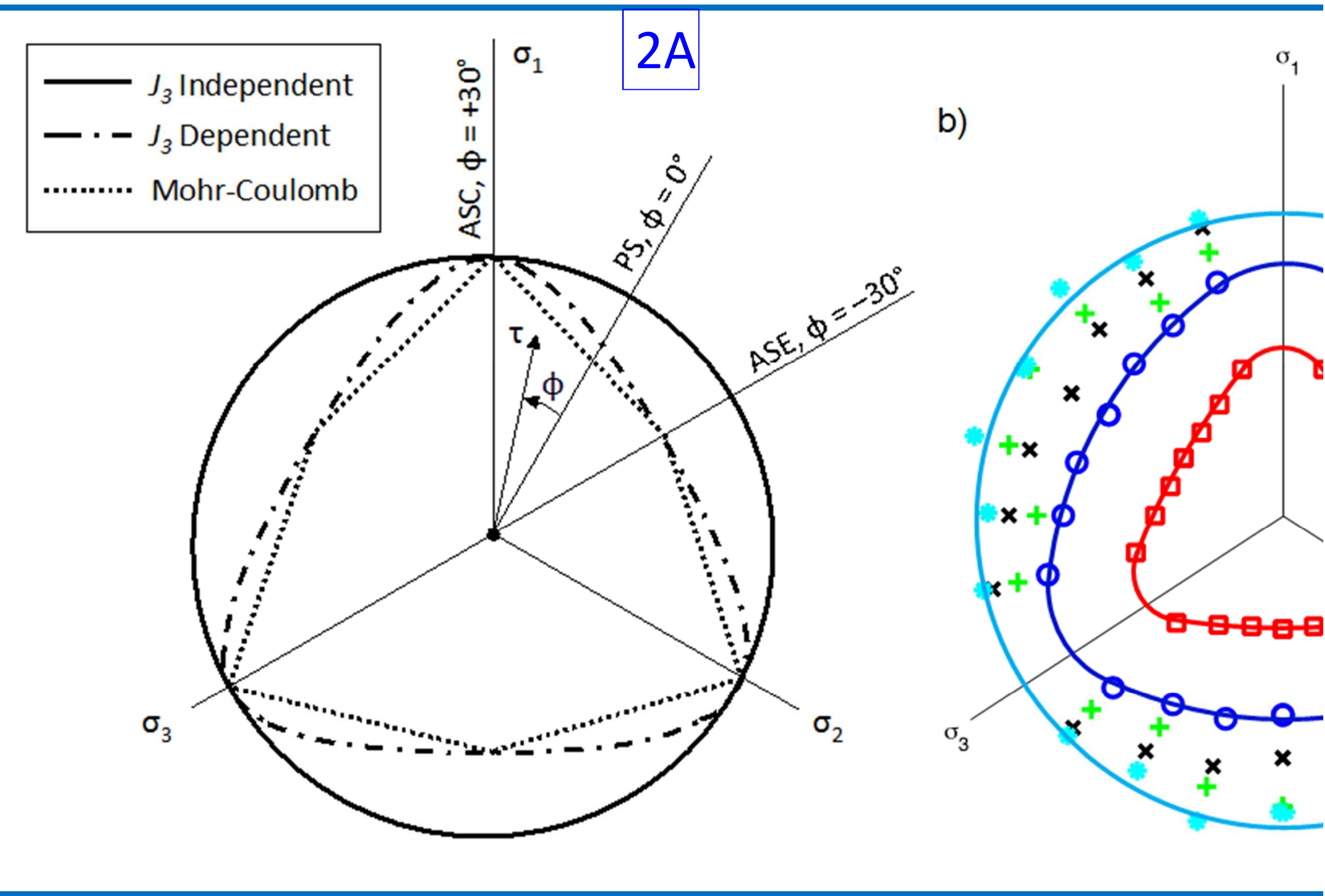
## Experimental Methods:

### Conducted true triaxial tests to investigate influence of $\sigma_2$ on failure

- Used smaller true triaxial apparatus at Sandia (Wawersik et al. 1997)
- Performed tests at constant mean stress,  $\sigma$ , and constant Lode angle,  $\phi$ .
  - Mean stresses: 30, 60, 90, 120, and 150 MPa
    - Assessed  $J_3$  dependence of failure
    - Enabled extraction of bulk and shear moduli from unload loops
  - Lode angles: 30° (ASC), 14.5°, 0° (PS), -14.5°, -30° (ASE)
    - Ranged from axisymmetric compression (ASC) to axisymmetric extension (ASE); determines the stress path in the Pi plane
    - Constant Lode angle,  $\phi$ , is equivalent to constant  $N_{II}$
- Evaluated strain localization predictions (Rudnicki & Rice 1975)
- Recorded acoustic emissions (AE); located AE within specimen
  - Plane fit through AE to locate band spatially and temporally (Fig. 3)
  - Onset of localization corresponds to drop in fitting error
  - AE band angle,  $\theta$ , agrees with angle measured on jacket (Table 1)

## Bulk Specimen Response:

- Typical mechanical responses ( $\sigma_1$  vs.  $\varepsilon_1$ ) are shown in Fig. 1:
  - 60 MPa constant mean stress tests at five Lode angles (Fig. 1A)
  - 0° (PS) Lode angle tests at five mean stresses (Fig. 1B)
- Mean stress dependence (Fig. 1B):
  - Low mean stress: stress peak and drop; shear band forms
  - High mean stress: stress plateau; compaction band or no band
- Lode angle dependence (Fig. 1A): moving from ASC to ASE:
  - Sharper stress peak
  - Lower stress required to initiate failure
- Failure depends on  $\sigma_2$  (see Fig. 2; octahedral plane)
  - Low mean stress: failure surface is a rounded triangle
  - High mean stress: failure surface is circle
  - Failure depends on third invariant of deviatoric stress,  $J_3 = \sigma_1' \sigma_2' \sigma_3'$



## Localization Conditions:

Predicted band angle,  $\theta$ , Rudnicki & Rice (1975) (see Eqn. 4) depends on:

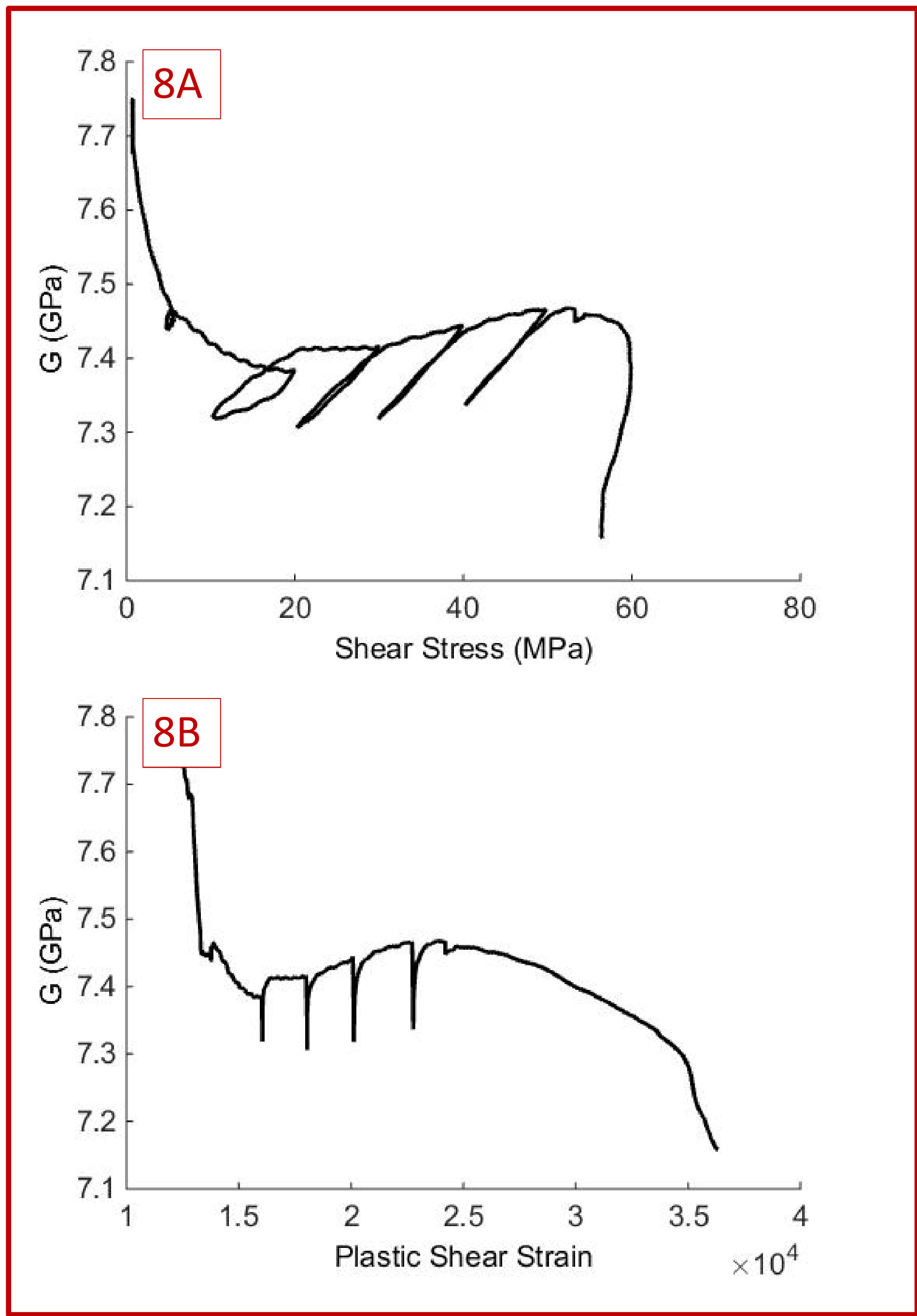
- Lode angle, through parameter  $N_{II}$
- Friction factor,  $\mu$ , which is yield surface slope
- Dilation coefficient,  $\beta = -d^p \varepsilon / d^p \gamma$
- Poisson's ratio,  $\nu$ , for elastic unloading

## Strain Separation:

- Elastic bulk and shear moduli,  $K$  and  $G$ , are
  - Stress dependent (Fig. 8A)
  - Plastic strain dependent (Fig. 8B)
- Total strain and strain increment (Eqn. 5)
- Strain separates into four parts (Eqn. 6):
  - A: elastic strain at constant modulus
  - B: strain due to stress dependent moduli
  - C: strain due to plastic strain dependent moduli
  - D: plastic strain
- Plot four strain components (Fig. 7)

## Localization Predictions:

- Calculate increment of inelastic strain : C + D
- Plot yield surface; determine friction factor,  $\mu$
- Determine  $\beta$  from plot of  $d^p \varepsilon$  vs.  $d^p \gamma$
- Calculate predicted band angle,  $\theta$  (Table 1)



$$\theta = \frac{\pi}{4} + \frac{1}{2} \arcsin \left[ \frac{\frac{2}{3}(1+\nu)(\beta + \mu) - N_{II}(1-2\nu)}{\sqrt{4-3N_{II}^2}} \right]$$
$$N_I = \frac{(\sigma - \sigma_3)}{\tau}, N_{II} = \frac{(\sigma - \sigma_2)}{\tau}, N_{III} = \frac{(\sigma - \sigma_1)}{\tau}$$

$\mu$  – Friction Factor,  $\beta$  – Dilation Coefficient,  $\nu$  – Poisson's Ratio

$$\gamma^t = \gamma^e + \gamma^p$$
$$d\gamma^t = d\left(\frac{\tau}{G(\tau, \gamma^p)}\right) + d\gamma^p$$
$$d\gamma^t = \frac{d\tau}{G} - \frac{\tau}{G^2} \left( \frac{\partial G}{\partial \tau} d\tau + \frac{\partial G}{\partial \gamma^p} d\gamma^p \right) + d\gamma^p$$

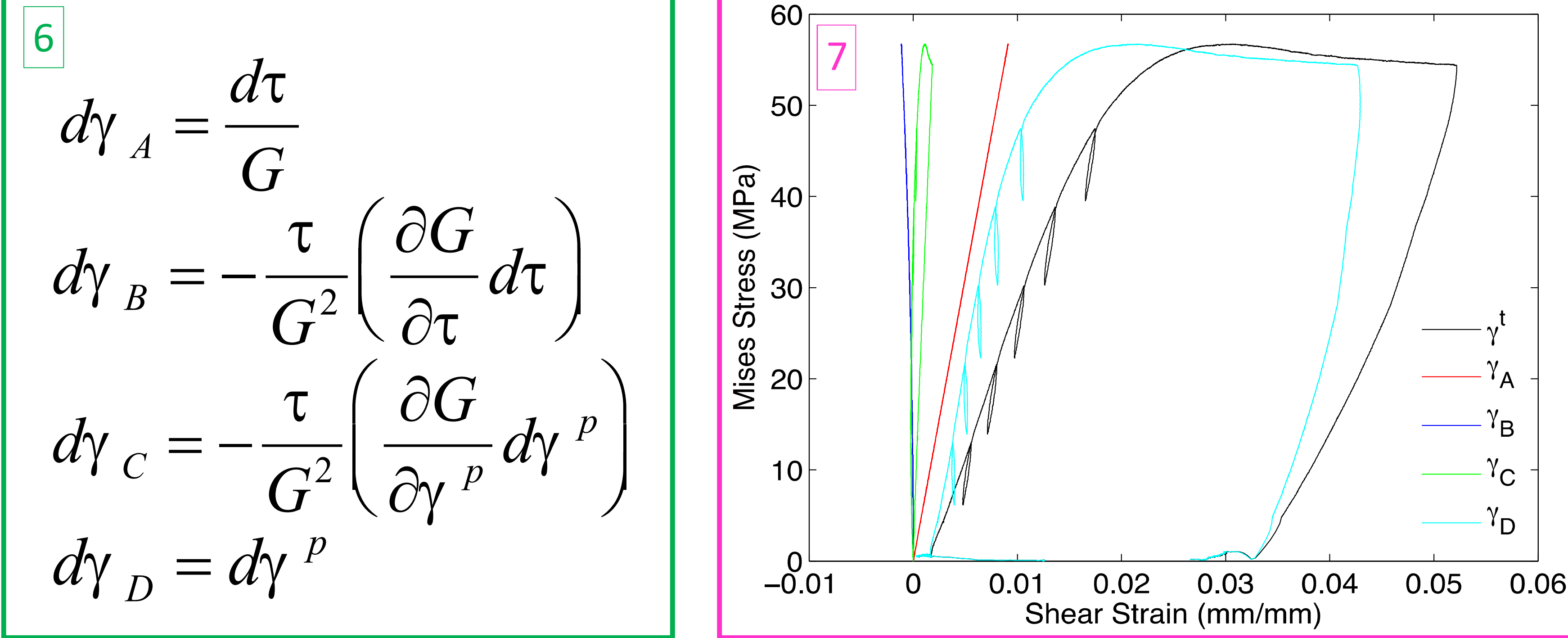


Table 1

Stress State	Mean Stress (MPa)	$\beta$	$\mu$	Predicted $\theta$	AE $\theta$	Measured $\theta$	Response Type
ASC	30	0.76	0.56	59	Conj. Bands	55-60	Shear
ASC	60	0.23	0.31	48	23	30-35	Shear
ASC	90	0.01	0.09	42	10-23	NA	CL
ASC	120	-0.29	0:-0.3	37:33	5-15	NA	CL
ASC	150	-0.66	-1.1:-1.3	3:0	NL	NA	NL
PS	30	0.09	0.94	57	58	61-80	Shear
PS	60	0.55	0.80	62	63	64	Shear
PS	90	0.08	0.67	54	54	58	Shear
PS	120	-0.23	0:-0.7	42:33	NL	NA	NL
PS	150	-0.75	-1.5:-4.4	15:0	16-25	NA	CL
ASE	30	0.76	0.85	80	51	65	Shear
ASE	60	0.65	0.49	68	NA*	70	Shear
ASE	90	0.04	0.13	54	41	46	Shear
ASE	120	-0.17	0:-1.9	50:23	Conj. Bands	45	Shear
ASE	150	-0.21	-1.8:-6	25:0	10-25	NA	CL

## Conclusions:

- Mechanical response and failure (band orientation and failure surface) depend on  $\sigma_2$
- Elastic moduli evolve with stress and plastic strain
- Strain separation was used to determine onset of yield and yield surfaces
- Reasonable agreement between localization predictions using strain separation and experimental results for Castlegate sandstone



Ingraham, MD, Issen, KA, Holcomb, DJ, "Response of Castlegate sandstone to true triaxial states of stress" *J. Geophys. Res. Solid Earth*, **118** (2013) p. 536, doi:10.1002/jgrb.50084.

Ingraham, MD, Issen, KA, Holcomb, DJ "Use of acoustic emissions to investigate localization in high-porosity sandstone subjected to true triaxial stresses" *Acta Geotechnica*, **8** (2013) p.645, doi: 10.1007/s11440-013-0275-y.

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