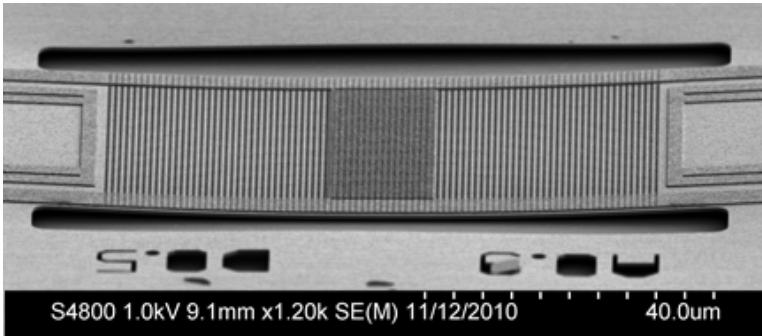


Exceptional service in the national interest



Metallic Inclusions in Phononic Crystals for Thermoelectric Applications

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Motivation: Thermoelectrics

Motivation

➤ Efficient Thermoelectrics:

- » Cooling
- » Power generation



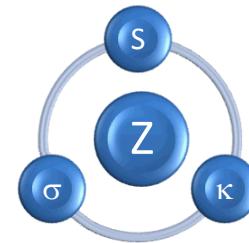
➤ Applications:

- » Sensors in space applications
- » Industrial waste heat recovery
- » Energy scavenging → solar thermophotovoltaics
- » Integrated microprocessor cooling for high-performance computing



Objective: High-ZT

ZT – Thermoelectric Figure-of-Merit



The ZT-Problem:

$$ZT = S^2 \sigma / \kappa T \rightarrow \text{need } S \uparrow, \sigma \uparrow, \kappa \downarrow$$

➤ The S - σ Problem:

- » S is related to the electronic **entropy** (disorder/scattering) in the system
- » σ is a measure of how fast we can propagate a current → decreases with increasing disorder
- » Mott Equation:

$$S = \frac{1}{\sigma} \frac{d \ln(\sigma(\varepsilon))}{d\varepsilon} \bigg|_{\varepsilon=\varepsilon_F}$$

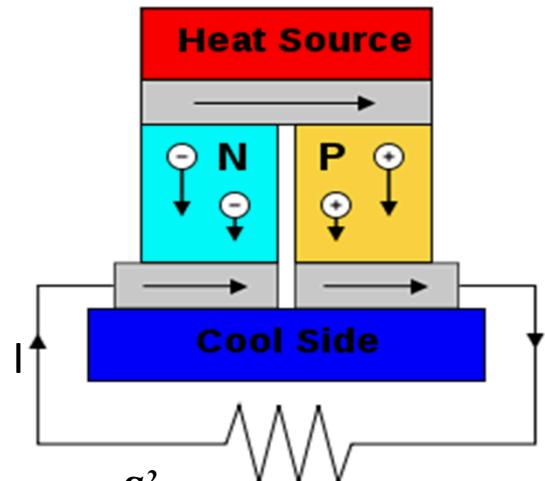
$$\rightarrow S \uparrow \Leftrightarrow \sigma \downarrow$$

➤ The κ - σ Problem:

- » In semiconductors, " κ " is dominated by the phonon contribution
- » κ reduced by increasing phonon scattering
 - » At the limit, this leads to an increase in electron scattering

$$\rightarrow \kappa \downarrow \Leftrightarrow \sigma \downarrow$$

TE Generator



$$ZT = \frac{S^2 \sigma}{\kappa}$$

S = Seebeck coefficient

σ = electrical conductivity

κ = thermal conductivity

T = temperature

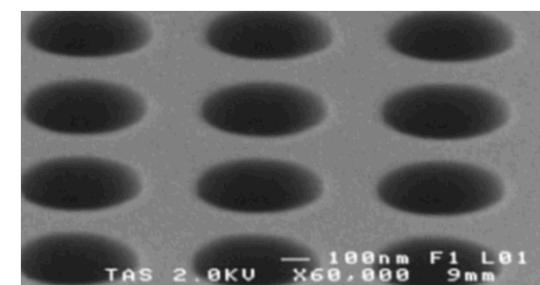
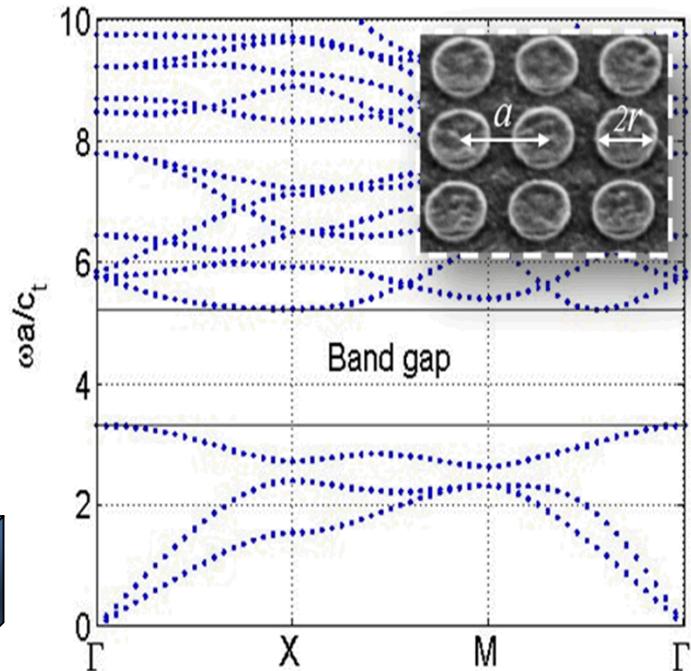
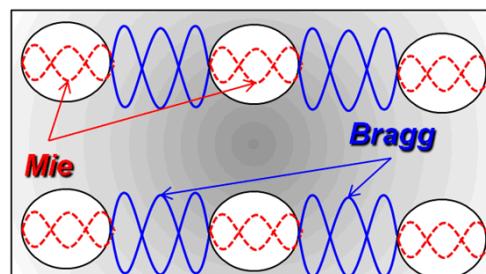
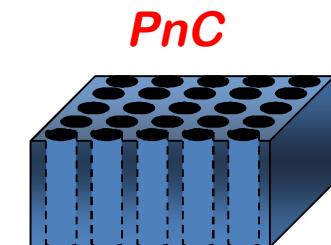
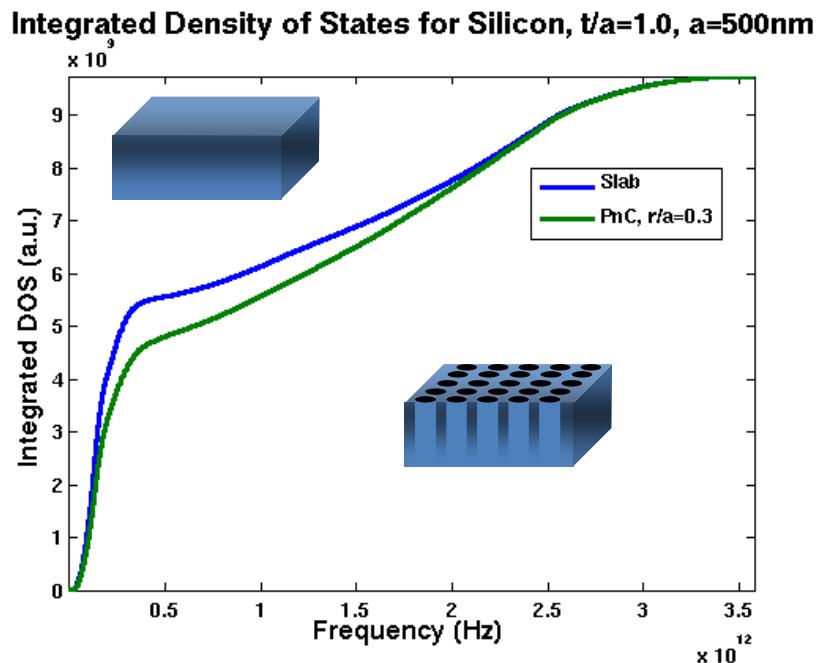
$$\eta_{max} = \frac{T_H - T_C}{T_H} \frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + \frac{T_C}{T_H}}$$

Phononic Crystals

Decoupling κ and σ

What is a phononic crystal (PnC)?

- » Periodic arrangement of elastic scattering centers in a matrix material that exhibits both incoherent and Mie and Bragg resonant scattering
- » Requires sufficient mechanical impedance mismatch

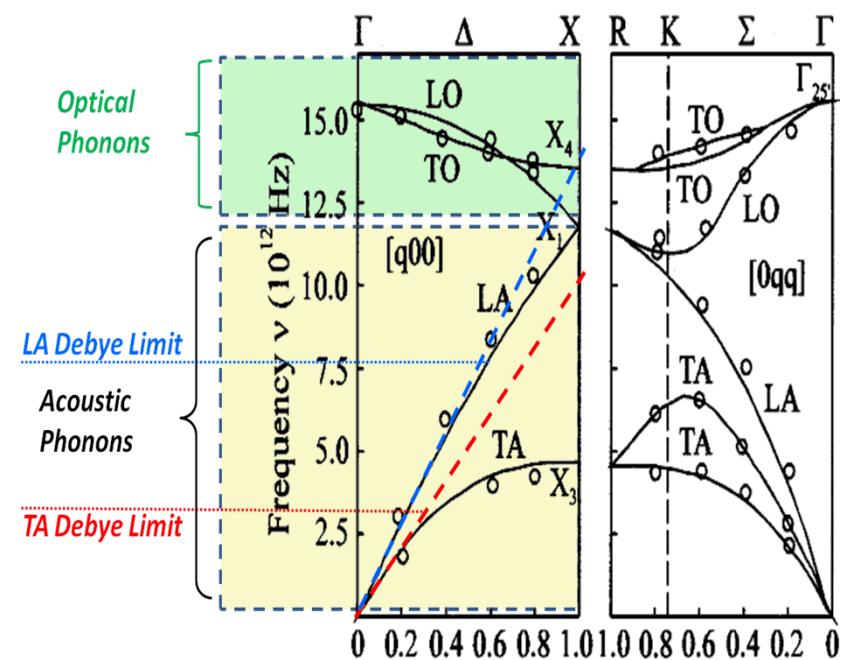
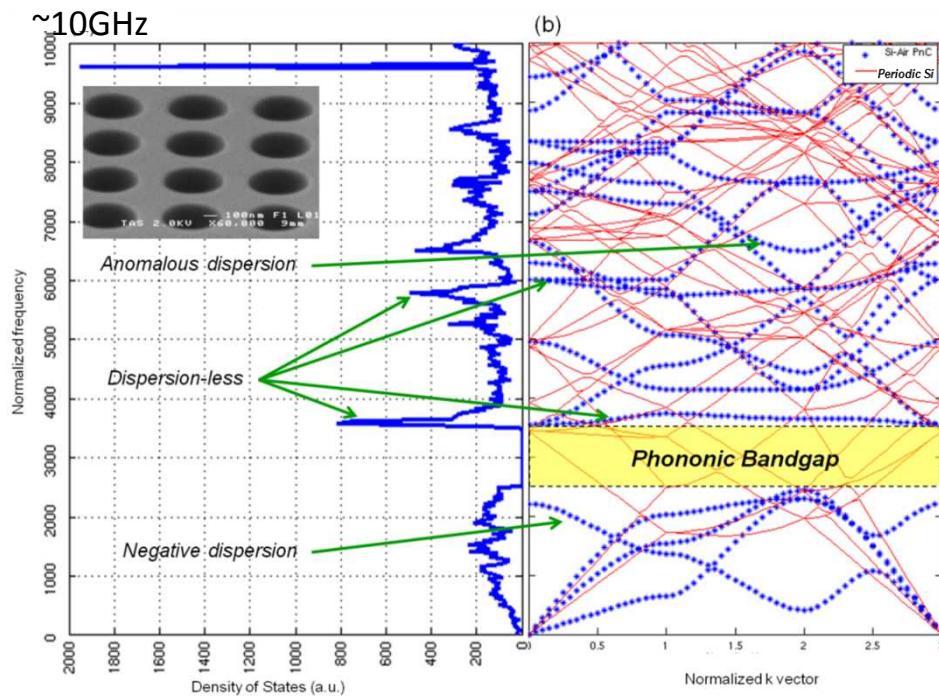


Phononic Crystals

Coherent Phonon Scattering

Thermal transport in phononic crystals

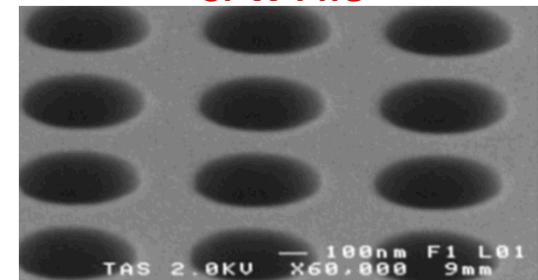
- » Coherent (lattice) scattering in addition to material removal and incoherent (boundary) scattering
- » Periodicity → Real (as oppose to artificial) zone folding
- » Flat Bands → Reduced group velocity, modified density of states
- » Negative Bands → Backward propagation (backward scattering)



De-Coupling κ and σ in PhCs

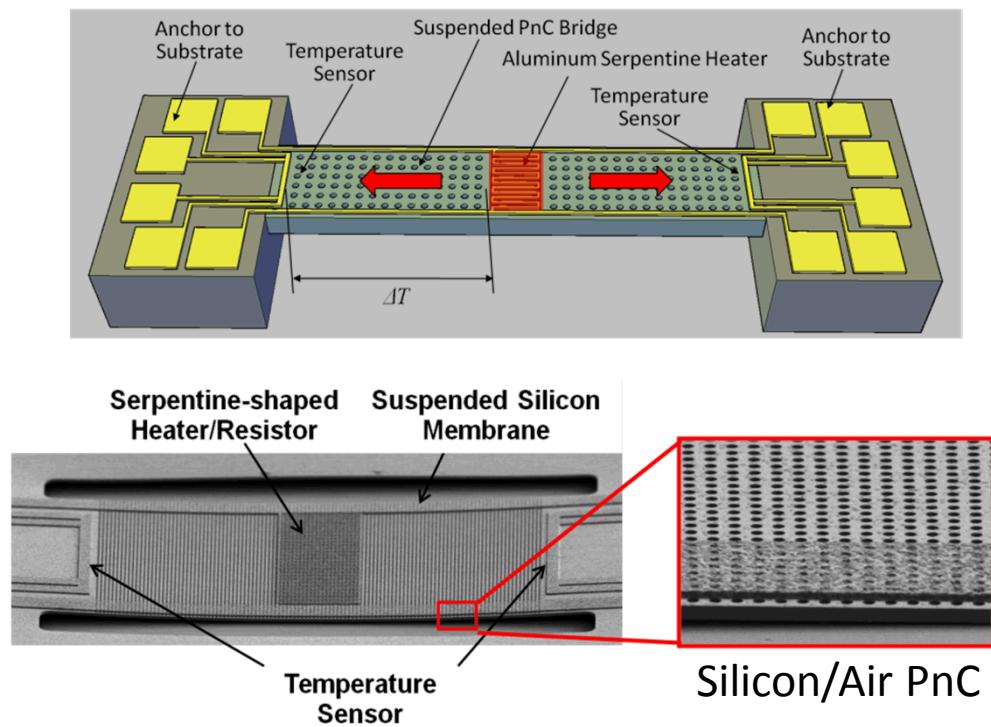
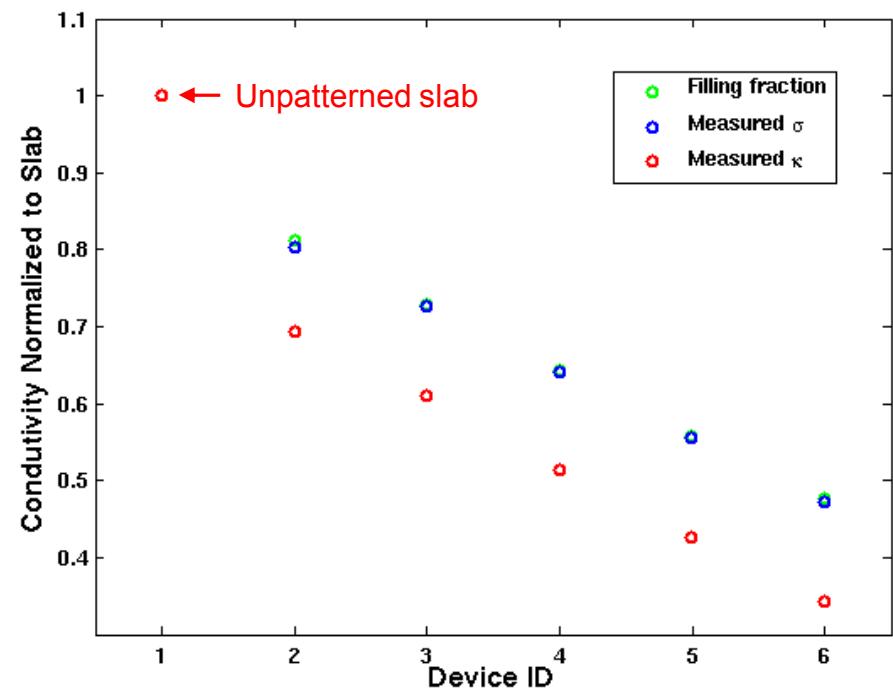
Si-Air PnCs

Si-W PnC



How do phononic crystals affect κ ?

- » Modification of phonon dispersion " $\omega(k)$ ", which is related to the phonon group velocity and influences the phonon scattering lifetime
- » Incoherent scattering due to the interface at each inclusion
- » Redistribution of the phonon density of states as compared with bulk



PnC Thermal Conductivity

Callaway-Holland Model

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Thermal conductivity given by Callaway-Holland model:

$$\kappa = \frac{1}{6\pi^2} \sum_j \int_k \frac{\hbar^2 \omega_j^2(k)}{k_B T^2} \frac{\exp\left[\frac{\hbar\omega_j(k)}{k_B T}\right]}{\left(\exp\left[\frac{\hbar\omega_j(k)}{k_B T}\right] - 1\right)^2} v_j^2(k) \tau_j(k) k^2 dk$$

$$\frac{1}{\tau_j(k)} = \frac{1}{\tau_{Umklapp,j}} + \frac{1}{\tau_{impurity,j}} + \frac{1}{\tau_{boundary,j}}$$

To modify κ , we need to change:

- Dispersion " $\omega(k)$ "
- Minimum feature size " L "

$\omega(k)$ is the phonon dispersion
 $v(k) = \partial\omega(k)/\partial k$ is the phonon group velocity
 $\tau(k)$ is the phonon scattering lifetime
 k is the wavevector
 $j = 1, 2, 3$ (1 longitudinal and 2 transverse modes)
 L is the limiting dimension (smallest feature size)

$$\frac{1}{\tau_{Umklapp,j}} = B T \omega_j^2(k) \exp\left[\frac{C}{T}\right]$$

$$\frac{1}{\tau_{impurity,j}} = D v_j^4(k)$$

$$\frac{1}{\tau_{boundary,j}} = \frac{L}{v_j(k)}$$

where B, C, D , and E are constants determined by fitting κ to experimental data

PnC Thermal Conductivity

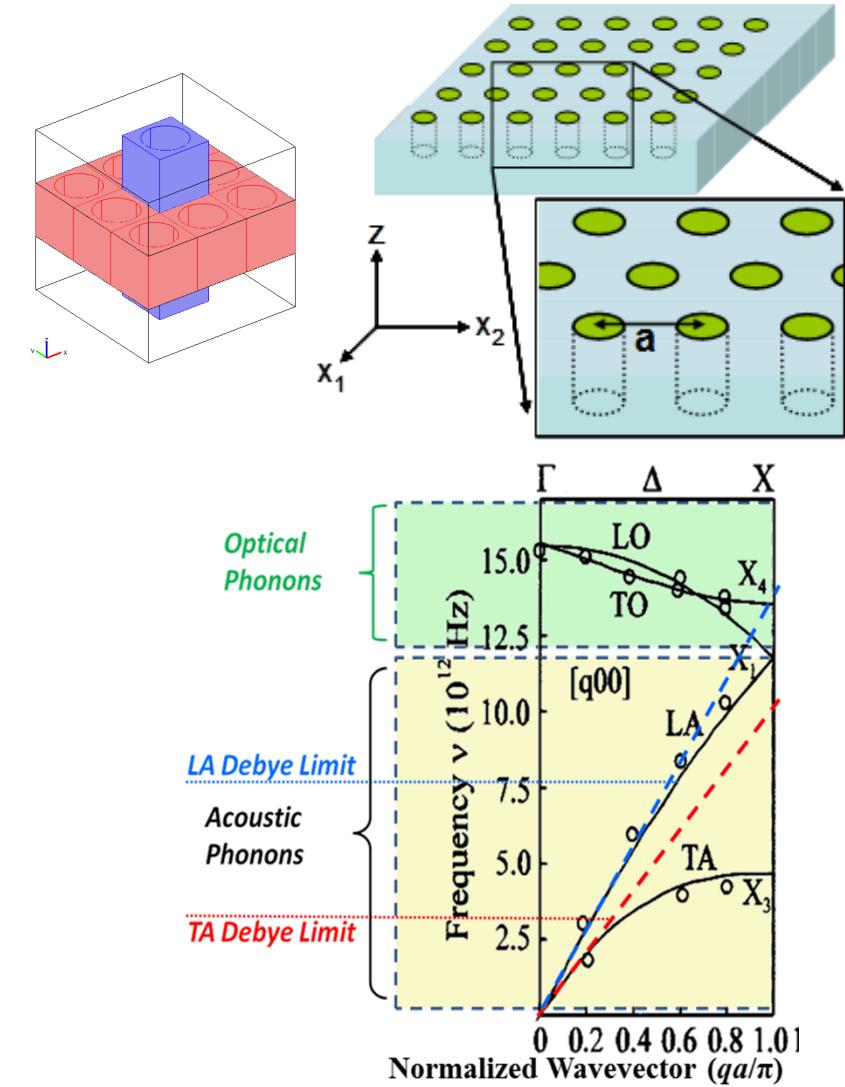
Hybrid LD + PWE Approach

Plane-wave expansion (PWE) combined with lattice dynamics

- » Continuum mechanics-based PWE theory breaks down at nanometer length scales
- » Lattice dynamics/molecular dynamics modeling becomes computationally intensive at the same limit
- » Simulation tool developed for calculating κ that supplements continuum mechanics with corrections from lattice dynamics

PWE dispersion:

- » Quantitatively valid in the Debye-limit portion of the dispersion
- » Qualitatively valid in the linear/slow portion of the material dispersion
- » Must calculate >45000 bands for room-temperature κ in Si



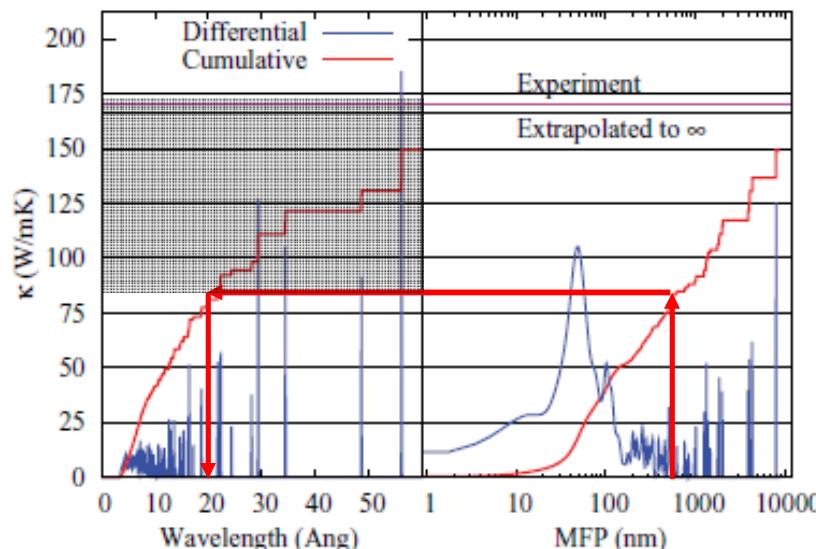
PnC Thermal Conductivity

Threshold Mean Free Path

Prediction of κ using our hybrid model

- » Dispersion data of PnCs performed up to $\sim 2\text{THz}$ using plane-wave expansion (PWE) used directly to calculate κ
 - » Threshold MFP (TMFP) uses phonon MPF data to distinguish the phonon population as coherent or incoherent
 - » Phonons with a MFP of **2 μm and longer** experience coherent PnC scattering

→ Si MFPs span from 1nm to 100 μm at T_{room}



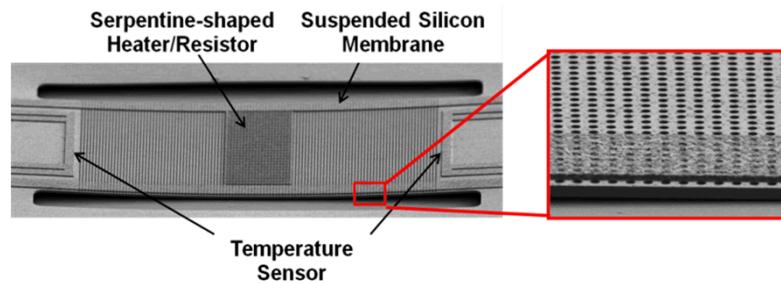
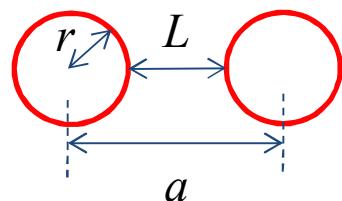
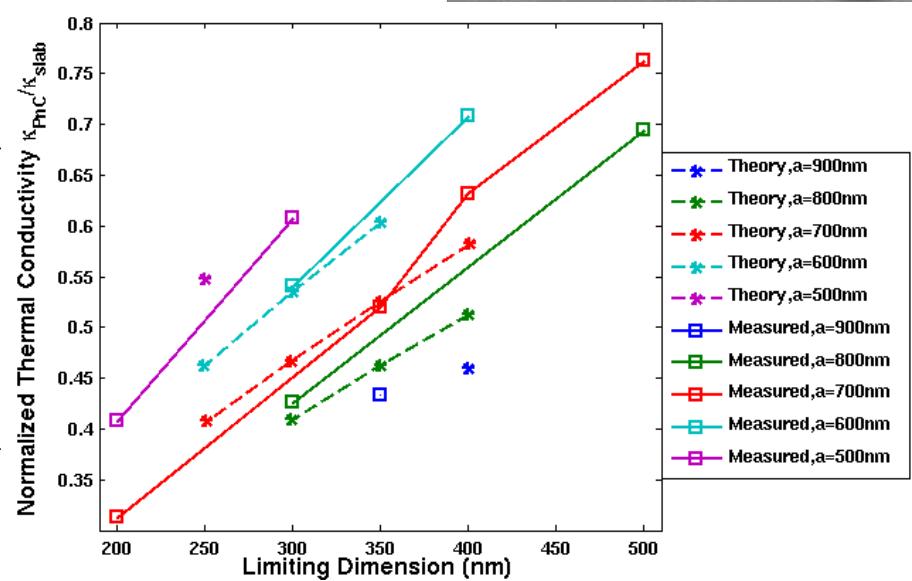
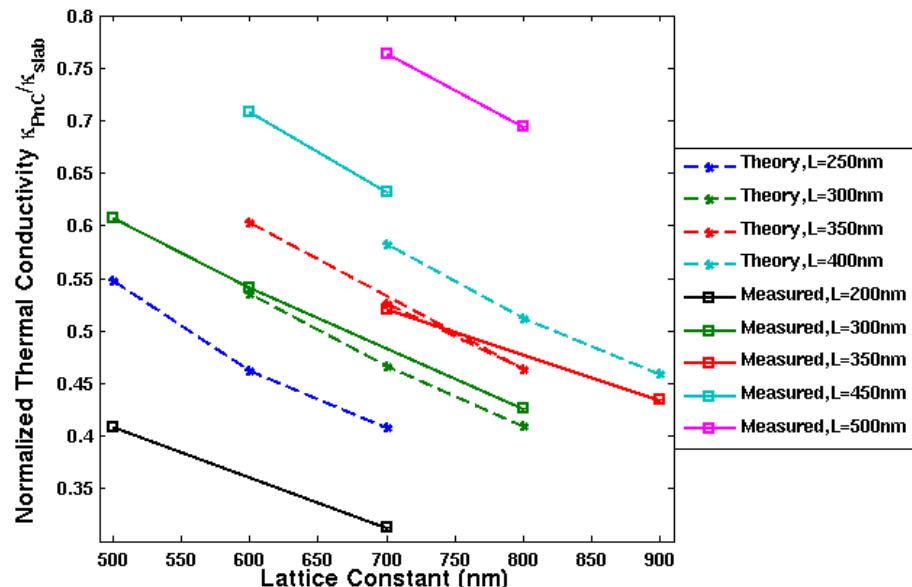
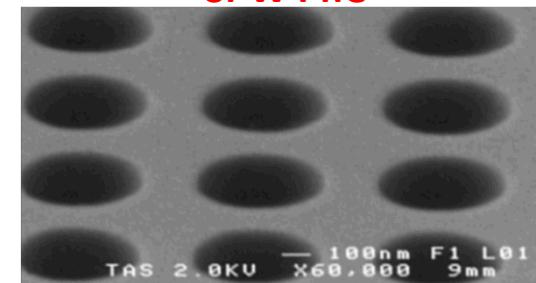
Gang Chen, et. al., PRB **84**, 085204 (2011)

PnC Thermal Conductivity

Verification of Hybrid Model

Coherent vs. Incoherent Phonon Scattering in Si/Air PnCs:

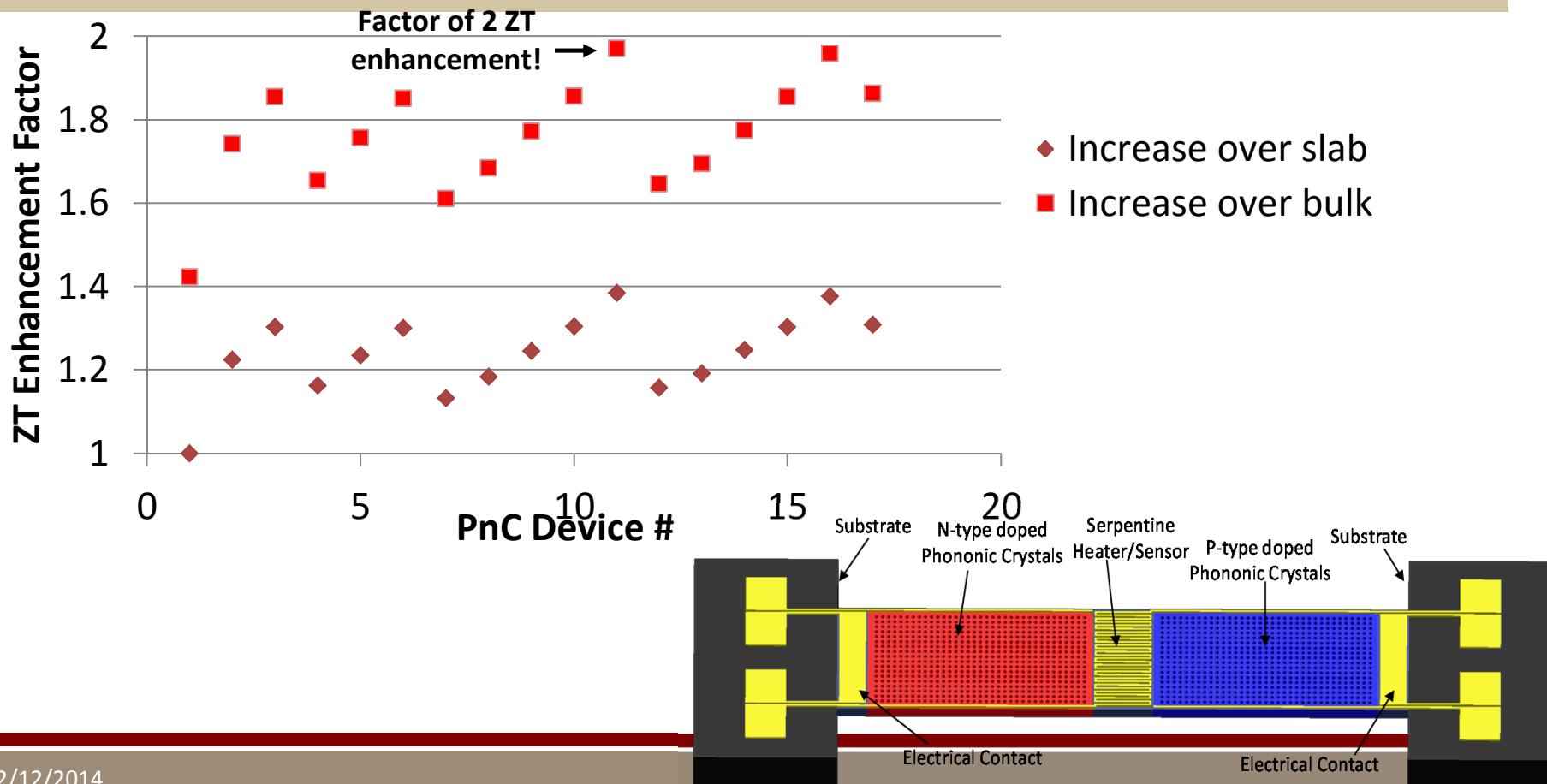
- » Theoretical predictions of κ reduction match well with measured data
 - » Fabricated parameters varied slightly from what was specified
 - » TMFP = 2 μ m was assumed for all PnCs



ZT Prediction

Overall System Performance

- » Enhancement in ZT by a factor of 2x versus bulk Si
 - » 1.4x Enhancement as compared to an unpatterned slab
- » Applicable to any material system
 - » ZT must be limited by high κ , and have a κ that is dominated by phonon transport (i.e. most semiconductors)



Concurrent Engineering of σ and κ

Si-Metal PnCs

Si-W PnC

 Sandia
National
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Concurrent σ Enhancement and κ Reduction:

- » Si-metal PnCs to reduce κ with simultaneous σ enhancement
 - » Is σ actually enhanced, and is κ still reduced without the porosity factor?

→ Measure thermal and electrical conductivity of existing Si-W samples

- Effect on σ :

- Increased doping:
 - » Decreases the interface potential width → electron tunneling

- Ohmic contact at interface:

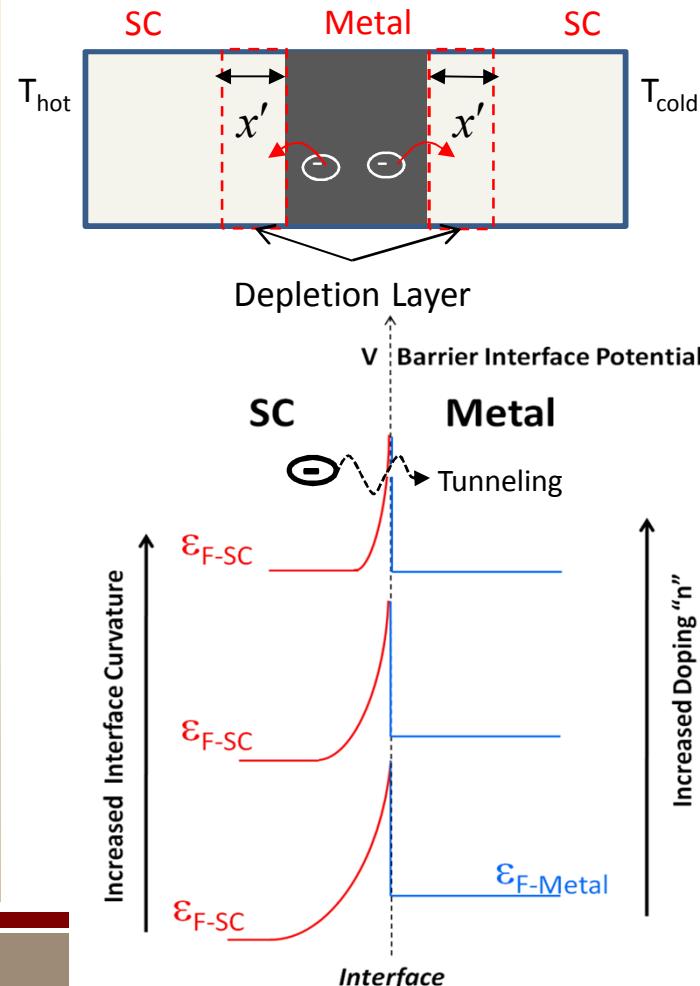
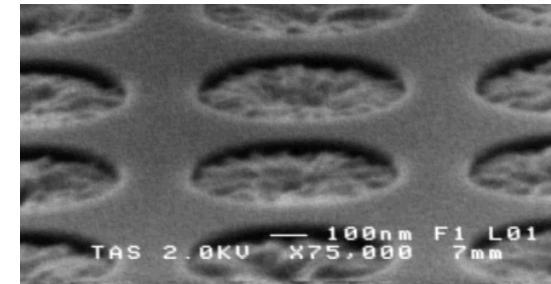
$$\sigma_{PnC+W} \approx \sigma_{Si} \frac{1+ff_W}{1-ff_W}$$

- Effect on κ :

- Solid-solid PnC rather than solid-air:

- » Mechanical impedance mismatch is still very high
- » Scatterer thermal conductivity is no longer negligible
- » Boundary thermal resistance must now be considered

$$\kappa_{PnC+W} \approx \kappa_{Si} \frac{C - ff_W}{C + ff_W}, C = 1 + 2 * \frac{\kappa_W}{\kappa_{Si}}$$

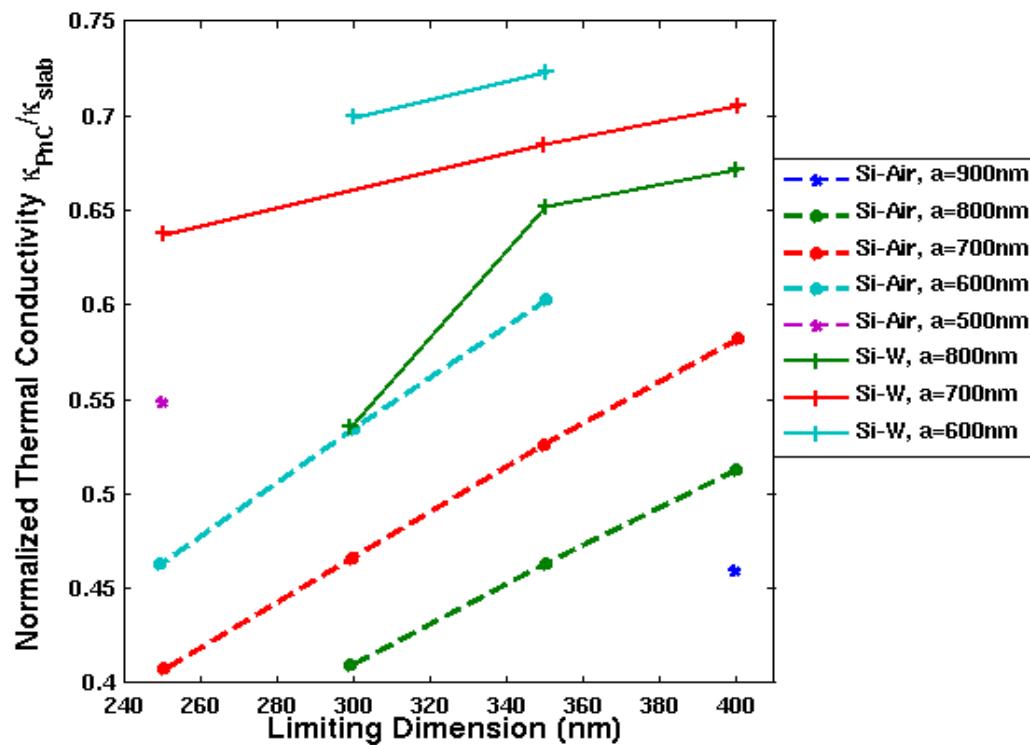


Concurrent Engineering of σ and κ

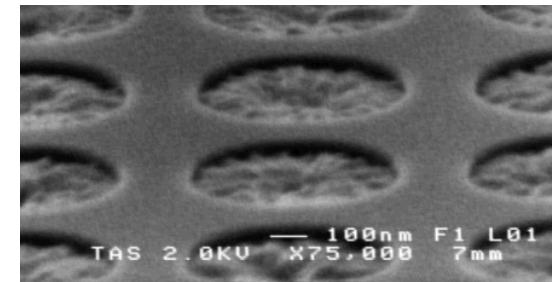
Theoretical Prediction

Concurrent σ Enhancement and κ Reduction:

- » Compared with Si-air, Si-W PnCs are predicted to have:
 - » ~15% less thermal conductivity reduction, but
 - » 1.7x – 3.8x enhancement in electrical conductivity



Si-W PnC



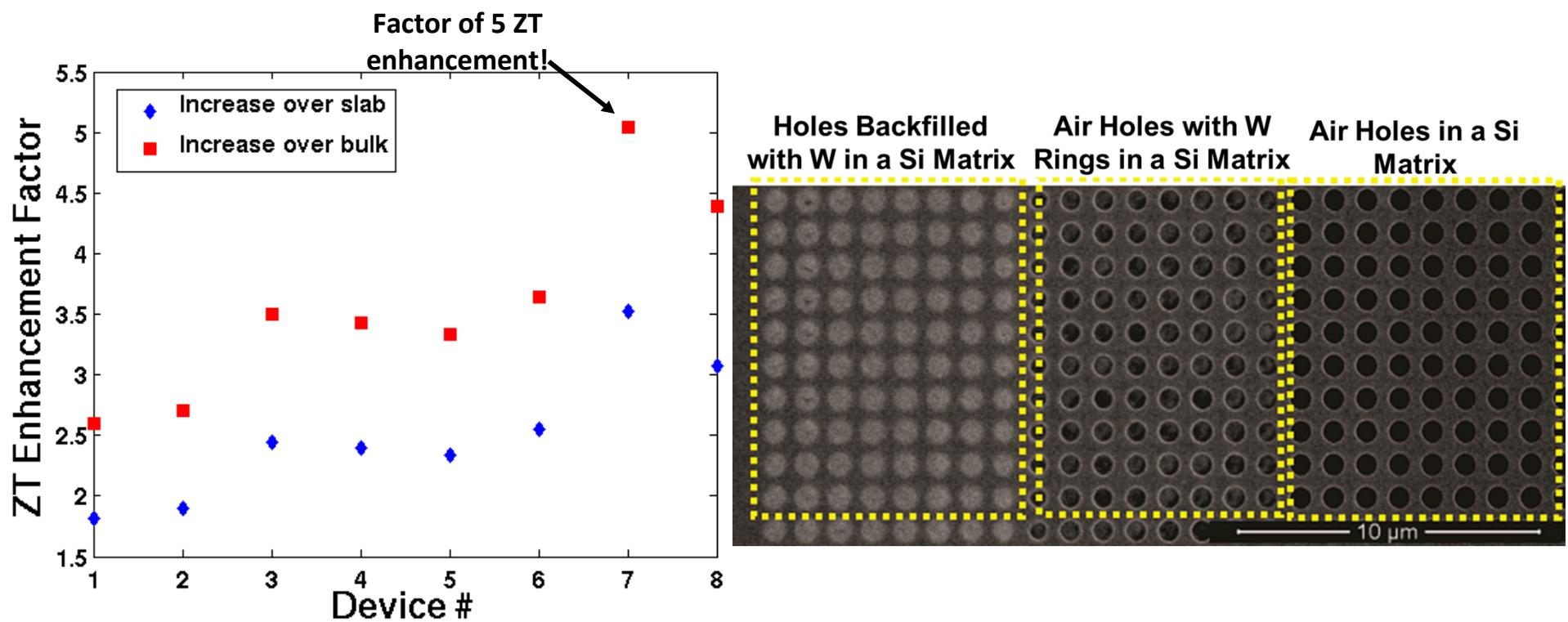
κ and σ in Si-metal PnC

ZT Prediction

Si-W PnC Performance

Concurrent σ Enhancement and κ Reduction:

- » Assuming Seebeck coefficient remains constant → 3.5x ZT enhancement compared to an unpatterned slab (~5x compared to bulk)
- » Si-metal-air PnC may enhance ZT even more by reducing κ further



Conclusions

- We have shown the first experimental observation of decoupling of electrical and thermal conductivity in PnCs, and theoretical concurrent engineering of the conductivities in a Si-metal PnC for a **factor of 5 enhancement of ZT**.
- We also introduced the concept of a threshold mean free path in conjunction with a hybrid model, representing one of the **most accurate models for thermal transport** in periodic micro-scale devices.
- Our approach enables us to use the thermal conductivity as a macroscopic metric for **inferring the average phonon coherence length** in our PnC samples.
- This work may hold the key to **unprecedented thermoelectric performance**; using PnC topologies to further suppress phonon transport of microscale porous samples while enhancing electron transport.