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Quantum Information Processing
An Overview

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1. Demands of Wireless Communication
2. Information Processing
    - Digital Information
    - Quantum Information
3. Quantum Algorithms
4. Cryptography
    - Confidentiality
    - Authentication
    - Signature
5. Quantum Cryptography
    - *Quantumness*: Relevant Laws
    - Achieving Perfect Secrecy
    - Quantum Key Distribution (QKD) Purpose
    - BB84 - A Quantum Cryptography Protocol
6. Final Remarks
7. References

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Demands of Wireless Communications

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1. Higher effective data rates: 1G (*circa 1981*) $\rightarrow$ 2G (*circa 1991*)$\rightarrow$ 3G (*circa 2001*)$\rightarrow$ 4G (*circa 2012*)$\rightarrow$ 5G (*maybe 2020*).

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2. Higher capacity in terms of number of simultaneous users.

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2. Higher capacity in terms of number of simultaneous users.
3. Secure communications for privacy.

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Computation 101

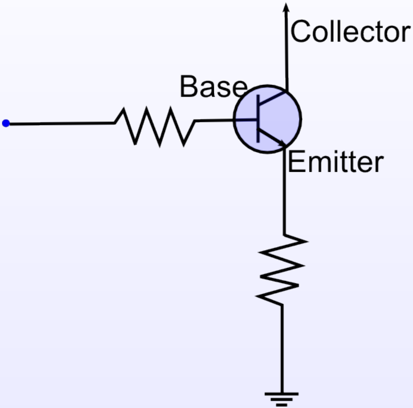

Figure 1: Transistor circuit: *increasing voltage at base, exponentially increases current flow from collector to emitter; the converse is also true...we have a switch!*

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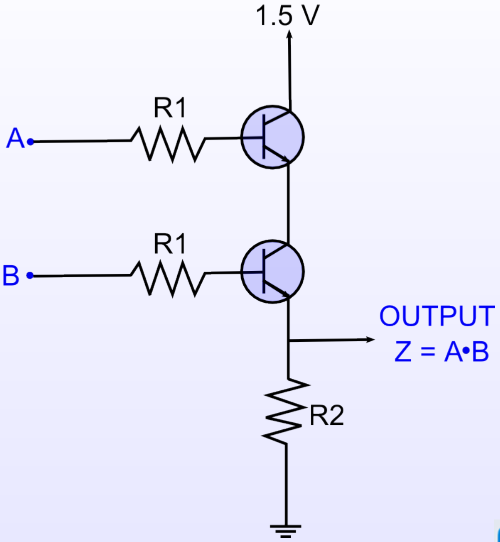

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Computation 101

BASIC UNIVERSAL GATES (generally, irreversible)

AND gate			OR gate			NOT gate	
A	B	Q	A	B	Q	A	Q
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

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1-BIT ADDER

input (two 1-bits)		output (2-bit)		
x	y	b_1	b_2	decimal value
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	0	2

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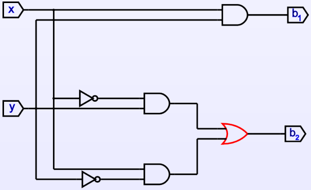

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Quantum computing 101: *A compare and contrast approach*

	Classical	Quantum
Basis of information	Bits: 0 or 1 2-level system	Qubits: $\|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ infinitely possible vectors or states, e.g., $\|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\|\phi\rangle = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$
Gates	AND, OR, NOT generally irreversible	Bit-flip $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Idle rotation $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ reversible operations
Can transport (qu)bits	Yes	*Yes*
Can clone (copy) (qu)bits	Yes	No
Effect of measurement	Nothing	Destroys original qubit

Potential Realizations of Qubits

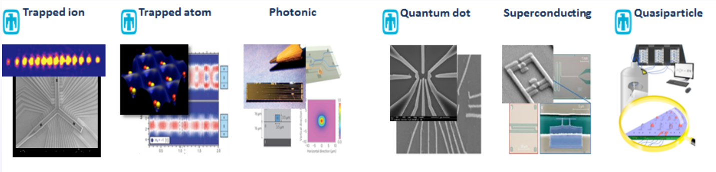

Figure 2: Source: Technical Overview presentation on AQUARIUS Grand Challenge

- Physical realization of qubits; *think of vacuum tubes and transistors for bits*
  - Quantum dots (*non-mobile*)
  - Ion traps (*mobile*)
  - Neutral atom laser (*non-mobile?*)
  - Photons (*mobile*)
  - Superconducting flux (*non-mobile*)

Models of Quantum Computing

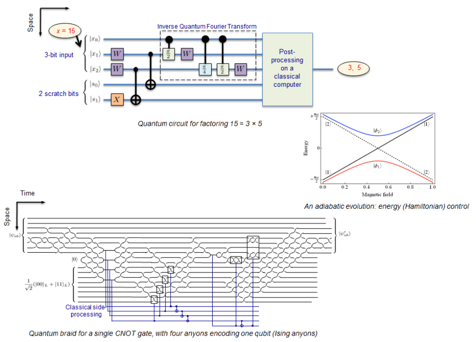

Figure 3: Source: Technical Overview presentation on AQUARIUS Grand Challenge

- Models of quantum computing
  - Circuit or network model
  - Adiabatic quantum computing model
  - One-way (Cluster state) computing model

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Quantum Algorithms

- *Grover's Algorithm*: Existence in an Unsorted Database.
- *Shor's Algorithm*: Factorization of Composite Numbers.
  - Extremely hard **classically**. Best classical solution takes exponential time in the number of bits to run.
  - Relatively easy to solve **quantum-wise** using *Shor's method*.

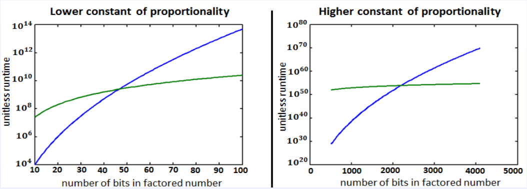

Figure 4:

- *Hallgren's Algorithm*: Solution to *Pell's* equation ($x^2 - dy^2 = 1$, $\{x, y, d\} \in \mathbb{Z}^+$ and d is a non-square positive integer).

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Information Confidentiality

Purpose is to obfuscate information.

- Universally used:
  1. bank transactions,
  2. investment account management,
  3. sensitive e-mails, ...
- Desired property:
  1. *impossible* to decipher (decrypt) by an unwanted entity,
  2. should stand the test of time, and
  3. be easy enough to be *implementable* today.
- Decent solutions: block cipher vs. stream cipher, secret-key vs. public-key, ...

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`blahblah` is my password.
Can encrypt as
`cmbicmbi` jt nz qbttxpse.

Is this good?

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`blahblah` is my password.

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Is this good? No, it is **horrible!** Susceptible to *Chosen-Plaintext Attack*

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Authentication: *They are who we thought they were!*

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- Validates the source of transmitted data.
- This is implemented over a classical channel, even in *quantum crptography*.
- Three main techniques exist:
  1. Use of *message authentication code* (MAC) - amenable to secret-key ciphers;
  2. Use of public key infrastructure (PKI) - amenable to public-key ciphers;
  3. Use of *universal families of hash functions* - amenable to quantum cryptography.

Signature: This is James Bond and I approve this message

- It is okay to eavesdrop, but it is not okay to forge my signature.
- Achievable with a form of encryption, *private-key encryption*.
- **Alice** signs a message. **Bob** receives the message and decrypts with *Alice's public-key*; this confirms Alice's signature.
- **Eve** can listen in the middle and even extract the message, but she cannot forge the signature

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Perfect Secrecy in Communications

- Can only be achieved if key size is greater than or equal to message size.
- Call $P = \{p : p$ is in the set of all plaintext$\}$ and $C = \{c : c$ is in the set of corresponding ciphertext$\}$, then if

$$Pr[P = p|C = c] = Pr[P = p]$$

  we say that we have perfect secrecy.
- Alice and Bob are legit users, while Eve is the eavesdropper. Eve should be able to listen/view cipher message all she wants, but not decipher it.

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  we say that we have perfect secrecy.
- Alice and Bob are legit users, while Eve is the eavesdropper. Eve should be able to listen/view cipher message all she wants, but not decipher it.
- Perfect secrecy of information **cannot** be achieved with key lengths less than message length.
  - Today's ciphers use key lengths much smaller than message lengths!
  - AES-256 uses a key length of 256 bits, message block size of 128 bits, but the same key for a long message.
  - RSA (cipher for SSH) uses keys of size 1024, 2048, and 4096 bits, but much longer messages.

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So, aren't current cryptographic techniques sufficient?

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- Current classical cryptographic systems are mostly *cat-and-mouse* solutions...propose solutions based on current attacks.
- Strong presumptions made on solvability of currently hard problems, like *modulo-factoring* for the famous RSA-cipher system.

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Relevant Laws of Quantum Physics

1. It is impossible to clone a qubit, the equivalent of classical bits. This limitation is mathematically proven and does NOT rely on technological limitation.

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1. It is impossible to clone a qubit, the equivalent of classical bits. This limitation is mathematically proven and does NOT rely on technological limitation.
2. Observing the state of a qubit destroys its previous state; hence, **we cannot distinguish between non-orthogonal states**:
   - Classically, we only have 1 and 0's...we can distinguish these.
   - Quantum-wise, we have $|1\rangle, |0\rangle, |+\rangle, |-\rangle$, ... in fact infinite representative qubits!

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Cloning qubits is impossible: *A No-go Theorem*

- One can make multiple copies of an *a priori* known qubit, but...
- One cannot make a gadget that makes copy of any arbitrary input qubit

Proof.

If such a gadget existed, then it should do the following: $|\psi\rangle \mapsto |\psi\rangle \otimes |\psi\rangle \equiv |\psi\psi\rangle$. Let such a gadget be represented by the linear operator, $\mathcal{C}$. The requirement for linearity of the operator comes from Schrödinger's equation. Then,

$$\mathcal{C}\,|\psi\rangle = |\psi\psi\rangle$$

By the same token, we expect

$$\mathcal{C}\,|0\rangle = |00\rangle \tag{1}$$
$$\mathcal{C}\,|1\rangle = |11\rangle \tag{2}$$
$$\mathcal{C}\,(|0\rangle + |1\rangle) = (|00\rangle + |11\rangle) \tag{3}$$

The last line comes from the linearity property of the operator $\mathcal{C}$, which contradicts the expected result of

$$\begin{aligned}\mathcal{C}\,(|0\rangle + |1\rangle) &= (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)\\ &\neq (|00\rangle + |11\rangle)\end{aligned}$$

Thus, no such gadget can exist; we call this the *No-cloning Theorem*. $\square$

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More details on Qubit Observation

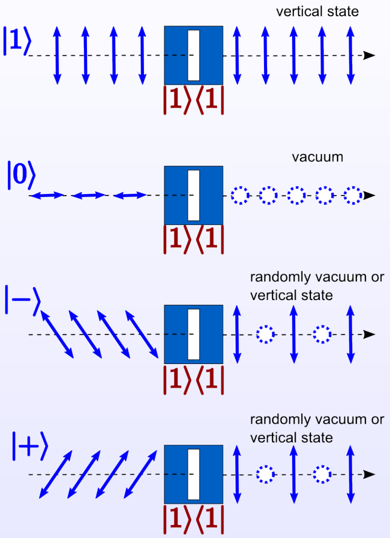

Figure 5: photon qubits being measured by polarizer

Perfect Secrecy Revisited

- One-time pad with key length at least as long as message: assume plaintext is 0110 and random key is 1010, then ciphertext can be 1100. Can't decipher plaintext from this!

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Perfect Secrecy Revisited

- One-time pad with key length at least as long as message: assume plaintext is 0110 and random key is 1010, then ciphertext can be 1100. Can't decipher plaintext from this!
- Cannot reuse cipher key or Eve can decipher message by performing XOR on consecutive message.
- Requires true random keys. Pseudo-random numbers prevalent on today's computers is not sufficient; they are deterministic unfortunately. TRNG generators exist:
  1. Hotbits - rely on radioactive decay, but slow rates $<$ 1Kbits/sec.
  2. Protego's SG100 EVO-USB - rely on resistive circuit elements and yields about 16 Mbits/sec per module.
  3. Quantum measurement of qubits.

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Quantum Cryptography is really Quantum Key Distribution

- We are not encrypting *qubits*, we are encrypting *bits* in plaintext.
- One-time pad needs constantly changing keys equal to message length or more.
- QKD achieves safe secret sharing using (a) quantum channel and (b) classically authenticated channel.

A Quantum Cryptography Protocol - BB84

- Alice will send messages to Bob. Eve is the eavesdropper.
- Alice and Bob authenticate to each other, using classical channel but maybe with universal hash function technique.
- Alice chooses four *non-orthogonal* qubit states.
- Bob measures these states, with some errors.
- Alice and Bob negotiate error and decide whether to discard or to correct and use distilled key.

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- Alice and Bob negotiate error and decide whether to discard or to correct and use distilled key.
- If Eve observes the channel, she will likely cause errors that both Alice and Bob will detect.
- In addition, Eve cannot store the qubits for future use in deciphering messages (*No-cloning principle*).

BB84 - Pictoral Depiction

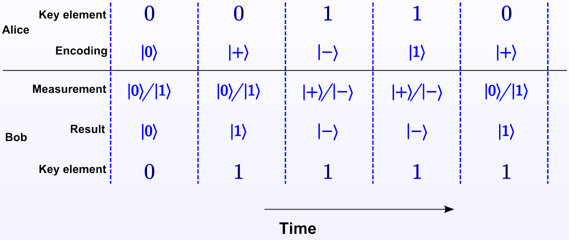

Figure 6:

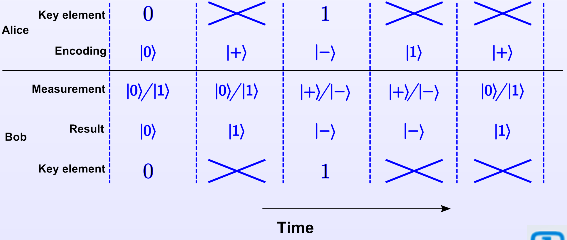

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Figure 7: Redrawn from G.V. Assche's textbook [2].

Final Remarks

- Quantum science shows promises for alternative way of information processing.
- QKD aides secret sharing of cryptographic keys based on information-theoretic limits not computational constraint. When combined with a one-time padding cipher, one can attain perfect secrecy.
- Commercial companies providing quantum key distribution (QKD) systems are:
    1. **id Quantique** in Geneva, Switzerland,
    2. **MagiQ Technologies** in New York, USA, and
    3. **QuintessenceLabs** in Australia.

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Questions

Thank you for being here!

Q & A

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.

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[3] U. M. Maurer, "Authentication theory and hypothesis testing," *Information Theory, IEEE Transactions on* vol. 46, no. 4, pp. 1350 - 1356, 2000.

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