

# Design, Fabrication, and Characterization of High Density Silicon Photonic Components

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# Notes

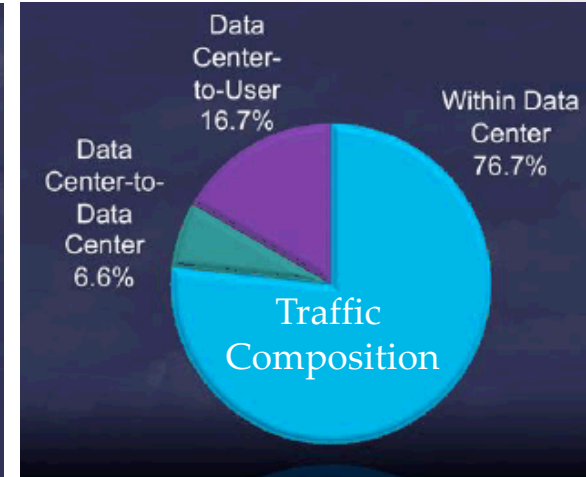
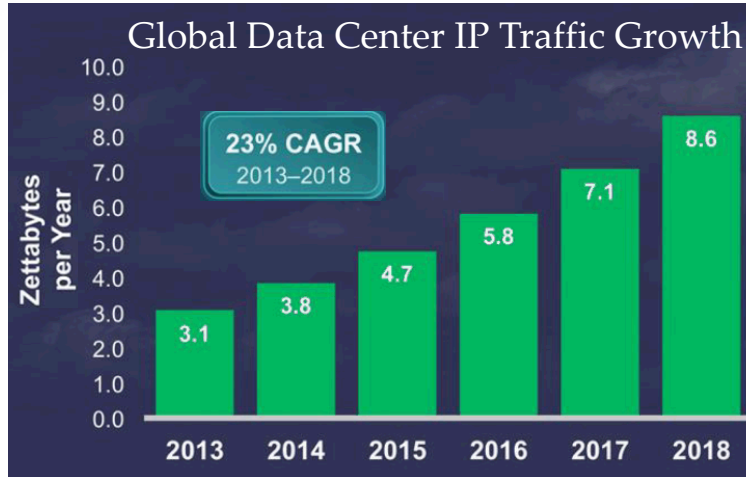
1. Link at bottom to supplementary slide w/links to all slides
2. Footer with page number, Sandia logo and OpSci logo
3. Section in footer, lower left
4. In RLP section, change  $\alpha_{\text{TOT}}$  to  $\alpha_{\text{DUT}}$
5. Make all fonts same, choose one w/o fine features
6. Make sure each slide highlights bottom line
7. SiPh slide – focus on CMOS 7, III-V integration, group IV photonics,
8. Show TE only, but say so
9. Add intro/conclusion slides to each section
10. Remove curve fitting slides
11. Add 'other work' slide
12. NOS – other 3D approaches, mention modification of crossing geometry as future work
13. Xmode - Note FDTD, add schematic, explain units, power input, wavelength
14. XT – importance of geometric mean
15. Synchronize gap/interlayer thickness verbage
16. XS – measurement difficulties & how data presented.
17. IPLM – rename 'Loss Measurement Overview', distinguish/describe two kinds of loss.
18. RLPCal – add image showing inserted structure loss estimation
19. RLPref – note correspondence to taper length rather than taper length (plot legend)
20. SPKtrans – make result jump out



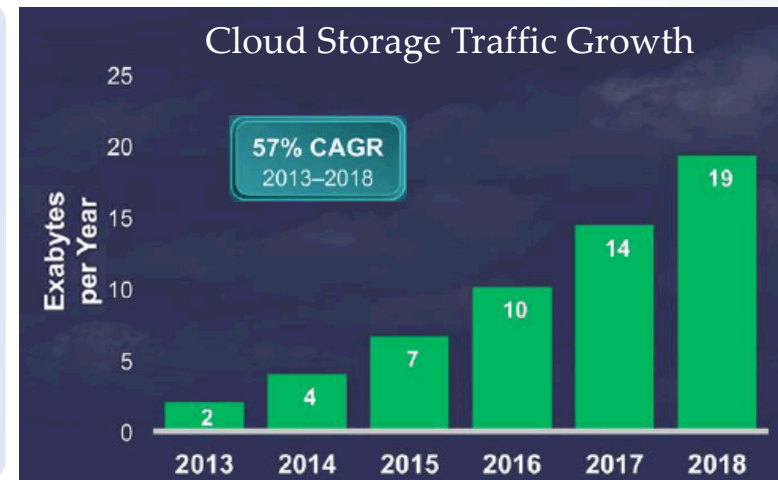
# Data Demands

Teleconferencing • Web Browsing • Video/Music streaming • Data storage

Pop. Pen.



- Petaflop machines require 5 to 10MW of power to operate with ~25% of that devoted to the computer network.
- Exaflop machines could require GW of power to operate.
- The Palo Verde Nuclear Generation Site generates 3.3GW.
- Photonic devices could reduce power requirements by two orders of magnitude compared to electronic components.
- This would bring Exaflop machine power requirements back to the MW regime.

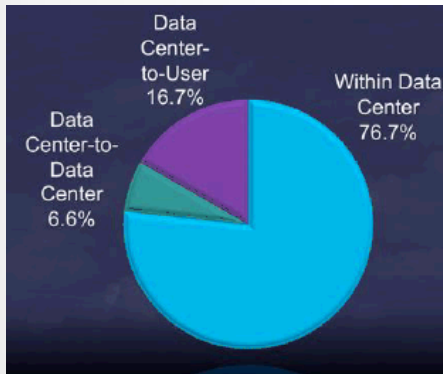


Connected vehicle safety applications • High frequency stock trading • Email/IM

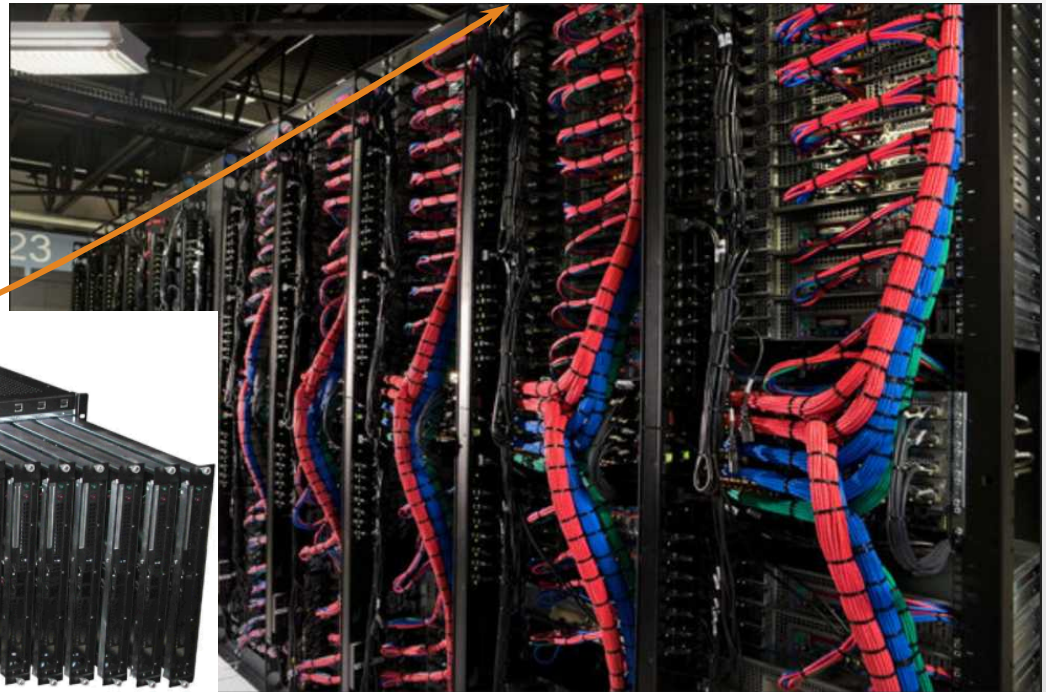


# High Performance Computers & Data Center Networks

- Data center comprised of multiple racks of multiple blade servers all of which are network connected.
- 75% of total traffic is within the data center.



HPC / Data Center



Server Rack

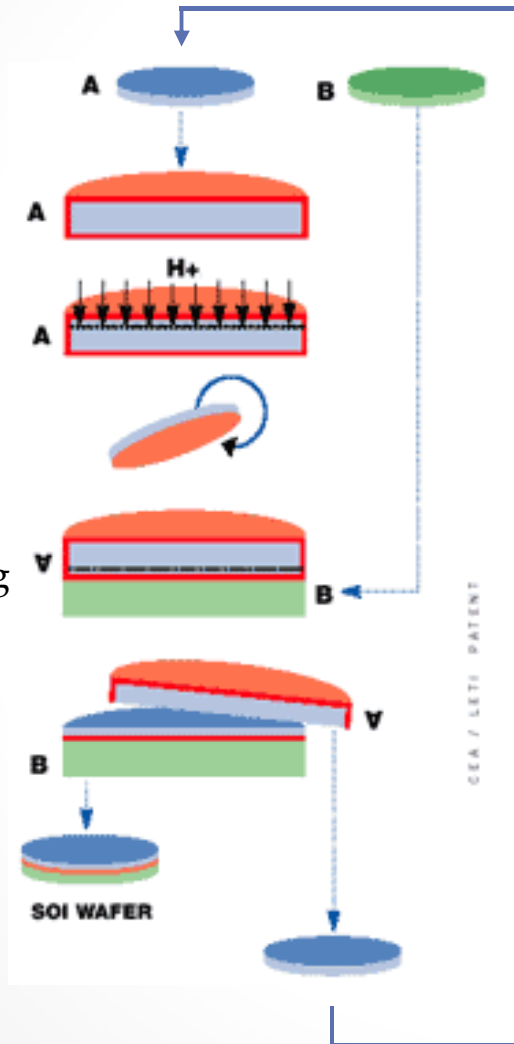
Blade Server



# Silicon Photonics

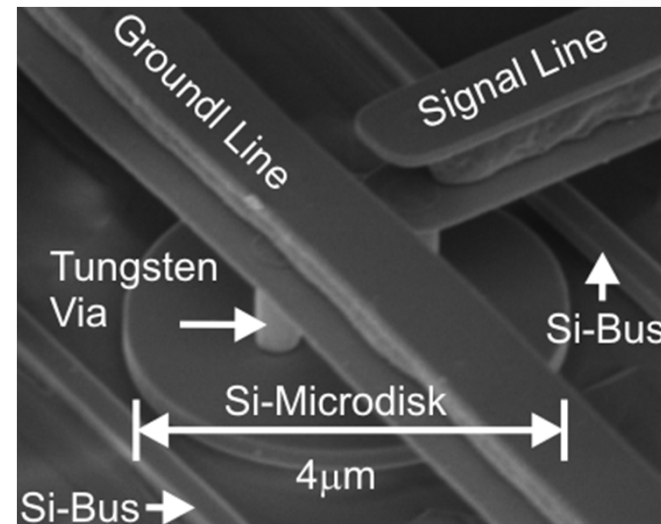
## SOItech SmartCut™ Process

1. Initial silicon
2. Thermal Oxidation
3. H<sup>+</sup> implantation
4. Cleaning & bonding
5. Splitting
6. Annealing & CMP  
Touch polishing



## Why SOI?

- CMOS front-end compatible.
- High refractive index contrast ( $\Delta n \sim 2.0$ ).
- Device layer can be doped
  - Monolithic integration of electronics/photronics.
- Low absorption loss at telecom wavelengths.



M.R. Watts, Opt. Express **19**, 21989-22003 (2011)



# Outline

## Scope of Thesis

- Investigation of waveguide crossings in multi-layer optical interconnection networks.
- Analysis a novel insertion loss measurement technique – the racetrack resonator loss platform.
- Statistical behavior of signals in rough waveguides and resonators due to Rayleigh scattering.

## Defense Outline

Data Demands	Silicon Photonics	<ul style="list-style-type: none"><li>• Intro to waveguide crossings</li><li>• Planar comparables</li><li>• The NOS crossing</li><li>• NOS optical modes</li><li>• Crosstalk estimation</li><li>• NOS crosstalk data</li><li>• NOS loss data</li><li>• Loss tradeoff analysis</li><li>• Example design optimization</li><li>• Conclusion</li></ul>	<ul style="list-style-type: none"><li>• Intro to loss measurement</li><li>• Racetrack loss platform</li><li>• Resonator transmission</li><li>• Loss calibration</li><li>• Width taper test case</li><li>• Reference resonator data</li><li>• Propagation loss data</li><li>• Width taper data analysis</li><li>• Example use cases</li><li>• Conclusion</li></ul>	<ul style="list-style-type: none"><li>• Speckle in rough waveguides and resonators.</li><li>• Intro to waveguide roughness</li><li>• Random phasor sums</li><li>• Quadratic EDF testing</li><li>• Reflected intensity</li><li>• Transmitted intensity</li><li>• Estimating roughness size</li><li>• Resonance splitting</li><li>• Conclusion</li></ul>	Conclusion
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# Waveguide Crossings in OEICs

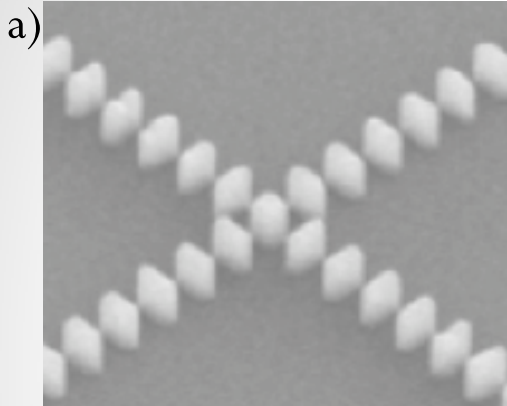
- Challenges in designing complex photonic interconnection networks
  - Each waveguide crossing causes attenuation and crosstalk.
  - Number of waveguide crossings scales with number of channels.
  - Number of crossings traversed varies by channel.
  - On-chip area scales with number of channels.
- No perfect planar solution exists.
- Multi-layer crossings suggested, but performance data not measured.

Here, we will explore the design space surrounding a CMOS compatible silicon nitride over SOI 3D optical layer for photonic interconnection networks and compare the results with single layer approaches.

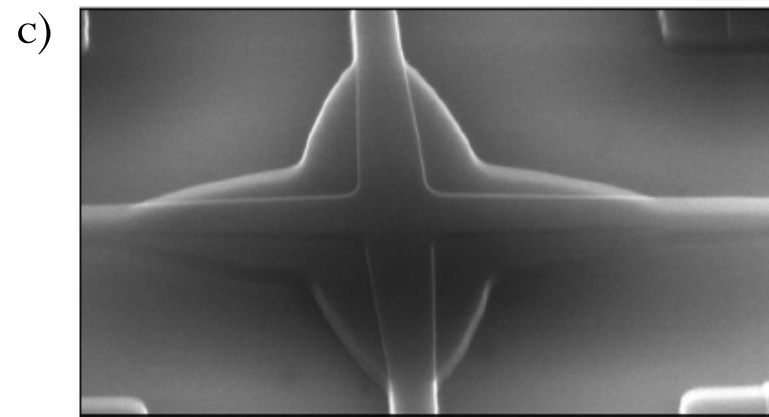
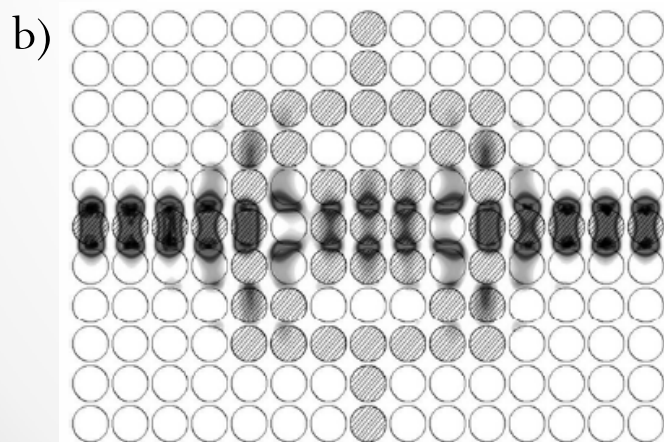
- Review of competing planar technologies.
- Description of developed multi-layer solution.
- Presentation of crosstalk and loss data as a function of layer separation.
- Discussion of loss tradeoff between vertical transitions and waveguide crossing.
- Example optimization of a simplistic optical interconnect.



# SOI Planar Waveguide Crossings

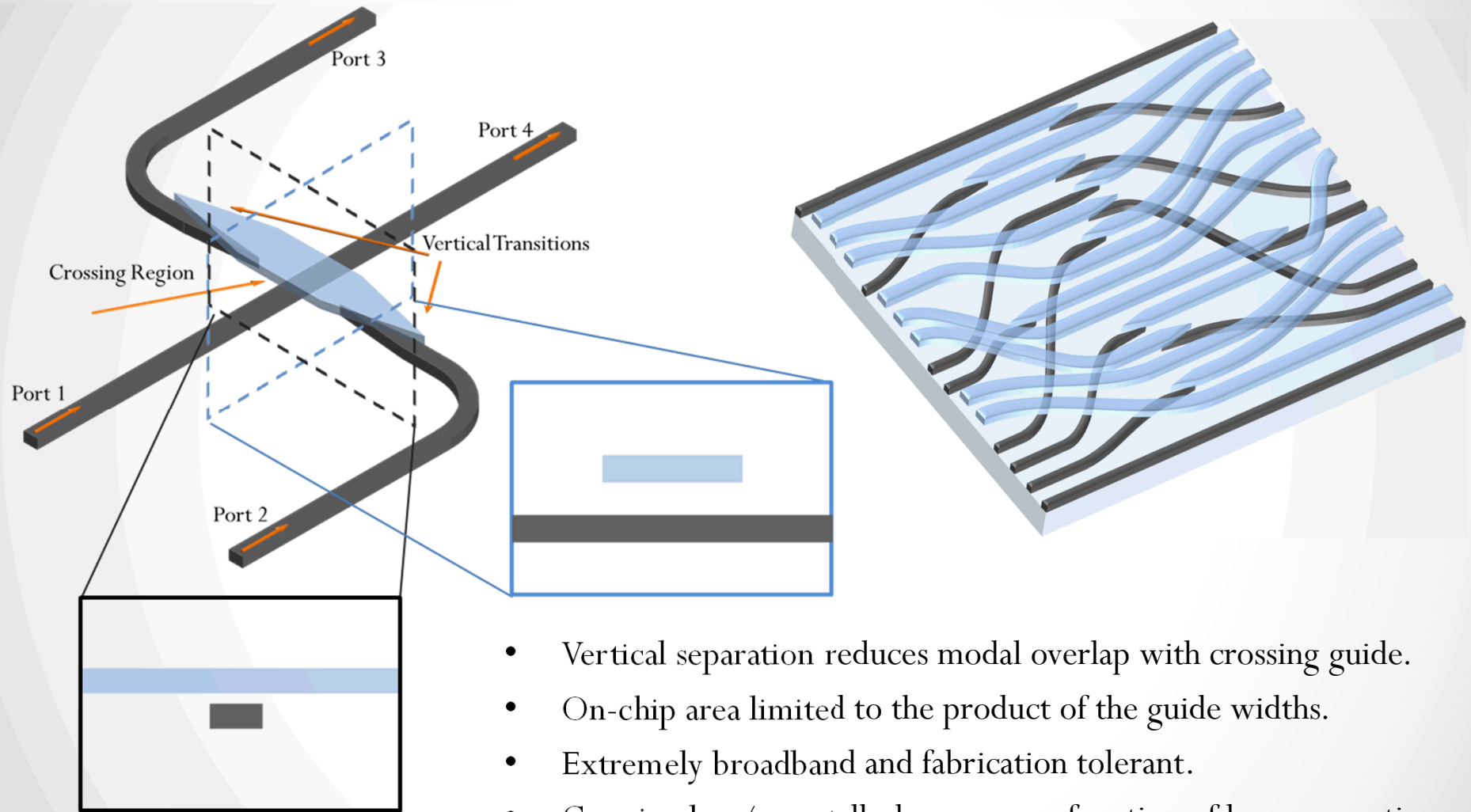


	$< 0.1$ dB Insertion Loss	$< -40$ dB Crosstalk	$< 5 \mu\text{m}^2$ On-chip Area	Broad Response (C-Band)	Fabrication Tolerant	Reference
Subwavelength Grating <sup>a</sup>	✓	✓				P. J. Bock, Opt. Exp. <b>18</b> , 16146-16155 (2010).
Photonic Crystal <sup>b</sup>	✓	✓				S. Lan, Opt. Lett. <b>27</b> , 1567-1569 (2002).
Anti-resonant	✓	✓	✓			S. G. Johnson, Opt. Lett. <b>23</b> , 1855-1857 (1998).
Wavefront Matching	✓					Y. Sakamaki, IEEE PTL <b>18</b> , 2005-2007 (2006).
Multimode Interference			✓	✓	✓	H. Chen, IEEE PTL <b>18</b> , 2260-2262 (2006).
Mode Transformation <sup>c</sup>		✓	✓	✓	✓	W. Bogaerts, Opt. Lett. <b>32</b> , 2801-2803 (2007).



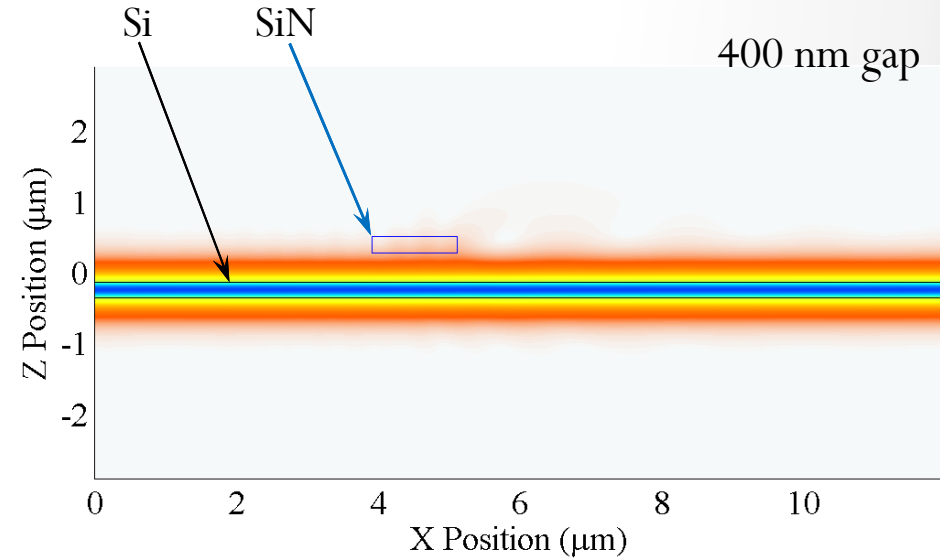
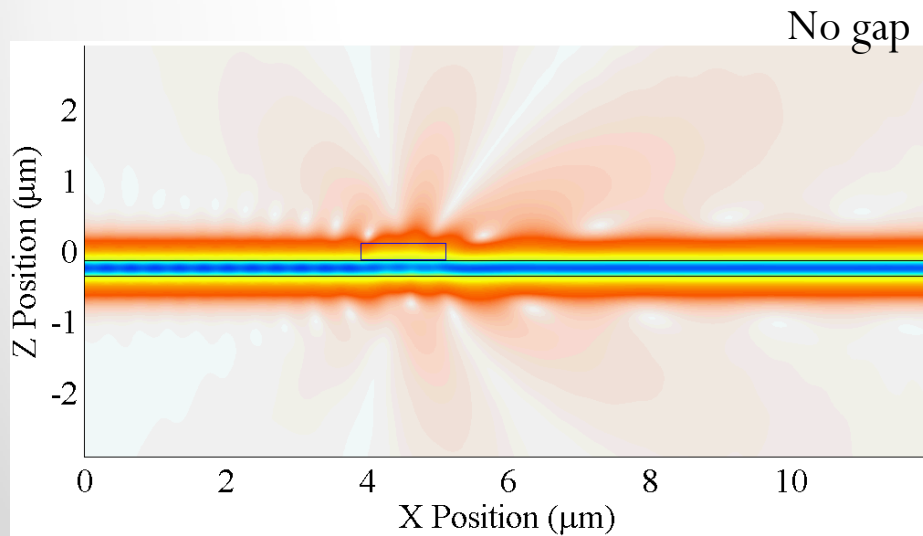
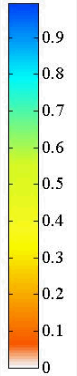
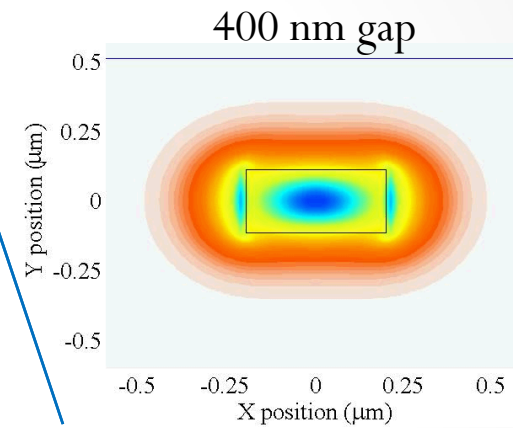
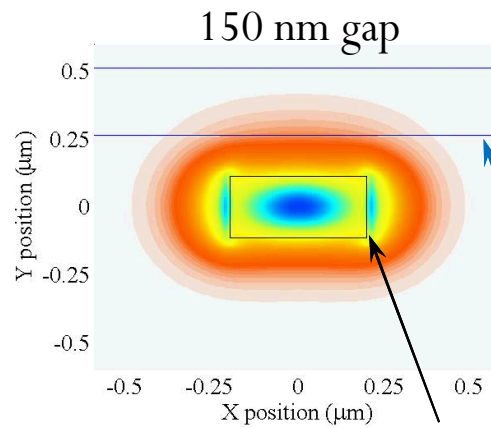
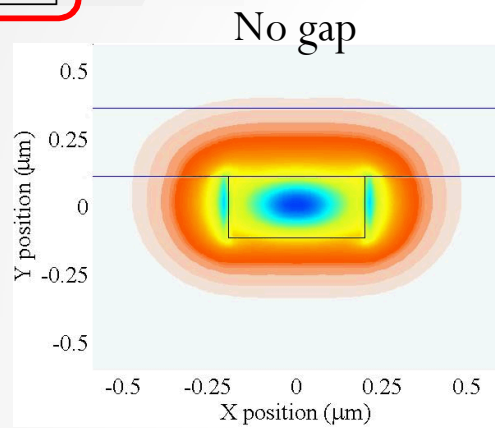
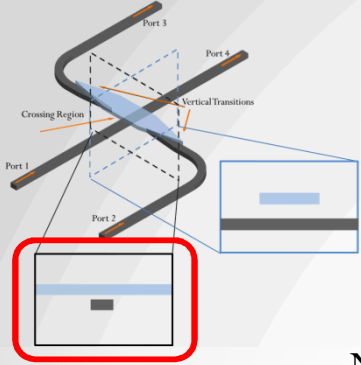


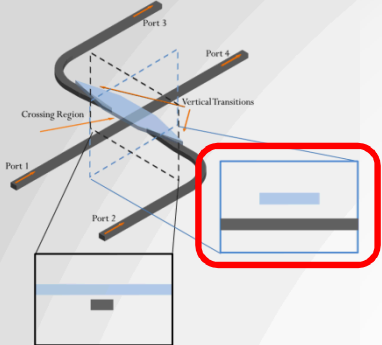
# SiN over SOI (NOS) Waveguide Crossings



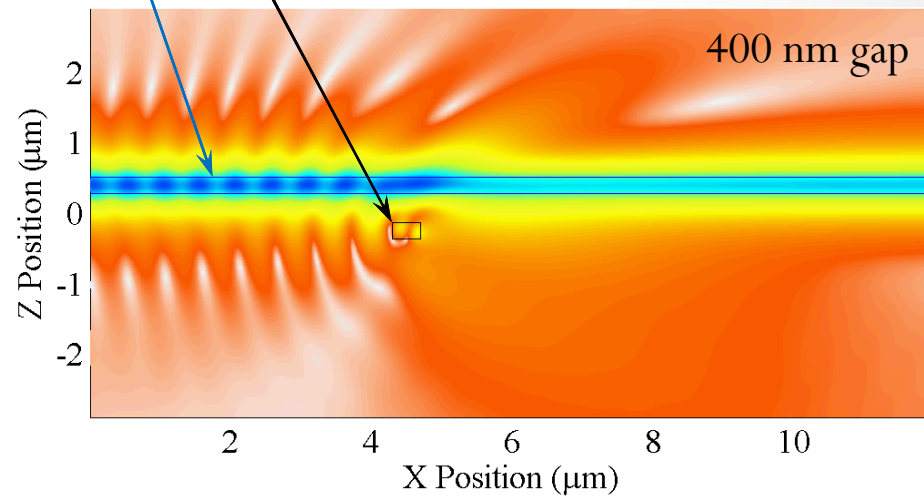
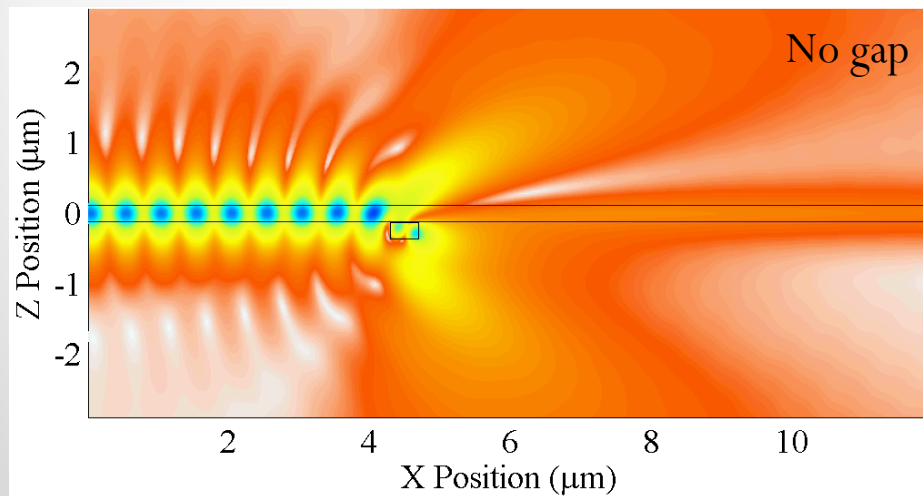
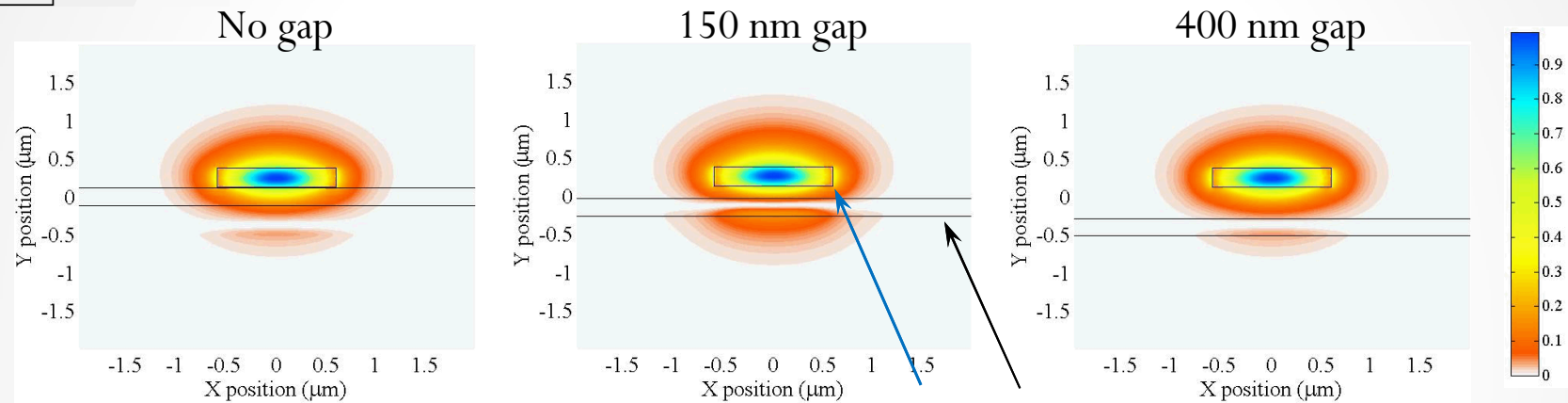
- Vertical separation reduces modal overlap with crossing guide.
- On-chip area limited to the product of the guide widths.
- Extremely broadband and fabrication tolerant.
- Crossing loss/crosstalk decrease as a function of layer separation.
- Vertical transition loss increases as a function of layer separation.

# Crossing Mode – SOI, TE





# Crossing Mode – SiN, TE



# Measuring Crosstalk

Externally Measured (port-to-port) Crosstalk

$$EXT_1 \approx \frac{P_{13}}{P_{14}} = \frac{\eta_1 \eta_3 L_1 L_3 \kappa_{13} P_0}{\eta_1 \eta_4 L_1 L_4 \kappa_{14} P_0} = \frac{\eta_3 L_3 \kappa_{13}}{\eta_4 L_4 \kappa_{14}}$$

$$EXT_2 \approx \frac{P_{24}}{P_{23}} = \frac{\eta_2 \eta_4 L_2 L_4 \kappa_{24} P_0}{\eta_2 \eta_3 L_2 L_3 \kappa_{23} P_0} = \frac{\eta_4 L_4 \kappa_{24}}{\eta_3 L_3 \kappa_{23}}$$

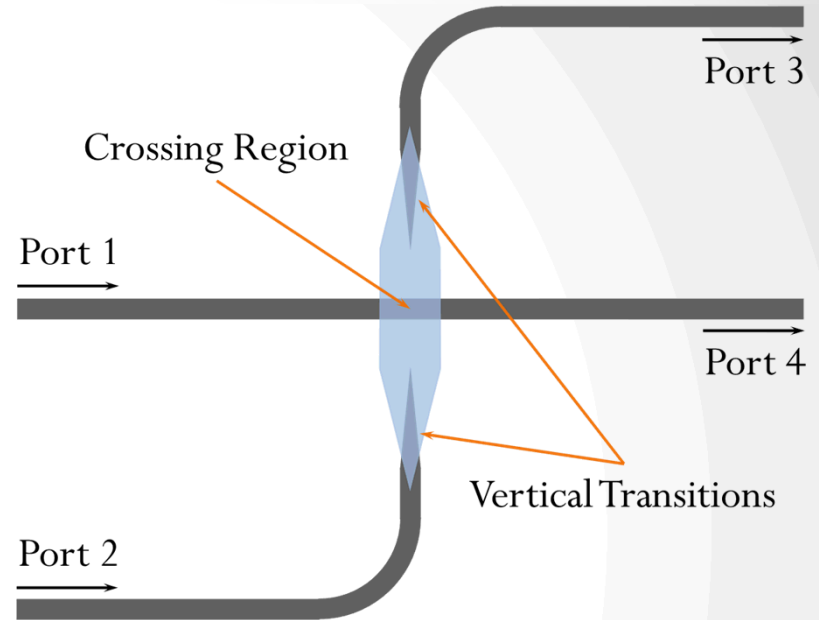
Crosstalk Caused by the Crossing

$$XT_1 = \frac{\kappa_{13}}{\kappa_{13} + \kappa_{14}} \approx \frac{\kappa_{13}}{\kappa_{14}} \quad \text{and} \quad XT_2 = \frac{\kappa_{24}}{\kappa_{23} + \kappa_{24}} \approx \frac{\kappa_{24}}{\kappa_{23}}$$

Geometric Mean

$$EXT_1 \times EXT_2 \approx \frac{\eta_3 L_3 \kappa_{13}}{\eta_4 L_4 \kappa_{14}} \times \frac{\eta_4 L_4 \kappa_{24}}{\eta_3 L_3 \kappa_{23}} = \frac{\kappa_{13}}{\kappa_{14}} \frac{\kappa_{24}}{\kappa_{23}} \approx XT_1 \times XT_2$$

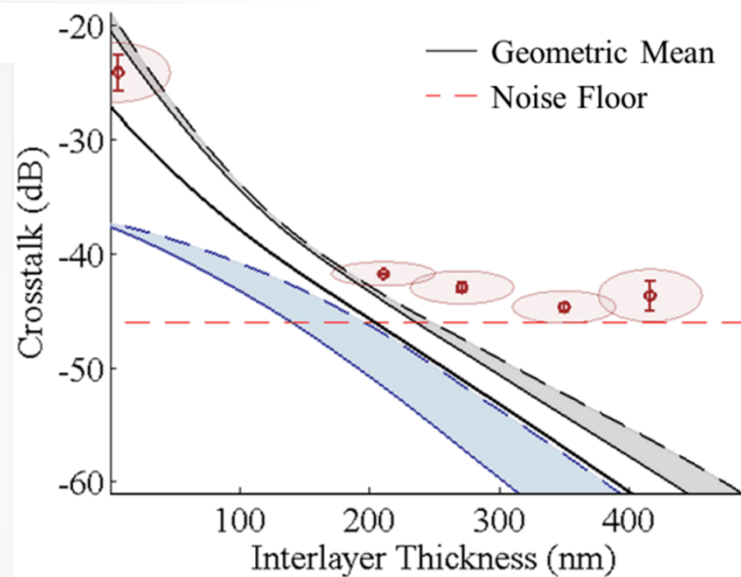
➤ The geometric mean of the externally measured crosstalk is an accurate approximation of the geometric mean of the device crosstalk.



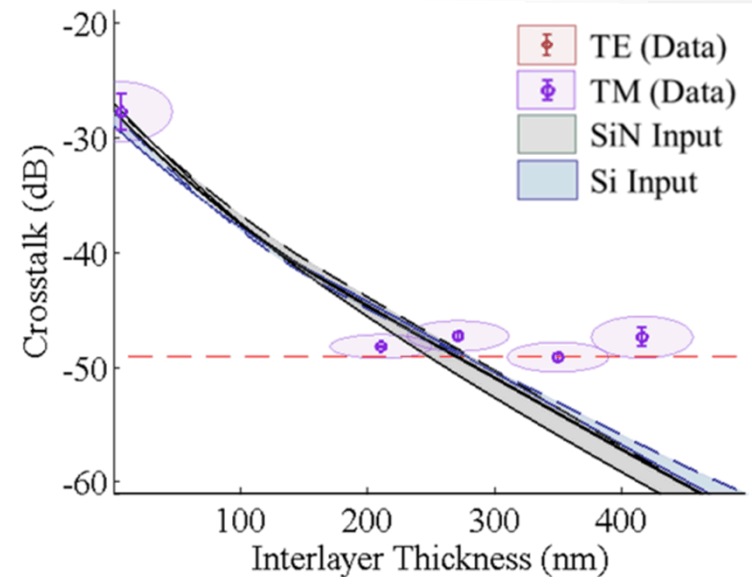
$\kappa_{ij}$	Power coupled from $i$ to $j$ by the crossing
$EXT$	Externally measured (port-to-port) crosstalk
$XT$	Crosstalk caused by the crossing
$\eta_i$	Fiber-to-waveguide coupling efficiency, port $i$
$L_i$	Path loss from port $i$ to the crossing
$P_0$	Input power
$P_{ij}$	Power measured in port $j$ for input port $i$

# Crosstalk

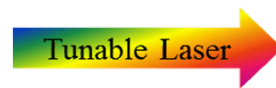
## TE Response



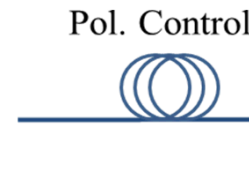
## TM Response



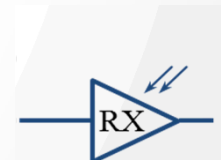
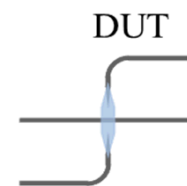
## Experimental Setup



Agilent 81600B



Thorlabs FPC560



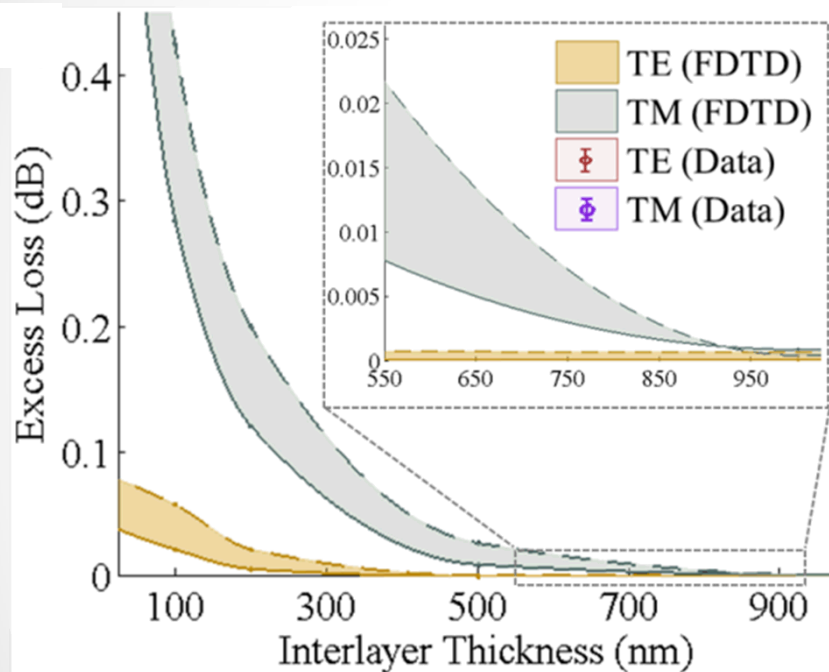
Agilent 81635A

- Solid line  $\rightarrow \lambda = 1500 \text{ nm}$
- Dashed line  $\rightarrow \lambda = 1600 \text{ nm}$
- $XT(\lambda) \rightarrow \sim \text{linear}$
- Scan step size  $\rightarrow \delta\lambda = 1 \text{ pm}$

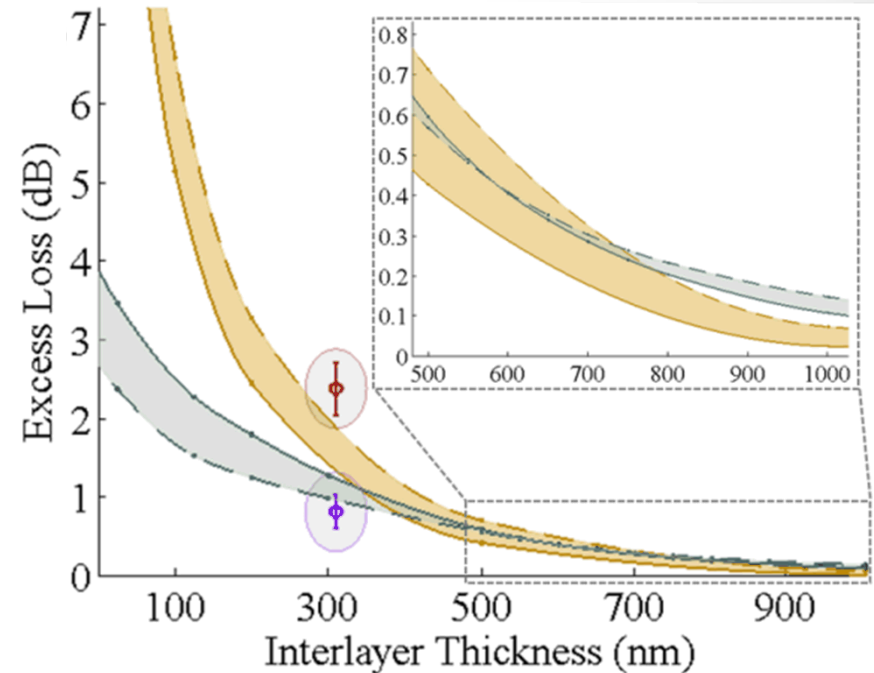


# Excess Loss

Si Input



SiN Input

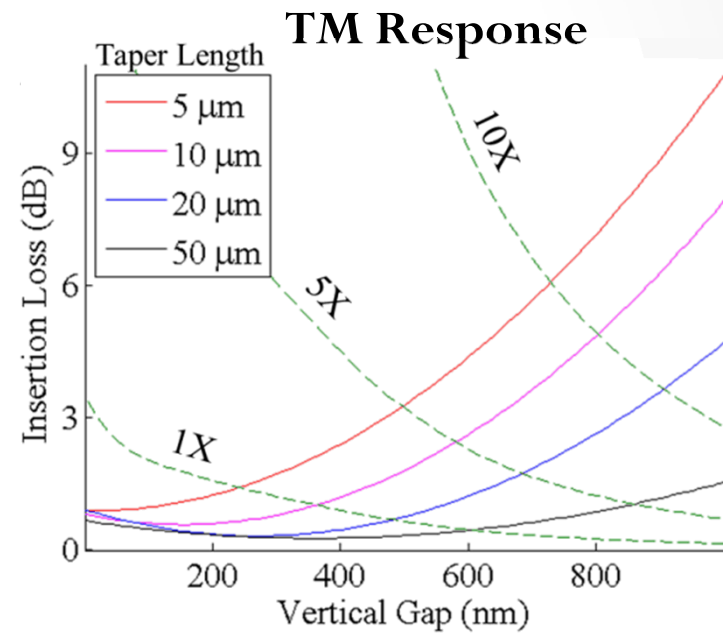
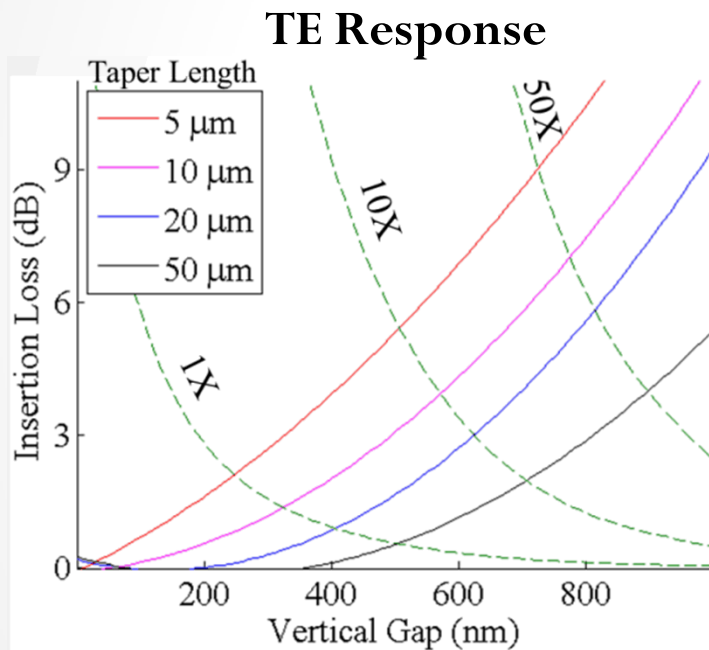


- Solid line  $\rightarrow \lambda = 1500 \text{ nm}$
- Dashed line  $\rightarrow \lambda = 1600 \text{ nm}$
- $XT(\lambda) \rightarrow \sim \text{linear}$
- Scan step size  $\rightarrow \delta\lambda = 1 \text{ pm}$



# Loss Tradeoff

- Crossing dominates loss for small gaps.
- Transition dominates loss for large gaps.
- Break-even point depends on transition length and number of crossing per transition.



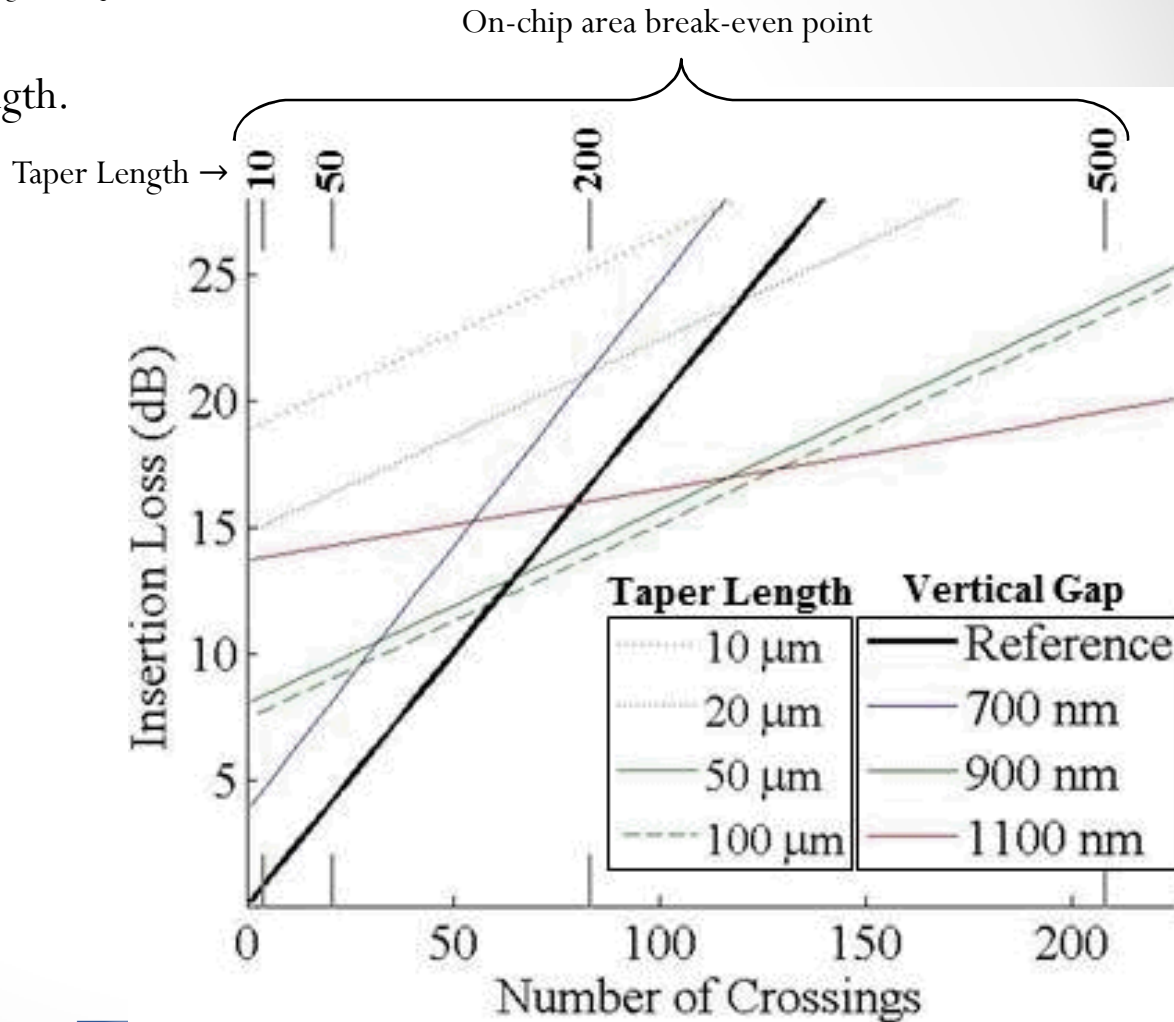
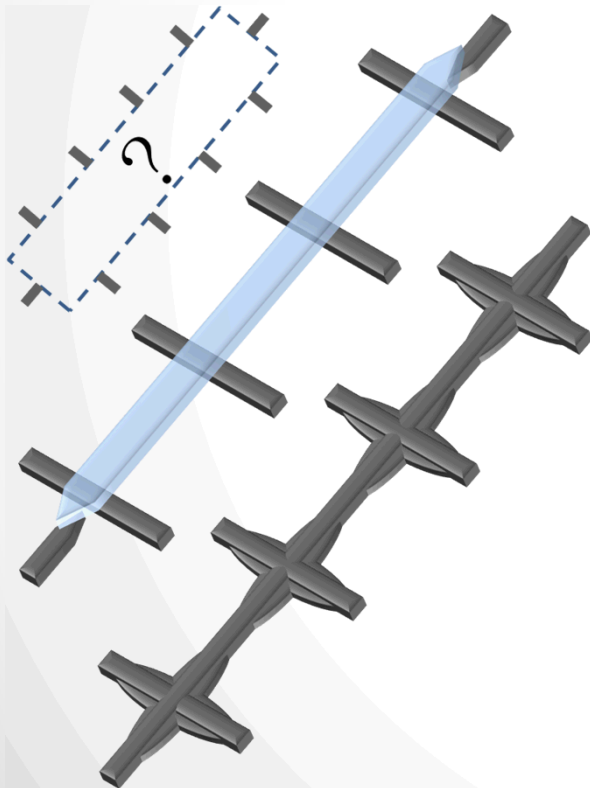
- |                                 |                               |
|---------------------------------|-------------------------------|
| ▪ Solid lines → transition loss | ▪ $\lambda = 1550 \text{ nm}$ |
| ▪ Dashed lines → crossing loss  | ▪ NX label → # crossings      |



# Example Optimization

## Example: Single waveguide with N crossings

- Compare to planar structure (W.Bogaerts, Opt. Lett. 32, 2801-2803 (2007)).
- Determine optimal vertical gap.
- Determine optimal transition length.



# NOS Crossing

## Conclusions

- Loss and crosstalk values for the SOI waveguide are exceptionally low.
- The SiN waveguide experiences over an order of magnitude larger insertion loss.
- The geometric mean of the externally measured crosstalk values is an accurate estimator of the geometric mean of the device crosstalk.
- Planar structures outperform the NOS crossings for optical paths with less than 75 crossings.
- NOS crossings require significantly lower on-chip area than efficient planar comparables.

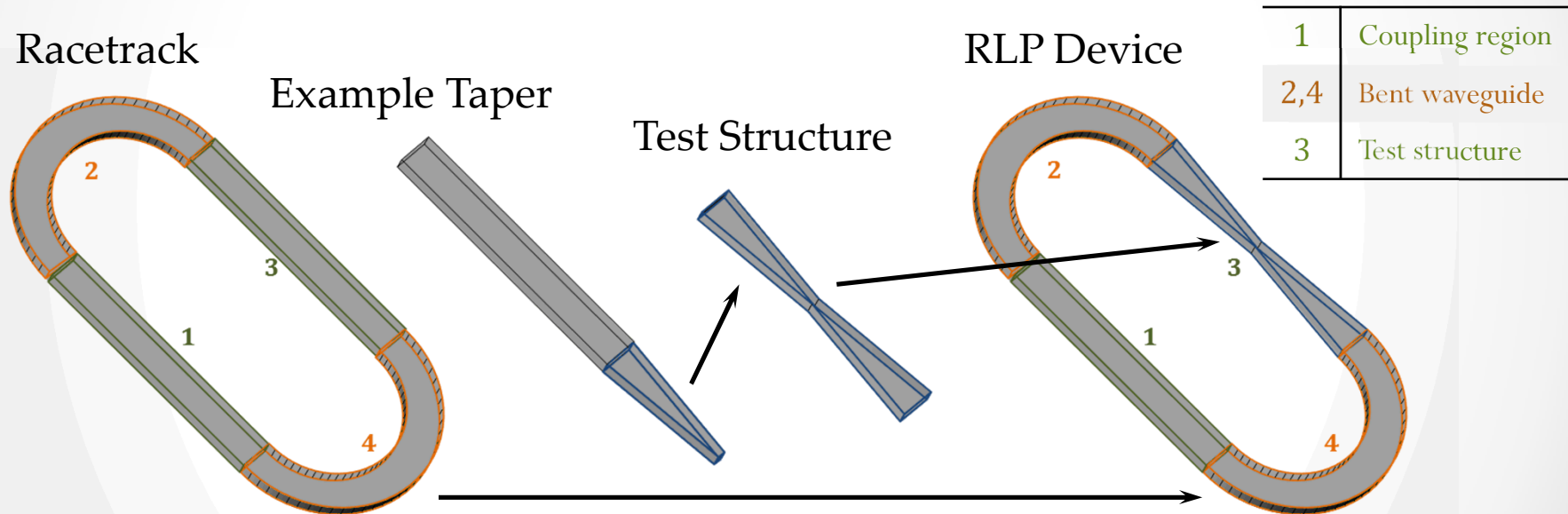
## Future Work

- Explore new materials for use as a secondary optical layer.
- Modify SiN guide geometry to enhance confinement.
- Modify crossing geometry to minimize modal interaction.



# Racetrack Resonator Loss Platform (RLP)

- Start with a standard racetrack resonator configuration.
- Replace section 3 with a mirror-image version of the test structure.
- Transmission function fitting yields insertion loss of the test device.
- Appropriate calibration yields insertion loss of the test structure.



➤ Enables accurate insertion loss measurement for compact, efficient photonic devices.

- Review -> Theory -> Calibration -> Experimental Results for lateral tapers



# Loss Measurement Overview

## Pros

## Cons

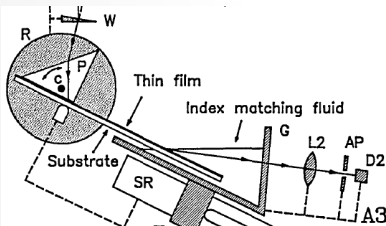
### Cut-back



- Easily implemented.
- Suitable for insertion and propagation Loss.

- Alignment error and fabrication variations affect loss estimate.
- Requires a large number of devices.

### Liquid Prism



- Single device required.
- Suitable for insertion and propagation loss.

- Requires a liquid with higher refractive index than the waveguide.

### Ring Resonator



- Single device required.
- Capable of measuring very low loss ( $< 1 \text{ dB/m}$ ).

- Susceptible to bend loss.
- Difficult to measure insertion loss.

# Resonator Transfer Function

- Contra-directional coupling with rate  $\gamma_\beta$  causes doublet formation.
- Response of doublet given by coherent addition of two singlets corresponding to standing wave modes with sinusoidal and cosinusoidal azimuthal dependence.

$$E^{c,s} = \frac{\kappa}{i(\Delta\omega \pm \gamma_\beta/2) - \gamma_\alpha^{c,s}/2} \quad T = | -1 + \frac{\kappa}{\sqrt{2}} (E^c + E^s) |^2$$

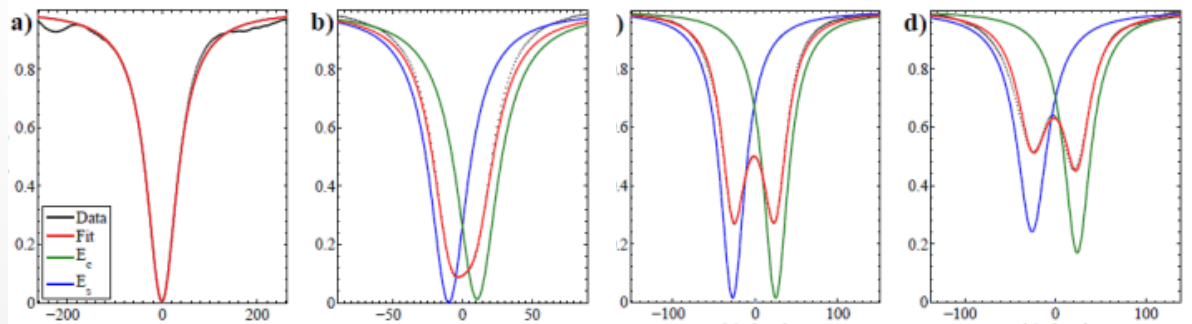
- Loss rate and loss coefficients are related by the  $Q$ -factor.

$$Q_\alpha = \frac{\omega_0}{\gamma_\alpha} = \frac{2\pi n_g L}{\lambda_0 \ln(\alpha)}$$

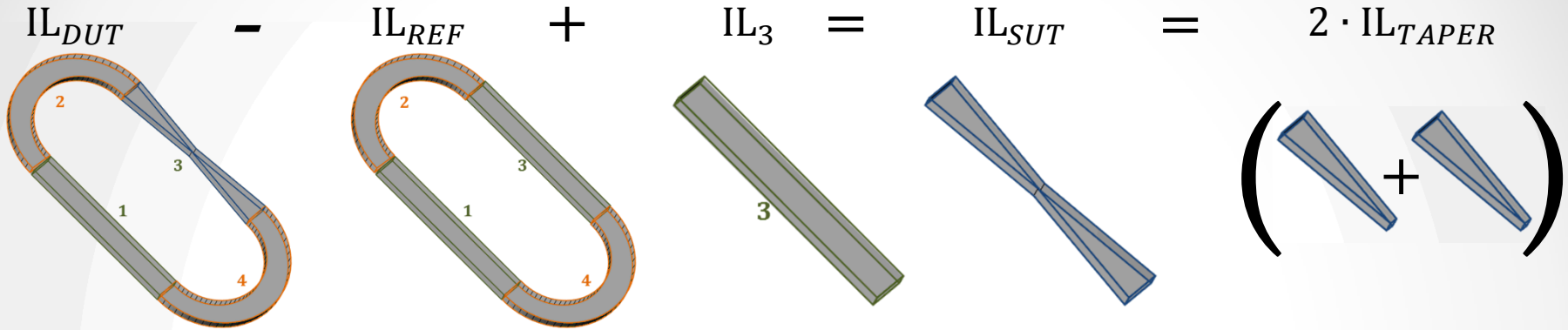
- Assuming doublet converges to singlet as  $\gamma_\beta \rightarrow 0$ , the device loss rate can be calculated.

$$\gamma_\alpha = \overline{\gamma_\alpha^{cs}} \left( 1 + \left( \delta_\alpha^{cs} / \overline{\gamma_\alpha^{cs}} \right)^2 \right) \quad \text{with} \quad \begin{cases} \overline{\gamma_\alpha^{cs}} = (\gamma_\alpha^c + \gamma_\alpha^s)/2 \\ \delta_\alpha^{cs} = (\gamma_\alpha^c - \gamma_\alpha^s)/2 \end{cases}$$

- Insertion loss is then determined from the loss coefficient via  $IL = 10\log_{10}(\alpha^2)$ .



# RLP – Loss Measurement Calibration




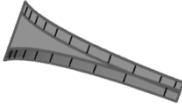
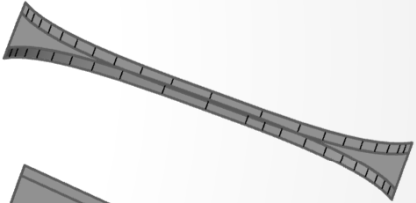

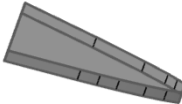
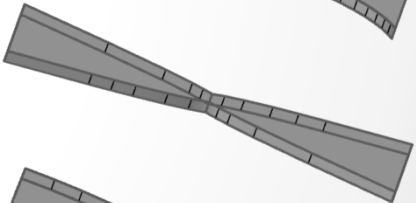

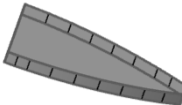
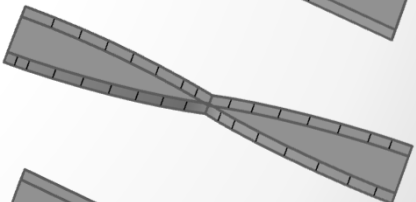


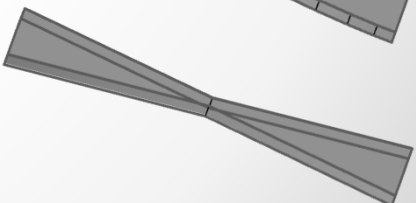
$$IL_{DUT} = IL_{SUT} + \sum_{i=1,2,4} IL_i + 4 \cdot IL_{STB} \quad IL_{REF} = \sum_{i=1}^4 IL_i + 4 \cdot IL_{STB}$$

Cal. #	$\hat{IL}_{SUT}$	$err(\hat{IL}_{SUT}) = IL_{SUT} - \hat{IL}_{SUT}$
0	$IL_{DUT}$	$err(\hat{IL}_{DUT}) + \sum_{i=1}^3 IL_i + 4 \cdot IL_{STB}$
1	$IL_{DUT} - IL_{REF}$	$err(\hat{IL}_{DUT}) - IL_3 + err(\hat{IL}_{REF})$
2	$IL_{DUT} - IL_{REF} + IL_3$	$err(\hat{IL}_{DUT}) + err(\hat{IL}_3) + err(\hat{IL}_{REF})$

➤ We will see that  $IL_3 \ll IL_{REF}$  in the example devices presented here.

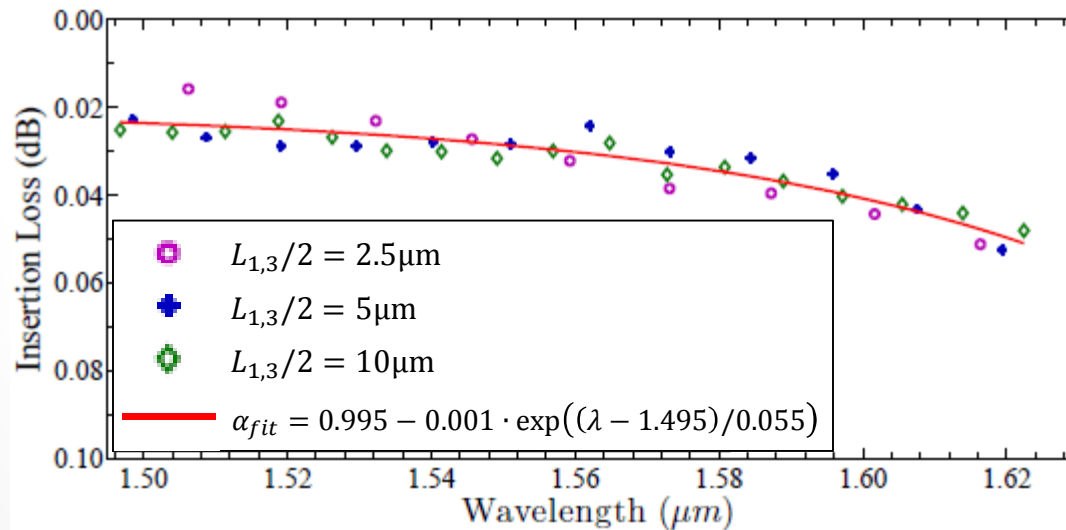
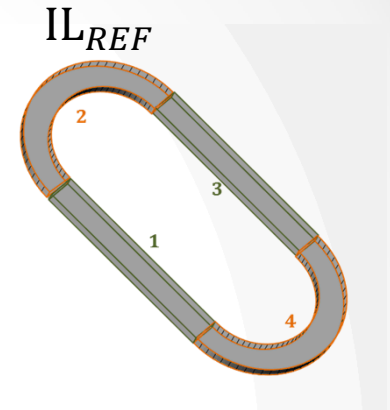
# RLP Test Case – Core Width Taper

- Test unit comprised of symmetric, mirror-image tapers.
- Taper from 400nm initial to 250nm final waveguide width
- Linear, Parabolic, Exponential, and linear  $n_{\text{eff}}$  taper functions.
- 2.5, 5, and 10  $\mu\text{m}$  long tapers.
- Bus width.
- Lateral separation.
- Reference resonators.

Function	$w(x)$	Taper		SUT
Exponential	$250 + 150e^{-5(1- x /L)}$			
Linear $n_{\text{eff}}$				
Parabolic	$250 + 150( x /L)^2$			
Linear	$250 + 150 \cdot  x /L$			

# Calibration - Reference Resonators

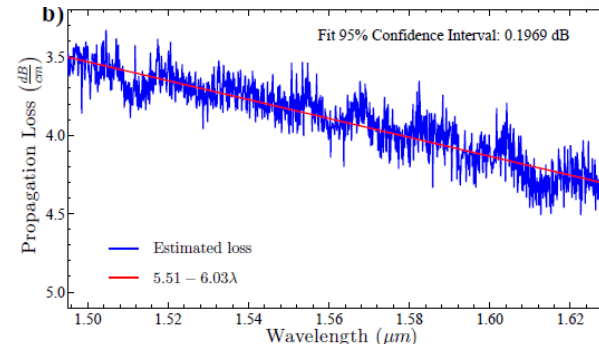
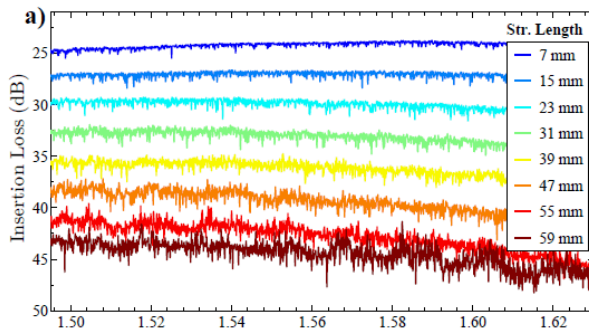
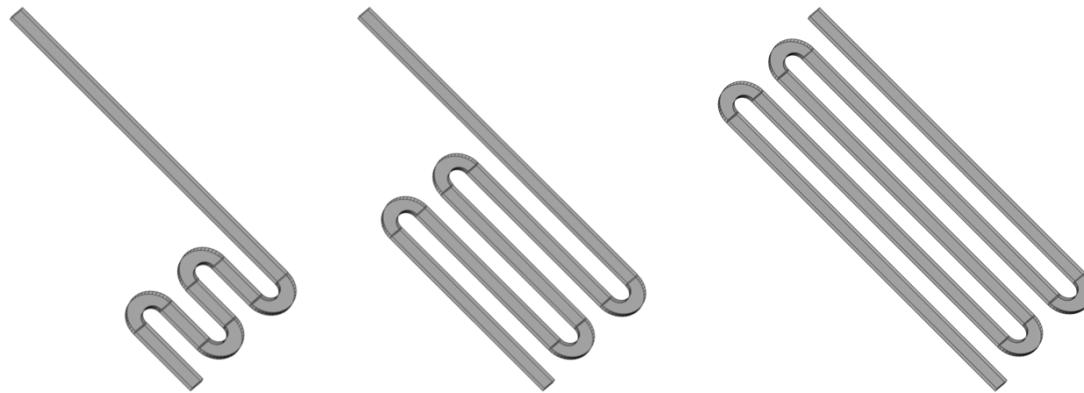
- Standard structure, no SUT.
- Confidence interval of 0.0063dB.
- Add resonator geometry





# Propagation Loss Estimation - Cutback

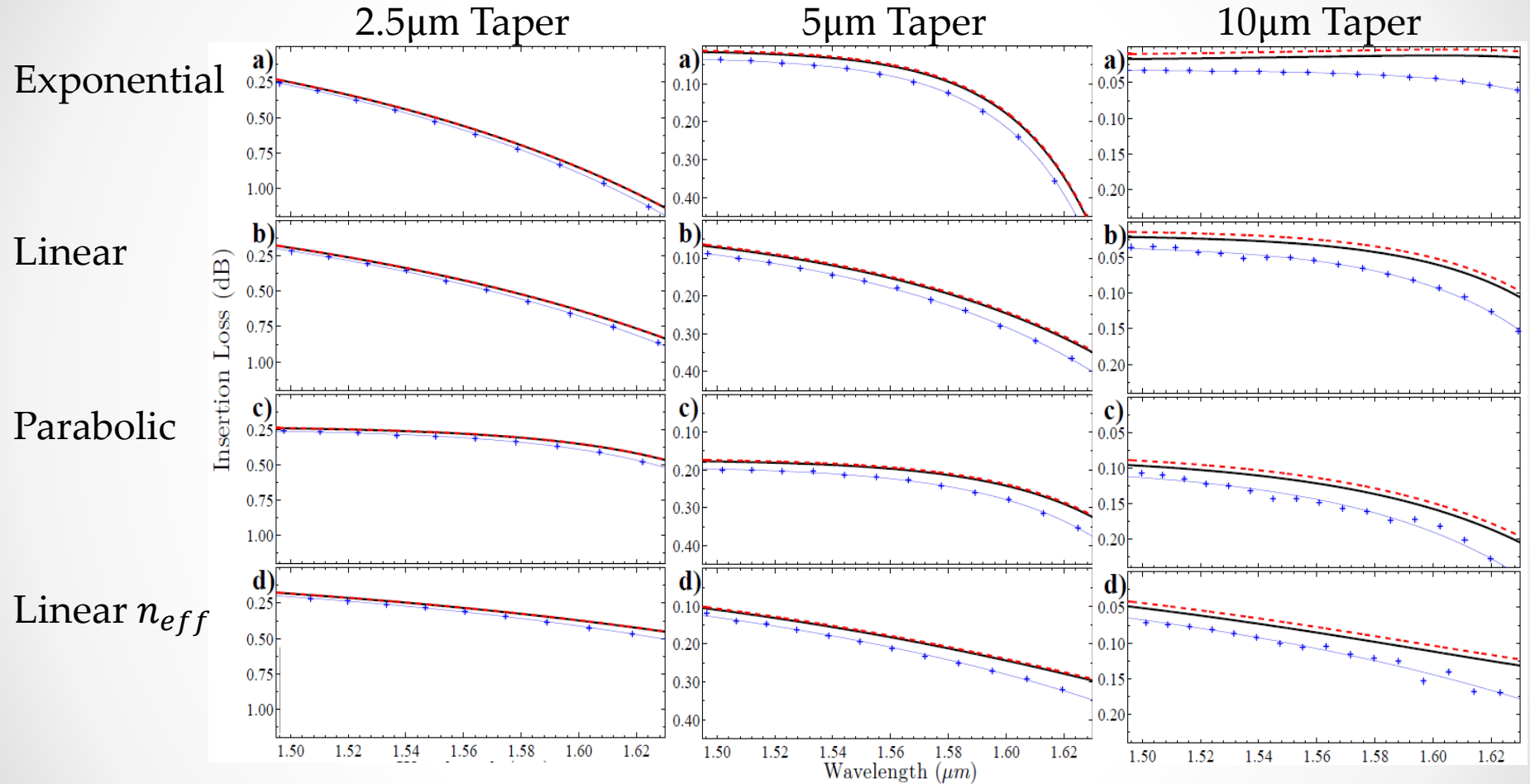
- S-bend structures used to vary total length of straight waveguide sections only.
- Propagation loss estimated by slope of linear fit to insertion loss as a function of straight length.



# Insertion Loss Data

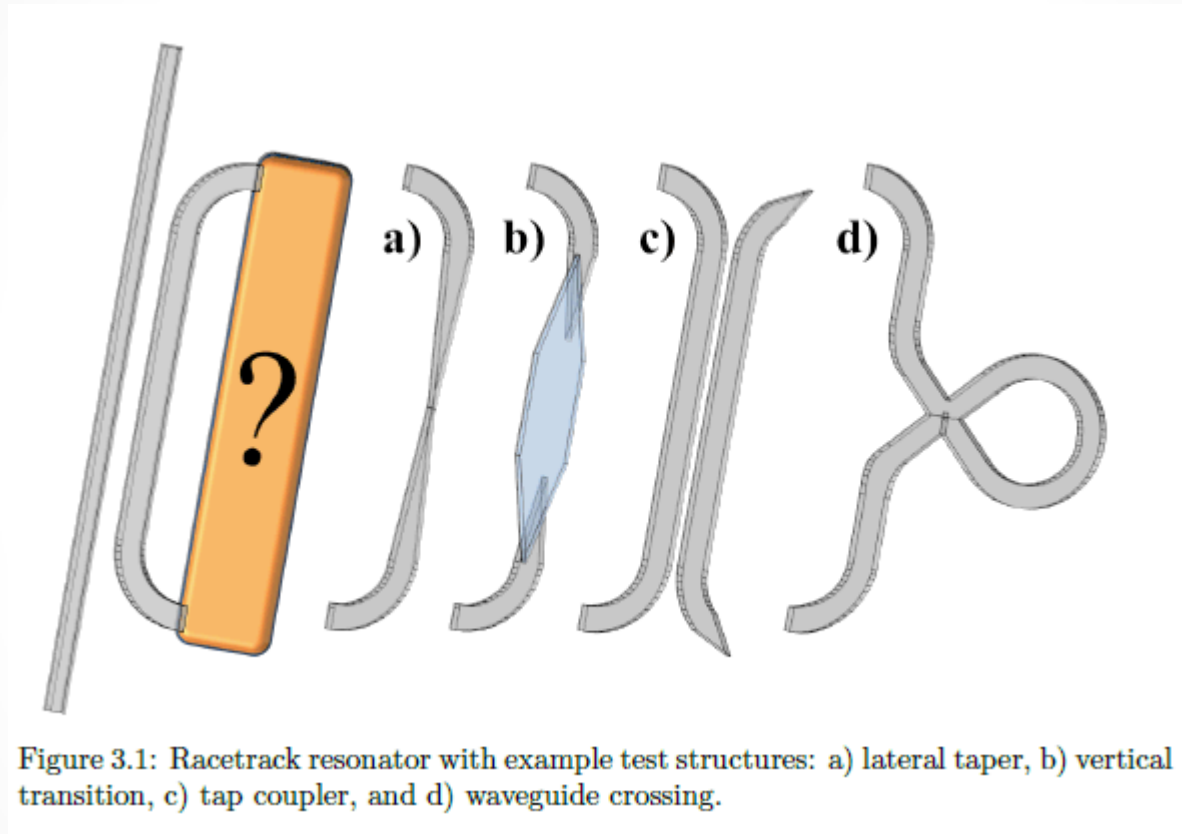
$\ast$   $IL_{DUT}$  (data)  
 $\text{—}$   $IL_{DUT}$  (fit)  
 $\text{---}$   $IL_{DUT} - IL_{REF}$  (fit)  
 $\text{—}$   $IL_{DUT} - IL_{REF} + IL_3$  (fit)

$\diamond$  Y-scale changes for  
 each taper length.



- Statements about which tapers perform in which regimes, trends, etc.
- Estimated error values.
- Change Legend entries to IL values

# Other Examples



# Racetrack Resonator Loss Platform

## Conclusions

- The RLP enables accurate analysis of the insertion loss of efficient, compact structures.
- The RLP is suitable for a variety of structures including waveguide tapers, vertical transitions, optical tap couplers, and waveguide crossings to name a few.
- In the presented SOI example, a confidence interval of 0.0063dB was obtained.
- Calibration by a reference resonator will likely be suitable in most cases.
- Secondary calibration by an estimate of  $IL_3$  provides exceptional accuracy.

## Future Work

- Implement in additional material systems.
- Apply technique to various test cases.
- Explore upper and lower loss measurement limits.
- Attempt to improve measurement error.



# Speckle in Rough Waveguides & Resonators

- Small roughness implies local mode is approximately that of the unperturbed waveguide.
- Scattering occurs when this mode impinges on a dielectric perturbation causing.

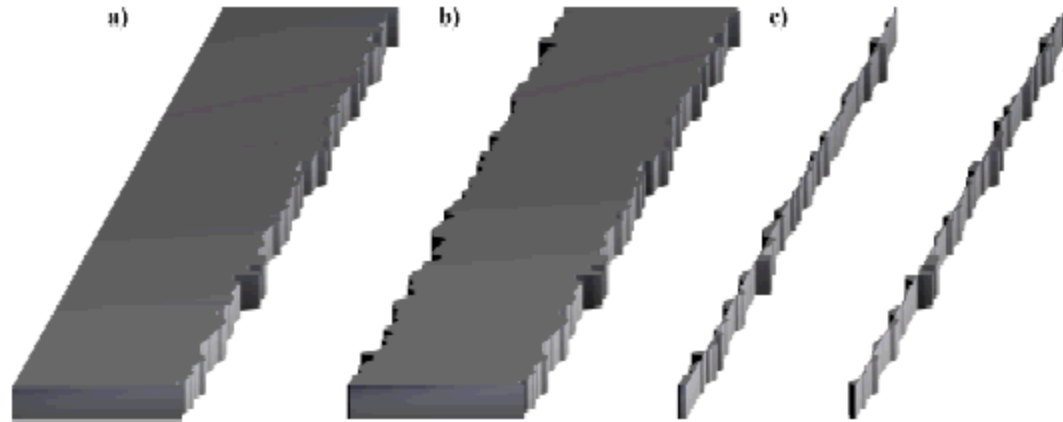


Figure 4.1: 3D representation of waveguides with (a) one and (b) two FISUR corrupted sidewalls along with (c) the sidewall roughness as a perturbation of the perfect waveguide.

$$K_{\mu\nu}(z) \approx f(z) \hat{K}_{\mu\nu}$$

$f(z) \rightarrow$  deviation from ideal waveguide boundary

$$\hat{K}_{\mu\nu} = \frac{i\omega\epsilon_0\delta\epsilon}{4P} \int_{-d}^d dy \left[ E_{\mu t}^* \cdot E_{\nu t} + \gamma_K \left( H_{\mu x}^* \frac{\partial E_{\nu z}}{\partial y} + \frac{\partial H_{\mu y}^*}{\partial x} E_{\nu z} \right) \right]$$

$$\gamma_K = \begin{cases} 1/(\omega\epsilon_0 n_g)^2, & f(z) > 0 \\ 0, & f(z) = 0 \\ -1/(\omega\epsilon_0)^2, & f(z) < 0 \end{cases}$$



# Scattering in Rough Waveguides

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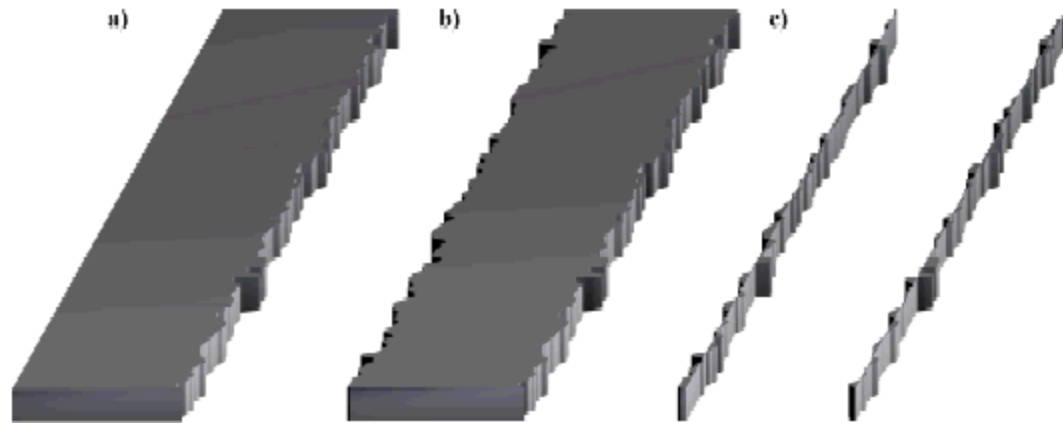


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# Random Phasor Sum

- Following Goodman[REF] and Dainty[REF], the distributions on the amplitude ( $A$ ), intensity ( $I$ ), and measured intensity ( $I'$ ) may be derived.
  1. Assumes a large number of statistically independent fields ( $\mathbf{a}_k = |a_k|\exp(i\phi_k)$ ).
  2. Assumes the joint distribution on the real (r) and imaginary(i) components is Normal.
- Condition(1) satisfied by sufficient propagation length.
- Morichetti, et.al. [REF] demonstrated that rough waveguides obey (2) regardless of geometry, polarization, etc.

$$\mathbf{a} = \frac{1}{\sqrt{N}} \sum_{k=1}^N |a_k| e^{i\phi_k} \quad p_{RI}(r, i) = \frac{1}{2\pi\sigma_{RI}^2} \exp\left[\frac{-(r^2 + i^2)}{2\sigma_{RI}^2}\right]$$

$$p(|A|) = \frac{2|A|}{\langle I \rangle} \exp\left(\frac{-|A|^2}{\langle I \rangle}\right), \quad |A| \geq 0$$

$$p(I) = \frac{1}{\langle I \rangle} \exp\left(\frac{-I}{\langle I \rangle}\right), \quad I \geq 0$$

$$p(I') = \left(\frac{n_0}{\langle I \rangle}\right)^{n_0} \frac{I'^{(n_0-1)} \exp(-I' n_0 / \langle I \rangle)}{\Gamma(n_0)}, \quad I' \geq 0$$



# Quadratic EDF Testing

- Effectively OLS applied to the EDF.
- Zero mean, Normally distributed residuals a strong indicator of appropriate fit function.
  - Lack of bias indicates appropriate distribution shape found.
- Some distributions we will need are presented as well.



# Statistical Behavior – Reflected Intensity

- Reflected intensity is simply a random phasor sum.
- Image from Morichetti, distribution, PDF & CDF plots.



# Statistical Behavior – Transmitted Intensity ( $T$ )

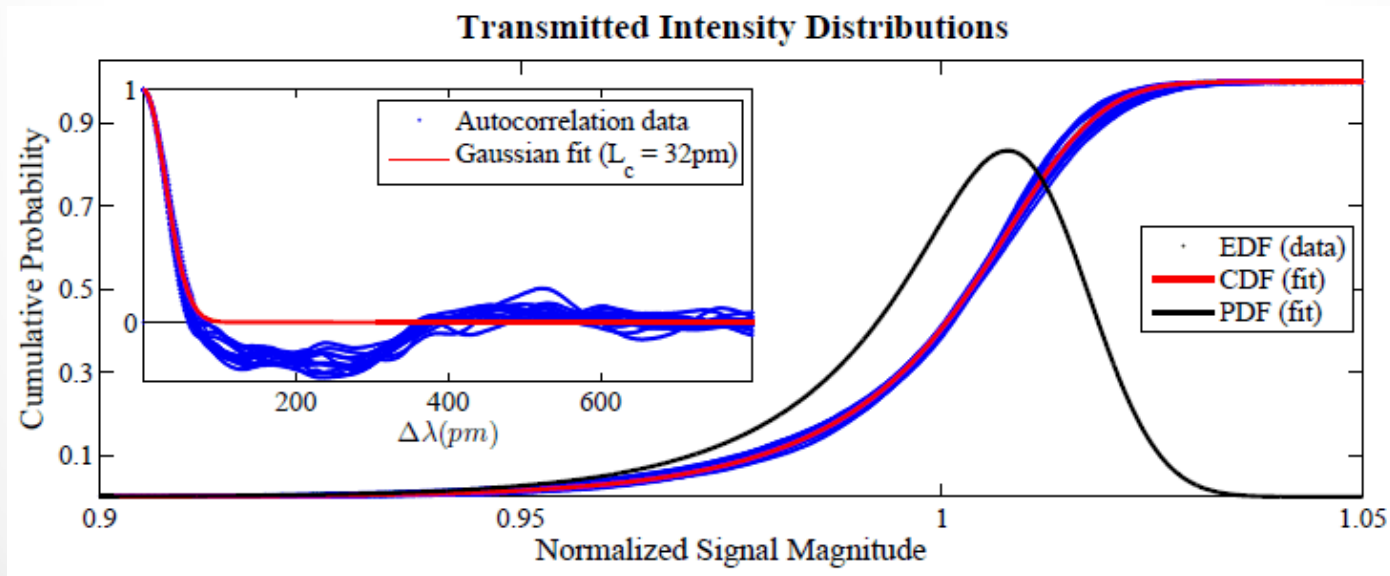
- Conservation of energy yields transmitted intensity.
  - Distribution on forward scattered fields converges to Normal.
  - Lumping loss and reflected fields together ( $S' = \alpha' - R'$ ) and performing a probability transformation give  $p(S')$ .

$$p(S') = \left( \frac{n_r}{\langle R \rangle} \right)^{n_r} \frac{(\bar{\alpha} - S')^{(n_r-1)} \exp(-(\bar{\alpha} - S')'n_r / \langle R \rangle)}{\Gamma(n_r)}, \quad S' \leq \alpha'$$

- Inclusion of additive white noise ( $N'$ ) produces the final relation on  $T'$ .

$$p(T') = p(S') * p(\delta'_\alpha) * p(N')$$

- PDF with fit, CDF with fit, correlation function



# Estimating Roughness Size from $T'$

- Assume fit to transmission data noise provides an estimate of  $\langle R \rangle$  and  $n_r$ .
- Link between  $\langle R \rangle$  and  $f(z)$  then obtained by the following equations

$$|a_k| = |f(z_k)|\hat{K}_{+-}(z_k) \quad \text{and} \quad \langle R \rangle = \langle |a_k|^2 \rangle = 2\sigma_{RI}^2$$

- Expressions can then be derived for  $p(|a_k|)$

Low Permittivity Contrast, one rough boundary

$$|a_k|_{l,1} = |\kappa f_1(z_k)|$$

$$p(|a_k|_{l,1}) = \text{HN}(0, \kappa\sigma_{SW})$$

Low Permittivity Contrast, two rough boundaries

$$|a_k|_{l,2} = |\kappa f_1(z_k)| + |\kappa f_2(z_k)|$$

$$p(|a_k|_{l,2}) = \text{HN}(0, \kappa\sigma_{SW}) * \text{HN}(0, \kappa\sigma_{SW})$$

High Permittivity Contrast, one rough boundary

$$|a_k|_{h,1} = |C_{1+}| + |C_{1-}|$$

$$p(|a_k|_{h,1}) = \text{HN}(0, s_+) * \text{HN}(0, s_-)$$

High Permittivity Contrast, two rough boundaries

$$|a_k|_{h,2} = |C_{1+}| + |C_{1-}| + |C_{2+}| + |C_{2-}|$$

$$p(|a_k|_{h,2}) = \left( \text{HN}(0, s_+) * \text{HN}(0, s_-) \right) * \left( \text{HN}(0, s_+) * \text{HN}(0, s_-) \right)$$

Variable Definitions

$$C_{n+} = \kappa_+ f_n(z_k) U(f(z_k))$$

$$C_{n-} = \kappa_- f_n(z_k) \left( 1 - U(f(z_k)) \right)$$

$$s_{\pm} = \kappa_{\pm} \sigma_{SW}$$

- ❖ Estimate of  $\sigma_{SW}$  highly depends on the quality of the loss estimate
- ❖ These results have yet to be validated experimentally.





# Statistical Behavior of Resonance Splitting (1)

- Multiplying energy rate equations by  $i\hbar$  yields.

$$i\hbar \frac{d}{dt} \begin{bmatrix} a^+ \\ a^- \end{bmatrix} = \hbar (\gamma_t/2 - \Delta\omega) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a^+ \\ a^- \end{bmatrix}$$

- Same form as TDSE and inherent adherence to requirements of said equation allow us to write.

$$\Psi \rightarrow \begin{bmatrix} a^+ \\ a^- \end{bmatrix} \quad H^0 \rightarrow \hbar (\gamma_t/2 - \Delta\omega) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Introducing contra-directional coupling

$$H = H^0 + H' = \hbar \left( (\gamma_t/2 - \Delta\omega) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{\gamma_\beta}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right)$$

- Energy eigenvalues of  $E^c$  and  $E^s$  can be found

$$E^c = -\hbar (\gamma_t/2 - \Delta\omega + \gamma_\beta/2) \quad \text{and} \quad E^s = -\hbar (\gamma_t/2 - \Delta\omega - \gamma_\beta/2)$$

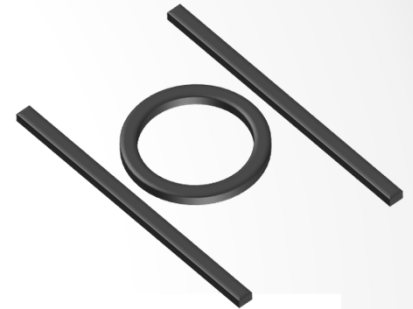


# Statistical Behavior of Resonance Splitting (2)

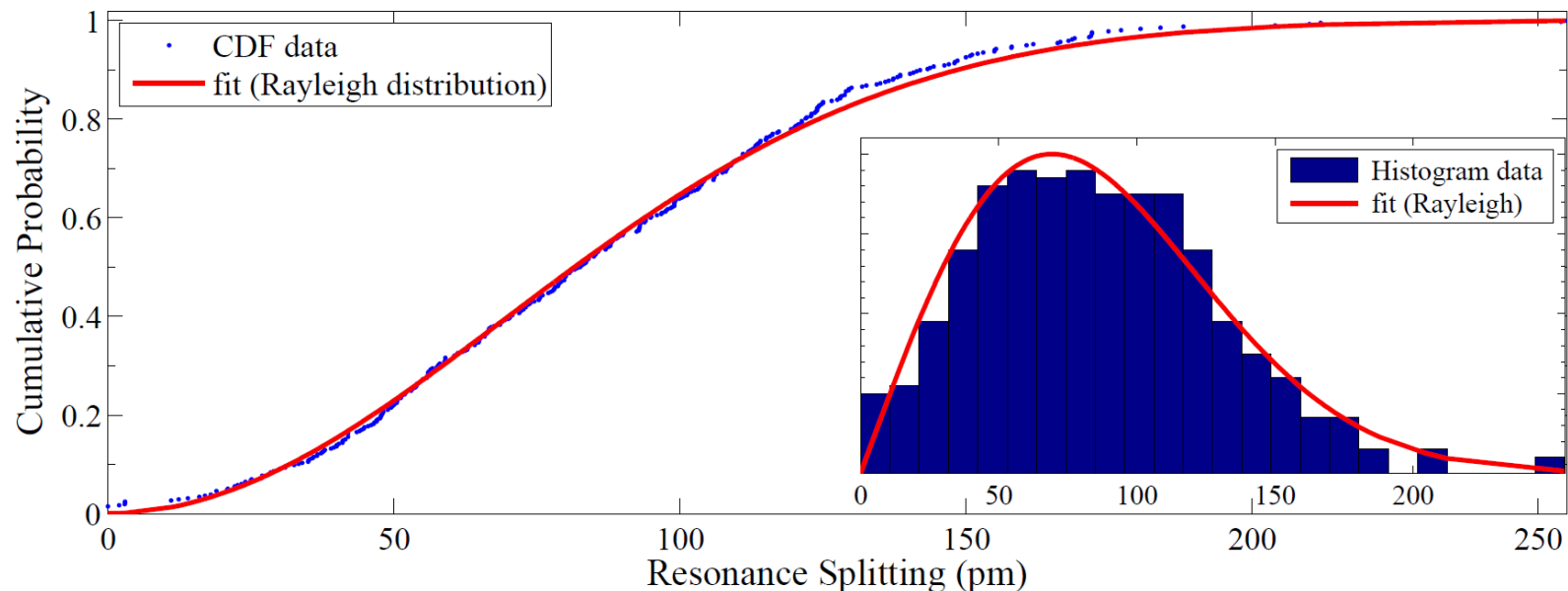
- Our previous derivation shows that  $\Delta\omega_{c,s} = \gamma_\beta$
- Via LLN,  $\gamma_\beta$  and, subsequently,  $\Delta\omega_{c,s}$  follow a Rayleigh distribution.

$$p(\Delta\omega_{c,s}) = \frac{2\Delta\omega_{c,s}}{\langle\Delta\omega_{c,s}\rangle} \exp\left(-\frac{\Delta\omega_{c,s}^2}{\langle\Delta\omega_{c,s}\rangle^2}\right)$$

- 418 resonances from eleven resonators fit
  - four port ring resonators with 50 $\mu\text{m}$  diameter and 400nm ring width
  - coupling gap and bus waveguide width varied



**Resonance Splitting Distribution**



# Scattering in Rough Waveguides & Resonators

## Conclusions

- Scattering in rough waveguides caused loss and distributed contra-directional coupling.
- Reflected field of a passive waveguide can be represented as a random phasor sum.
- Transmitted measured intensity follows a smoothed Gamma distribution.
- A theoretical link between roughness size and transmitted intensity was derived but has yet to be experimentally validated.
- Resonance splitting caused by scattering induced contra-directional coupling follows a Rayleigh distribution.
- Theory dictates that very few resonances with zero splitting should be observed indicating that many doublets are incorrectly identified as singlet resonances.

## Future Work

- Validate theoretical link between  $T'$  and  $\sigma_{SW}$ .
- Explore enhancement of contra-directional coupling via modification of scattering center geometry.
- Explore minimization of radiation mode coupling via modification of scattering center geometry.



# Other Work

- Design of ring resonator based Optical Add/Drop Multiplexer (OADM).
- Study of curve fitting in the presence of speckle noise.
  - Heteroskedastic, correlated noise violates Gauss-Markov.
- Development of a loss envelope removal algorithm utilizing morphological erosion.
- Development of a noise-robust resonance locating algorithm.
- Development of a perturbative approach to split resonance fitting.
  - Reduce six parameter fit with a large number of local minima in the 6D error function to a series of three parameter fits with a single global minima.



# Acknowledgements

- University of Arizona, College of Optical Sciences
- Sandia National Labs, Org. 1765 Applied Photonic Microsystems



# Questions

