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Adaptive particle methods for global geophysical flow

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Introduction

We investigate the use of Lagrangian particle methods for global atmospheric modeling applications. Particle position is given by the flow map

$$\frac{\partial}{\partial t}\boldsymbol{x}(\boldsymbol{a},t) = u(\boldsymbol{x}(\boldsymbol{a},t),t), \quad \boldsymbol{x}(\boldsymbol{a},0) = \boldsymbol{a}, \qquad (1)$$

where $\boldsymbol{a}$ is a Lagrangian parameter and t is time. For our applications, the fluid velocity $\boldsymbol{u}$ is tangent to the sphere.

A consequence of the flow map is that the material derivative becomes an ordinary time derivative. Hence, any distribution $q(\boldsymbol{x},t)$ that satisfies the advection equation

$$\frac{Dq}{Dt} = \frac{\partial}{\partial t}q(\boldsymbol{x},t) + \boldsymbol{u}\cdot\nabla q(\boldsymbol{x},t) = 0, \qquad (2)$$

also satisfies

$$\frac{d}{dt}q(\boldsymbol{x}(\boldsymbol{a},t),t) = 0 \Rightarrow q(\boldsymbol{x}(\boldsymbol{a},t),t) = q(\boldsymbol{x}(\boldsymbol{a},0),0) \qquad (3)$$

Equation (3) states that advected tracers are constant following the flow. Our numerical method focuses on solving (1) and (3) instead of (2), as is done by most current operational and research models.

The sphere is discretized in space by a set of M particles arranged in a collection of N panels that cover the sphere. The particles approximate (1), $\boldsymbol{x}_j(t) \approx \boldsymbol{x}(\boldsymbol{a}_j,t)$.

Tracer transport

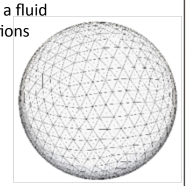
Tracer transport problems define a fluid velocity $\boldsymbol{u}(\boldsymbol{x},t)$ and initial conditions for a tracer $q(\boldsymbol{x},t)$ [1].

The ODEs we solve are

$$\frac{d}{dt}\boldsymbol{x_j}(t) = \boldsymbol{u}(\boldsymbol{x_j}(t),t),$$

$$q_j = q(\boldsymbol{\alpha_j}),$$

for $j = 1,\ldots,M.$



Barotropic vorticity equation

The absolute vorticity $\omega(\boldsymbol{x},t) = \zeta(\boldsymbol{x},t) + f(\boldsymbol{x},t)$ satisfies (2), where $\zeta = \nabla\times\boldsymbol{u}$ is relative vorticity and $f(\boldsymbol{x},t) = 2\Omega z/R$ is the Coriolis parameter. Velocity is given by the Biot-Savart law [2] and the ODES we solve are

$$\frac{d}{dt}\boldsymbol{x}_j(t) = -\frac{1}{4\pi R}\sum_{\substack{k=1\\k\neq j}}^{N}\frac{\boldsymbol{x}_j\times\boldsymbol{x}_k}{R^2-\boldsymbol{x}_j\cdot\boldsymbol{x}_k}\zeta_k A_k,$$

$$\frac{d}{dt}\zeta_j(t) = -\frac{2\Omega}{R}\frac{\partial}{\partial t}z_j(t), \quad j=1,\ldots,M.$$

Remeshing

Like any Lagrangian method, the arrangement of our particles deforms over time due to particle motion which can degrade accuracy (figure 1). We introduce a new remeshing technique, called *inverse interpolation of the flow map*, avoid this. We use the deformational flow tests of [3] and the moving vortices test from [4] demonstrate remeshing in this Lagrangian Particle Method (LPM) and to compare results with the commonly used Lin-Rood scheme [5].

Figure 2 shows error convergence from test case 1 in [3], where the tracer is a pair of Gaussian hills. Remeshing by inverse-interpolating the flow map performs much better in both tracer error (a) and tracer integral error (b) compared to the standard practice of remeshing by direct tracer interpolation.

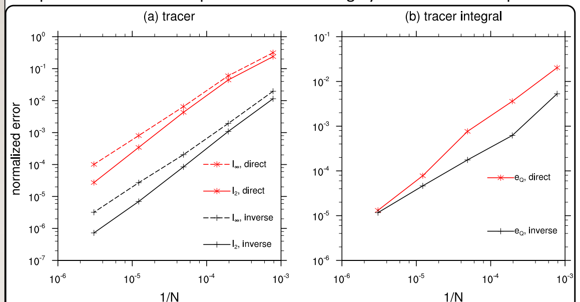


Figure 2. Gaussian hills test error analysis.

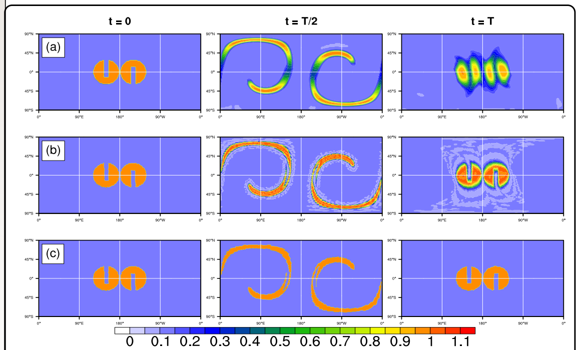


Figure 3. Advection of a discontinuous tracer by reversing deformational flow [3]; (a) Lin-Rood; (b) LPM, remeshing by direct tracer interpolation; (c) LPM, remeshing by inverse interpolation of the flow map.

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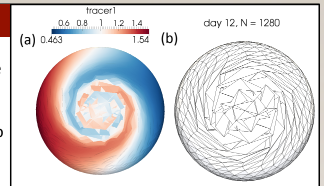

Figure 1. Error due to particle distortion; (a) passive tracer from the moving vortices test [4] day 12; (b) panels.

Figure 3 shows a discontinuous tracer advection test from [3] for (a) Lin-Rood with N = 16200 grid cells, (b) LPM with remeshing by direct interpolation, of the tracer and (c), LPM with remeshing by inverse-interpolation of the flow map.

Adaptive refinement

Particles may be added or removed from a computation adaptively at each remeshing step until a chosen set of criteria are met over the whole sphere.

Figure 4 shows the Gaussian hills test at day 6 (maximum deformation) using two criteria designed to resolve local features of the tracer.

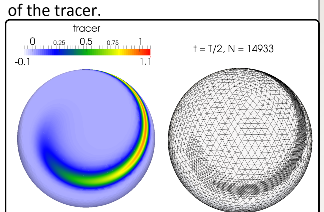

Figure 4. Gaussian hills test, day 6; (a) tracer, (b) adaptively refined panels.

Figure 5 shows the moving vortices test case with refinement criteria chosen to resolve the flow. Figure 6 shows error analysis of an adaptive mesh compared to a uniform mesh.

The adaptively refined computations produce smaller error and use a smaller number of particles.

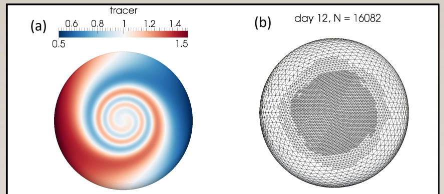

Figure 5. Moving vortices test, day 12; (a) tracer, (b) panels. Inverse-interpolation of the flow map combined with adaptive refinement.

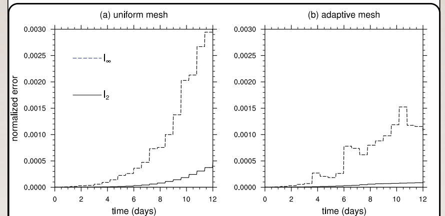

Figure 6. Moving vortices test; (a) uniform set of $N = 20480$ panels, (b) adaptively refined panels, $N \leq 16082.$

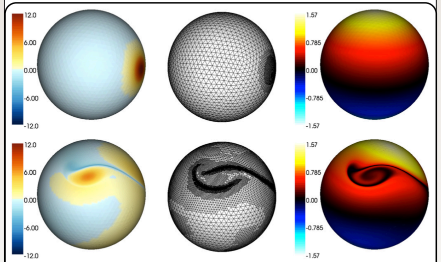

Figure 7. Simulation of a Gaussian vortex; (left) vorticity, (middle) panels, (right) passive tracer. Top row, day 0; bottom row, day 3.

References

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