

Application of Bayes' Theorem for Pulse Shape Discrimination

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Detection for Nuclear₁
Nonproliferation Group



Overview

- Background on PSD
- Applications and Techniques
- Bayes Theorem for PSD
- Results
- Future Work

PSD Background

- In organic scintillators the fraction of light in the slow component of the pulse depends on the incident particle
- This can be attributed to the variation in the density of triplet states produced along the track of the particle which in turn depends on rate of energy loss dE/dx

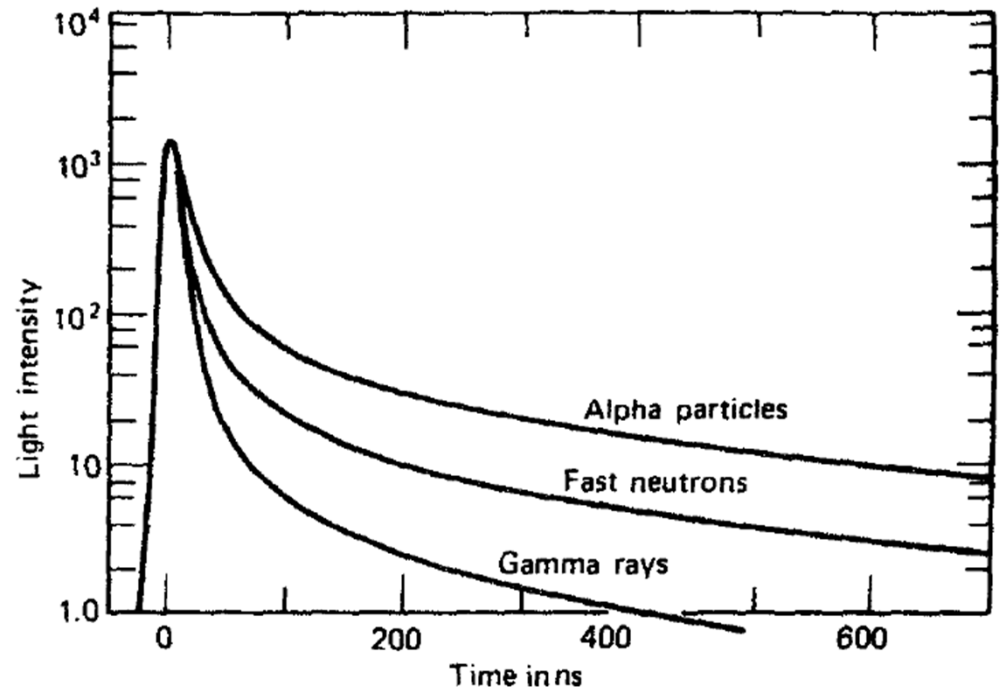
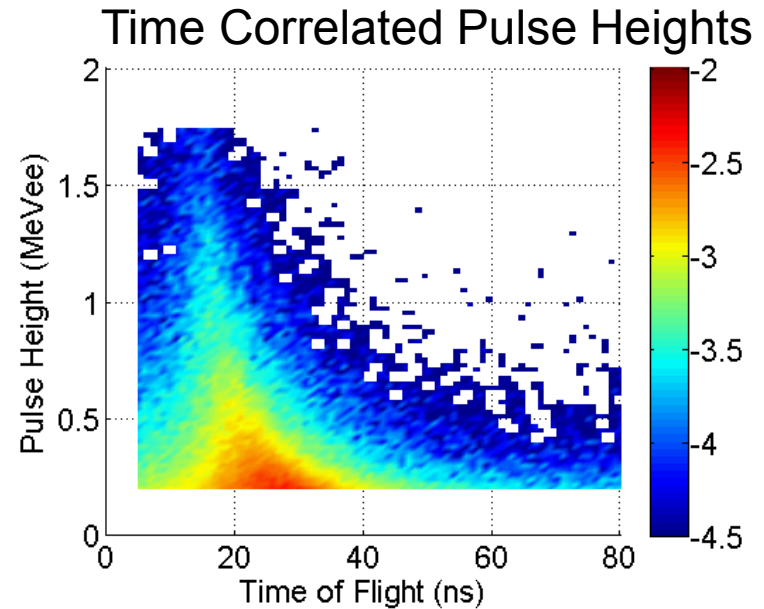
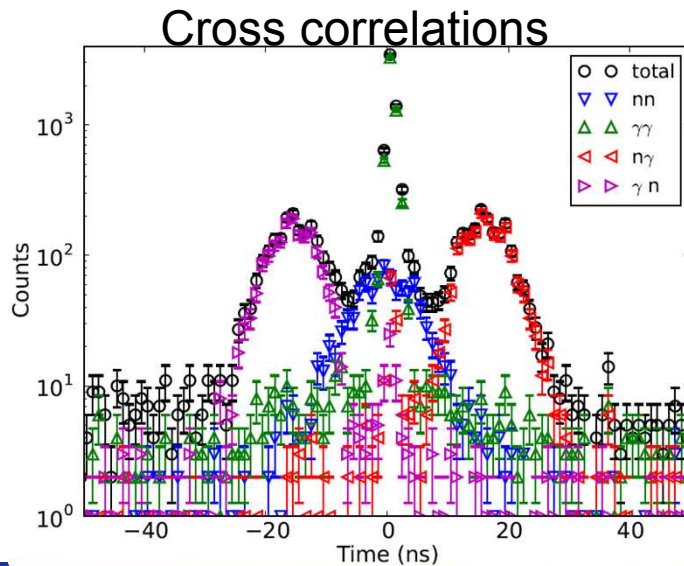


Figure courtesy of Glenn Knoll, *Radiation Detection & Measurement 3rd edition*, John Wiley & Sons

PSD Applications

- Nuclear safeguards, non-proliferation and forensics
- Replacing ^3He counters
- Fast cross correlations
 - Neutron-Neutron
 - Gamma-Gamma
 - Neutron-Gamma



- Learn about the basic physics of the fission process
- TCPH to characterize SNM through fission chain timing characteristics

Digital Evolution

- Digitization of pulse waveforms allows for comprehensive post-processing
- Previous research has focused on parameterizing pulse shapes
 - Charge integration
 - Pulse gradient
 - Template fitting
- This is the first of the two step process of characterizing gamma and neutron pulses

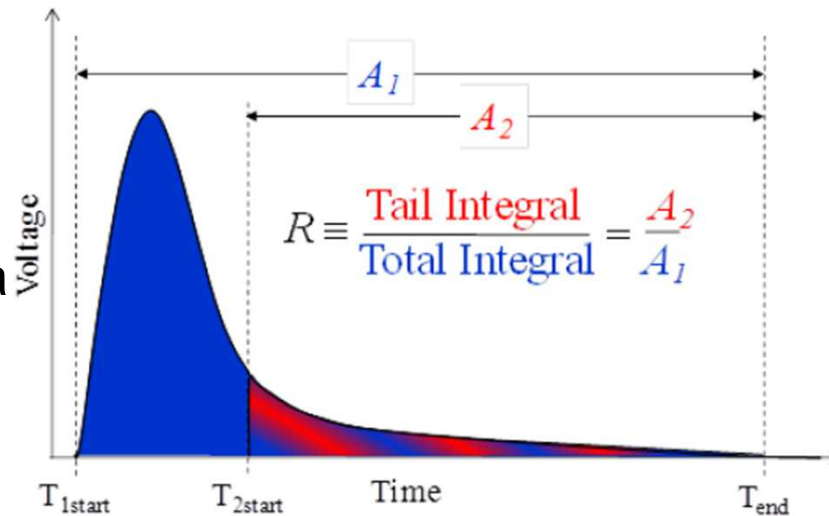
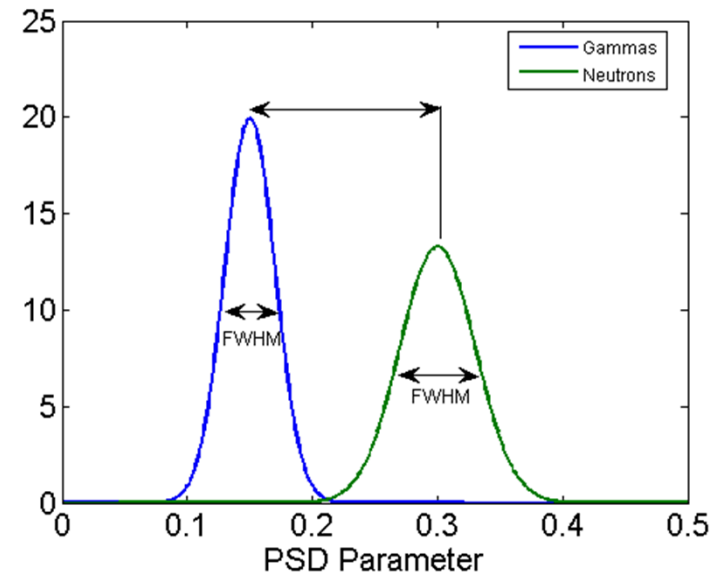
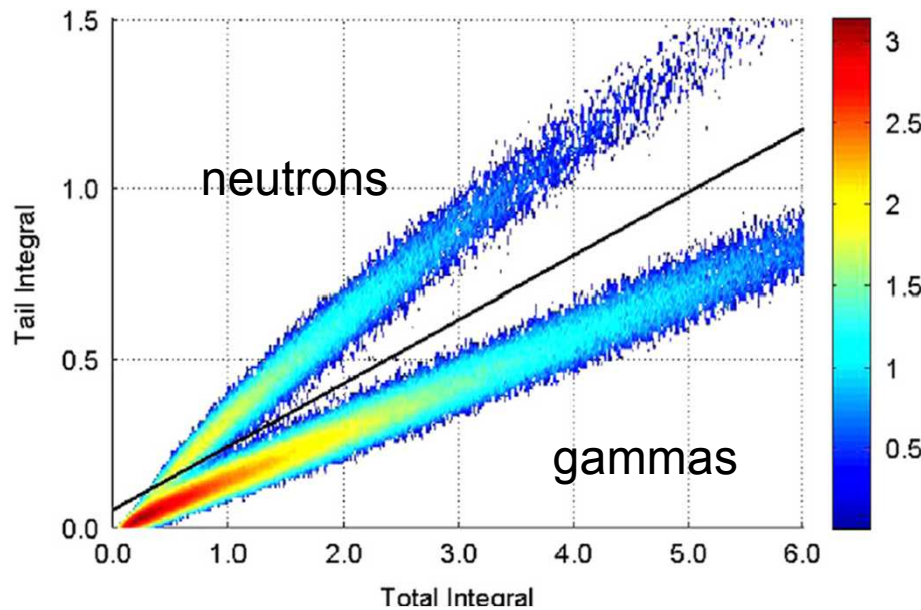


Figure courtesy of Eric Miller



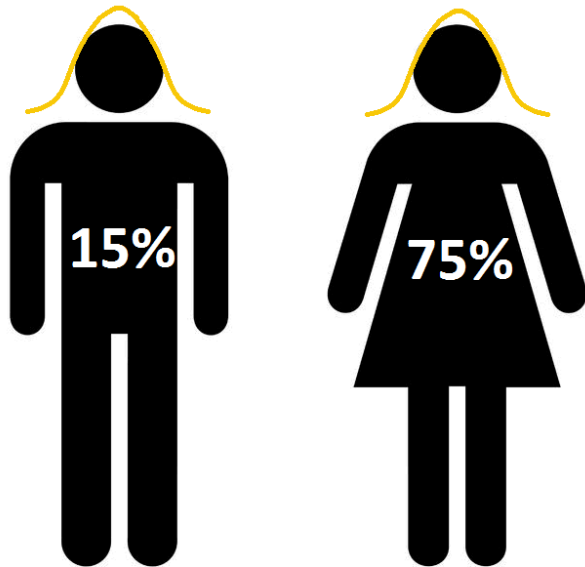
PSD Parameterization

- Performance of a PSD parameters is defined by some FOM, typically separation between peaks and their FWHM
- The neutron and gammas are separated by a demarcation line
- The performance at low energies is problematic



Bayes Theorem

- It is a way of calculating the posterior probability of a random event given relevant evidence
- Example: probability that a person is a woman provided they have long hair?

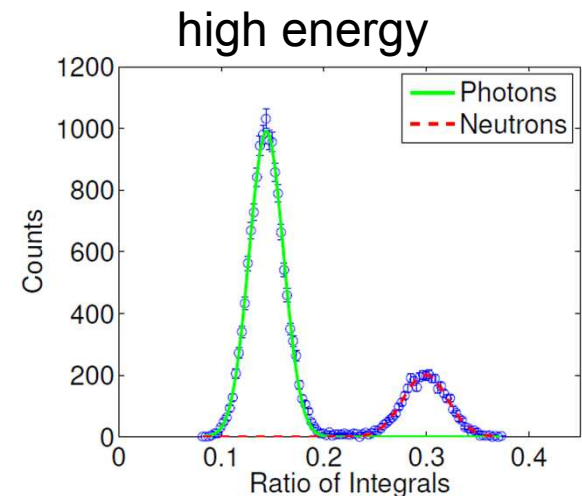
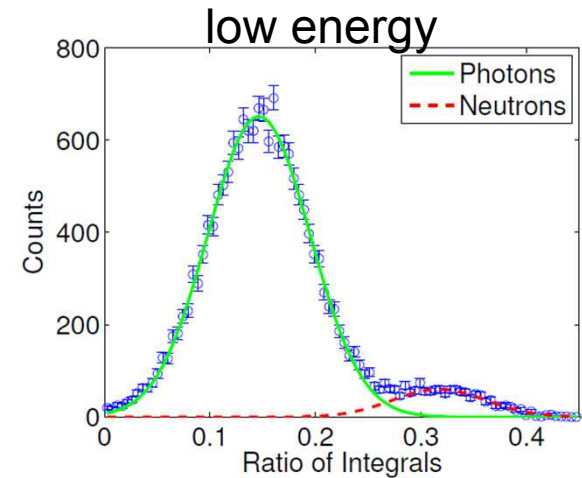
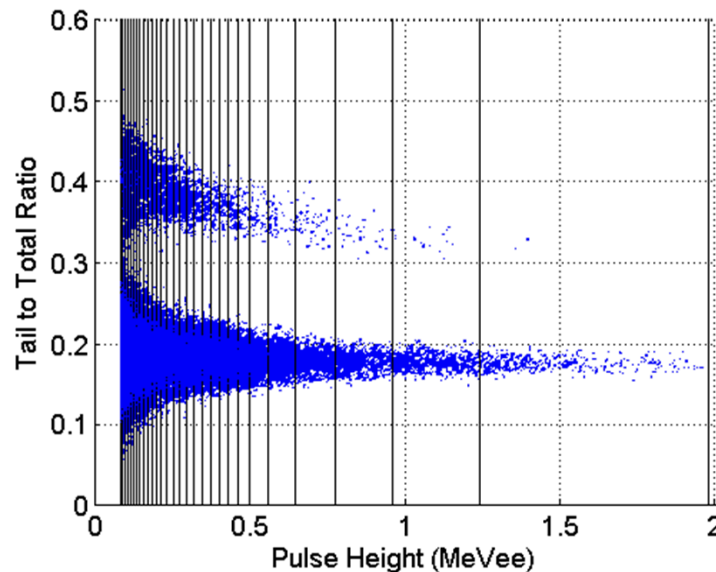


$$\begin{aligned}
 P(W|L) &= \frac{P(L|W)P(W)}{P(L|W)P(W) + P(L|M)P(M)} \\
 &= \frac{0.75 \times 0.50}{0.75 \times 0.50 + 0.15 \times 0.50} = \frac{5}{6}
 \end{aligned}$$

~83% probability that the person is a woman

Getting the Conditional Probabilities

- Dividing the tail to total ratio by pulse height
- Double Gaussian fit on each energy slide
- Get conditional probabilities from the normalized fits



Applying Bayes Theorem

Posterior probability: Gammas

Neutrons

$$P^E(\gamma|S) = \frac{f_\gamma^E(S)R_{\gamma/n}^E}{f_\gamma^E(S)R_{\gamma/n}^E + f_n^E(S)} \quad P^E(n|S) = \frac{f_n^E(S)}{f_\gamma^E(S)R_{\gamma/n}^E + f_n^E(S)}$$

S : PSD Parameter, ratio of tail to total integral

$R_{\gamma/n}$: The ratio of gammas to neutrons in the energy bin

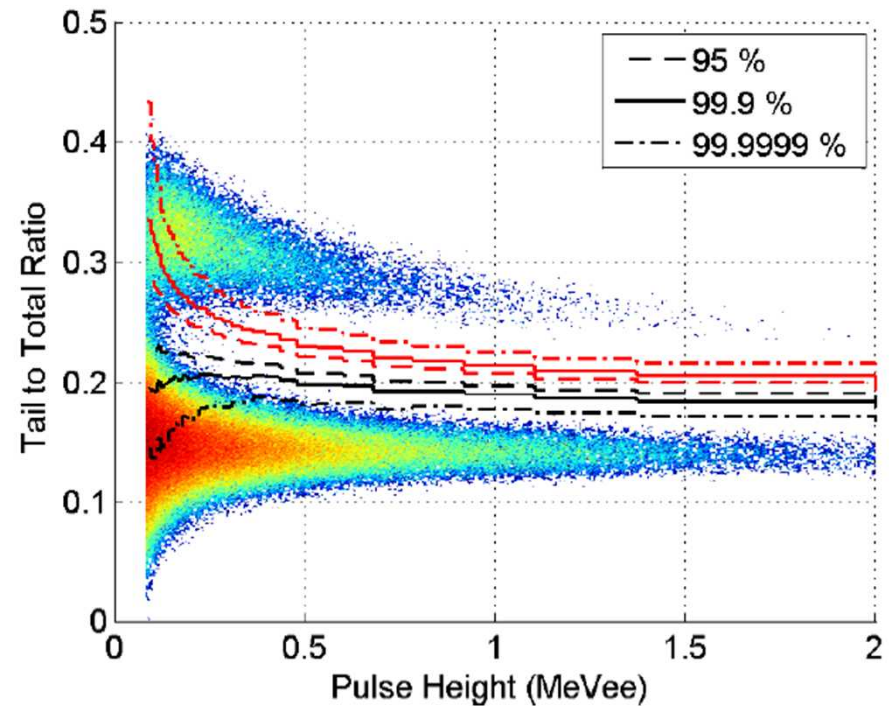
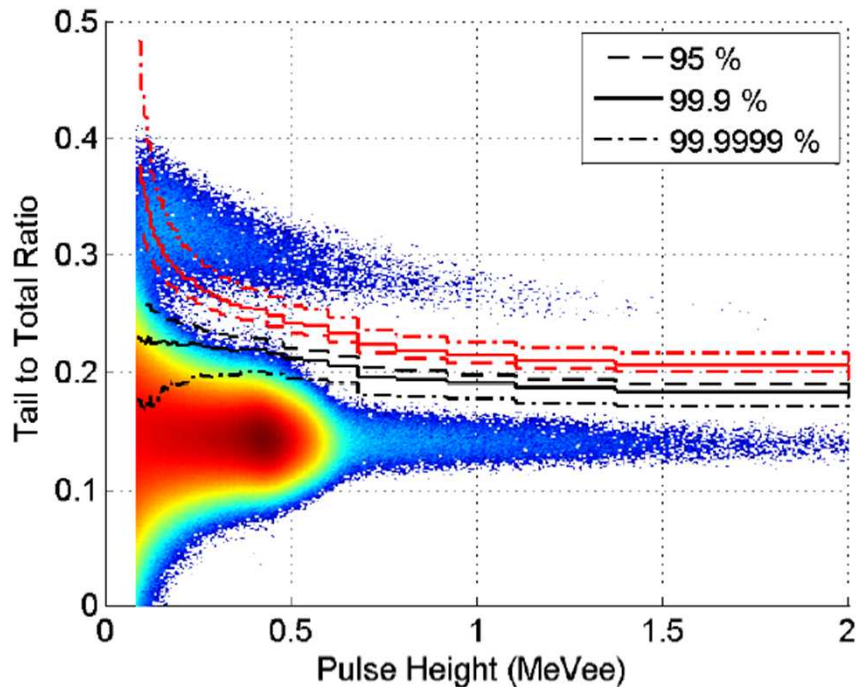
$f(s)_n$: Neutron Gaussian fit, $P(S|n)$

$f(s)_\gamma$: Gamma Gaussian fit, $P(S|\gamma)$

$$P(\gamma|S) \geq P_C \text{ or } P(n|S) \geq P_C$$

P_C : Confidence probability that the pulse came from a gamma or a neutron (i.e. 95%, 99%, 99.9999% ect.)

Applying Bayes Theorem



- The higher the confidence the harder the cuts and higher number of unclassified
- Different gamma/neutron ratios change the discrimination lines

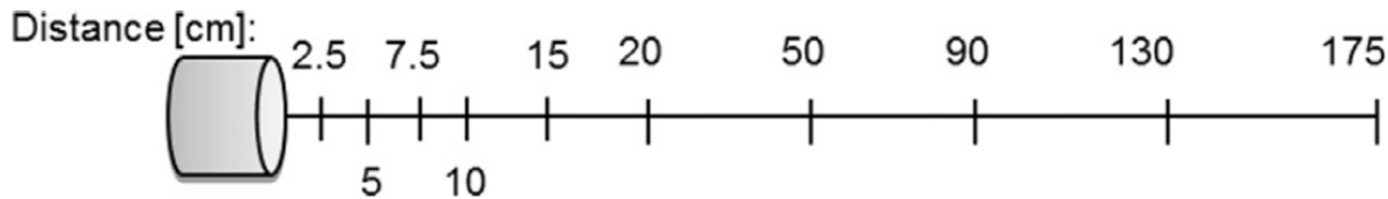
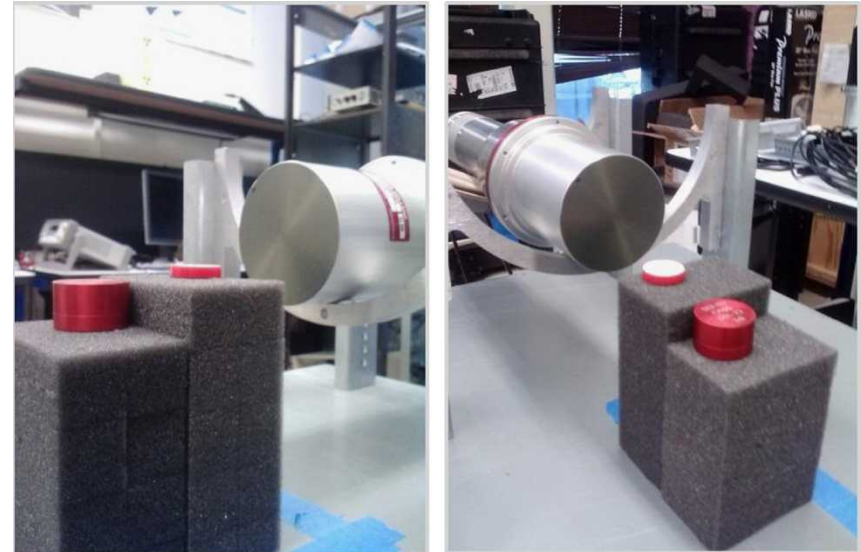
Correcting for cut out pulses

- For high confidence probabilities, the proportion of unclassified pulses will be significant, especially at low energies
- There are multiple ways to do this, one way is to take the number of gamma/neutron counts and divide by the portion of the Gaussian distribution in region

$$\gamma_{count}^* = \frac{\gamma_{count}}{\int_{-\infty}^{S_{\gamma}} f_{\gamma}(s)}$$
$$n_{count}^* = \frac{n_{count}}{\int_{S_n}^{\infty} f_n(s)}$$

Experimental Setup

- Single 3x3" EJ-309 detector
- CAEN V1720, 12-bit, 250-MHz digitizer
- Cf-252 source place 10 cm away
- Cf-137 source moved from 2.5 to 175 cm



Results

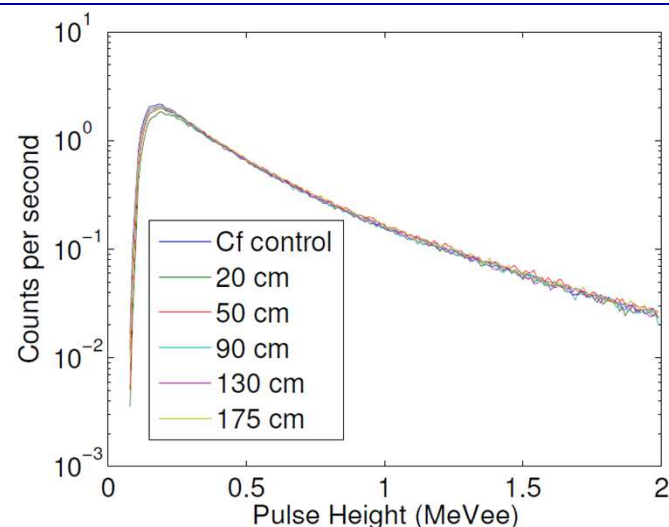
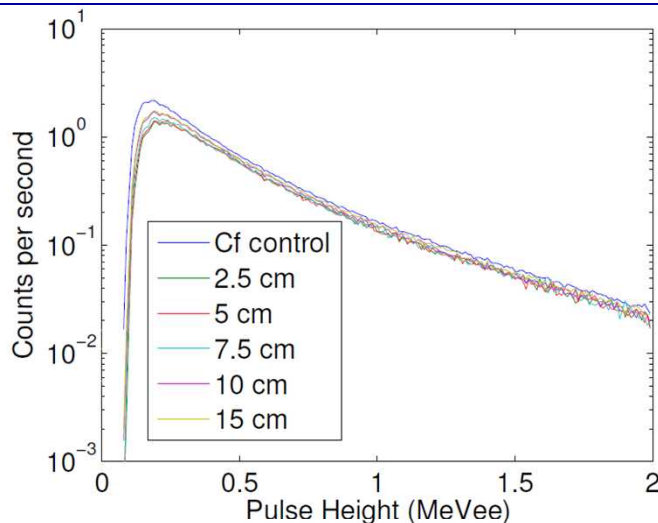
Distance (cm)	Total rate (s ⁻¹)	γ/n ratio	n count rate (s ⁻¹)	Error (%)	γ/n ratio*	n count rate* (s ⁻¹)	Error* (%)
2.5	64015	1057	58.4	-26.5	788.6	81.2	-18.4
5	42866	698.2	58.8	-25.9	524.0	81.7	-17.8
7.5	25944	406.2	60.7	-23.5	316.4	82.0	-17.5
10	18971	268.0	66.2	-16.6	208.9	90.5	-9.1
15	10651	145.2	68.1	-14.3	114.0	92.9	-6.6
20	6766	86.89	71.1	-10.5	69.84	95.9	-3.6
50	1833	20.20	76.3	-3.8	17.91	97.6	-1.9
90	899	9.169	75.1	-5.4	8.550	94.8	-4.7
130	769	7.383	76.7	-3.4	6.947	97.3	-2.2
175	669	6.094	78.6	-1.0	5.898	97.7	-1.8
²⁵² Cf alone	536	4.474	79.4	-	4.362	99.5	-

- Error: the difference between neutron count rate with Cf-252 alone case
- * The corrected values using Gaussian fits

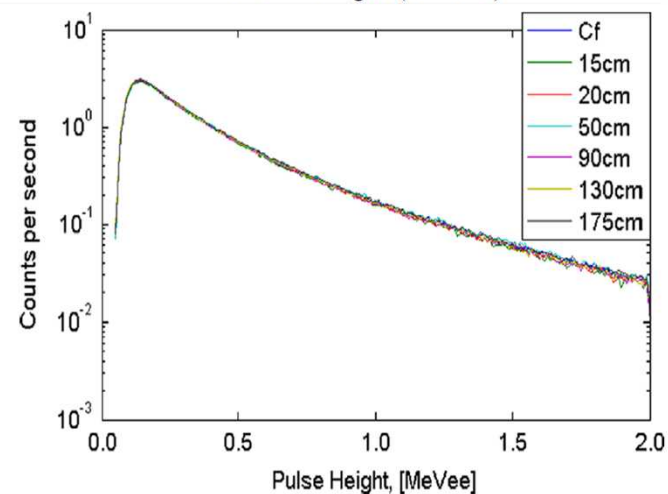
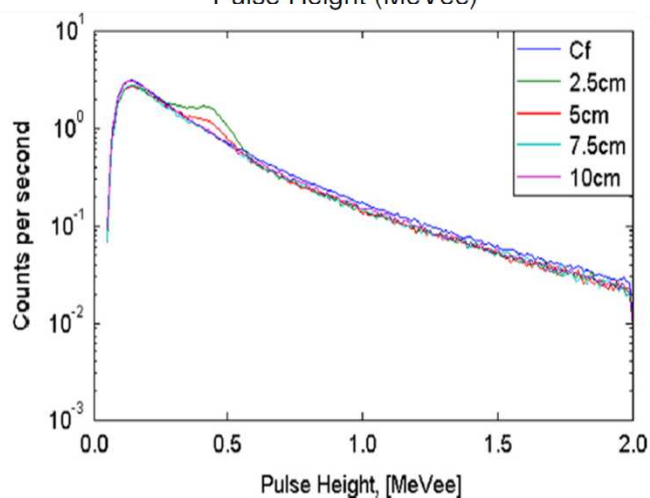


Pulse Height Distributions

Monterial et al.:



Kaplan et al.:



Future Work

- The problem with a mixing sources is the count rate increases if a specific gamma/neutron ratios are desired
 - Complications with double pulse cleaning
- Use TOF to separate neutrons and gammas using an AmBe source
- Sample from neutron and gamma distributions to achieve desired ratios, and test PSD performance



Acknowledgments

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