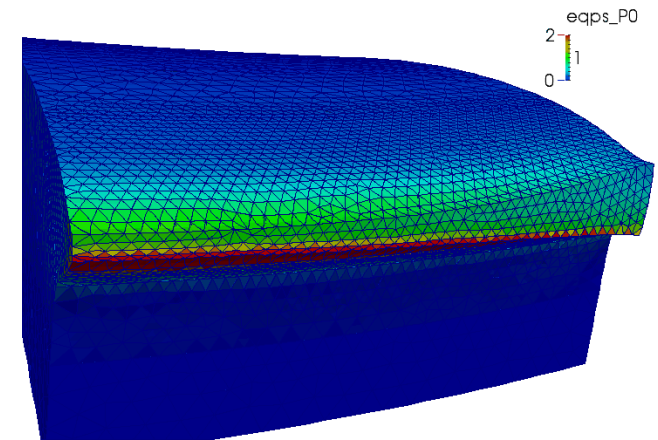
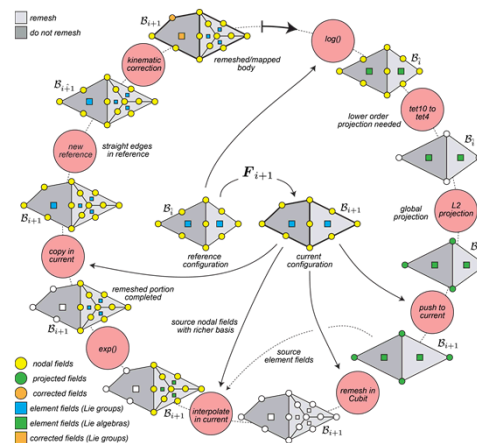
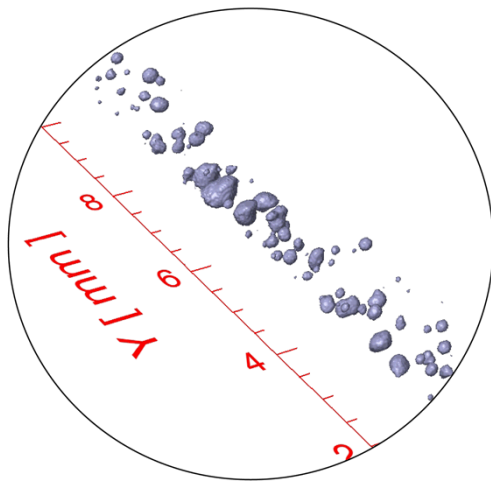


*Exceptional service in the national interest*



# Resolving the evolution of pore structures in 304-L laser welds

J. Foulk, M. Veilleux, J. Emery, J. Madison, H. Jin, J. Ostien, A. Mota  
SES, Purdue University, October 2, 2014

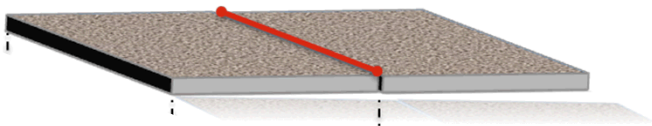
# Failure is the loss of load-bearing capacity

Austenitic stainless steels are extremely tough and damage tolerant

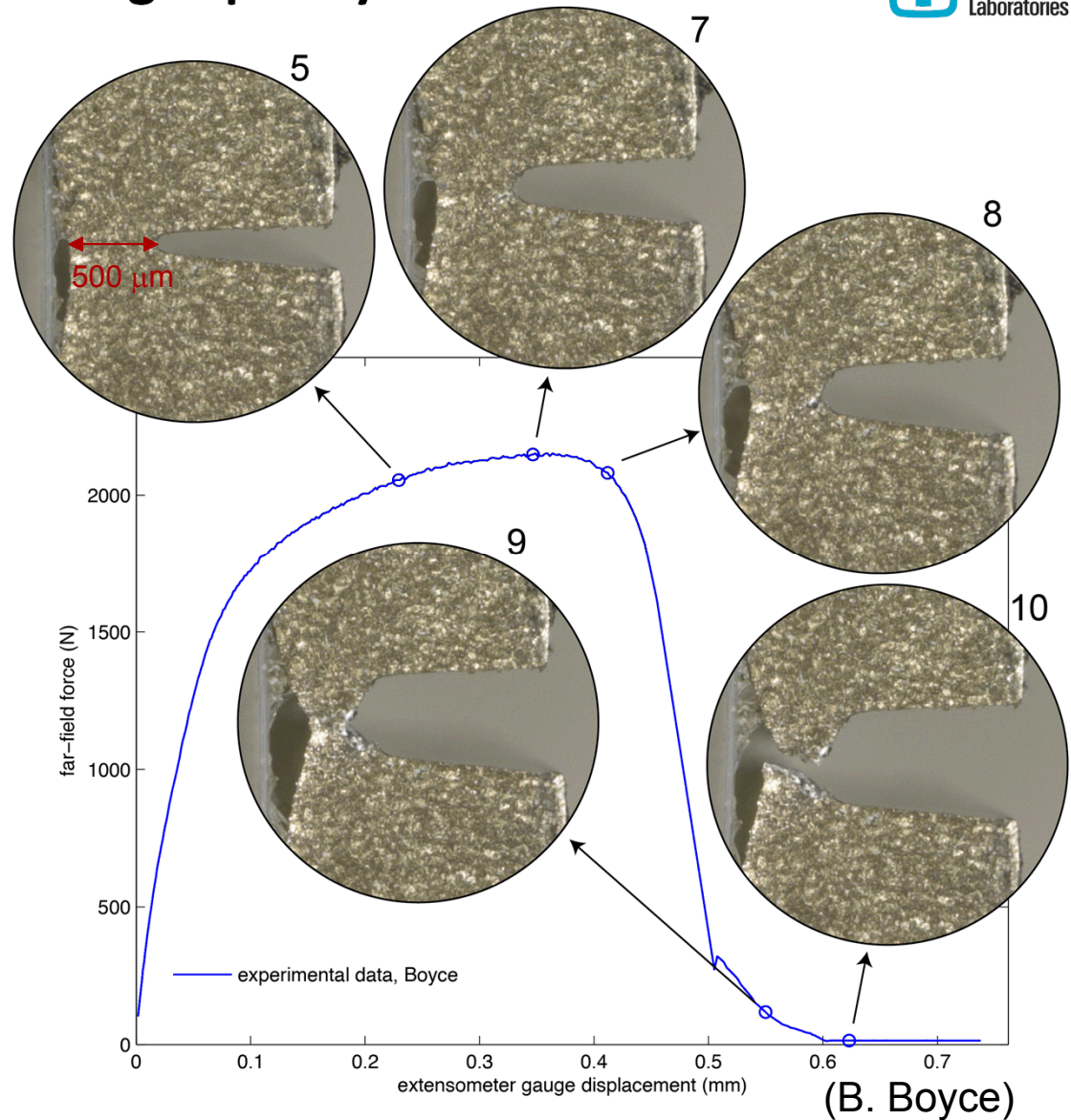
The failure of 304-L is a necking problem. Free surface creation is a 2<sup>nd</sup> order effect.

Hypothesis: Pore size and distribution can aid the necking process

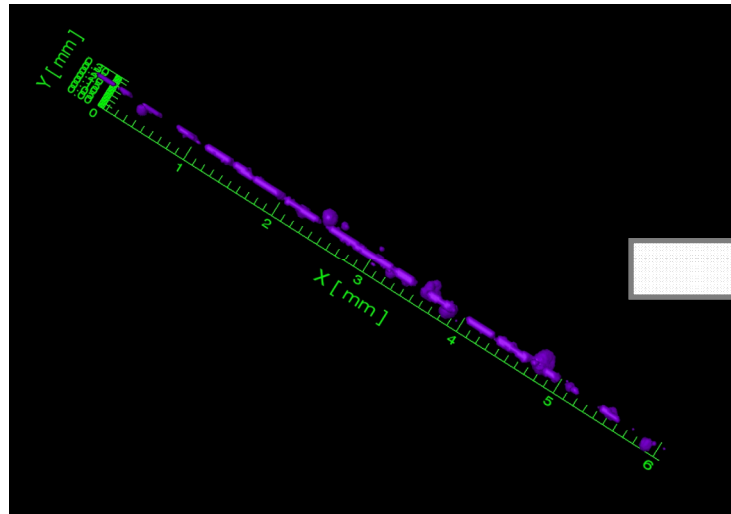
- $\mu$ -CT needed to probe initial and interrupted pore structures
- Remeshing/mapping needed to resolve the evolution of pore structure
- Homogenization not applicable



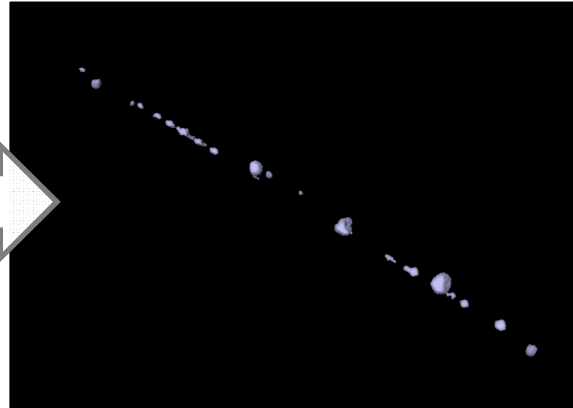
304-L butt weld



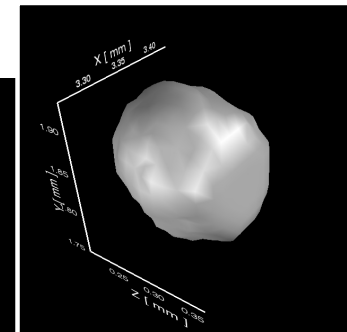
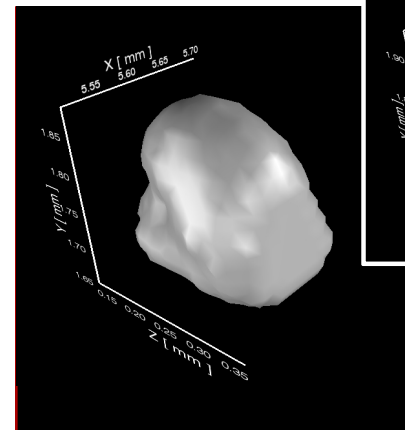
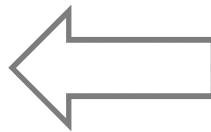
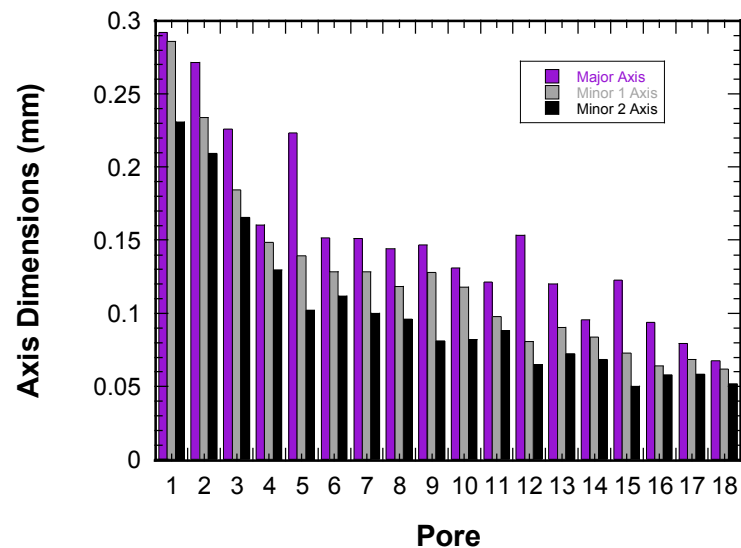
# Pores large relative to the ligament – homogenization n/a



*μ-Computed Tomography*

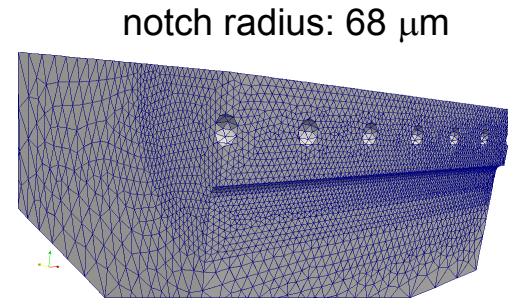
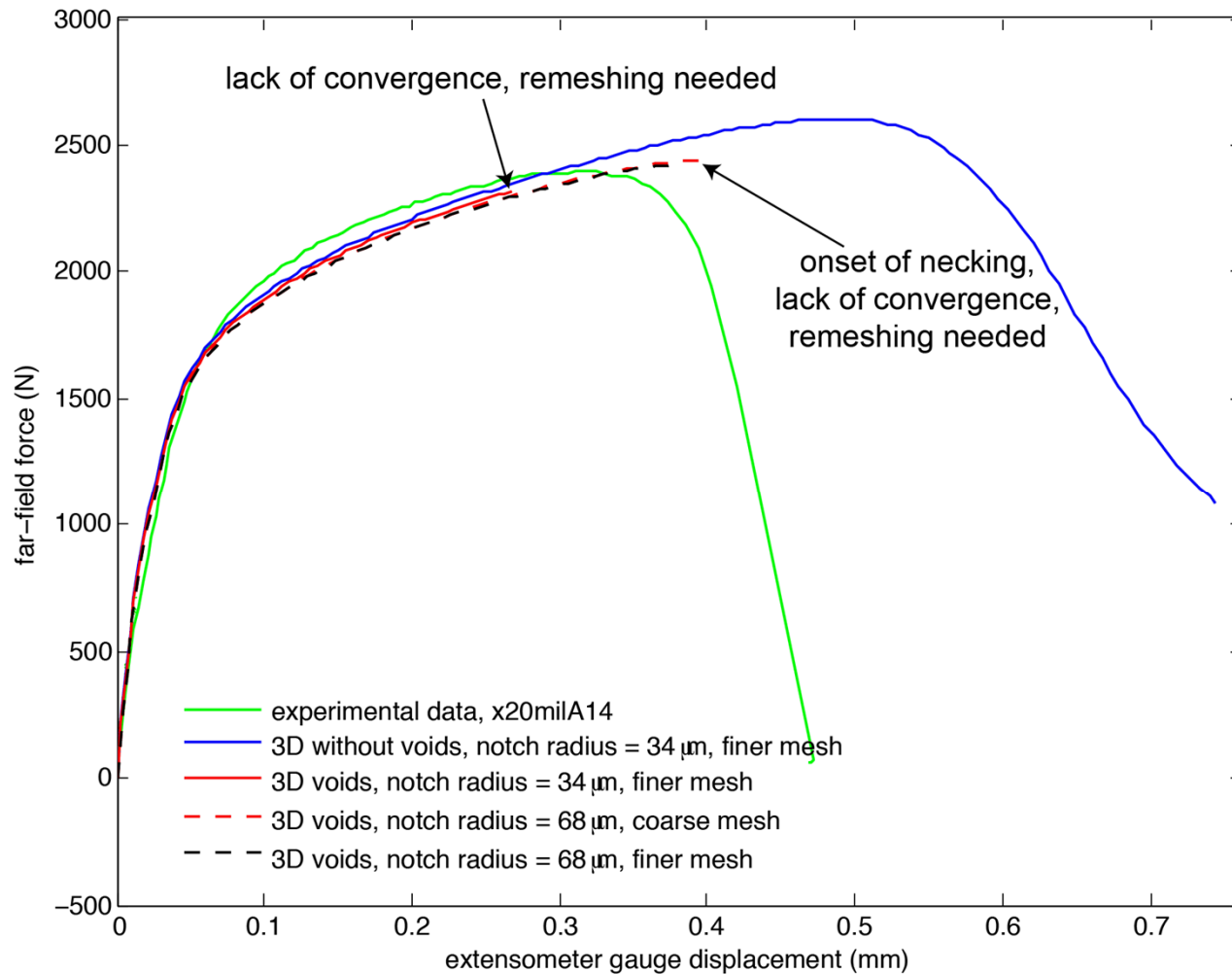


Magnification: 9X  
Voxel size: 14  $\mu\text{m}$   
Energy: 130 keV

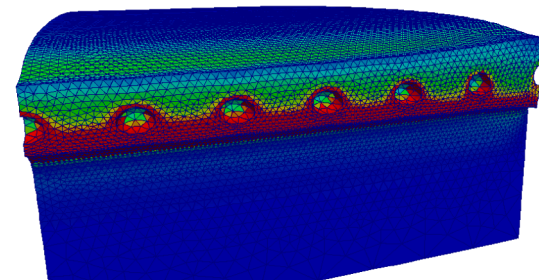


J. Madison, L. K. Aagesen, "Quantitative Characterization of Porosity in Laser Welds of Stainless Steel" SCRIPTA MATERIALIA (2012)

# Initial efforts w/pores problematic – remeshing needed



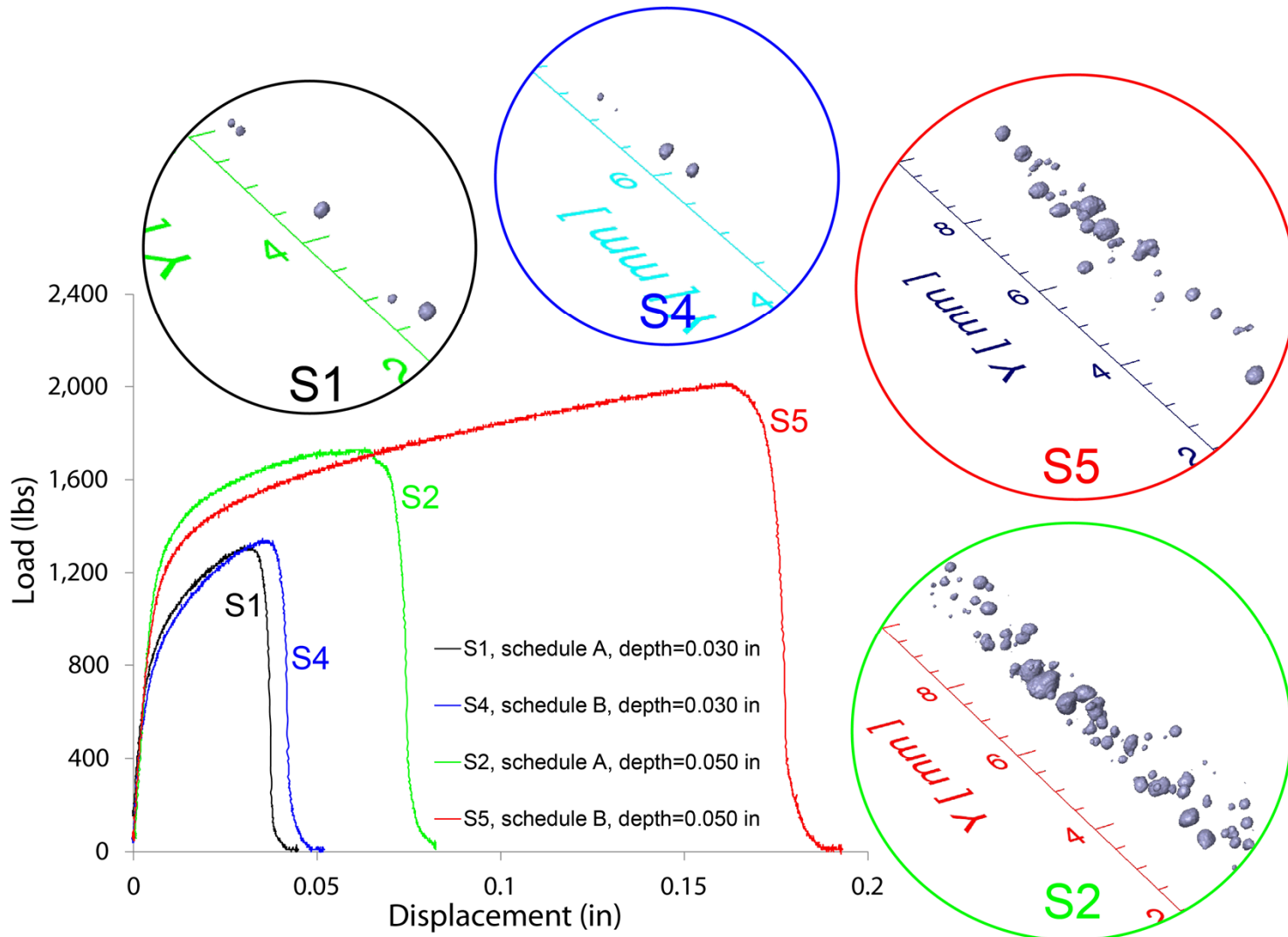
Onset of necking  
notch radius = 68  $\mu\text{m}$   
coarse mesh



*NOTE: Same constitutive model employed for cases with and without voids*

# Deeper-penetration welds provide additional motivation

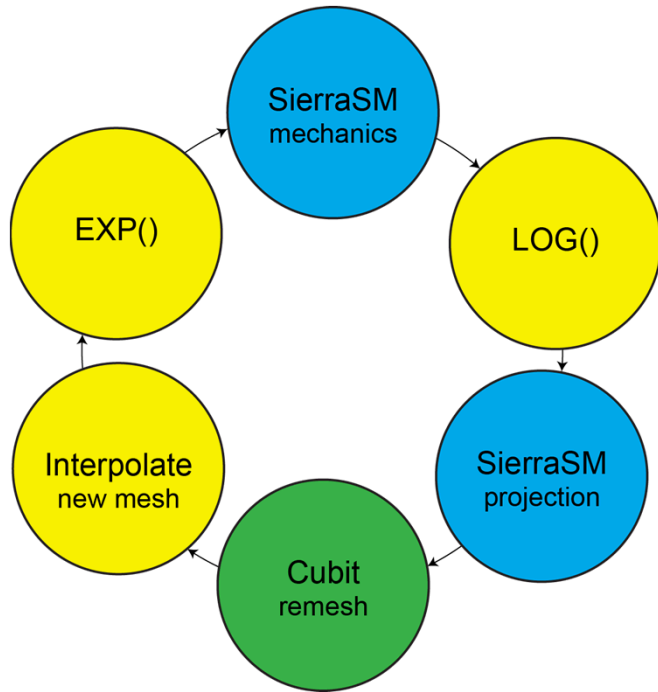
*Weld schedule impacts porosity. Porosity impacts performance.*



*Given the void structure (shape, size, location), can we predict these findings?*



# Our approach: mapLL ( $L_2$ + Lie Group/Algebra)



$$\Phi[\varphi, \bar{z}, \bar{y}] := \int_B W(\mathbf{F}, \bar{z}) dV + \int_B \bar{y} \cdot (\bar{z} - z) dV - \int_B \rho_0 \mathbf{B} \cdot \varphi dV - \int_{\partial_T B} \mathbf{T} \cdot \varphi dS$$

source field available at integration points

global field through projection

$\bar{z}_h(\mathbf{X}) := \lambda_\alpha(\mathbf{X}) \bar{z}_\alpha$

source field

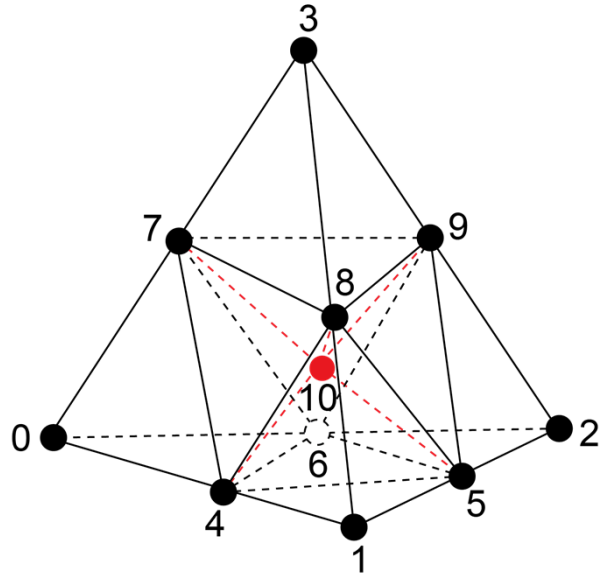
target field

$$\bar{z}_h = \lambda_\alpha \left( \int_B \lambda_\alpha \lambda_\beta \mathbf{I} dV \right)^{-1} \int_B \lambda_\beta z dV$$

- The variational principle naturally yields an optimal,  $L_2$  projection
- The spaces of variables (Lie algebra, Lie Group) are honored through LOG() and EXP()

# We use tetrahedral elements

*Motivated by prior work of Thoutireddy, et. al., IJNME (2002)*



$$\bar{B}_{aJ}(\mathbf{X}) = \lambda_c(\boldsymbol{\xi}) \left[ \int_{\Omega_0} \lambda_c \lambda_b dV_0 \right]^{-1} \int_{\Omega_0} \lambda_b N_{a,J} dV_0$$

$$\bar{B}_{aJ}(\mathbf{X}) = \lambda_c(\boldsymbol{\xi}) \left[ \int_{\Omega_\xi} \lambda_c \lambda_b dV_\xi \right]^{-1} \int_{\Omega_\xi} \lambda_b \frac{\partial N_a}{\partial \xi_k} dV_\xi \left( \frac{\partial \xi_k}{\partial X_J} \right)$$

$$\bar{B}_{aJ}(\mathbf{X}) = \bar{L}_{ak}(\boldsymbol{\xi}) \xi_{k,J}$$

$$\bar{L}_{ak}(\boldsymbol{\xi}) = \lambda_c(\boldsymbol{\xi}) M_{cb}^{-1} \int_{\Omega_\xi} \lambda_b \frac{\partial N_a}{\partial \xi_k} dV_\xi$$

$$\bar{L}_{ak}(\boldsymbol{\xi}) = \lambda_c(\boldsymbol{\xi}) M_{cb}^{-1} \sum_{S=0}^{11} \frac{\partial N_a}{\partial \xi_k} \int_{E_S} \lambda_b dV_\xi$$

*IDEA: Chain rule!*

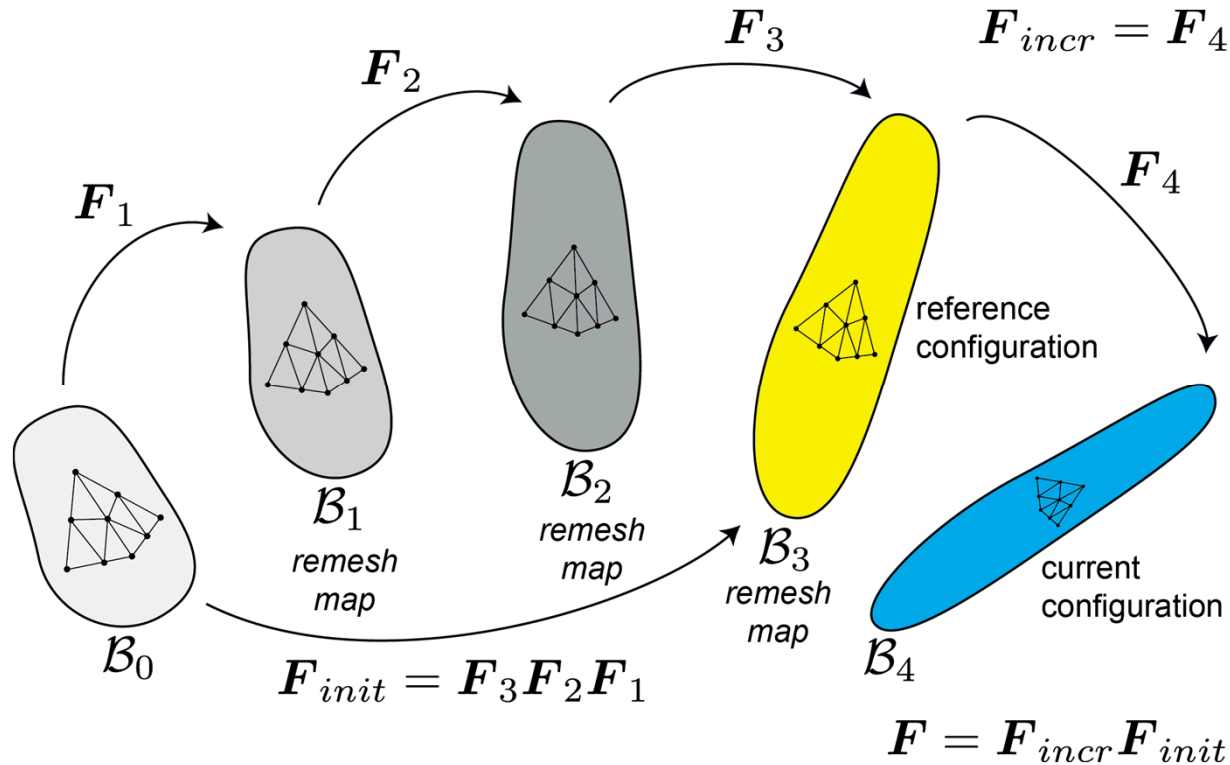
$$\frac{\partial N_a}{\partial X_J} = \frac{\partial N_a}{\partial \xi_K} \left( \frac{\partial X_J}{\partial \xi_K} \right)^{-1}$$

$$\bar{B}_{aJ}(\mathbf{X}) = \bar{L}_{ak}(\boldsymbol{\xi}) \left( X_{Jb} \bar{L}_{bk}(\boldsymbol{\xi}) \right)^{-1}$$

$$\bar{L}_{ak}(\boldsymbol{\xi}) = \begin{pmatrix} \frac{1}{8}(-17 + 20\xi_1 + 20\xi_2 + 20\xi_3) & \frac{1}{8}(-17 + 20\xi_1 + 20\xi_2 + 20\xi_3) & \frac{1}{8}(-17 + 20\xi_1 + 20\xi_2 + 20\xi_3) \\ -\frac{3}{8} + \frac{5\xi_1}{2} & 0 & 0 \\ 0 & -\frac{3}{8} + \frac{5\xi_2}{2} & 0 \\ 0 & 0 & -\frac{3}{8} + \frac{5\xi_3}{2} \\ -\frac{35}{12}(-1 + 2\xi_1 + \xi_2 + \xi_3) & \frac{1}{12}(-4 - 35\xi_1 + 5\xi_2 + 10\xi_3) & \frac{1}{12}(-4 - 35\xi_1 + 10\xi_2 + 5\xi_3) \\ \frac{1}{12}(-1 + 5\xi_1 + 40\xi_2 - 5\xi_3) & \frac{1}{12}(-1 + 40\xi_1 + 5\xi_2 - 5\xi_3) & -\frac{5}{12}(-1 + \xi_1 + \xi_2 + 2\xi_3) \\ \frac{1}{12}(-4 + 5\xi_1 - 35\xi_2 + 10\xi_3) & -\frac{35}{12}(-1 + \xi_1 + 2\xi_2 + \xi_3) & \frac{1}{12}(-4 + 10\xi_1 - 35\xi_2 + 5\xi_3) \\ \frac{1}{12}(-4 + 5\xi_1 + 10\xi_2 - 35\xi_3) & \frac{1}{12}(-4 + 10\xi_1 + 5\xi_2 - 35\xi_3) & -\frac{35}{12}(-1 + \xi_1 + \xi_2 + 2\xi_3) \\ \frac{1}{12}(-1 + 5\xi_1 - 5\xi_2 + 40\xi_3) & -\frac{5}{12}(-1 + \xi_1 + 2\xi_2 + \xi_3) & \frac{1}{12}(-1 + 40\xi_1 - 5\xi_2 + 5\xi_3) \\ -\frac{5}{12}(-1 + 2\xi_1 + \xi_2 + \xi_3) & \frac{1}{12}(-1 - 5\xi_1 + 5\xi_2 + 40\xi_3) & \frac{1}{12}(-1 - 5\xi_1 + 40\xi_2 + 5\xi_3) \end{pmatrix}$$

*This is exact. Evaluate for your flavor of cubature.*

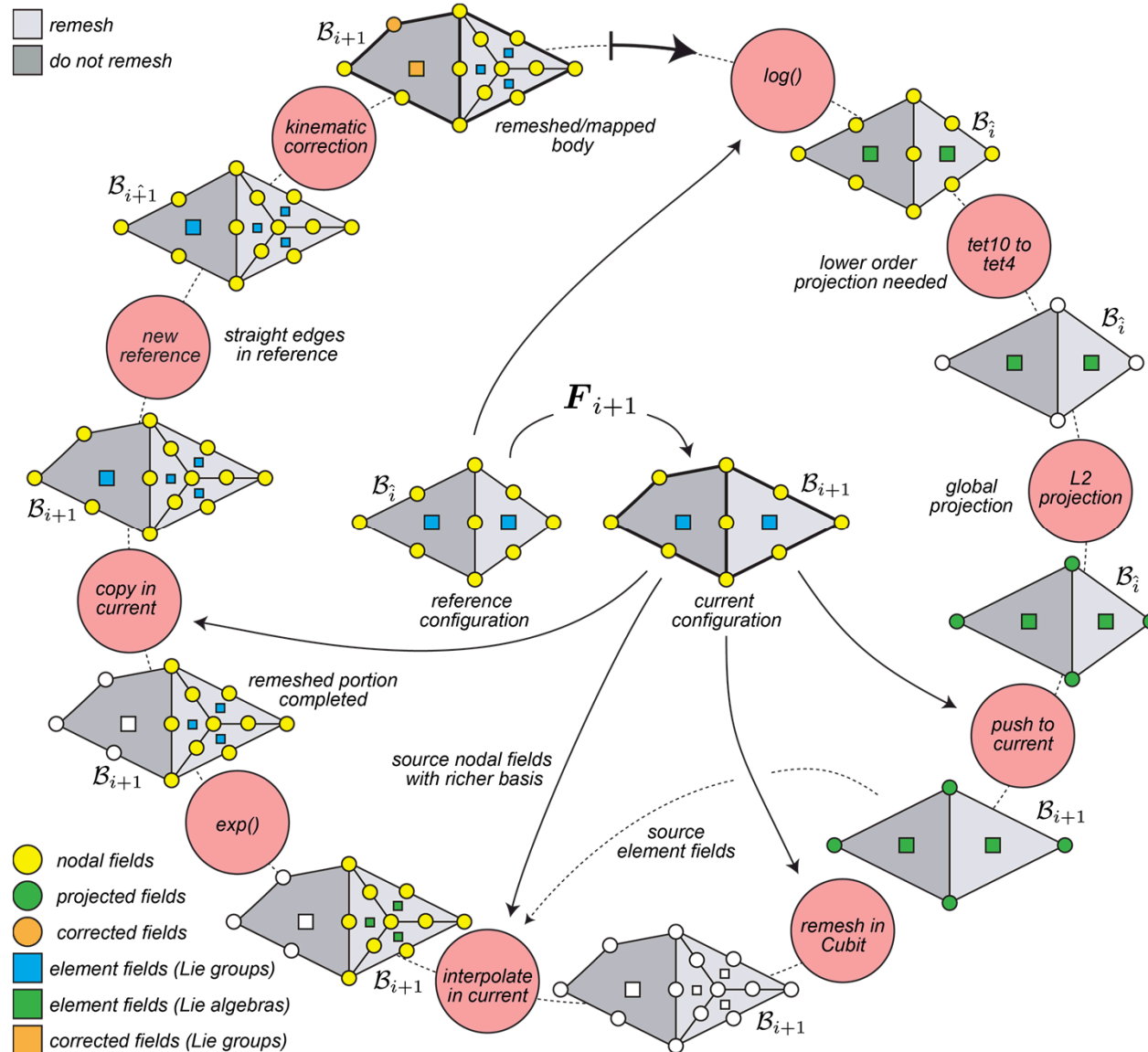
# We adopt a new reference configuration



- Prior work on hexahedral elements maintained the reference configuration
- Elements degrade in the reference configuration - T-L element integrate in reference
- We now adopt a new reference configuration and map  $F_{init}$  (which lives in a Lie Group)



# We accommodate local remeshing



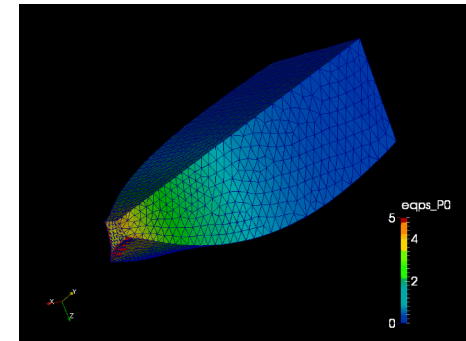
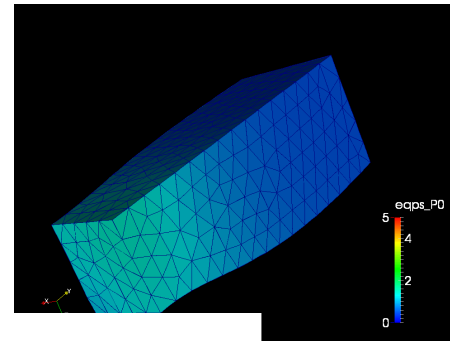
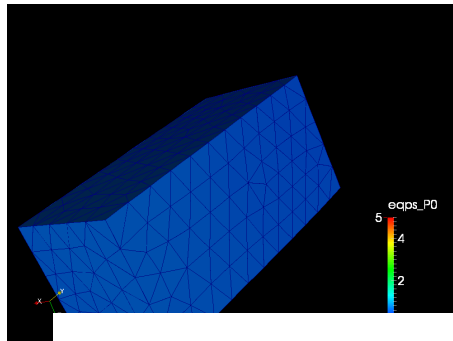
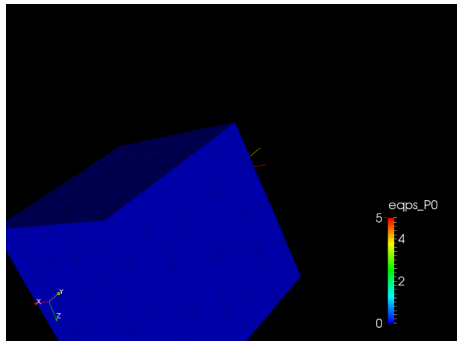
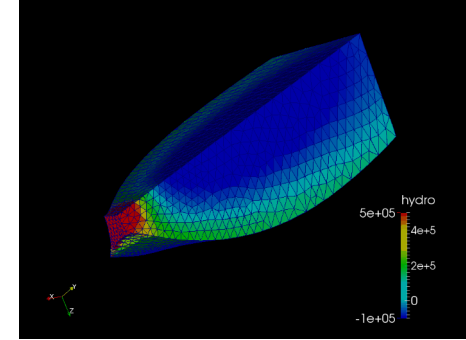
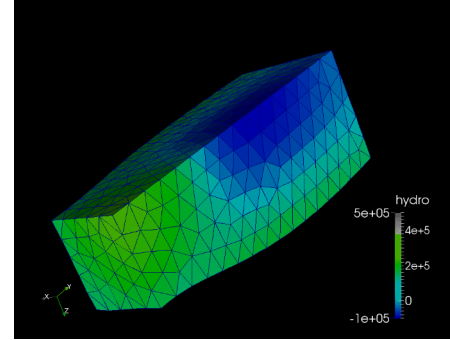
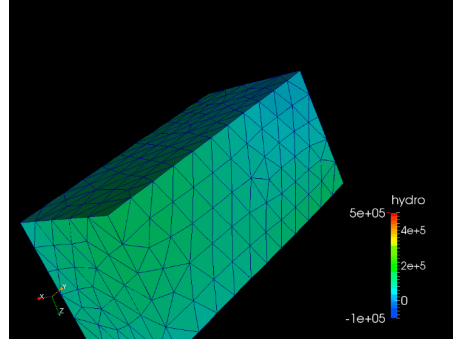
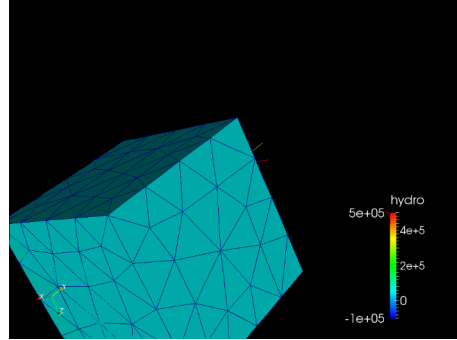
# Move over bar. Cubes can neck too.

initial configuration

117 maps

178 maps

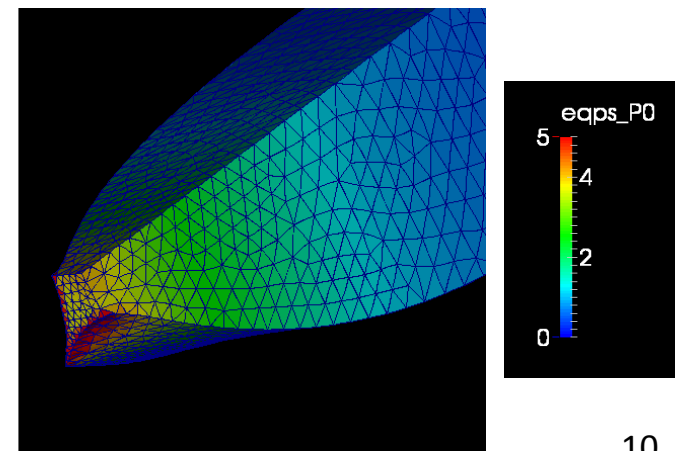
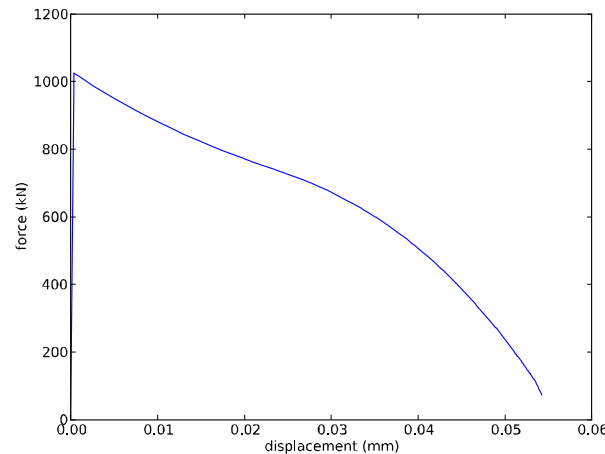
235 maps



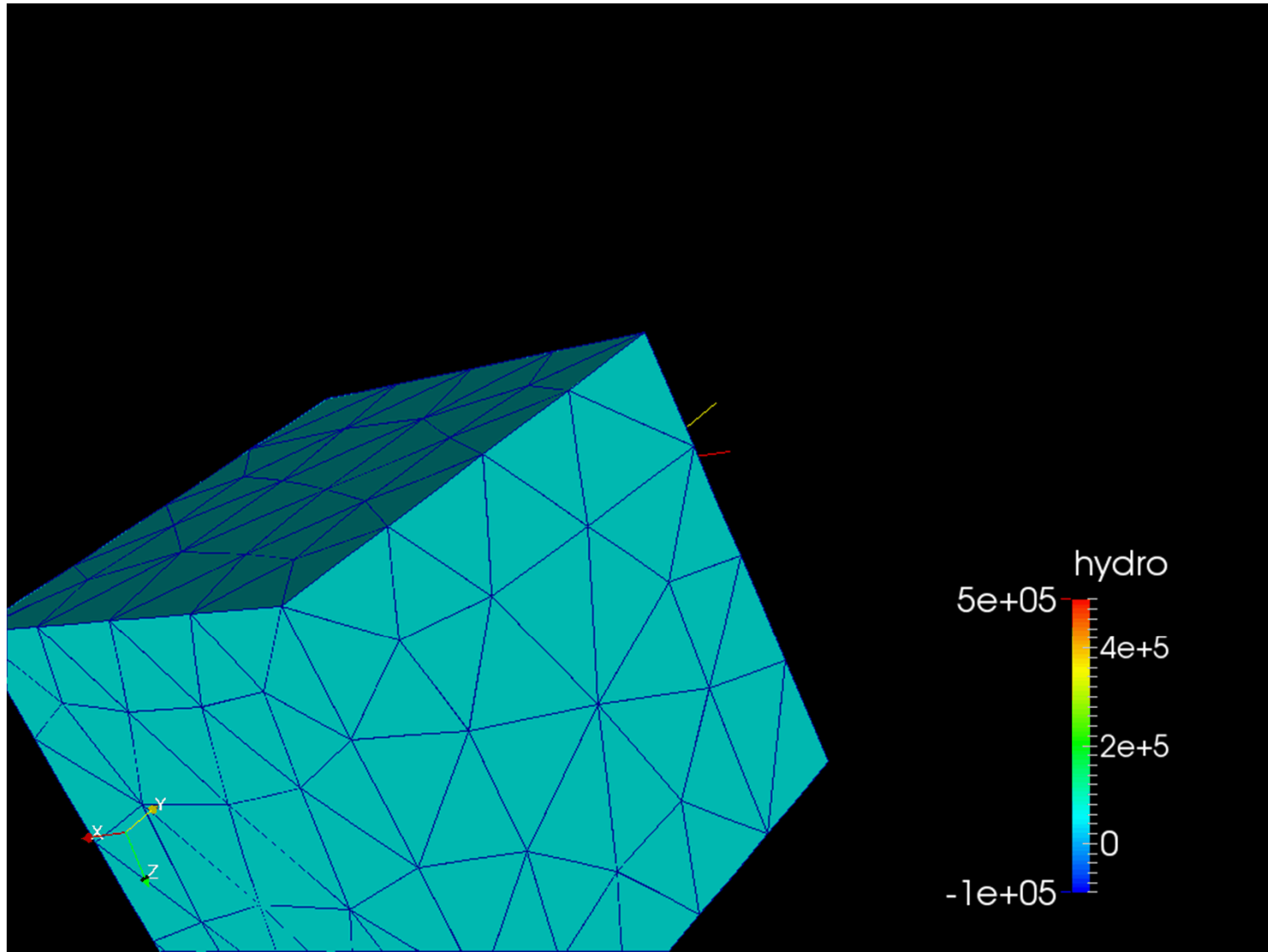
Unit cube w/symmetry.  
Pull “top” and keep 5  
elements at “waist”

235 remesh/map steps

Plastic strains > 500%  
Pressure is smooth

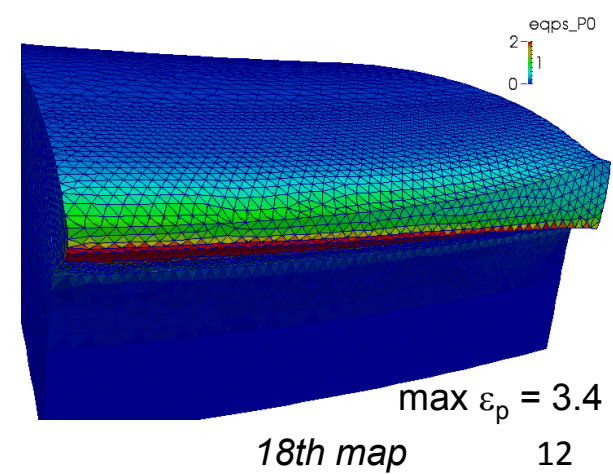
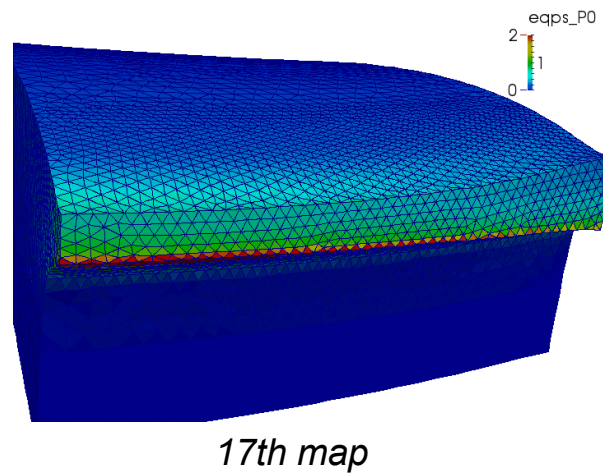
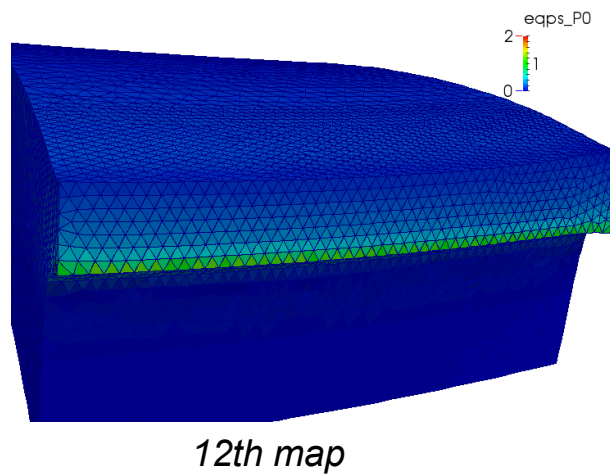
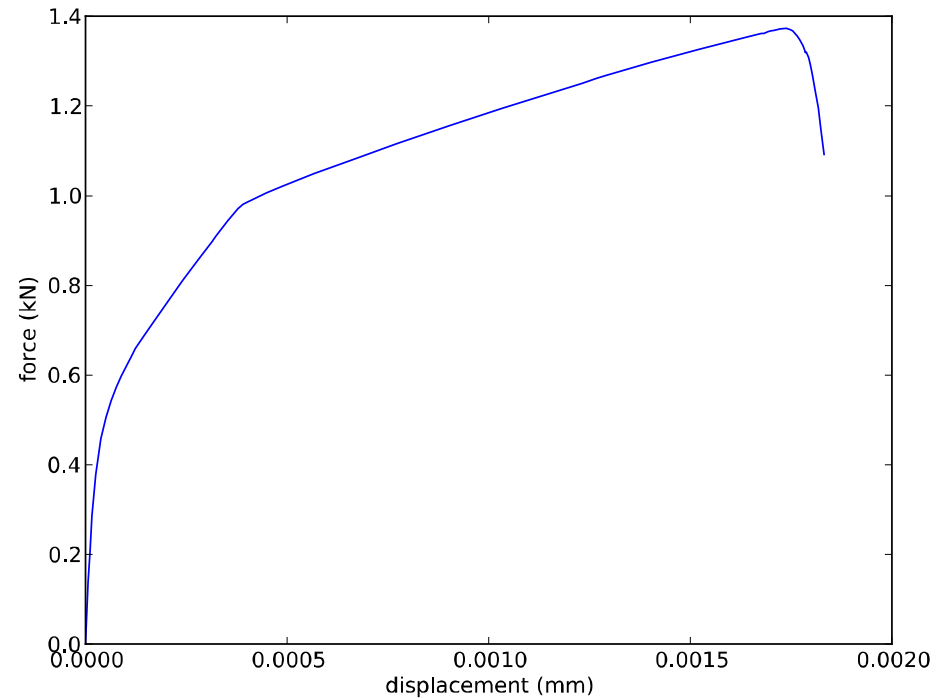
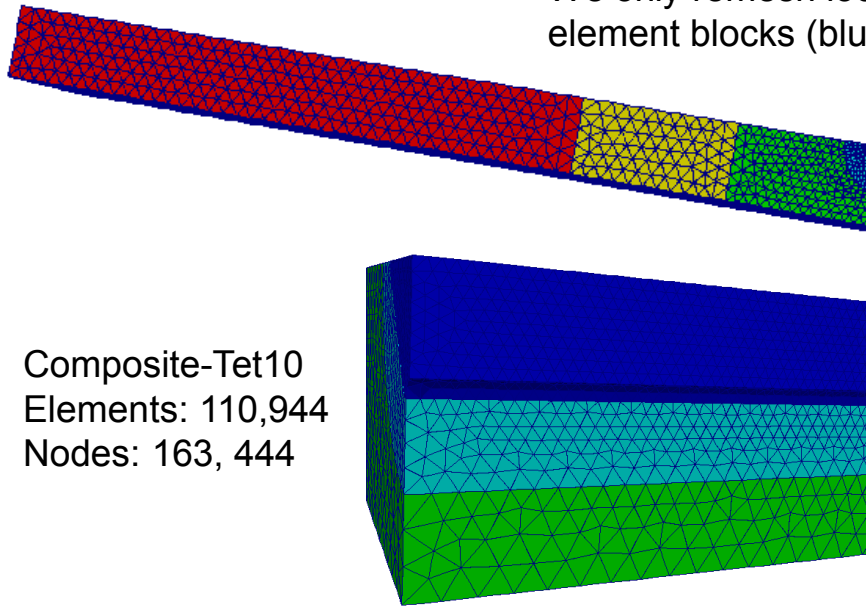


# Every frame is another circle of remeshing and mapping



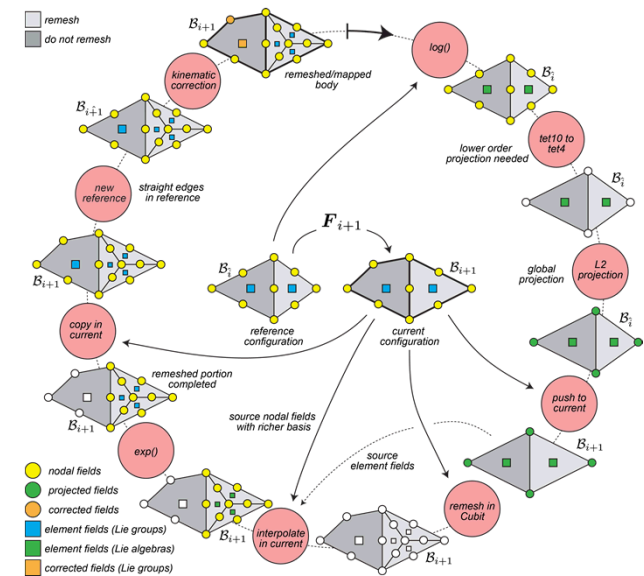
# Can we now model the loss of load-bearing capacity? Yes

We only remesh local  
element blocks (blue)

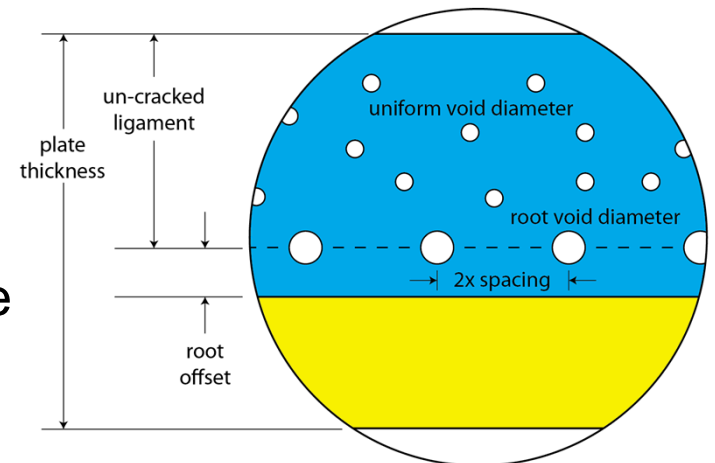


# Conclusions and Path forward

- *mapLL* ensures a sound theoretical basis
- Tetrahedral elements permit discretization
- Composite-tetrahedral elements resolves ISVs
- New reference configuration enables solution
- We are able to predict the load-bearing capacity

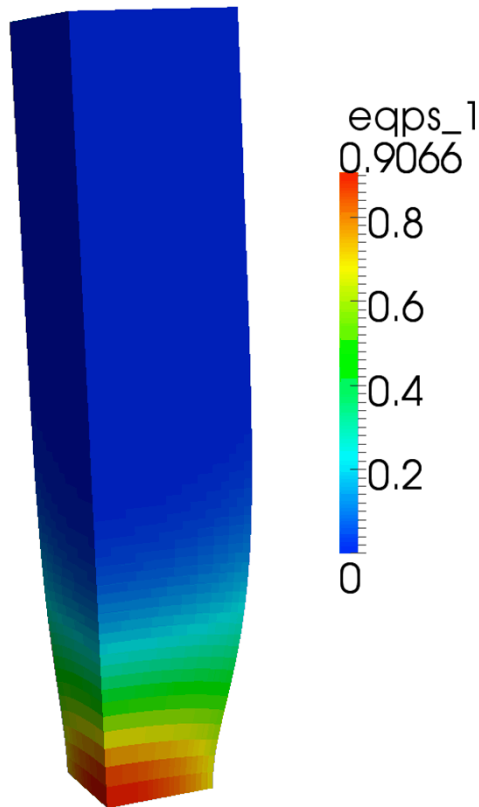


- Solidify remeshing/mapping
- Model idealized void configurations
- Connect void structure to weld performance

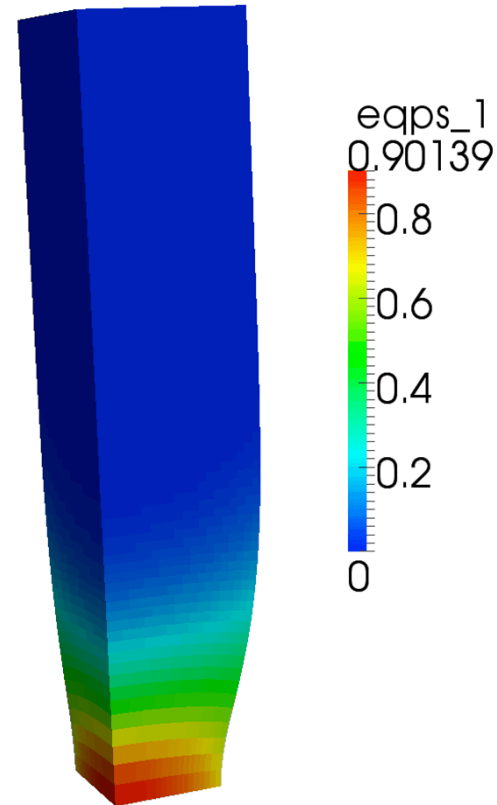


# Numerous remaps exhibits minimal “diffusion” of ISVs

*Equivalent plastic strain in fine mesh at one integration point per element at  $t = 0.25$*



no mapping

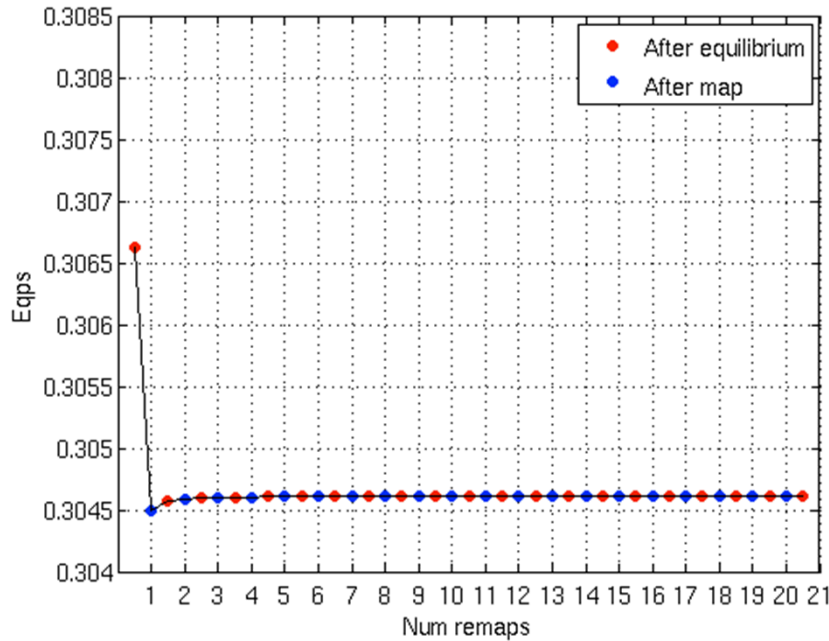


map 100 times between  
 $t = 0$  and  $t=0.25$



# Repeated mapping convergent in global and field quantities

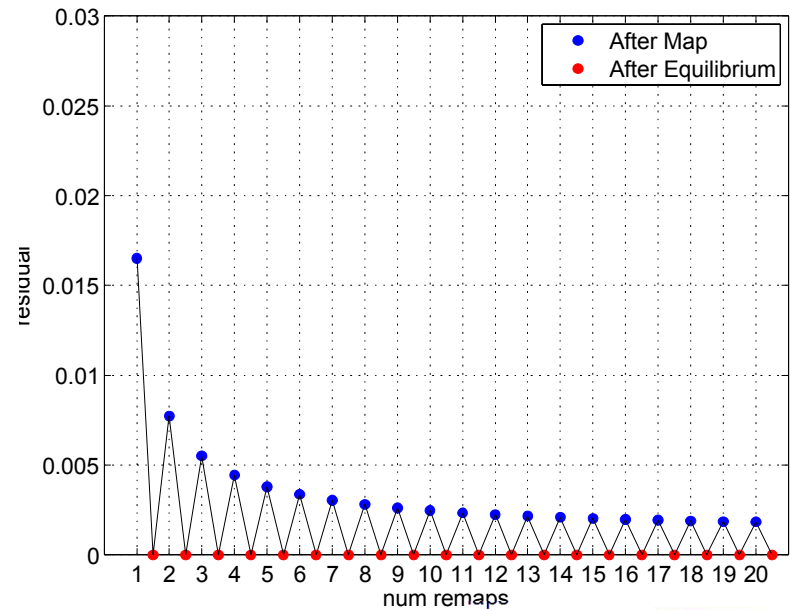
## Maximum plastic strain converges



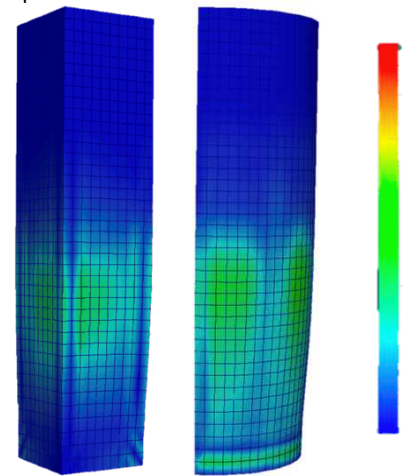
The system comes back into equilibrium rapidly, i.e. only a few iterations.

The residual after mapping may be an indication of the discretization error. Investigation into different levels of refinement are needed.

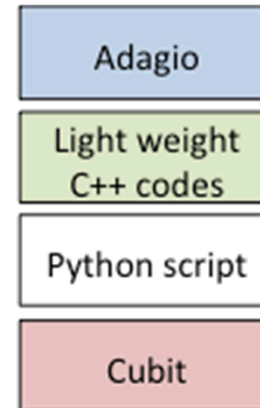
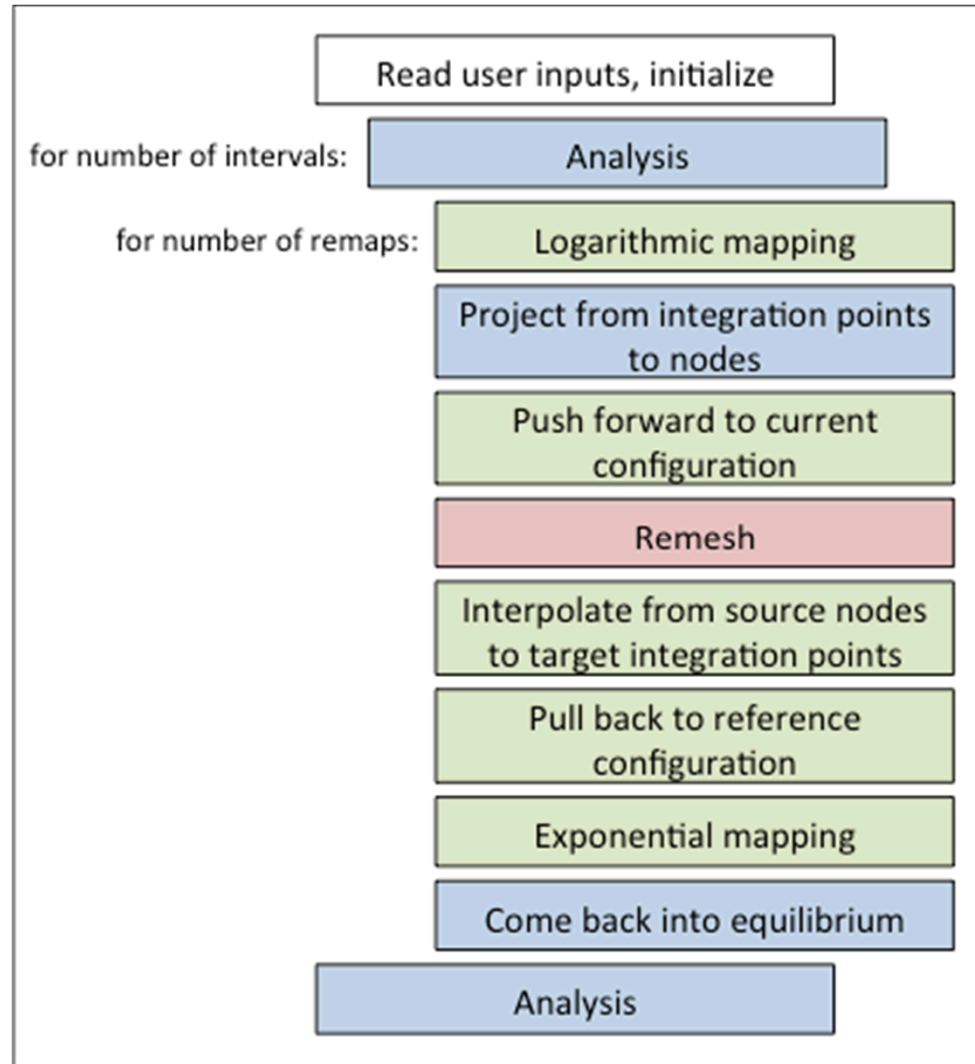
## Initial residual decreasing with additional mappings



Difference in magnitude of residual



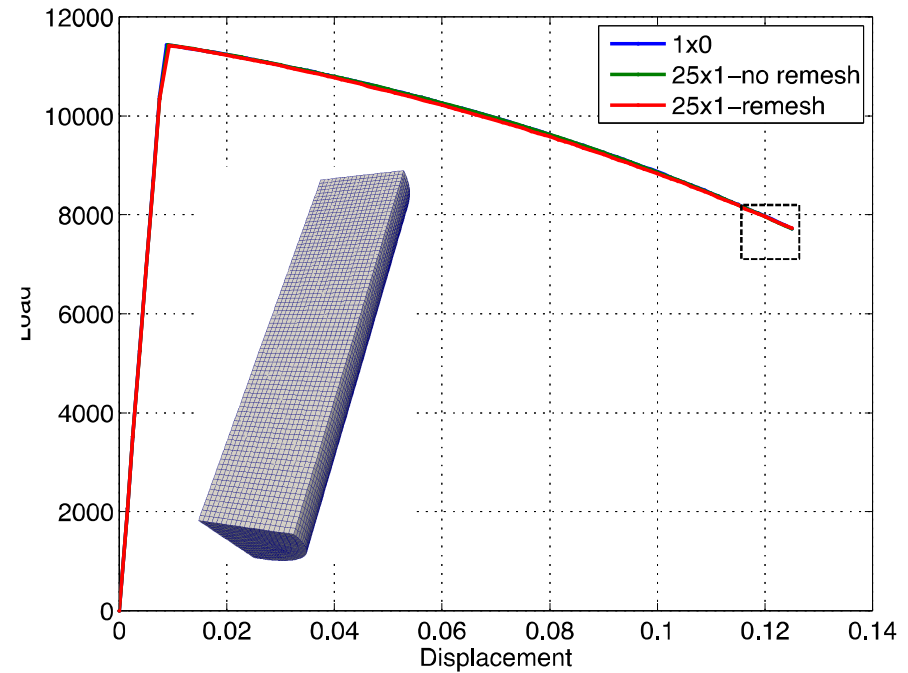
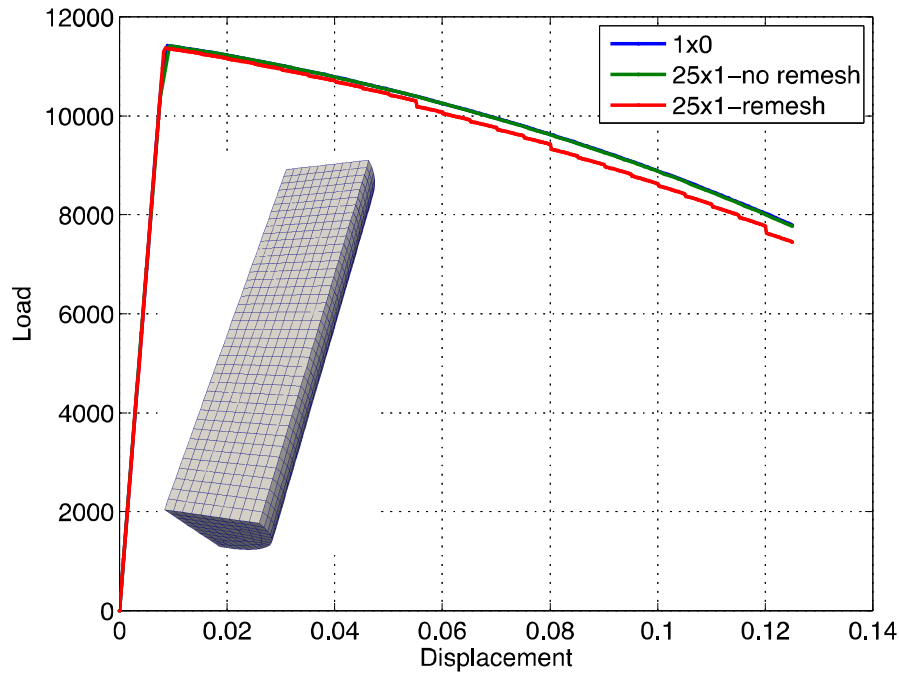
# Include remeshing in automated procedure



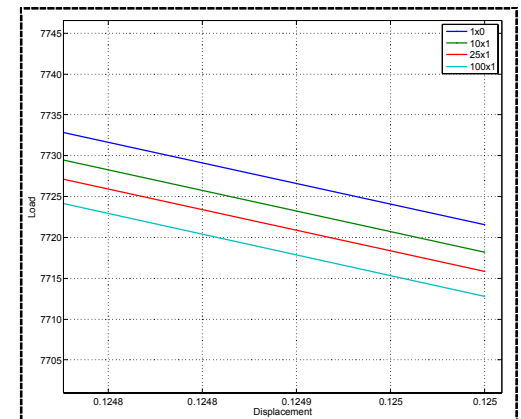
*NOTE: This scheme keeps the reference configuration*

# Converged meshes not sensitive to remeshing/mapping

25x refers to 25 mapping and remeshing procedures



*NOTE: Discretizations with resolved (converged) fields of internal state variables are less sensitive to the remeshing/mapping procedures.*



# Are we any closer to our goal? Yes and no.

*Hexes look great, but....*

- Meshing of arbitrary geometries requires tetrahedral elements
- Composite tet10 formulation not robust for isochoric motions
- Consistent projection for piecewise linear tet10 is flawed
- Total-Lagrange elements will require a new reference configuration

*We did not hesitate to address these fundamental issues (no shortcuts)*

- We will use tetrahedral elements. Period.
- Derive an analytical gradient operator for composite tet10
- Volume averaging  $J$  yields smooth pressure fields under isochoric motions
- Employ linear projection (tet4) for higher-order tetrahedral elements
- Establish a new reference configurations for T-L elements through  $\mathbf{F}_{init}$