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Resilient Iterative Linear Solvers via Skeptical Programming

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Overview

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- Questions?
- A Problem
- Myth of a Reliable Machine
- Reliability For Algorithms
  - Is the root cause of a fault important or is its manifestation?
- Skeptical Programming
  - All iterative ABFT schemes assume some form of detection… how?
- Questions?

Questions

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- Quantitative comparisons of ABFT schemes
  - Solve the same problem?
- Fault Models?
  - How critical are they? Granularity? (node, process, operation)
  - Fault rates? (we know so little)
- Reliable Programming
  - Data is often replicated (devices and hosts)
- Correct solution (research) vs performance
  - Bridging the gap… theory/math and systems

The Problem (well one of them)

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Function: Puppy = make_puppy(Dog, Dog)

The Problem (well one of them)

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The Problem (well one of them)

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Function: Puppy = make_puppy(Dog, Dog)

+

=

The Problem (Well one of them)

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The Problem (well one of them)

Two things happened:

1. You noticed a fault, therefore …
2. You can respond (correct the fault)

Detection and Correction

The Problem (well one of them)

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How to detect that a silent transient fault occured?

- Enforce fine-grained correctness (systems approach)
  - ✓ **Do calculations multiple times and vote**
  - × **Ignores numerical properties of algorithms**

The Problem (well one of them)

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How to detect?

- Enforce fine-grained correctness (systems approach)
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- Convergence (numerical error dampening)
  - ✓ **Iterative methods have proven properties to converge to a correct solution given preconditions/assumptions about the inputs**
  - × **Convergence promise invalidated if assumptions/preconditions are invalidated at any point.**

The Problem (well one of them)

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- Can we do algorithm-based fault tolerance while still exploiting the numerical properties?

Myth of a Reliable Machine

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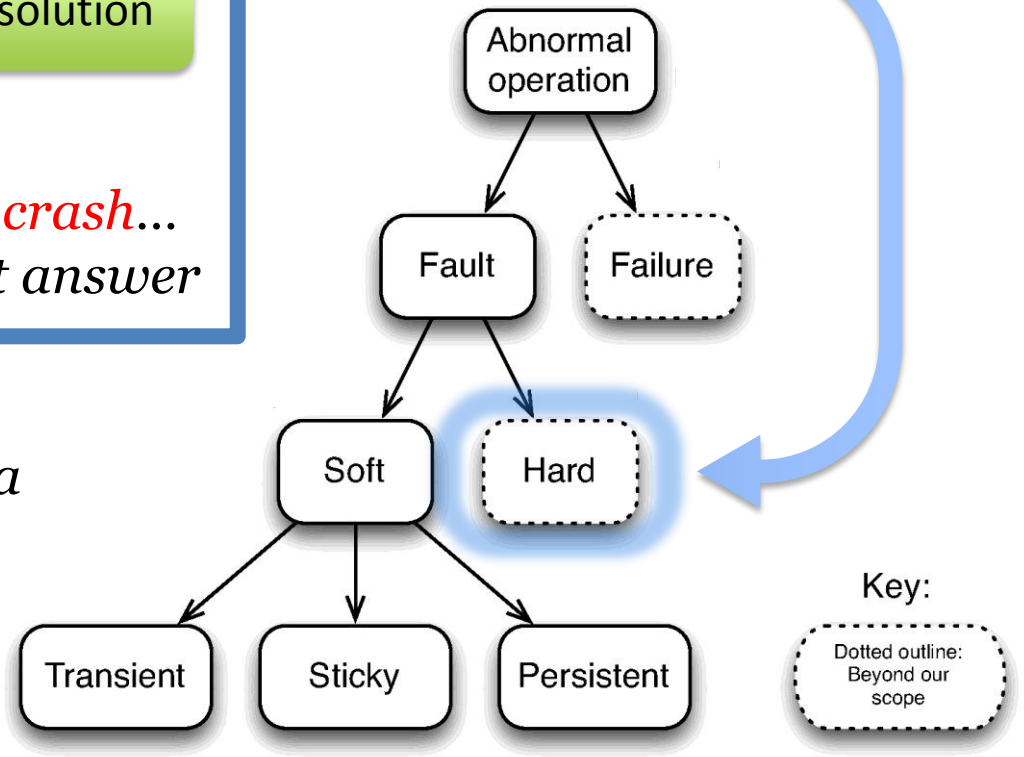
Current algorithms assume ***numerical reliability***

Reliable Machine

start → operations → solution

As long the application does not crash…
we will get the right answer

Hard Faults: Not in scope of this work



Machine reliability addressed via checkpoint/restart, replication, algorithm based approaches

Myth of a Reliable Machine

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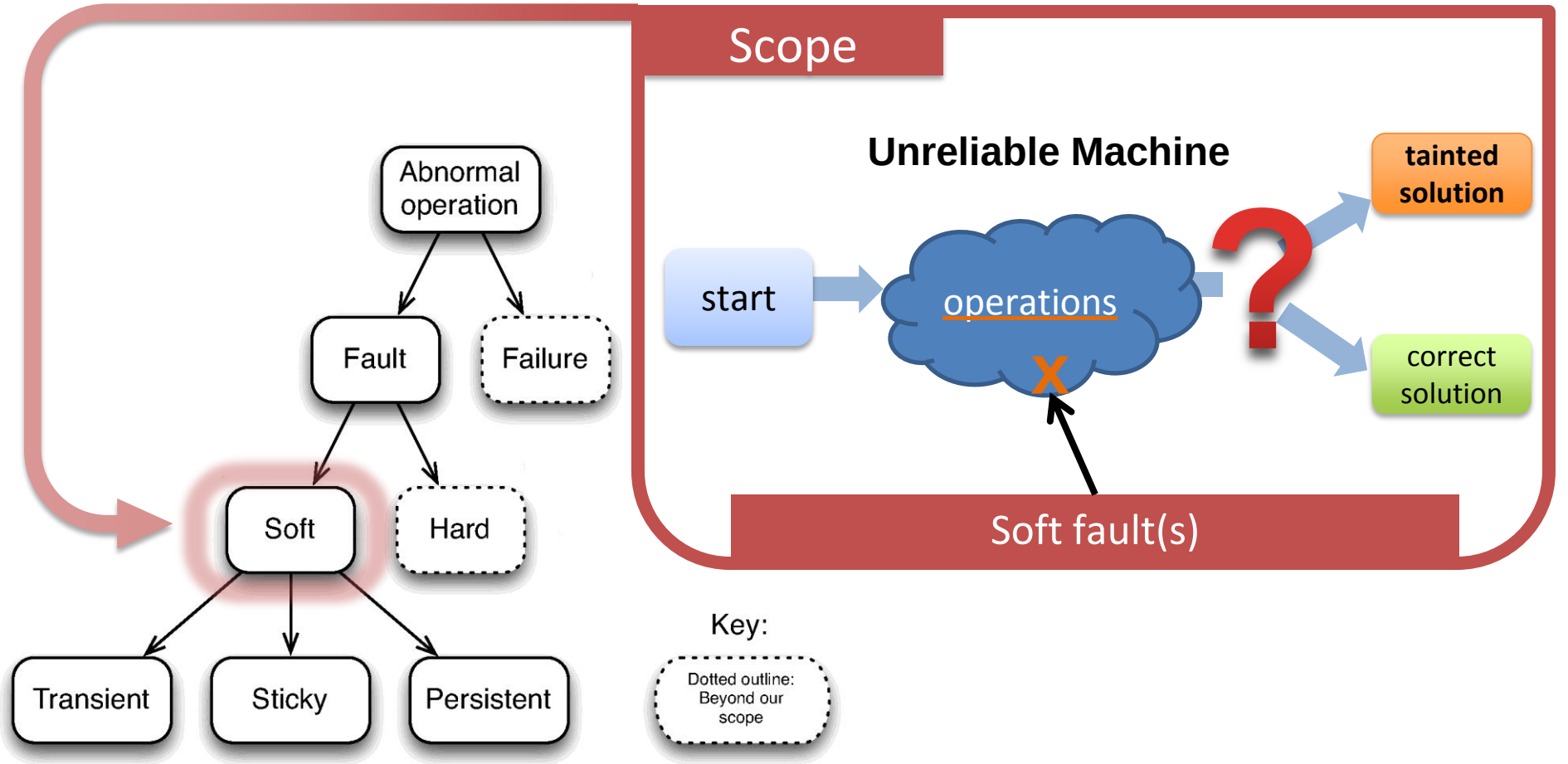
Numerical reliability:

numerical operations may be unreliable



Reliability for Algorithms

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Numerical Method = Theory + Data

(algorithm) (with assumptions)

- **Two Categories:**
  - **Data:** data required theoretically by the algorithm (Preconditions/Assumptions)

    E.g., for GMRES: A, x0, orthonormal subspace of A, right-hand side
  - **Meta-data:** "stuff" required to implement the algorithm

    E.g., loop counters, sparse data structures (indices), code

Regardless of who implements GMRES or in what language, the data can always be assumed (otherwise we would not be considering GMRES!)
Language and Implementation details may result in vastly different meta-data.

- Most extensible research: **focus on faults in data.**

Reliability for Algorithms

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- Result of silent fault in **data**:
  - **Silent Data Corruption (SDC)**
- Result of silent transient fault in **meta-data**
  - Faults in code:
    - Weird behavior
    - Perform wrong operation (e.g., add instead of divide) **= SDC** (output is “tainted”)
  - Faults in loop counters, indices (if no segfault/crash)
    - Operate on incorrect data **= SDC** (output is “tainted”)
    - Iterate too much or too little **= SDC** (output is “tainted”)
- Perhaps more

Key: corrupted data model *many* types of faults!

Big Idea: Bounded Error

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- Focus on ensuring **bounded error,** *rather than* ***detecting and correcting <u>all errors</u>*.**

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- Invariant checks are silly in a reliable environment
  - If unreliable – **invariants serve as cheap detectors** that the algorithm is in a **theoretically impossible state**.

Big Idea: Bounded Error

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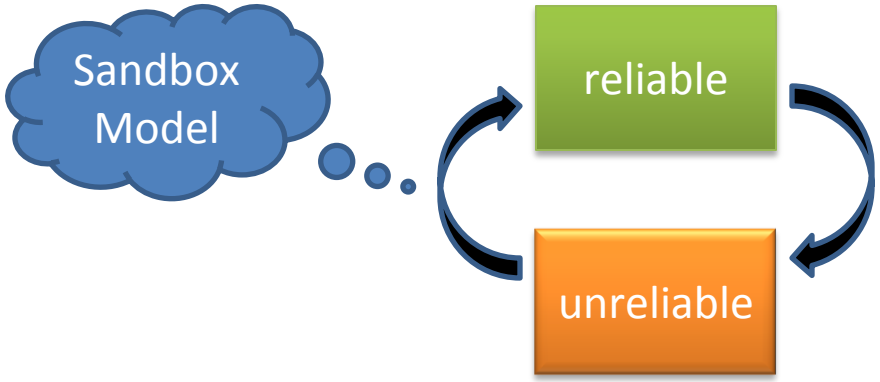
- Focus on ensuring **bounded error,** *rather than* ***detecting and correcting all errors*****.**
- Use preconditions/assumptions on inputs with algorithms' theoretical basis to derive invariants.
- Invariant checks are silly in a reliable environment
  - If unreliable – **invariants serve as cheap detectors** that the algorithm is in a **theoretically impossible state**.
- No promise to detect/correct all errors
  - One piece of the resilient algorithm pie
- We call this Skeptical Programming.
  - **Be skeptical of computations performed on an unreliable machine**

Reliability

Looking from the Reliable Set:

- **Sandbox Model**



Operate unreliably, but do not trust the results.

- **Selective Reliability:**

  What operations need to be reliable?

Universe of **operations** inherently ***unreliable***

Unreliable ops:
Preconditioner call (and all its operations)

Reliable ops:
Flexible GMRES, (currently all collectives)

Selective Reliability

Algorithm 1 Flexible GMRES (FGMRES)

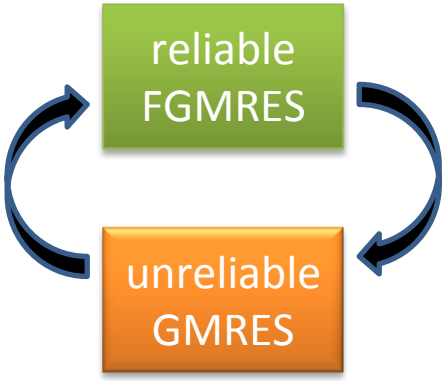
Input: Linear system $Ax = b$ and initial guess x_0
Output: Approximate solution x_m for some $m \geq 0$

1: $\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$ ▷ Unpreconditioned initial residual
2: $\beta := \|\mathbf{r}_0\|_2$, $\mathbf{q}_1 := \mathbf{r}_0/\beta$
3: **for** $j = 1, 2, \ldots$ until convergence **do**
4: Solve $\mathbf{q}_j = \mathbf{M}_j\mathbf{z}_j$ ▷ Apply current preconditioner
5: $\mathbf{v}_{j+1} := \mathbf{A}\mathbf{z}_j$ ▷ Apply the matrix $\mathbf{A}$
6: **for** $i = 1, 2, \ldots, k$ **do** ▷ Orthogonalize
7: $h_{i,j} := \mathbf{q}_i \cdot \mathbf{v}_{j+1}$
8: $\mathbf{v}_{j+1} := \mathbf{v}_{j+1} - h_{i,j}\mathbf{q}_i$
9: **end for**
10: $h_{j+1,j} := \|\mathbf{v}_{j+1}\|_2$
11: Update rank-revealing decomposition of $\mathbf{H}(1{:}j, 1{:}j)$
12: **if** $\mathbf{H}(j+1, j)$ is less than some tolerance **then**
13: **if** $\mathbf{H}(1{:}j, 1{:}j)$ not full rank **then**
14: Did not converge; report error
15: **else**
16: Solution is $\mathbf{x}_{j-1}$ ▷ Happy breakdown
17: **end if**
18: **else**
19: $\mathbf{q}_{j+1} := \mathbf{v}_{j+1}/h_{j+1,j}$
20: **end if**
21: $\mathbf{y}_j := \arg\min_y \|\mathbf{H}(1{:}j+1, 1{:}j)\mathbf{y} - \beta\mathbf{e}_1\|_2$
22: $\mathbf{x}_j := \mathbf{x}_0 + [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_j]\mathbf{y}_j$ ▷ Compute solution update
23: **end for**

M_j are the preconditioners:

$$\mathbf{z}_j = \text{gmres}(\mathbf{A}, \mathbf{q}_j)$$

M_j represents using GMRES as a preconditioner… inside FGMRES.



Skeptical Programming

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Algorithm 1: **GMRES**

for $l = 1$ to do
  $\mathbf{r} := \mathbf{b} - \mathbf{A}\mathbf{x}^{(j-1)}$
  $\mathbf{q}_1 := \mathbf{r} / ||r||_2$
  for $j = 1$ to *restart* do
    $\mathbf{w}_0 := \mathbf{A}\mathbf{q}_j$
    for $i = 1$ to j do
      $h_{i,j} := \mathbf{q}_i \cdot \mathbf{w}_{i-1}$
      $\mathbf{w}_i := \mathbf{w}_{i-1} - h_{i,j}\mathbf{q}_i$
    end
    $h_{j+1,j} := ||\mathbf{w}||_2$
    $\mathbf{q}_{j+1} := \mathbf{w}/h_{j+1,j}$
    Find $\mathbf{y} = \min ||\mathbf{H}_j\mathbf{y} - ||\mathbf{b}||\,\mathbf{e}_1||_2$
    Evaluate convergence criteria
    Optionally, compute $\mathbf{x}_j = \mathbf{Q}_j\mathbf{y}$
  end
end

Skeptical Programming

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Orthogonalization
Builds upper Hessenburg matrix (H)

Skeptical Programming

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    Evaluate convergence criteria
    Optionally, compute $\mathbf{x}_j = \mathbf{Q}_j\mathbf{y}$
  end
end

Theoretical Bounds on the Arnoldi Process

$$\|\mathbf{w}_0\| = \|\mathbf{A}\mathbf{q}_j\| \leq \|\mathbf{A}\|_2 \|\mathbf{q}_j\|_2$$
$$\|\mathbf{w}_0\| \leq \|\mathbf{A}\|_2 \leq \|\mathbf{A}\|_F$$

From isometry of orthogonal projections,

$$|h_{i,j}| \leq \|\mathbf{A}\|_F$$

- $h_{i,j}$ form Hessenberg Matrix
- Bound only computed once, valid for entire solve

Skeptical Programming

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Theoretical Bounds on the Arnoldi Process

$$\|\mathbf{w}_0\| = \|\mathbf{A}\mathbf{q}_j\| \leq \|\mathbf{A}\|_2 \|\mathbf{q}_j\|_2$$
$$\|\mathbf{w}_0\| \leq \|\mathbf{A}\|_2 \leq \|\mathbf{A}\|_F$$

From isometry of orthogonal projections,

$$|h_{i,j}| \leq \|\mathbf{A}\|_F$$

- $h_{i,j}$ form Hessenberg Matrix
- Bound only computed once, valid for entire solve
- **Identifies that something isn't right**

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Preconditioners

$$\tilde{\mathbf{z}}_i = \mathbf{M}^{-1}\mathbf{q}_i$$

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Fault:
subdomain permutes preconditioner's output
(preserves length)

X-axis:
subdomain also scales permuted output

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120 *inner* preconditioner applies using Flexible Gmres + Gmres + Additive Schwarz (with zero overlap) ILU(0) as subdomain solver (right preconditioning).

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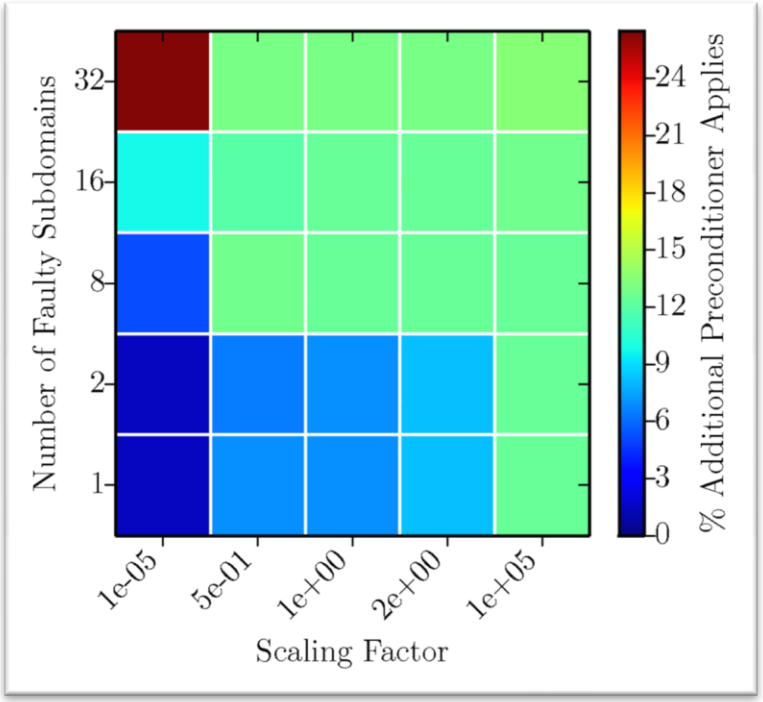
reliable

faulty

120 *inner* preconditioner applies using Flexible Gmres + Gmres + Additive Schwarz (with zero overlap) ILU(0) as subdomain solver (right preconditioning).

No communication with other subdomains

Mean percent additional preconditioner applies



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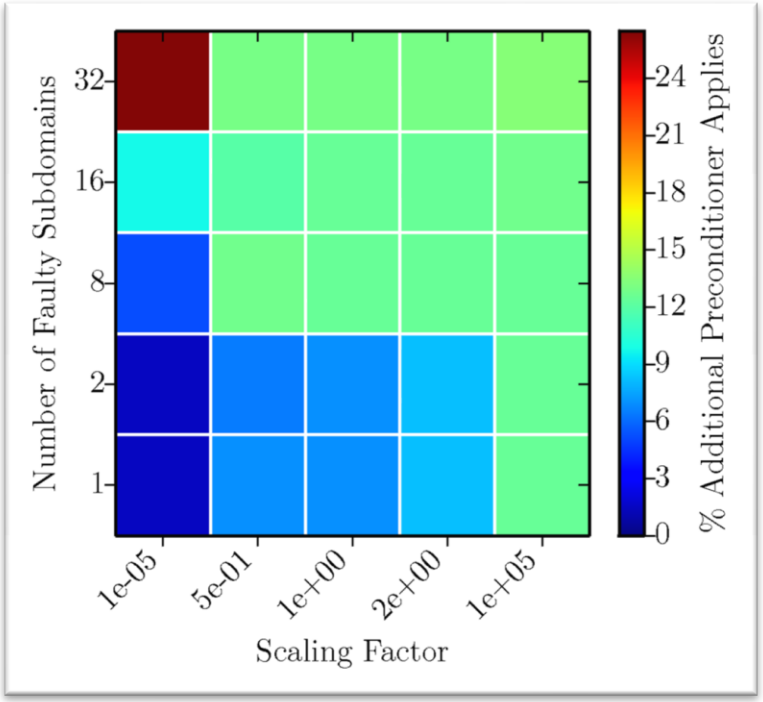

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Statistics from introducing a single fault at all possible (120) preconditioner applications.

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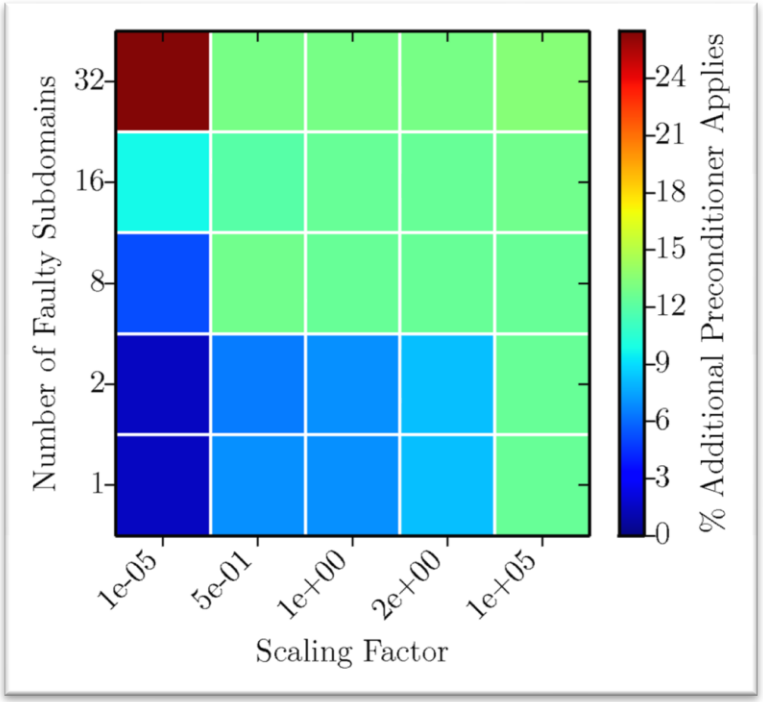

all subdomains faulty

1 faulty subdomain

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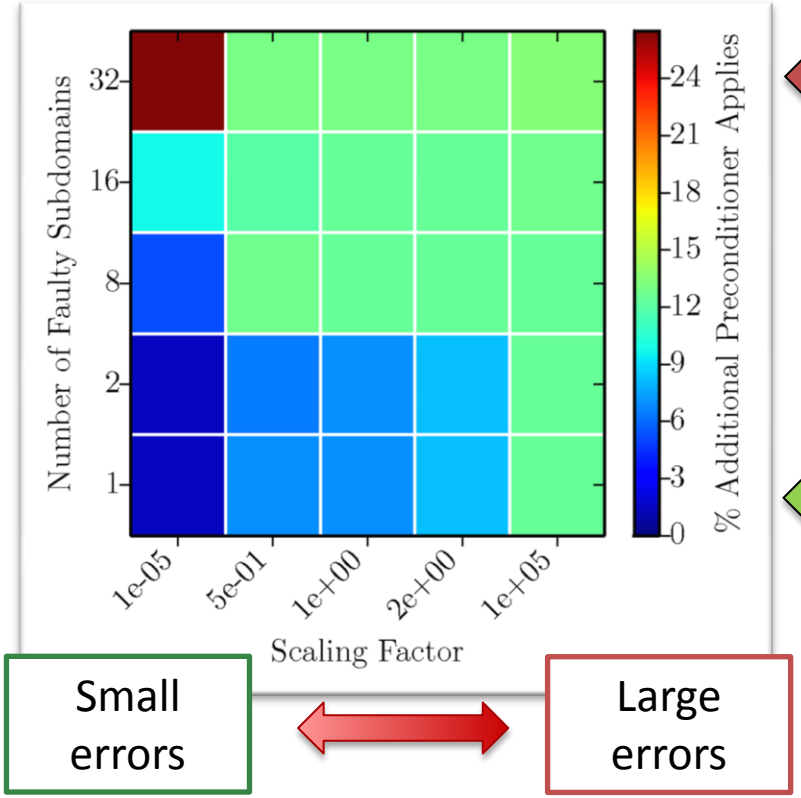

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Small errors

Large errors

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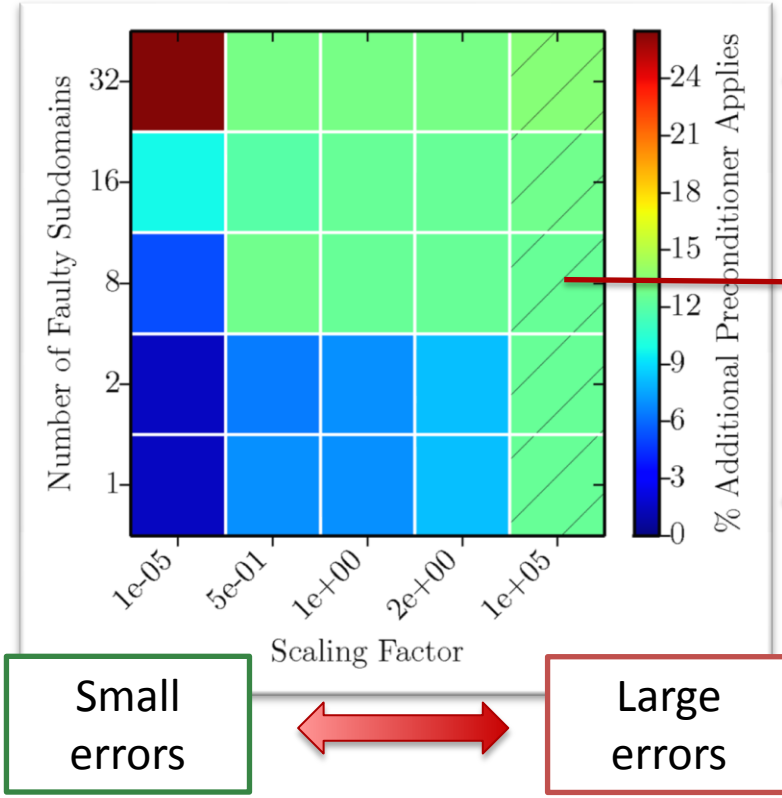

all subdomains faulty

Detected via norm bound

1 faulty subdomain

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Statistics from introducing a single fault at all possible (120) preconditioner applications.

Multiple Faults and Reactive FT

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- What to do when a fault is detected – options based on FT strategy

Solver Type	Restart	Abort Solve
GMRES		
Restarted GMRES	X	
Nested Solvers	X	X

Restarting:

theoretically bad for GMRES (you lose valuable state)

Aborting: inner solve exits abnormally

Both cases: (for right preconditioned solver)– must apply preconditioner to get solution. (This can be expensive!)

Residual checks (Ax-b), require preconditioner application

Multiple Faults and Reactive FT

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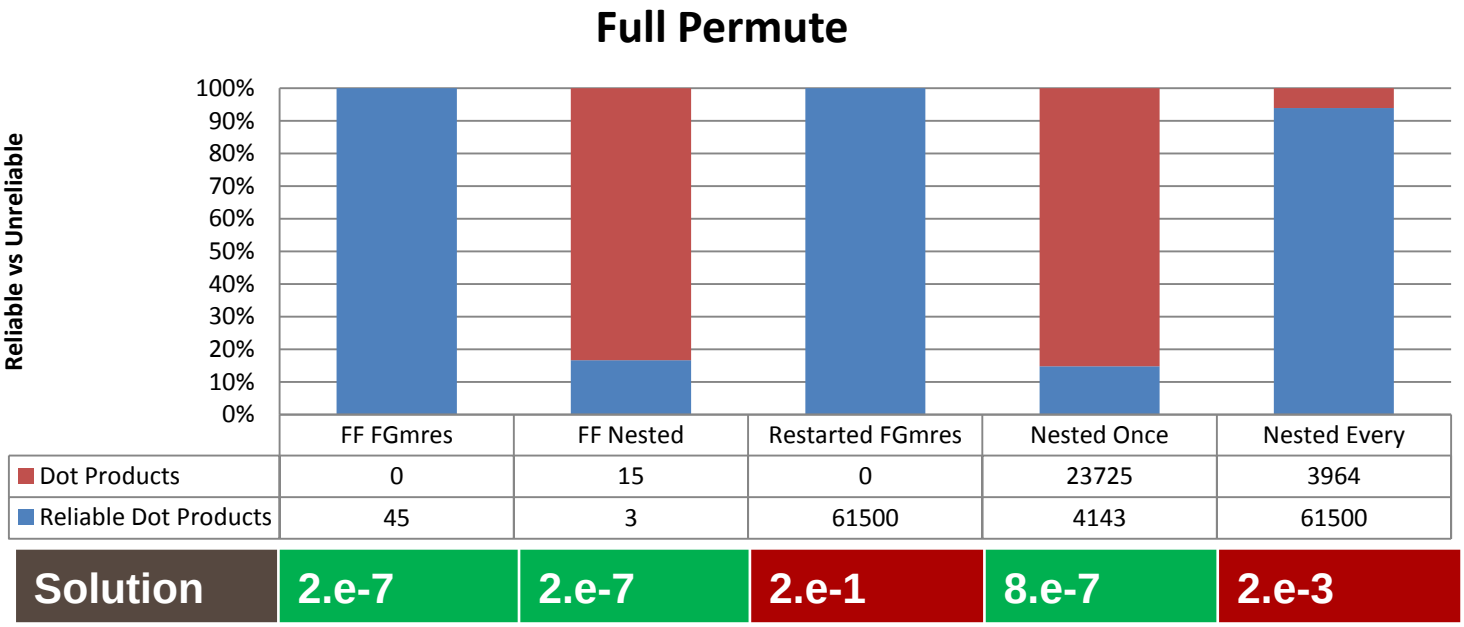
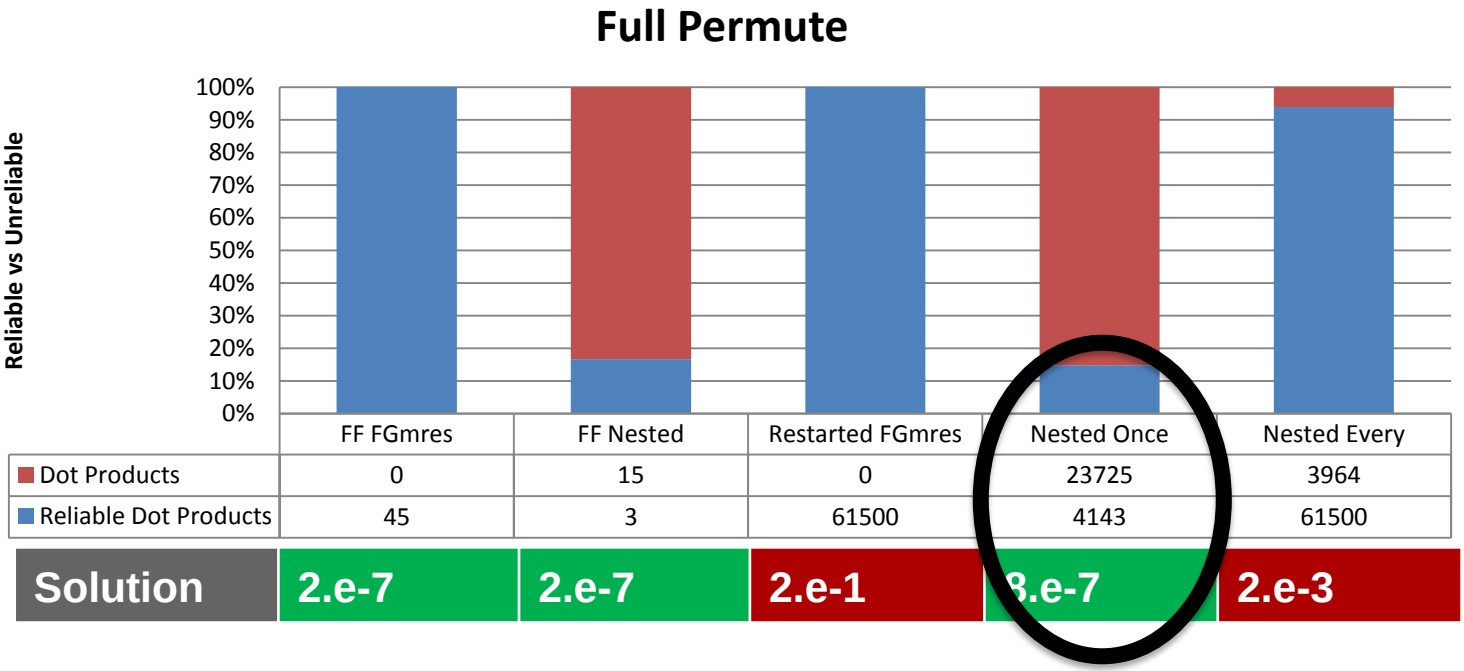
- Experiment, 10 MPI processes
  - at every preconditioner apply, single MPI rank is faulty
  - Fault: MPI process permutes (shuffles) all values in the output from it's preconditioner or swamps the min and max values in its output
  - Requirement: residual (||Ax-b||) less than 1x10^-6
  - Solver:
    - Reliable Restarted FGmres: Flexible GMRES, with reliable (triple modular redundancy) dot products and Sparse Mat-Vec (SpMV) – 3x cost in flops
    - Reliable Restarted FGmres + Gmres: Nested solver, e.g., FT-GMRES
  - Preconditioner :
    - 10 layer Multigrid – one of (if not) the best Poisson solvers
  - Solver + Preconditioner are widely used because of robustness and speed
    - Compares against a "good" solver

Multiple Faults and Reactive FT



	FF FGmres	FF Nested	Restarted FGmres	Nested Once	Nested Every
Dot Products	0	15	0	23725	3964
Reliable Dot Products	45	3	61500	4143	61500
Solution	**2.e-7**	**2.e-7**	**2.e-1**	**8.e-7**	**2.e-3**

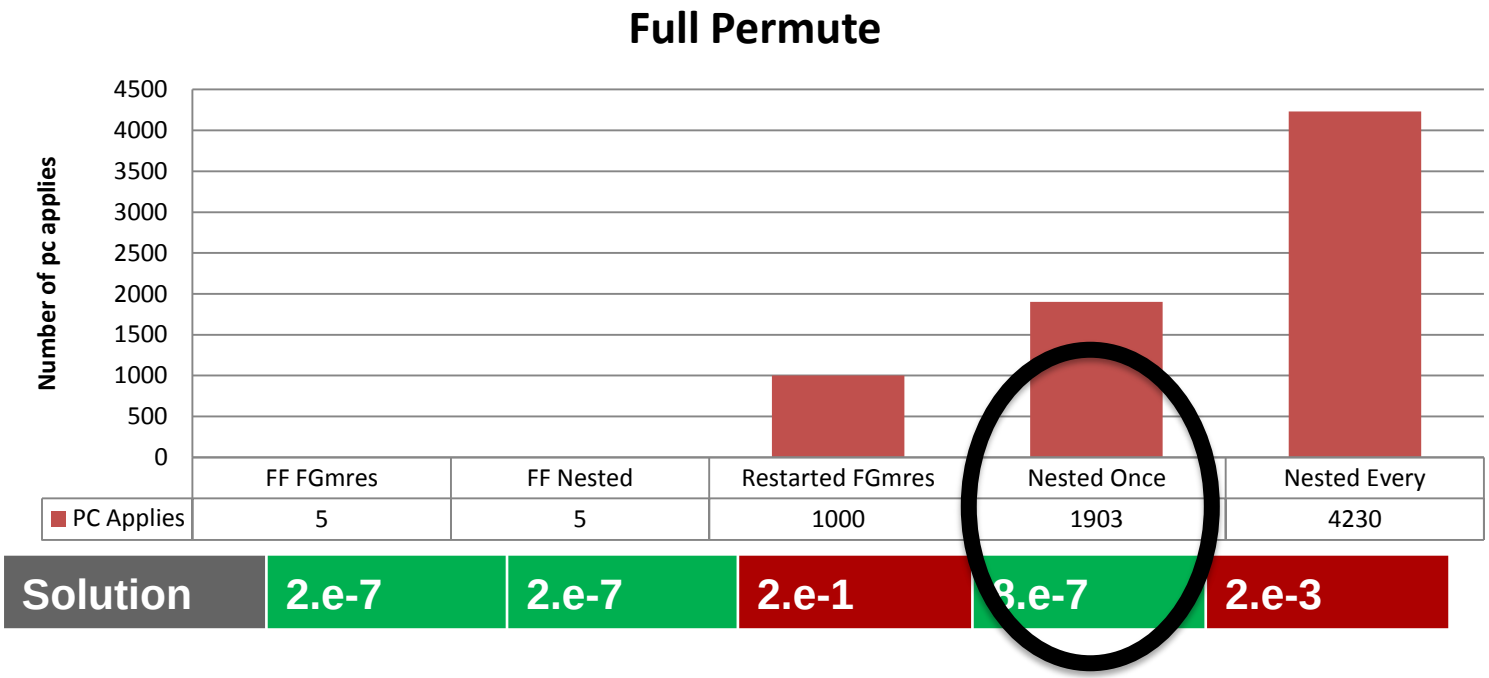
Multiple Faults and Reactive FT



Checks residual once, all work is “bad’, single checks ensures you do not go backwards

Total work equal to work done in failure free

Multiple Faults and Reactive FT



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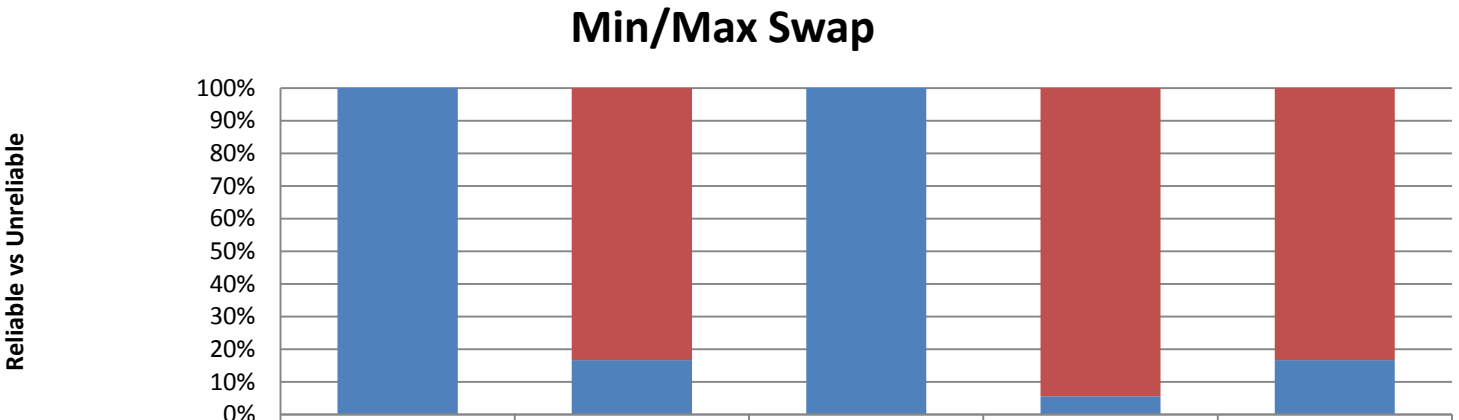
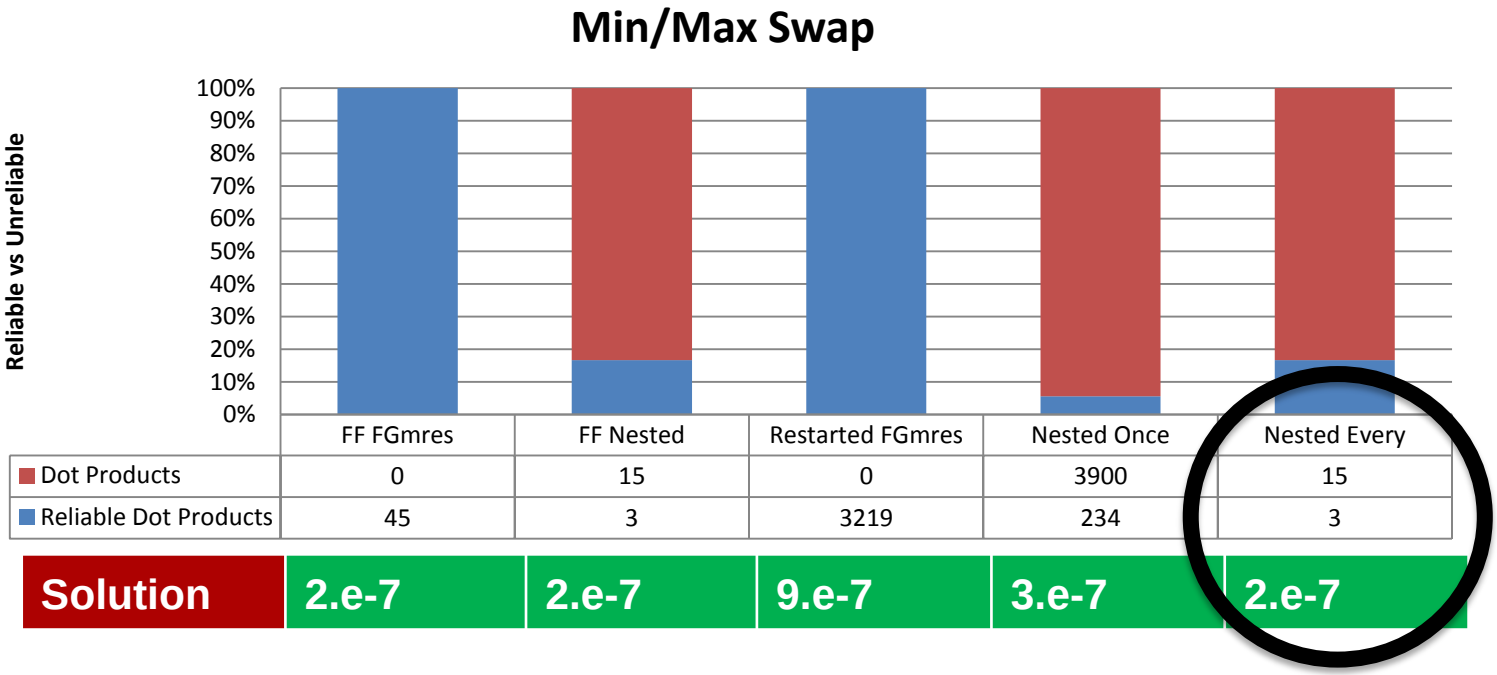


	FF FGmres	FF Nested	Restarted FGmres	Nested Once	Nested Every
Dot Products	0	15	0	3900	15
Reliable Dot Products	45	3	3219	234	3

Solution	2.e-7	2.e-7	9.e-7	3.e-7	2.e-7

Multiple Faults and Reactive FT



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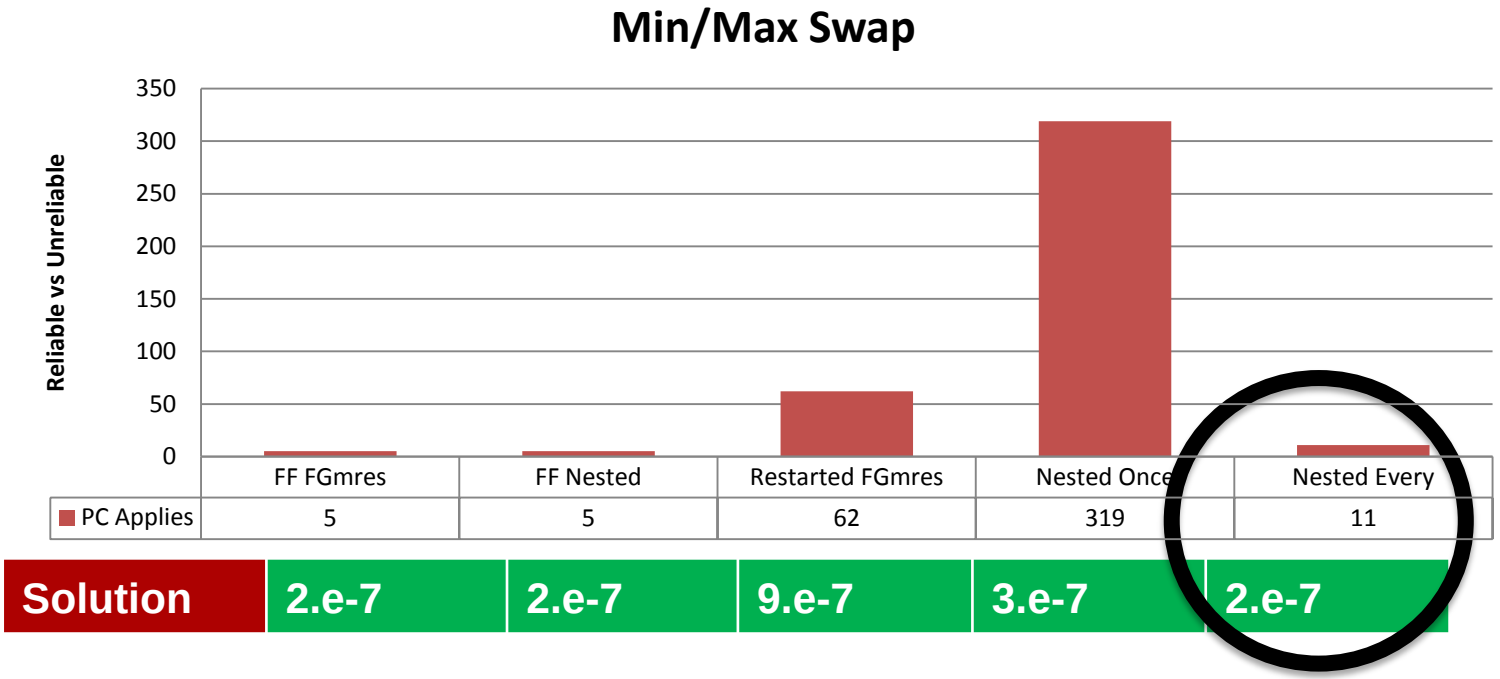
Solution	**2.e-7**	**2.e-7**	**9.e-7**	**3.e-7**	**2.e-7**

Checks residual every time, catches “bad” errors early
Total work equal to work done in failure free

Multiple Faults and Reactive FT

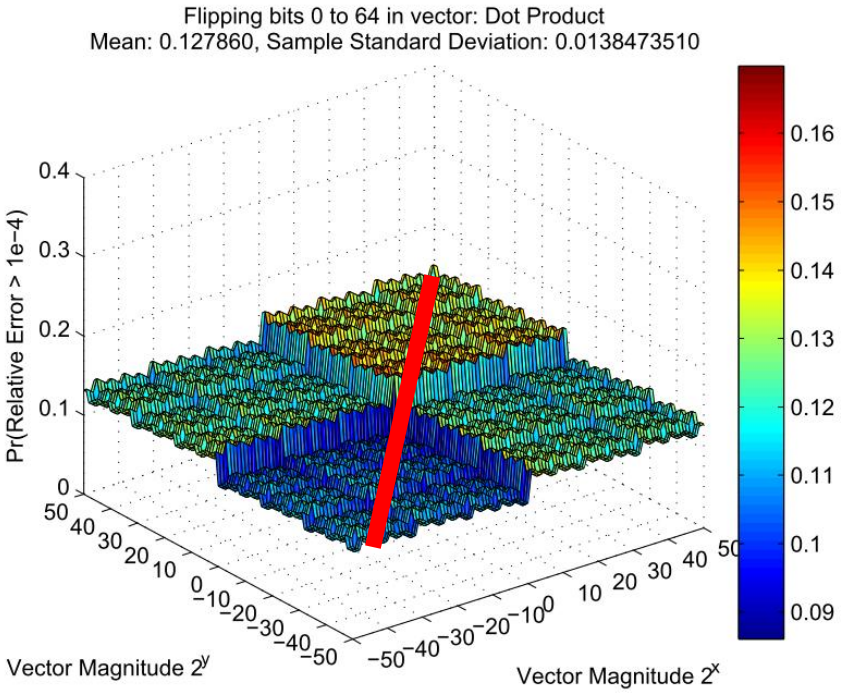


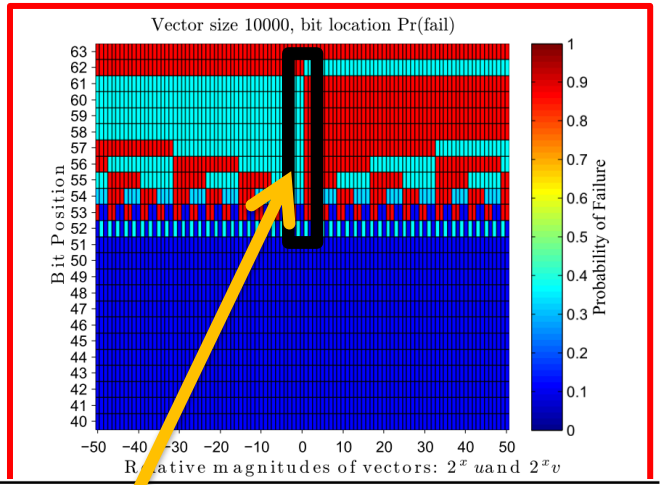
Checks residual every time, catches “bad” errors early
Total work equal to work done in failure free

James Elliott
Computer Science, North Carolina State Univ.

- Example: Dot products:

$$v = (-1)^{sign}\left(1 + \sum_{i=0}^{51} b_i 2^{i-52} \times 2^{e-1023}\right)$$



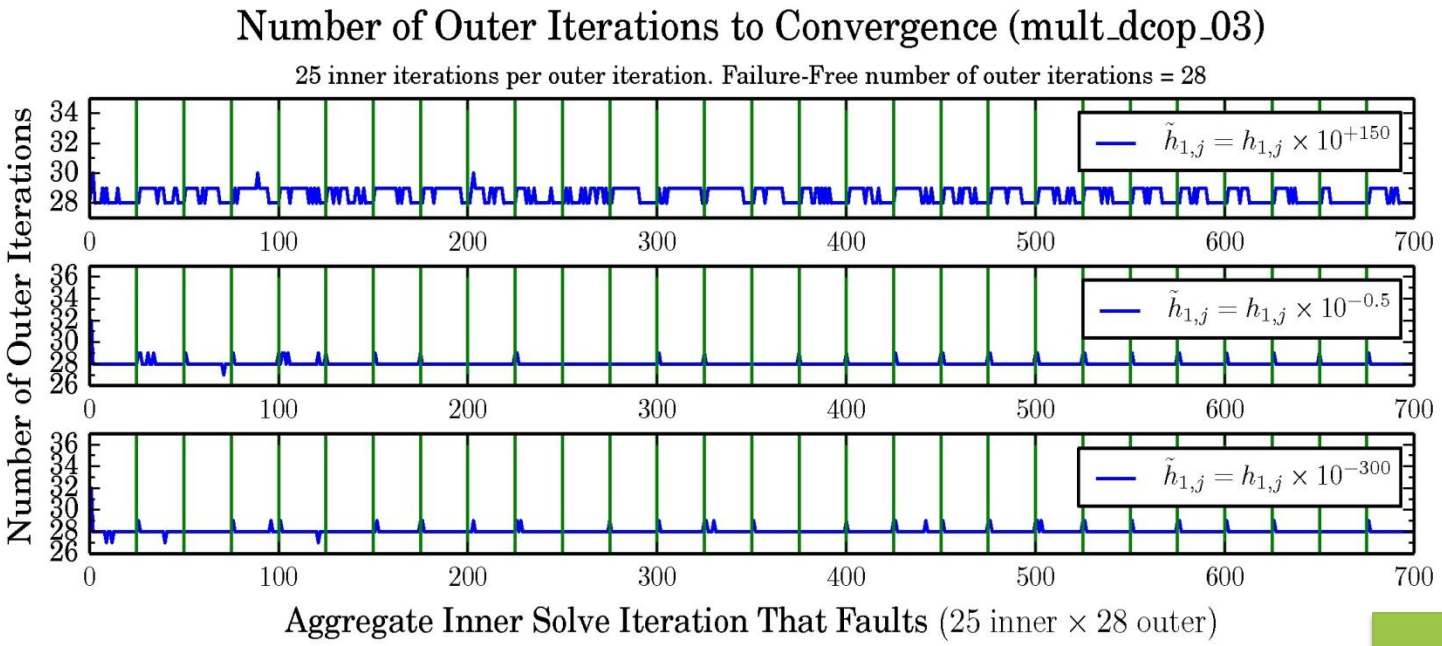


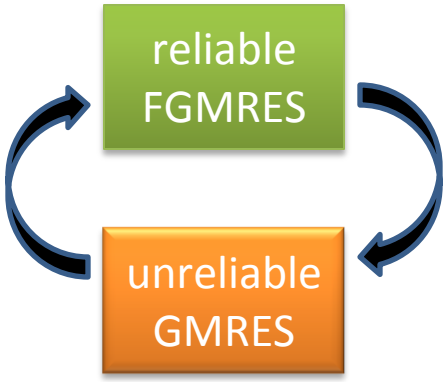
	Exponent Bits				
Base10	b_e	...	b_{52}	Bias Relation	Effective Exp.
2	10000000000			$2^{1024-1023}$	2^1
1	01111111111			$2^{1023-1023}$	2^0
.5	01111111110			$2^{1022-1023}$	2^{-1}

Figure shows probability of experiencing large error.

Due to floating point implementation, not all bit flips create large error – can model the probability of experiencing specific magnitudes of error.

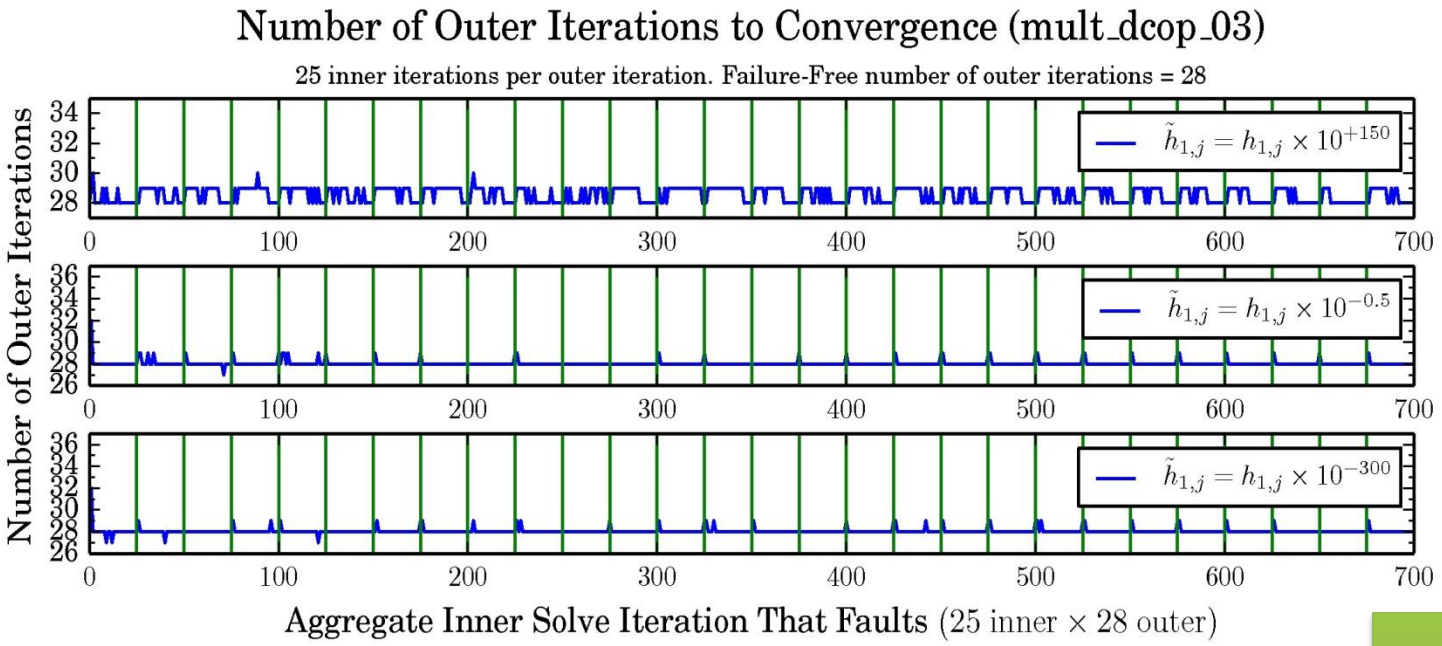
Experiments



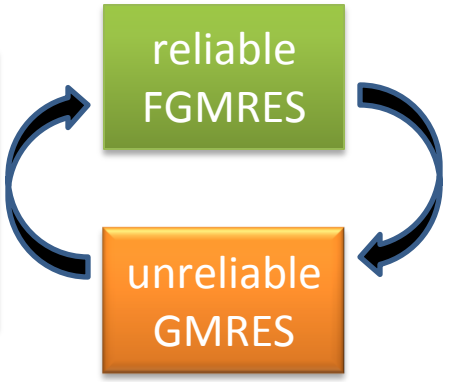


Prototype Design: Fault Tolerant GMRES iterative solver
Flexible GMRES (FGMRES) creates sandbox
Implemented using Trilinos

Experiments



Failure Free: **700 orthgonalization calls**
28 outer iterations to converge
25 inner iterations per outer
x-axis:
introducing single fault in x-th orthogonalization



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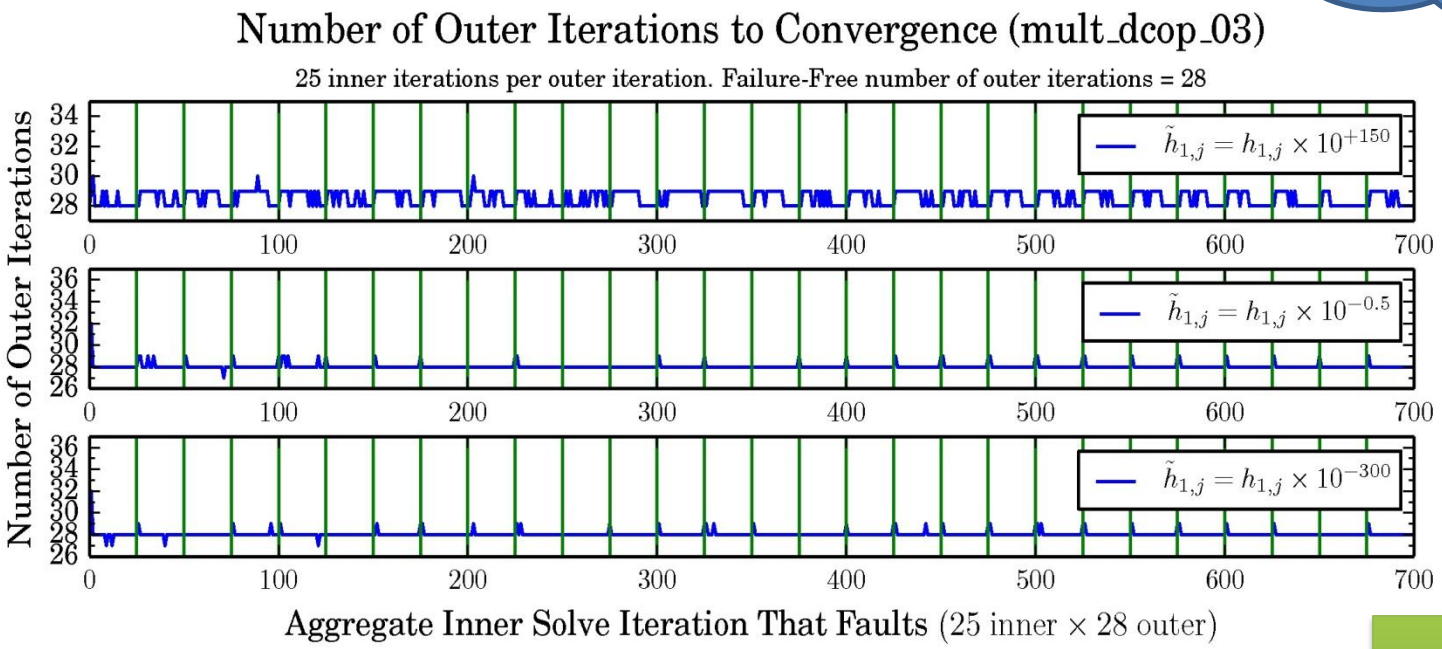
Experiments

700 experiments per fault type



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reliable FGMRES

unreliable GMRES

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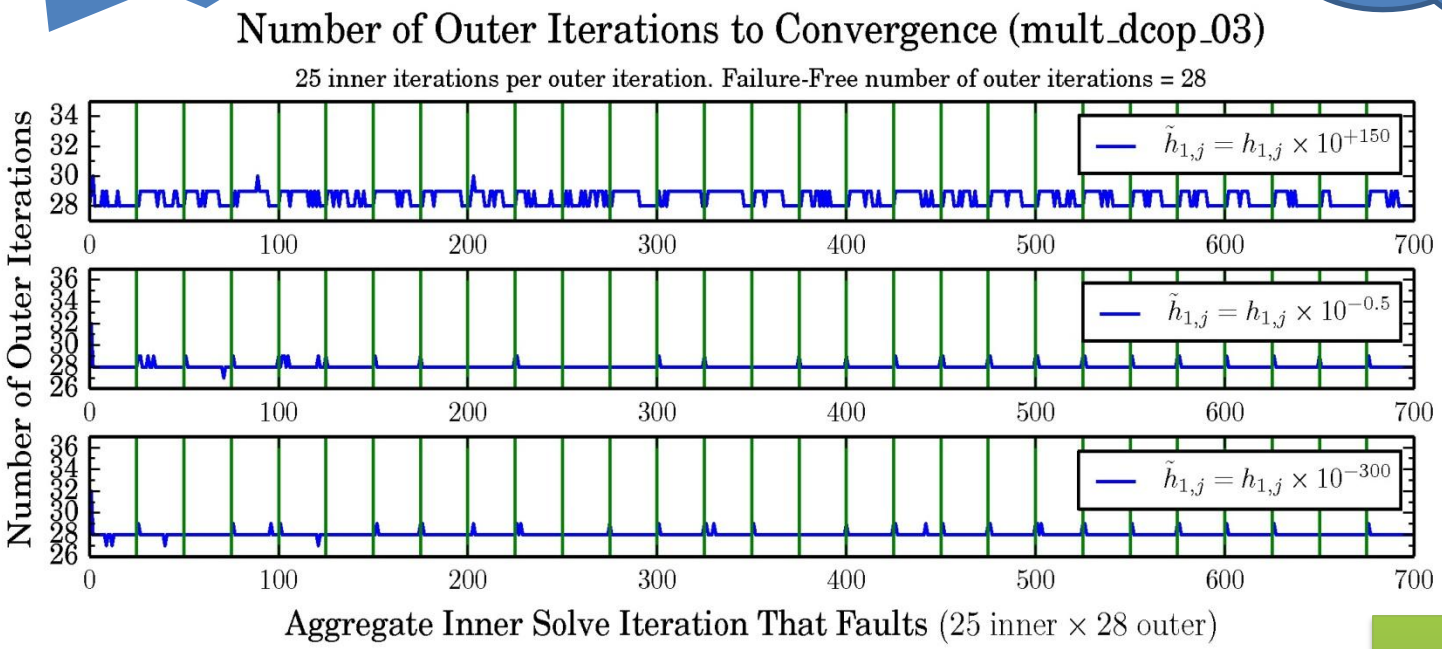
Inject ***single*** fault
Study impact

700
experiments per
fault type



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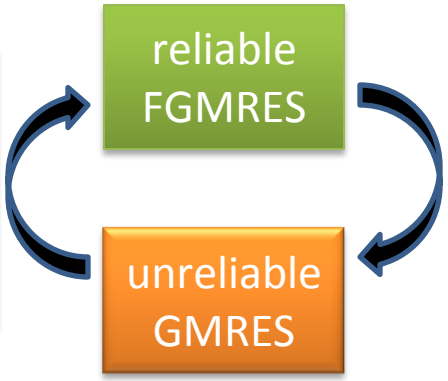

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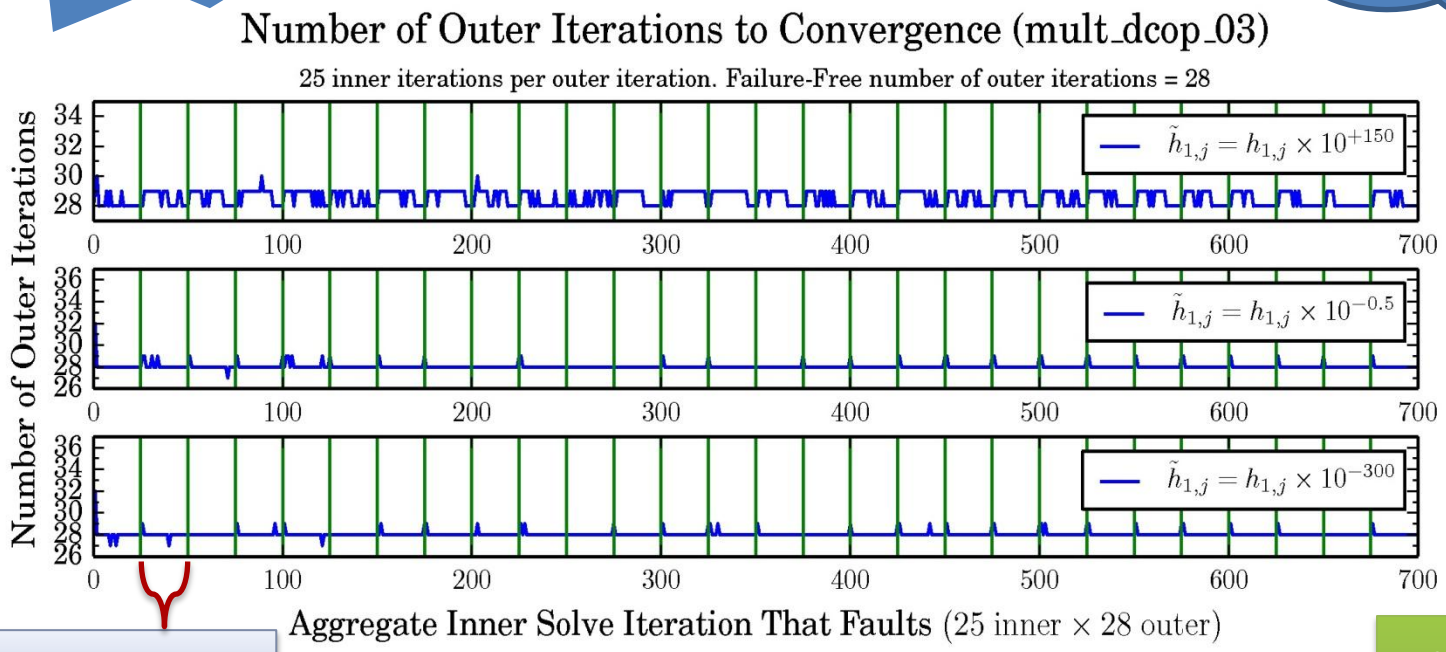

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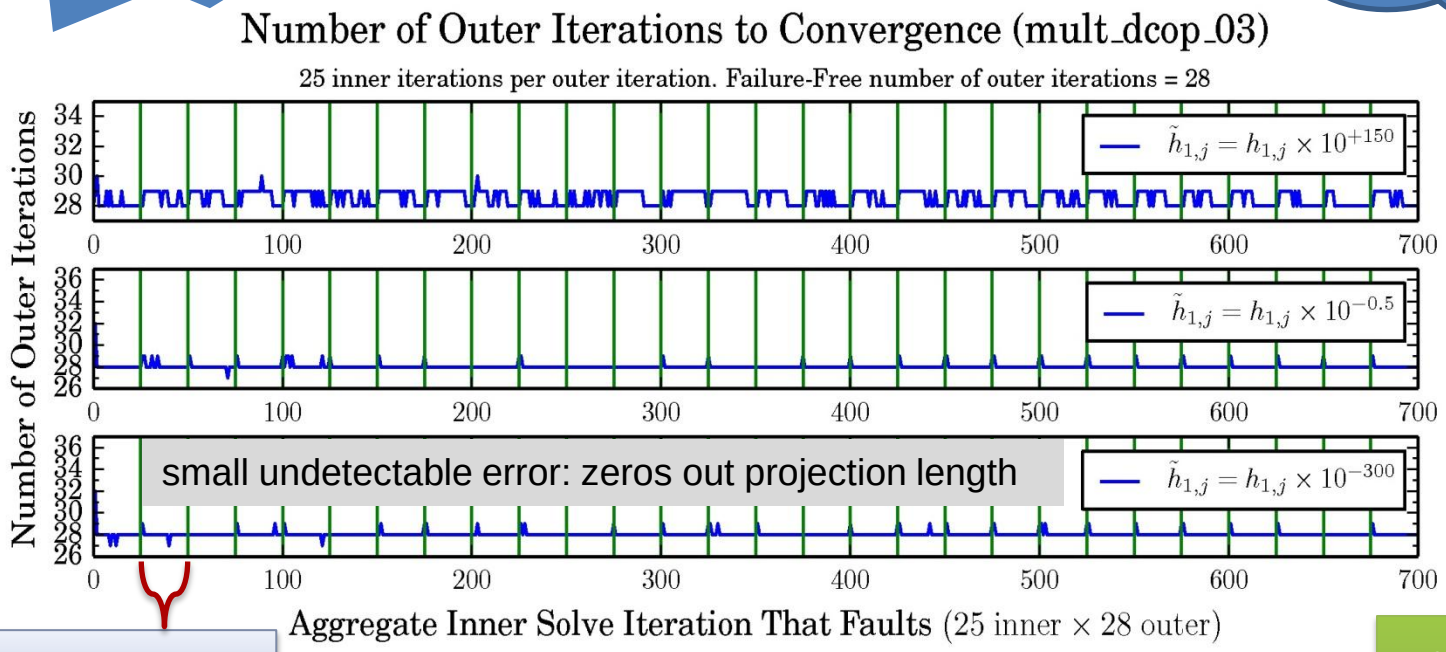

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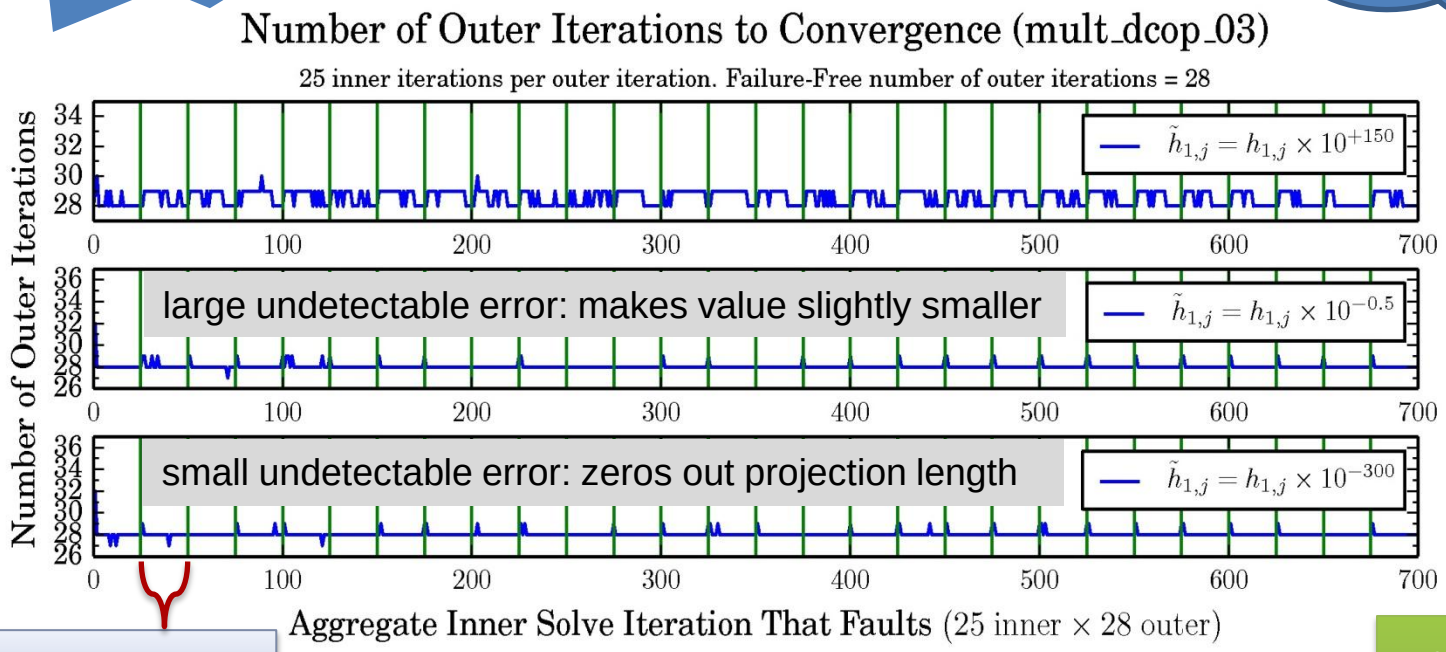

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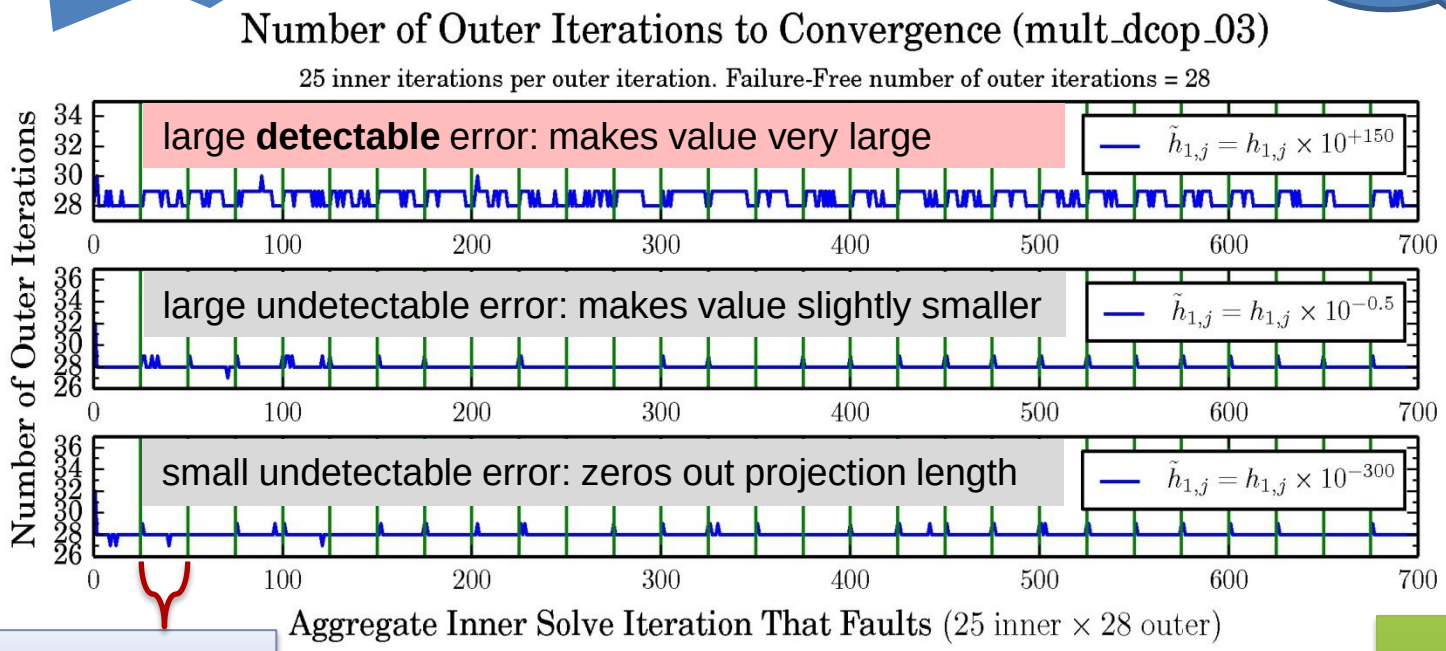

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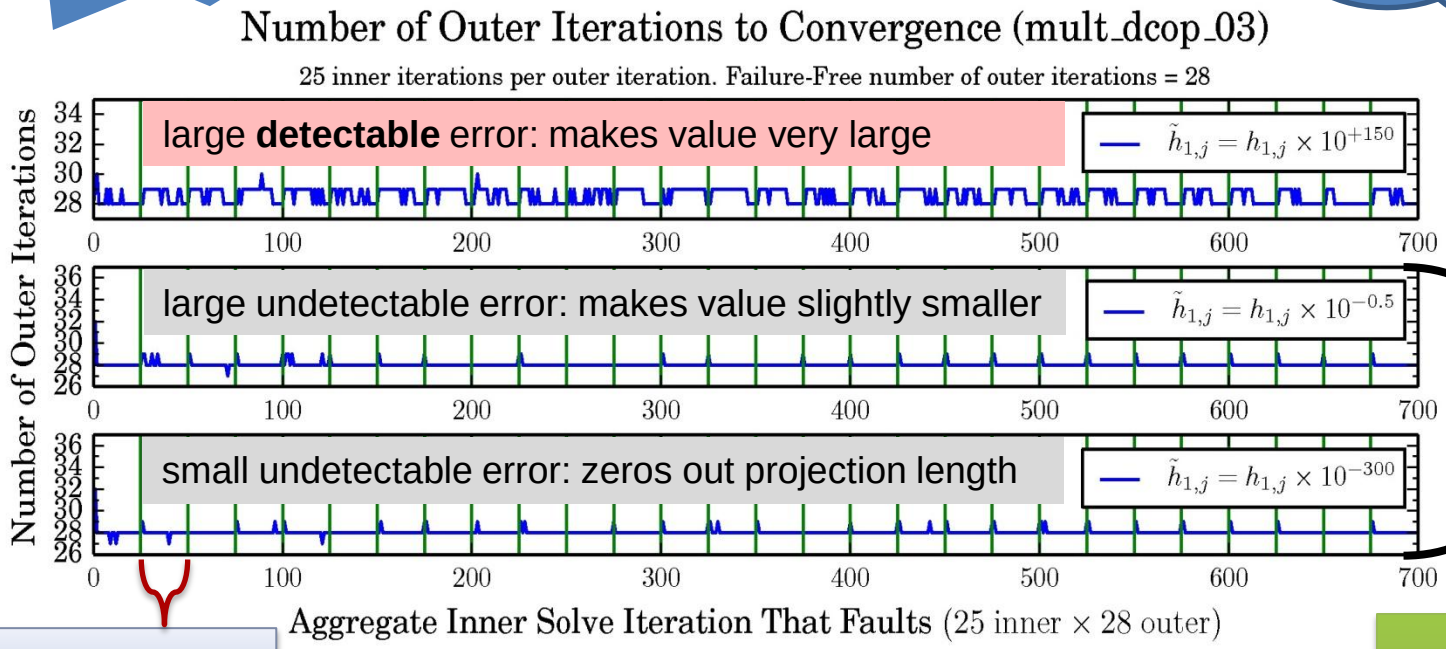

Smaller than bound

one inner solve

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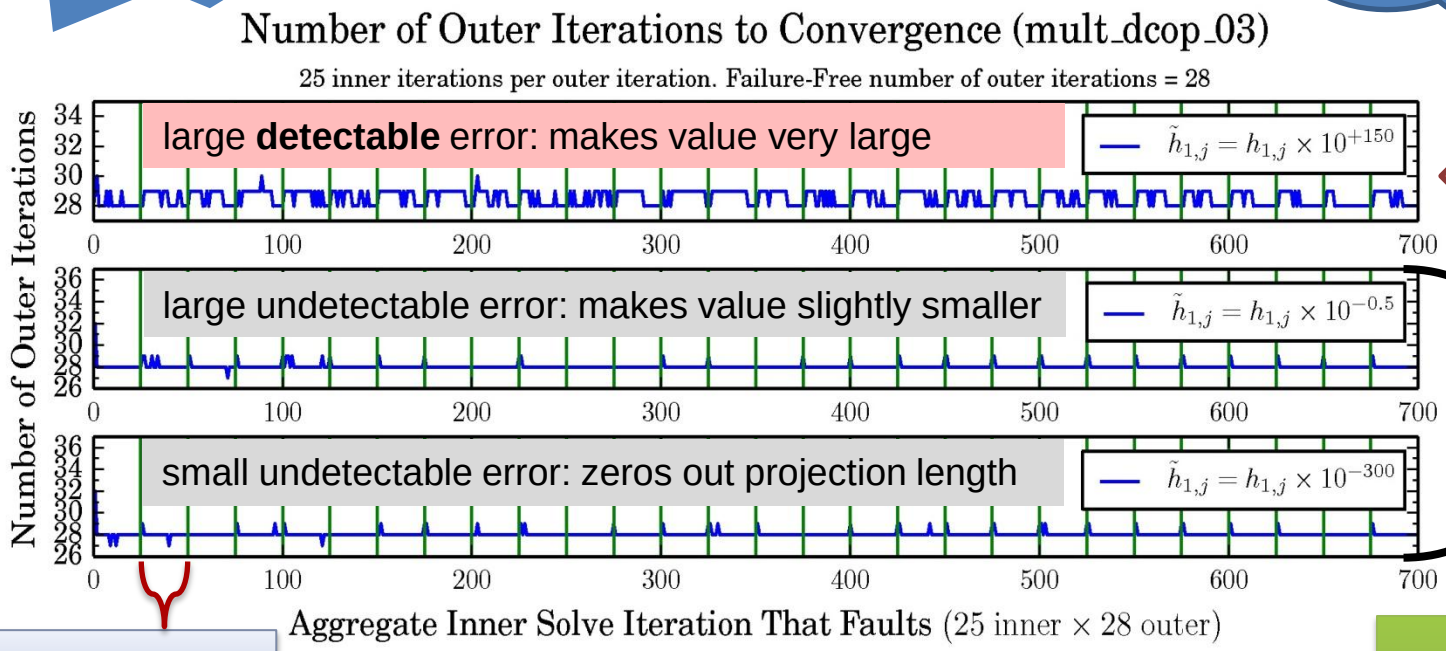

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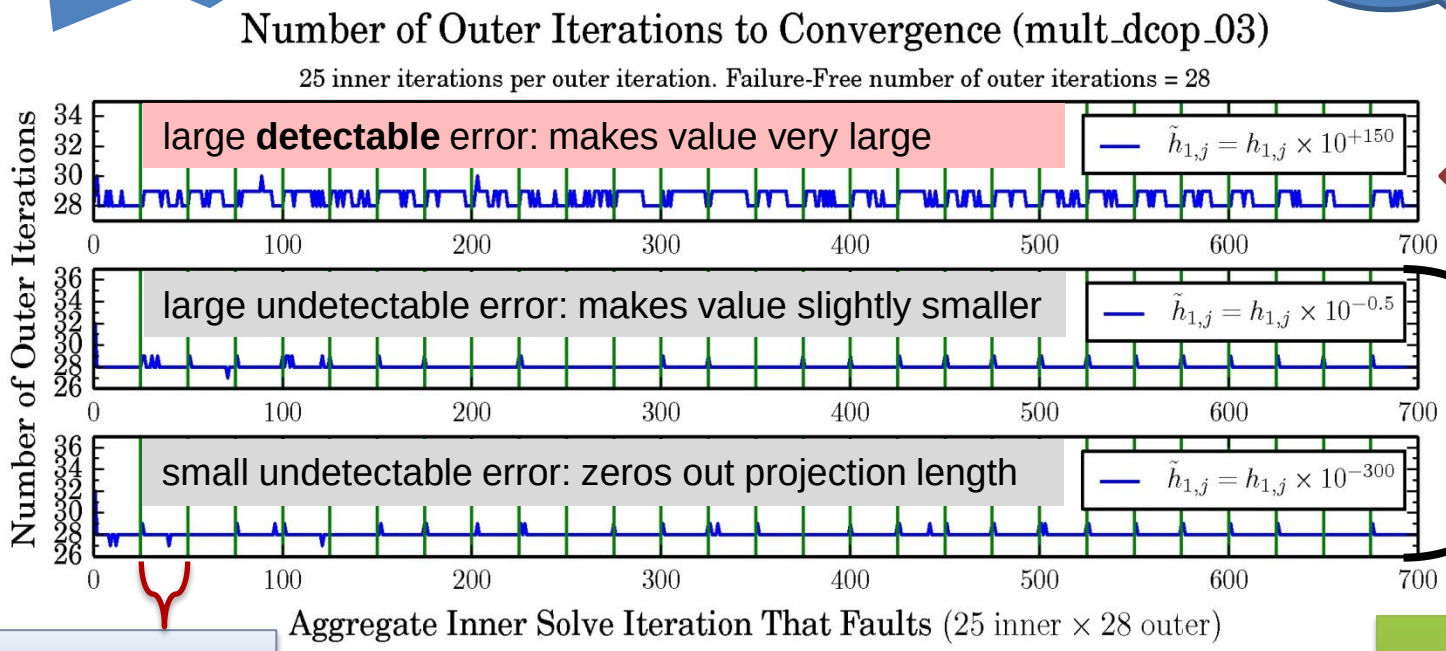

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unbounded error = high variability in time to solution
bounded error = low variability, in time to solution

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Conclusions

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- Enforcing bounds is cheap
- Enables numerical approaches to roll-through errors without “blowing up”

Future Work

- Link norm bound to Inexact Krylov convergence rate (push bounded faults back onto the matrix)
- Investigate link between reliable computations and redundancy (e.g., reorthogonalization vs redundant orthogonalization)

Questions/Comments: {jjellio,mhoemme}@sandia.gov mueller@cs.ncsu.edu

Papers: http://arxiv.org/find/cs/1/au:+Elliott_J

http://www4.ncsu.edu/~jjellio3/

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