

A Comparison of Alternating Direction Swept Remap and Intersection Remap in the X-FEM Alegra Code

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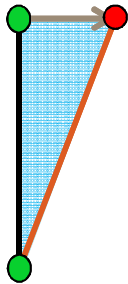
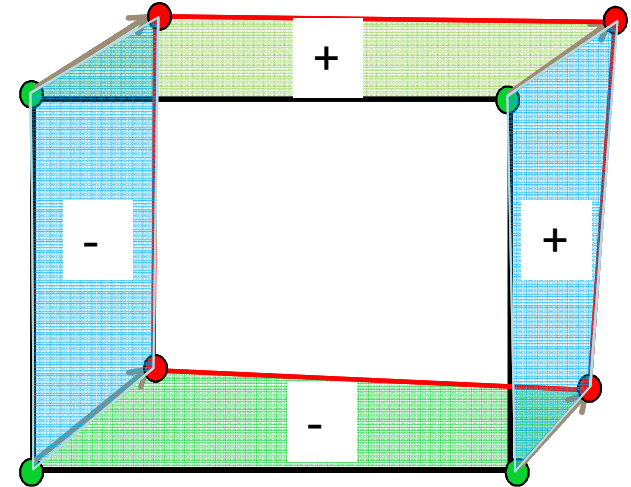
³ tevoth@sandia.gov

Talk Outline:

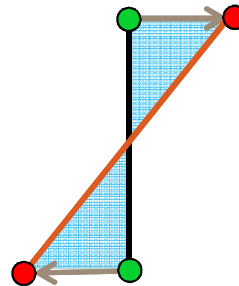
- Overview of swept remap
- Overview of intersection remap
- Differences between the two algorithms
- Comparison test problems
 - Translation at several mesh orientations
 - Expansion of a distribution
 - Translation and shear test
- Preliminary conclusions and future directions

Alternating-Direction, Swept Remap

- Nodal velocities
- End-of-Lagrangian step element
- Next step element
- Fluxes = area swept by an element face
- x fluxes, y fluxes
- Flux order alternates xy, yx, xy, ...
- Scalar flux description is simple, less descriptive
- 1D interpolation uses sweep's facial neighbors



1D interpolation ignores transverse 2nd and 3rd order variations in fluxed values

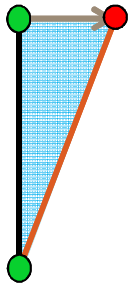


Scalar description misses finer detail of volume exchange

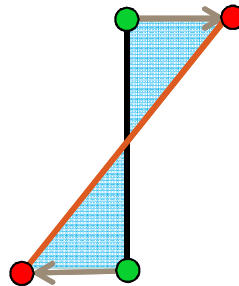
- Simpler algorithm that is computationally fast
- Due to two spatial sweeps, creating an intermediate state, higher dissipation

Intersection Remap

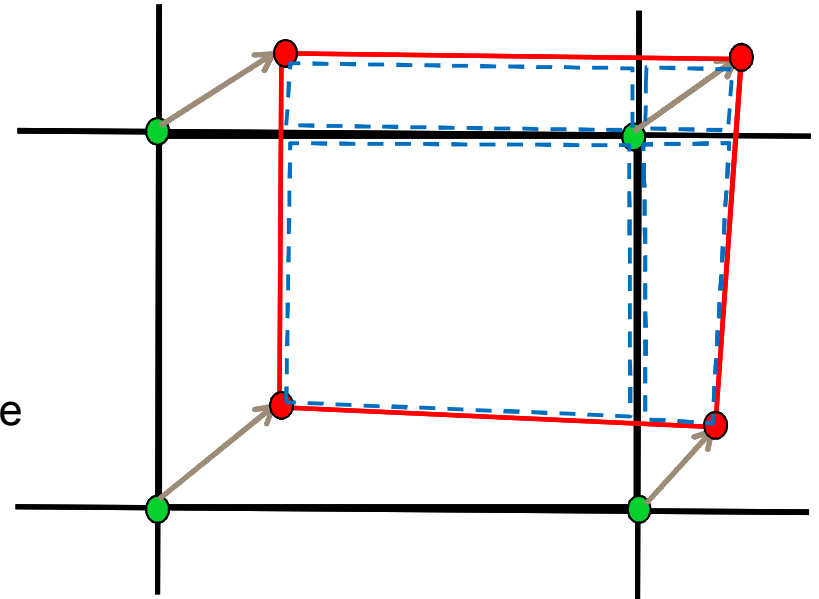
- Nodal velocities
- End-of-Lagrangian step element
- Next step element
- Fluxes = polygonal intersection meshes
- Flux polygons
- Single step
- polygonal flux description is more descriptive
- 2D interpolation uses all donor neighbors



2D interpolation
includes transverse 2nd
and 3rd order variations
in fluxed values



polygonal description
includes finer detail of
volume exchange



- Complex algorithm that is computationally slower
- Due to single remap step dissipation is reduced for smooth flows
- Dissipation may be increased in areas of sharp discontinuity due to monotonic limiting

Translation Test Problem Description

- Gaussian density distribution

$$\rho = 20 \exp^{-10 \left[(x_c - x_{disk})^2 + (y_c - y_{disk})^2 \right] / r_{disk}}$$

- Nodal velocities are specified in input (no momentum fluxing is used)

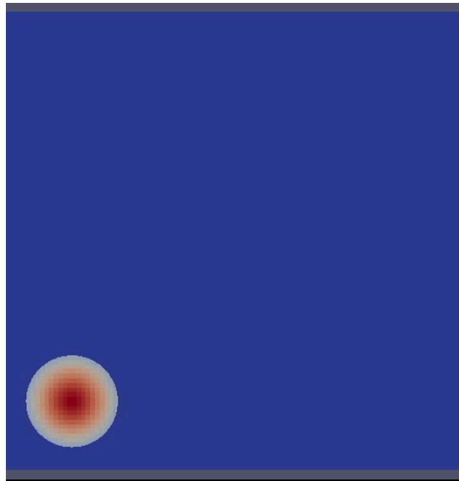
$$\vec{u} = u \cos \theta \hat{i} + v \sin \theta \hat{j}$$

where $u = 1.0$

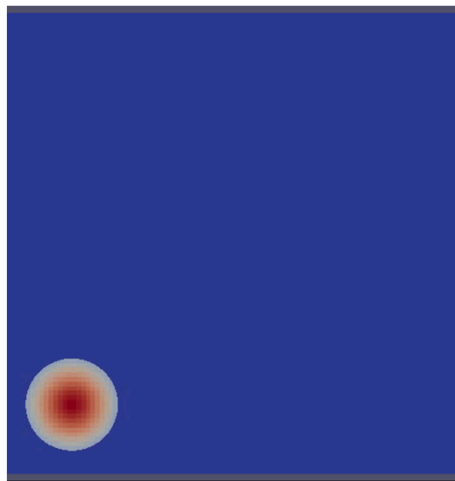
$$\text{if } (t > t_{mid}) u = -1.0$$

- Circle moves away from starting position and back
- Accuracy is measured by L1 error of each element $L_1 \text{Error} = \sum_{n \text{ cells}} V_{\text{element}} (\rho_{\text{end}} - \rho_{\text{start}})$
- Error compares the starting element density in each element with the problem's ending material density
- Second order interpolation is used for both swept and intersection remaps
- Fixed time step for all angles: maximum Courant number 0.4

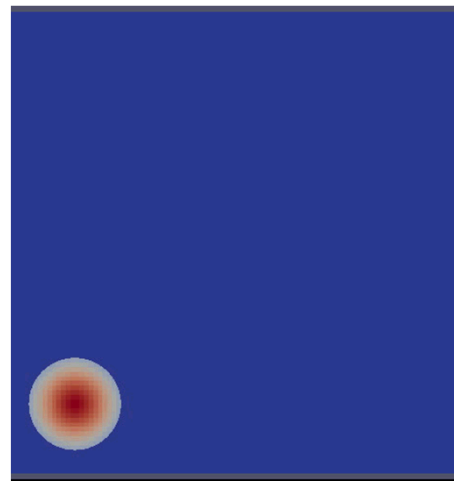
Swept Translation Test Problem



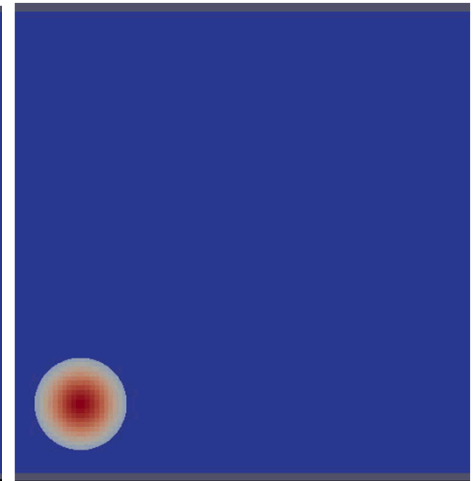
0 degrees



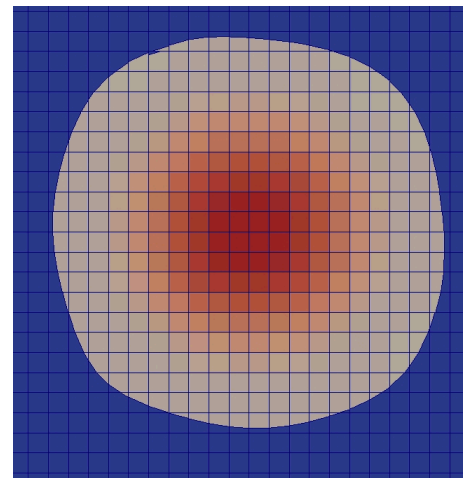
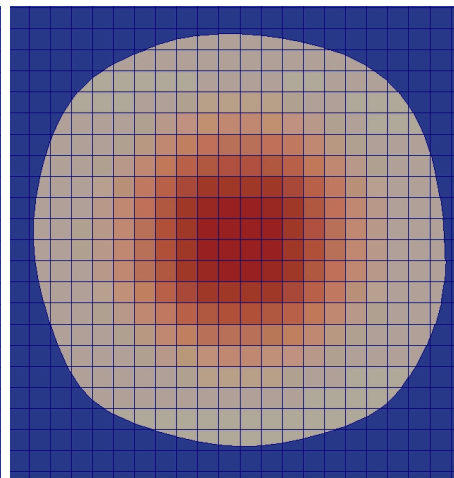
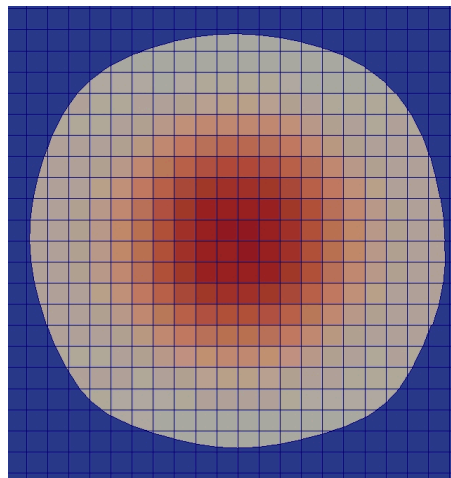
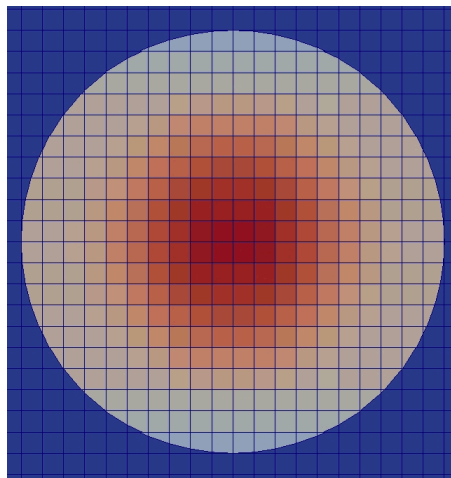
15 degrees



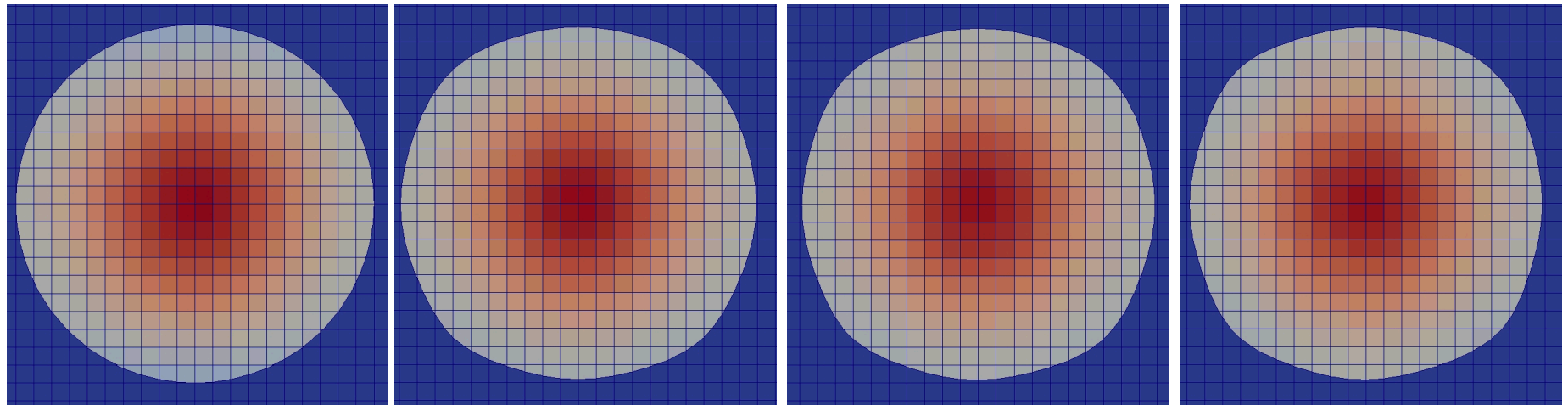
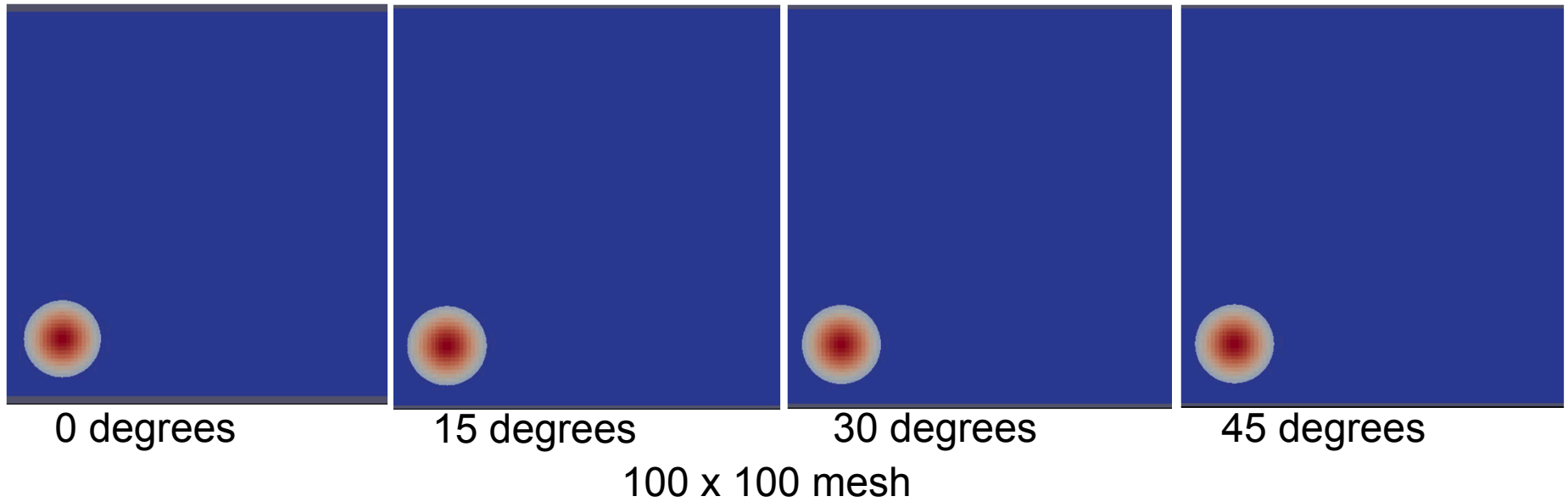
30 degrees



45 degrees

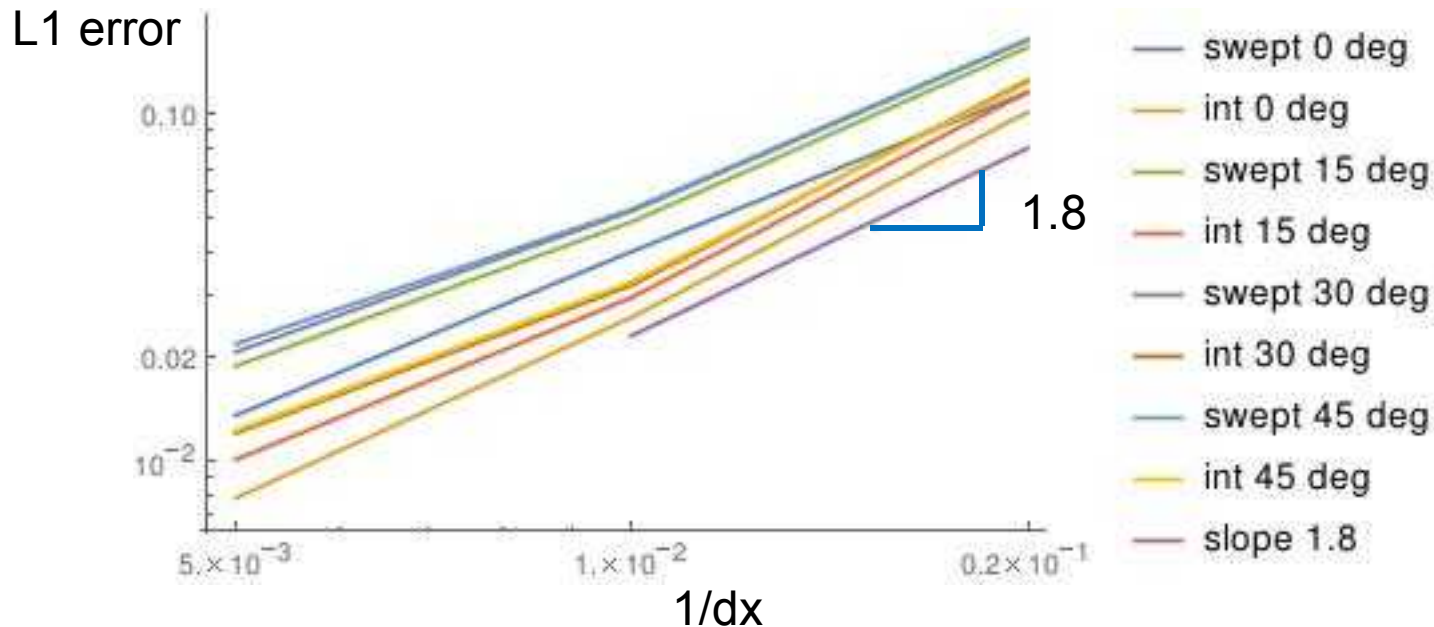


Intersection Translation Test Problem



Translation Test Problem Spatial Error

- Problems run with 3 levels of mesh refinement (1/50, 1/100, 1/200)
- Order of error is between 1.4 and 2.0
- Intersection remap errors are below the corresponding swept remap errors



Expansion Test Problem Description

- Gaussian density distribution

$$\rho = 20 \exp^{-10 \left[(x_c - x_{disk})^2 + (y_c - y_{disk})^2 \right] / r_{disk}}$$

- Nodal velocities are specified in input (no momentum fluxing is used)

$$\theta = \arctan(N_x, N_y)$$

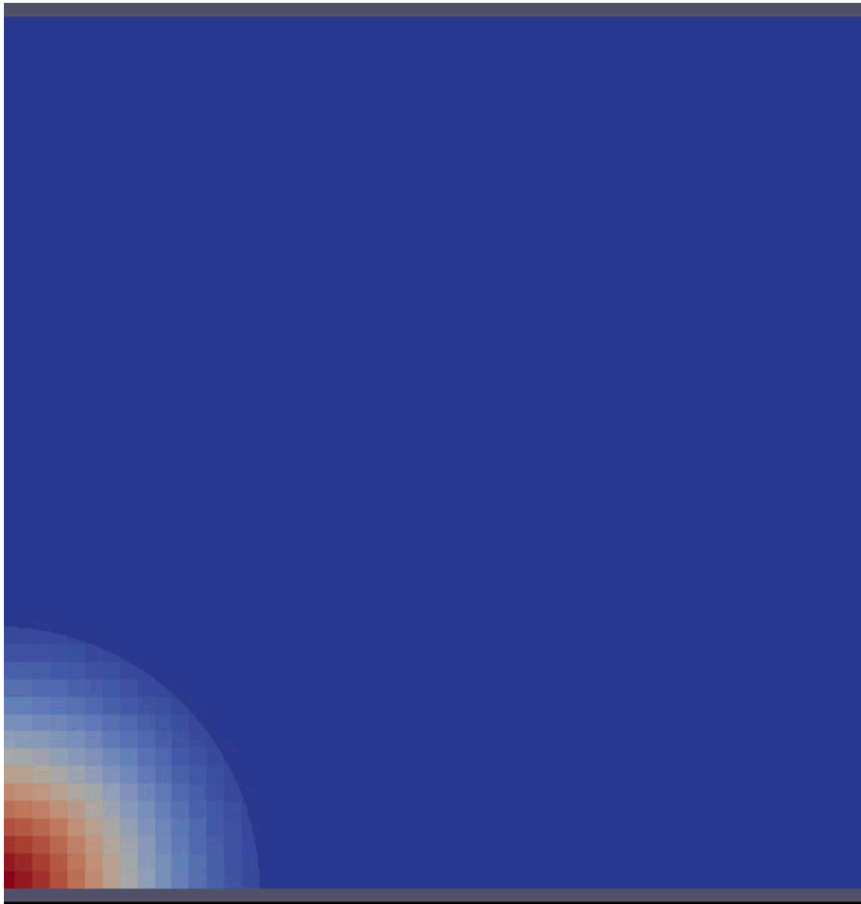
$$\vec{r} = \sqrt{N_x^2 + N_y^2} / 1.414 \vec{u} = u r \cos(\theta) \hat{i} + u r \sin(\theta) \hat{j}$$

where $u = 1.0$

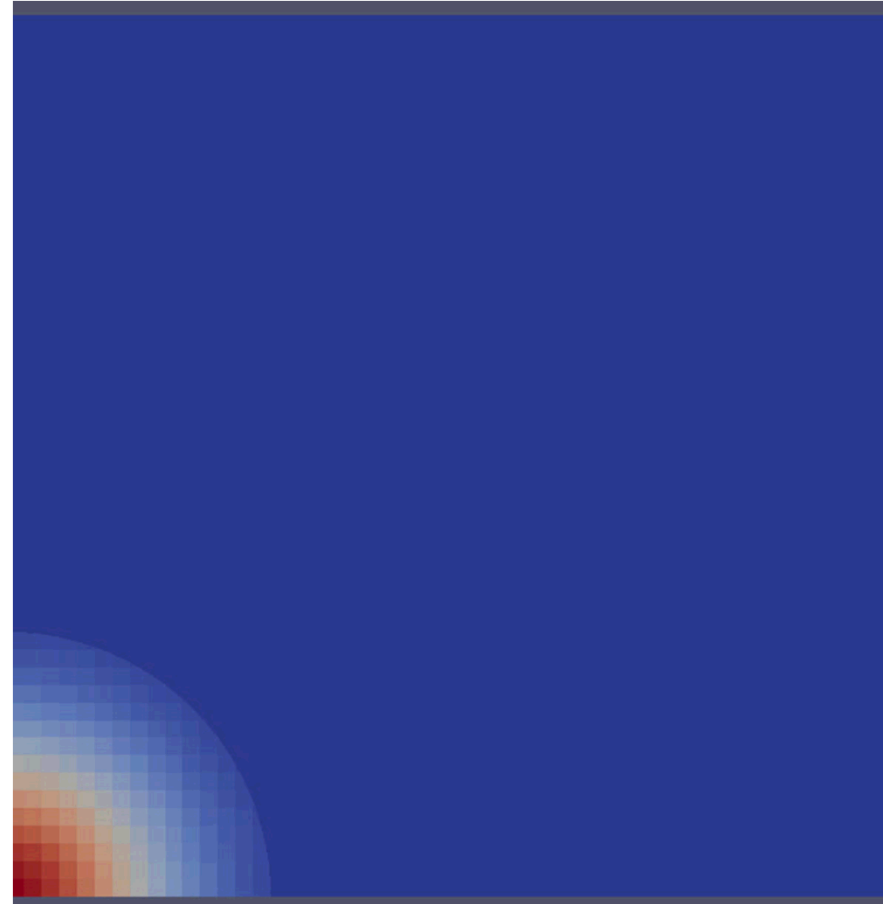
$$\text{if } (t > t_{mid}) u = -1.0$$

- Disk expands away from origin position and back
- Accuracy is measured by L2 error of each element
- Error compares the starting element mass in each element with the problem's ending material mass
- Second order interpolation is used for both swept and intersection remaps
- Fixed time step for all angles: maximum Courant number 0.4

Expansion Test Problem (50 x 50)



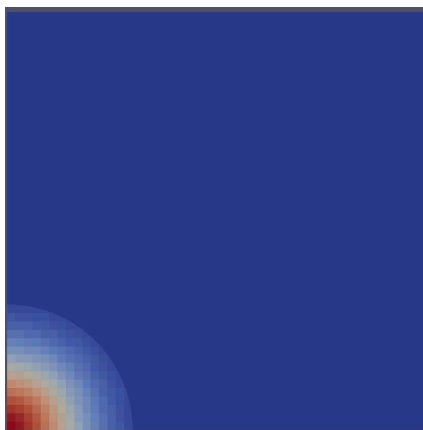
swept



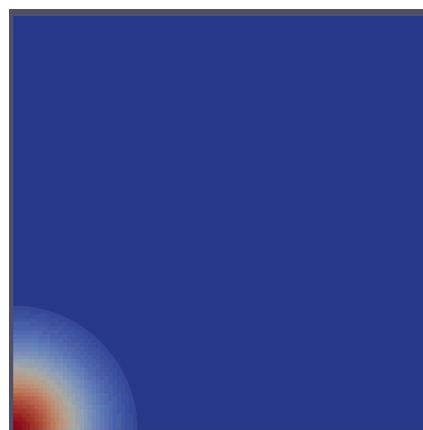
intersection

Expansion Test Problem, Swept

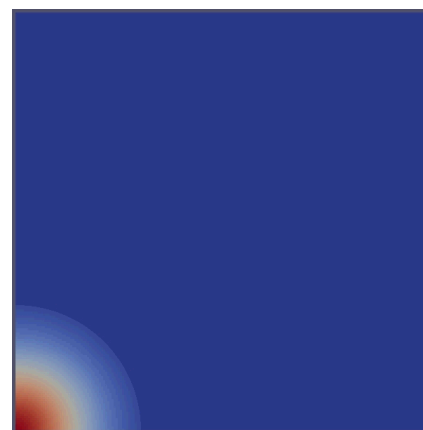
start



$Dx=1/50$

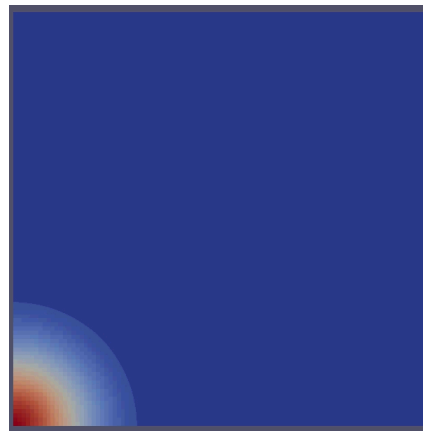
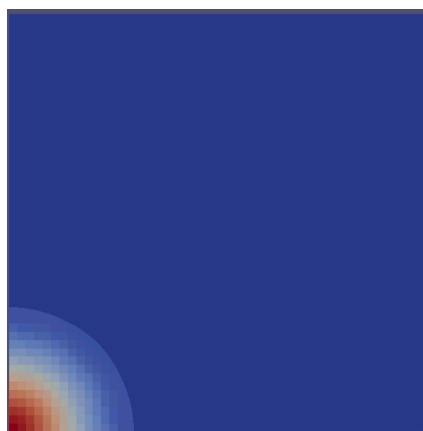


$Dx=1/100$



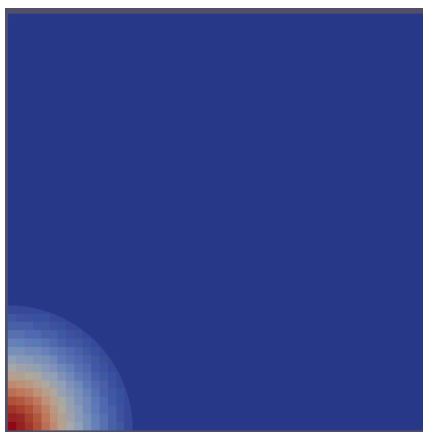
$Dx=1/200$

end

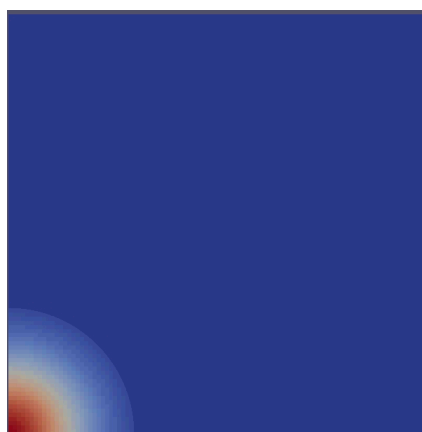


Expansion Test Problem, Intersection

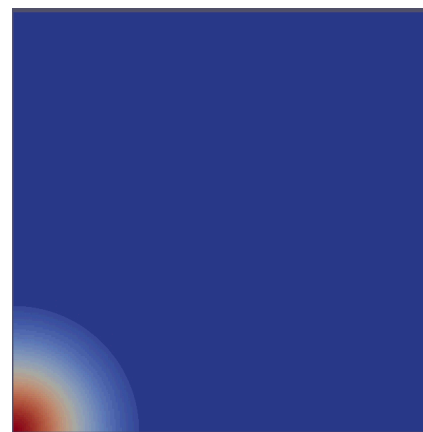
start



$Dx=1/50$

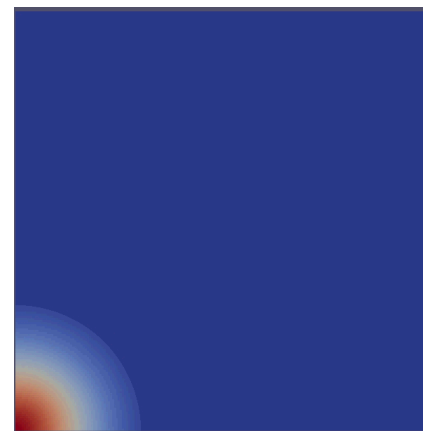
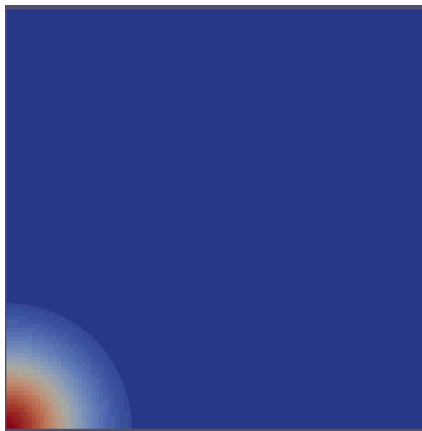
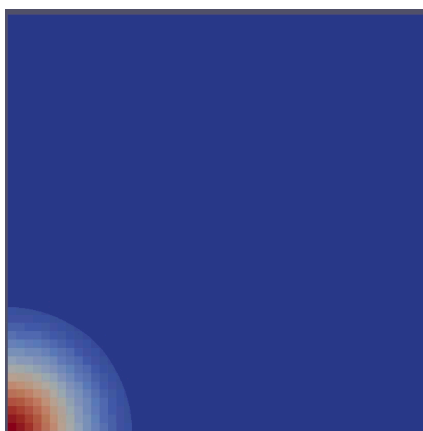


$Dx=1/100$



$Dx=1/200$

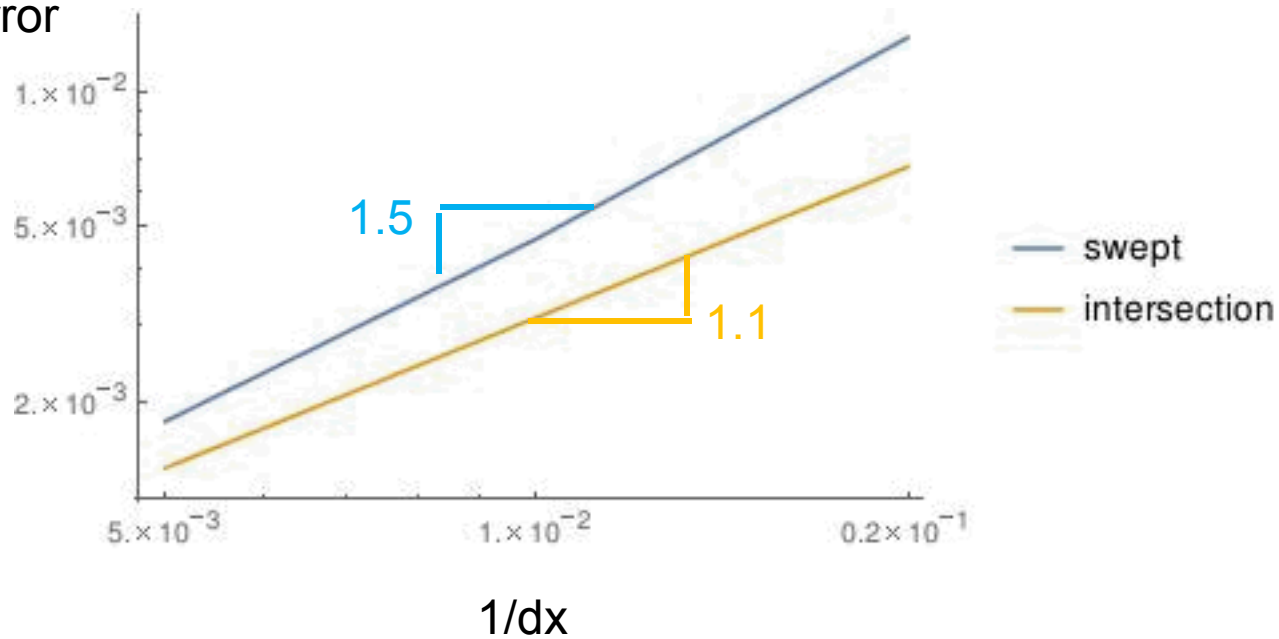
end



Expand Test Problem Spatial Error

- Problems run with 3 levels of mesh refinement (1/50, 1/100, 1/200)

L1 error



Distort Test Problem Description

- Gaussian density distribution

$$\rho = 20 \exp^{-10 \left[(x_c - x_{disk})^2 + (y_c - y_{disk})^2 \right] / r_{disk}}$$

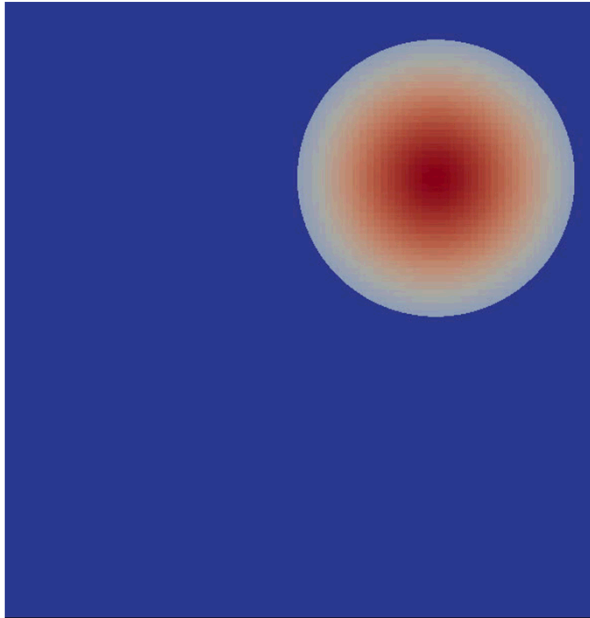
- Nodal velocities are specified in input (no momentum fluxing is used)

$$\vec{V}_x = 2 \sin(2\pi N_y) \sin^2(\pi N_x) \cos(\pi t / t_{end})$$

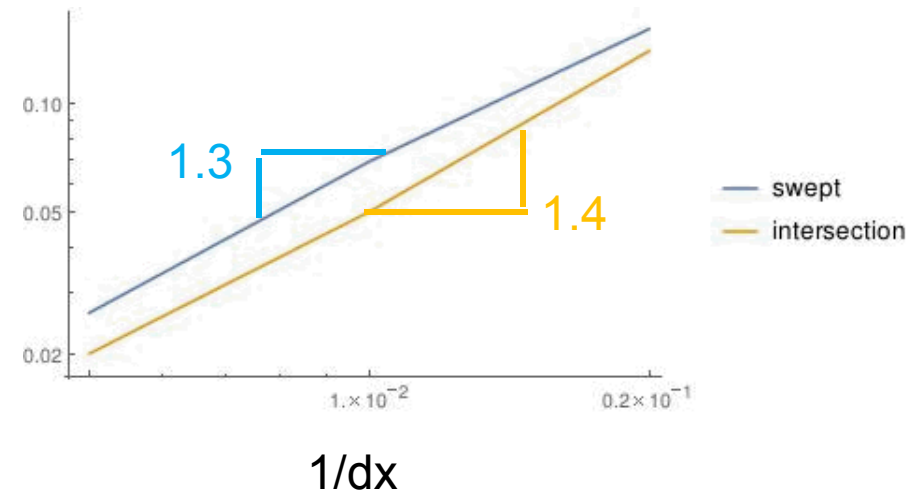
$$\vec{V}_y = -2 \sin(2\pi N_x) \sin^2(\pi N_y) \cos(\pi t / t_{end})$$

- Disk is translated and distorted until motion is reversed and moves back to starting position
- Accuracy is measured by L1 error of each element
- Error compares the starting element density in each element with the problem's ending material density
- Second order interpolation is used for both swept and intersection remaps
- Fixed time step for all angles: maximum Courant number 0.4

Distort Test Problem



L1 error



Intersection remap (200 by 200)

Preliminary Conclusions

- Intersection remap has lower error magnitudes than swept remap
- To conclusively demonstrate that the error reduction is due to the remap algorithm, both must use a common interpolation algorithm
- Both algorithms have approximately the same convergence rate; but the errors for intersection remap are always smaller.
- In translation tests, the significant portion of errors for both remap algorithms is due to curvature errors from under-resolved interface reconstruction