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Quantum Simulation: Classical Algorithms versus Analog Simulators

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Quantum simulation landscape

Fundamental limits of simulating quantum with classical?

exact quantum simulations
exact reference calculations
precise & accurate experiments
digital quantum simulation

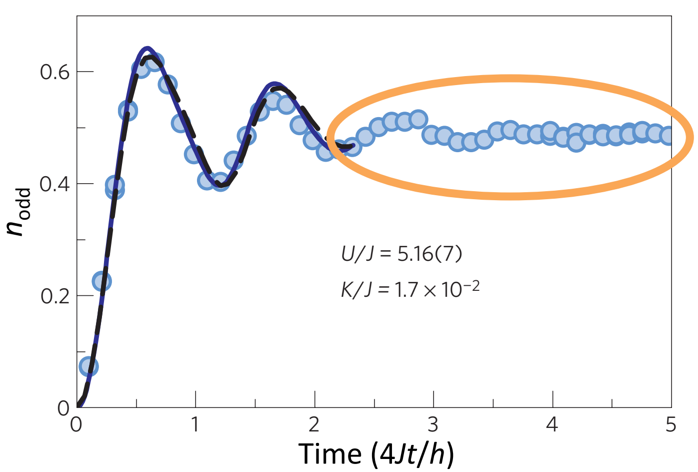
Fundamental limits of quantum power with noise?

approximate classical algorithms
density functional theory
tensor network states
quantum Monte Carlo

How to compare them?

Who wins?

analog quantum simulators
superconducting circuits (D-Wave)
cold atomic gases (I. Bloch)
trapped ion lattices (C. Monroe)



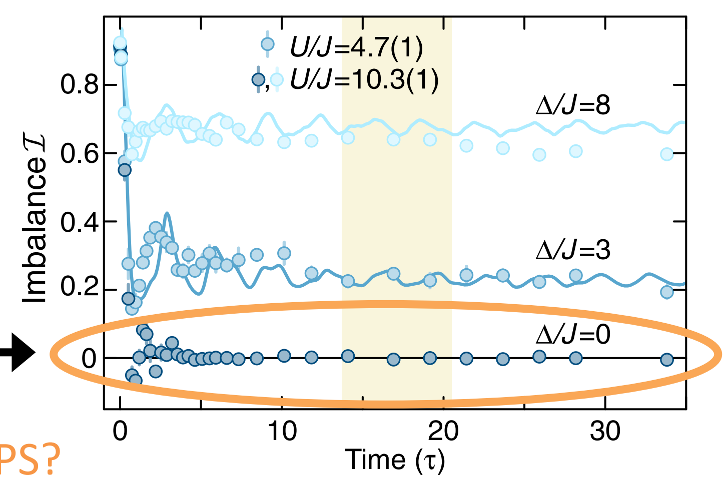
[Nat. Phys. **8**, 325, (2012)]

non-equilibrium time evolution in one dimension

← bosons

fermions →

analog "beats" MPS?



[arXiv:1501.05661 (2015)]

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matrix product states (MPS) & MaxEnt reconstructions

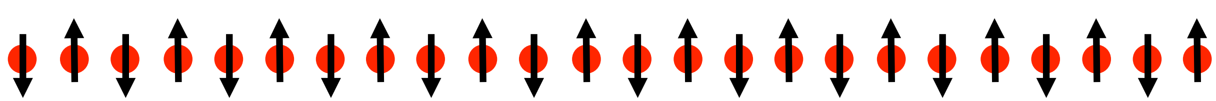
evolve 1D classical AF w/ Heisenberg Hamiltonian

$$H = -\sum_{i=1}^{n-1} J_i \vec{S}_i \cdot \vec{S}_{i+1}$$

random distribution of static Hamiltonian noise

$$J_i = J + \delta J_i$$

$$\delta J_i \in [-\epsilon, \epsilon]$$

At t=0,  (N=16,777,216)

Fair comparisons?

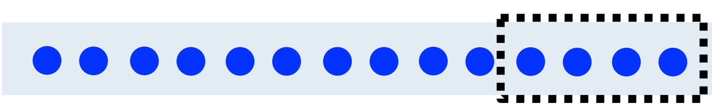
N unknown quantum states = N copies of ψ from exact simulator or N copies of ρ from noisy simulator ?

Quantum Chernoff bound: [Phys. Rev. Lett. **98**, 160501 (2007)]

Probability of wrong guess $\propto \exp(-cN)$ $\qquad c = -\log\langle\psi|\rho|\psi\rangle$

Measuring only a few spins at a time: $c \approx -\log \|\sqrt{\rho_1}\sqrt{\rho_2}\|_{\mathrm{tr}}^2$

fidelity



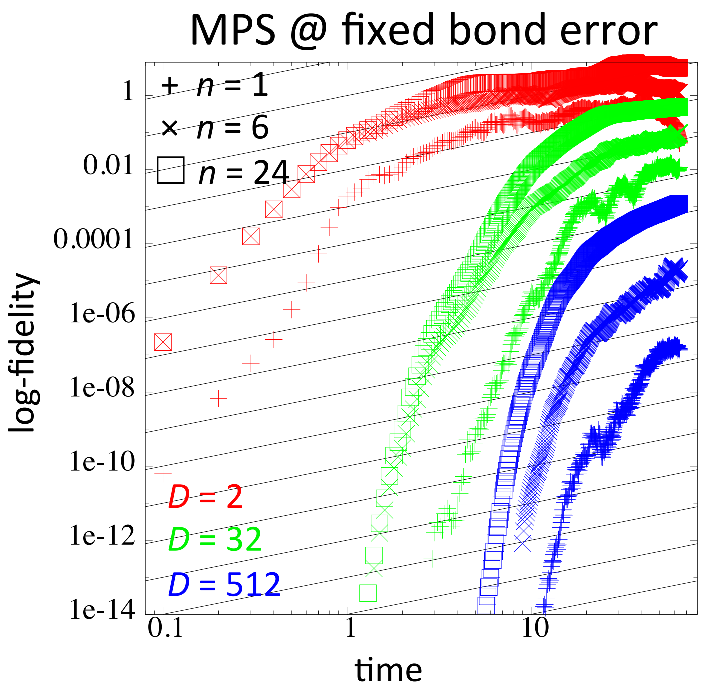
partition: *A* & *B*

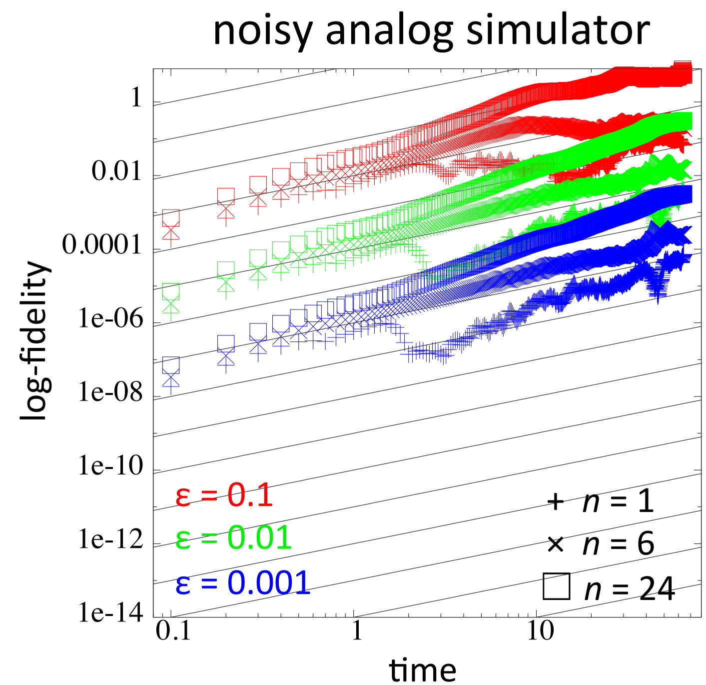
$\rho_1 = \mathrm{tr}_A|\psi\rangle\langle\psi|$ or $\rho_2 = \mathrm{tr}_A\rho$

partial trace

Does this make sense when we aren’t measuring a quantum state?

MPS vs. noisy simulator





MPS @ fixed bond error

bond dimension

MPS @ fixed bond dimension

log-fidelity

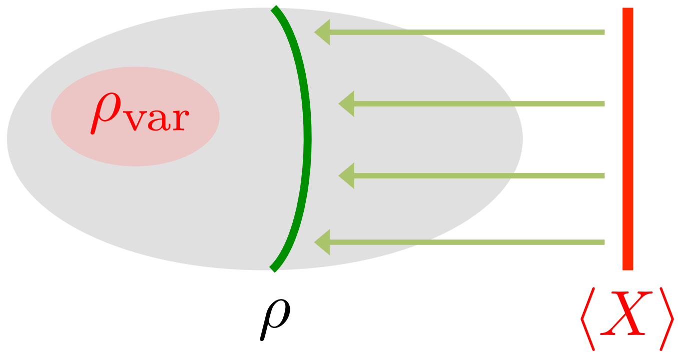
Different comparisons (fixed error vs. fixed dimension) suggest different conclusions!

MaxEnt reconstruction

With limited classical memory,

either
limit the form of
a quantum state

typical structure



or
limit knowledge about
a quantum state

important observables

Reconstruct the quantum state in the latter case,

maximum entropy principle [Jaynes]

Evolve the reconstructed state, [Tishby & Levine, Phys. Rev. A **30**, 1477 (1984)]

$$\frac{d}{dt}\langle X\rangle_t = ih_X\langle X\rangle_t + ih_Y\langle Y\rangle_t$$

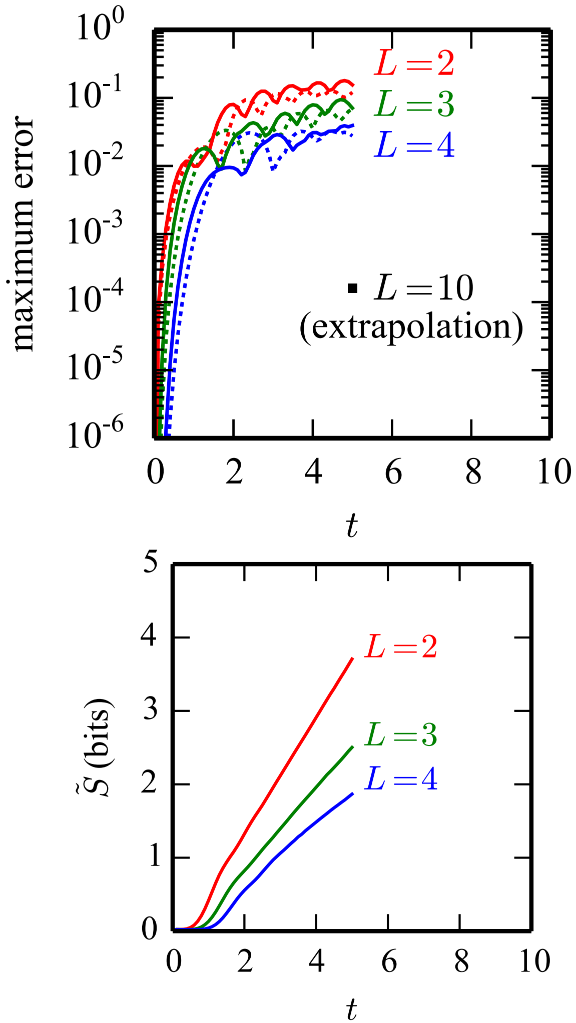
$$\langle Y\rangle_t = \underset{y}{\operatorname{argmax}}\ \tilde{S}(\langle X\rangle_t, y)$$

$$\underbrace{S(\rho) \leq \tilde{S}(\langle X\rangle, \langle Y\rangle)}_{\text{the hard part}}$$

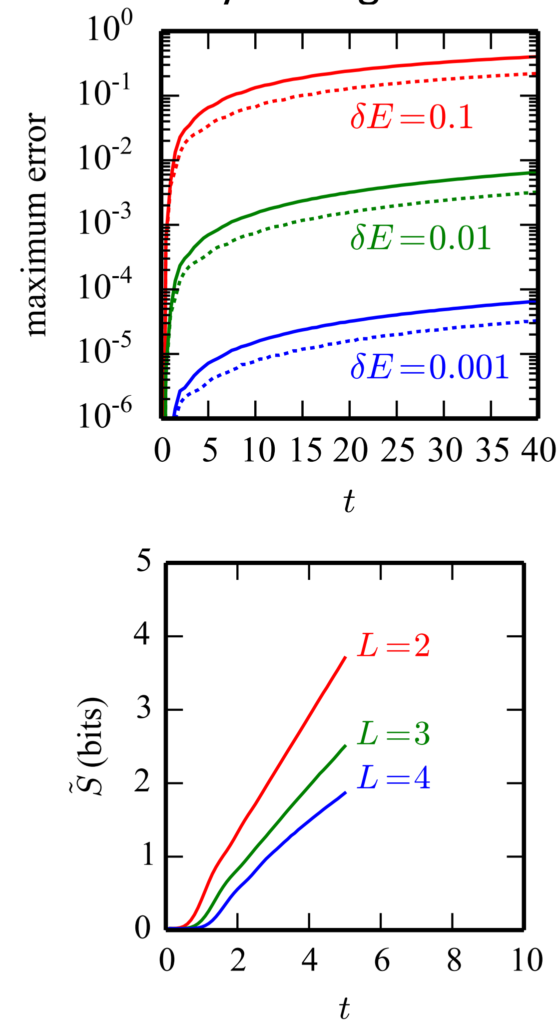
[Hastings & Poulin]

MaxEnt vs. noisy simulator

MaxEnt simulation



noisy analog simulator



Fundamentally similar (highly generic) behavior

Outlook

- Fair & *accessible* comparison of simulations?

  Expectation values as standard simulation output?

  Consistency between approximate simulations in absence of exact simulations?

- Axiomatic derivation of quantum MaxEnt

  Clarify the formal benefits of MaxEnt reconstruction

  Address discontinuity concerns [arXiv:1308.6126]

- Better entropy approximations

  Existing approximation limited to local 1D correlations (similar to MPS)

  New general-purpose convex entropy upper bound *in development*

Do specific tasks warrant special purpose analog quantum simulators (or are they merely steps on the path to digital quantum computers?)

ESCALATION! *more* experiments, *more* numerics, *more* comparisons!