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A NONLOCAL STRAIN MEASURE FOR DIC

D. Z. TURNER, R. B. LEHOUCQ, AND P. L. REU

ABSTRACT. It is well known that that the derivative-based classical approach to strain is problematic when the displacement field is irregular, noisy, or discontinuous. Difficulties arise wherever the displacements are not differentiable. We present an alternative, nonlocal approach to calculating strain from DIC data that is well-defined and robust, even for the pathological cases that undermine the classical strain measure. This integral formulation for strain has no spatial derivatives and when the displacement field is smooth, the nonlocal strain and the classical strain are identical. We submit that this approach to computing strains from displacements will greatly improve the fidelity and efficacy of DIC for new application spaces previously untenable in the classical framework.

1. INTRODUCTION

At the frontier of digital image correlation (DIC) technology lies a formidable challenge: dealing with discontinuous displacement fields (cracks) and steep strain gradients in the context of image noise. This pursuit is particularly difficult for a number of reasons, the primary of which include the fact that the mathematical framework used in conventional methods is not appropriate for this space of fields and that a delicate balance exists between filtering noise from the solution and smoothing out the underlying structure. Certainly, sophisticated alterations to conventional methods are possible to capture discontinuities, but we propose instead an alternative perspective that naturally incorporates this class of problems. This alternative approach is based on nonlocal vector calculus (NLVC).

For relevant background information regarding NLVC, the reader is referred to [1, 2, 3, 4] and the references therein. The main benefit of NLVC is that it provides a mathematically consistent measure of the rate of change for fields that may not be differentiable. Rather than using spatial derivatives, NLVC uses integral operators to express these rates of change. A basic example is the nonlocal gradient operator:

$$\tilde{\nabla}\mathbf{u}(\mathbf{x}) = \int_{-\infty}^{\infty} \mathbf{u}(\mathbf{x})\boldsymbol{\kappa}(\mathbf{y}-\mathbf{x})d\mathbf{y} \tag{1}$$

where $\boldsymbol{\kappa}(\mathbf{y}-\mathbf{x})$ is the kernel. The conditions on the kernel are simply that its integral over the domain is zero. Formally speaking, the kernel should be a distribution or delta function, but for implementation considerations we will engender an approximation of this operator over a finite support by using alternative kernel functions. The nonlocal gradient operator

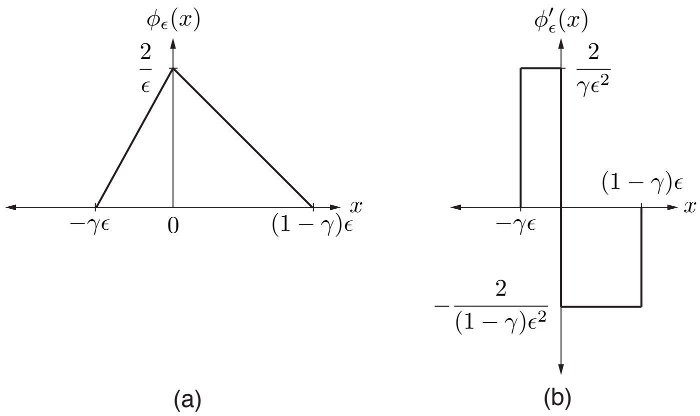

FIGURE 1. (a) Kernel function $\phi_\epsilon(x)$ and (b) its derivative that can be used to construct a multidimensional kernel $\boldsymbol{\alpha}(\mathbf{y}-\mathbf{x})$. In this case, γ is a parameter that can be used to skew the hat function.

then becomes

$$\tilde{\nabla}\mathbf{u}(\mathbf{x}) = \int_{-\epsilon}^{\epsilon} \mathbf{u}(\mathbf{x})\boldsymbol{\alpha}(\mathbf{y}-\mathbf{x})d\mathbf{y} \tag{2}$$

where $\boldsymbol{\alpha}(\mathbf{y}-\mathbf{x})$ is the kernel with finite support. An example of such a kernel is the derivative of a hat function as shown in Figure 1. In the discrete setting one may recognize that for regular grids, the above nonlocal operator takes the form of a convolution. This is the point of similarity with other methods for computing strain [5, 6]. In convolution form, the nonlocal gradient becomes

$$\tilde{\nabla}\mathbf{u}(\mathbf{x}) = \mathbf{c}(\mathbf{x}) \star \boldsymbol{\alpha}(\mathbf{y}-\mathbf{x}) \tag{3}$$

In the convolution form above, the kernel remains the same as in the integral operator of equation (2). The coefficients $\mathbf{c}(\mathbf{x})$ can be chosen depending on the desired properties of the resulting operator. If $\mathbf{c}(\mathbf{x})$ is chosen such that $\mathbf{c}(\mathbf{x}) = \mathbf{u}(\mathbf{x})$ the resulting operator will be interpolating, but will be subject to oscillations when the gradients are large. Alternatively, the coefficients can be solved for in the frequency domain such that the convolved gradient field is interpolating at the data points. This requires extra computational expense, but enables greater flexibility in the choice of $\boldsymbol{\alpha}(\mathbf{y}-\mathbf{x})$.

Regardless of the kernel choice or how the coefficients are computed, what has been outlined above is a general framework for computing gradients that does not require spatial derivatives. Also, we point out that the analyst can still choose the form of the strain measure (for example, Green-Lagrange or Almansi, etc.). We have merely defined a way to compute the necessary terms in each of these by evaluating the rates of the change of the displacement field.

2. Comparisons among existing methods

To make the presentation more clear, we will focus this work on casting the nonlocal approach in the context of existing methods so that the differences can be pointed out. Along the pathway from displacement data to strain, there are a number of techniques available. For the most part, the conventional methods involve curve fitting.

2.1. **Savitzky-Golay filtering.** One common approach involves the use of a *virtual strain gauge* to compute strain. While at first glance it may seem that there are a number of similarities between Savitzky-Golay (SG) inspired strain measures (like the virtual strain gauge) and the nonlocal approach, the two are in reality very different. SG filters are based on the idea that the underlying displacement field can be fit with a polynomial representation. The derivatives of the displacement field are then computed using the coefficients of the fitted polynomial. The similarity between the SG filter approach and the nonlocal approach comes from the convolution used to compute the strains (that includes the least-squares fit of the data on a regular grid). Although the form of the operator is similar to the nonlocal form, the basic assumption that the data be a continuous polynomial in form is not necessary in the nonlocal approach. The SG filter approach is limited to displacement fields that can be approximated by the polynomial basis of the filter. For example, a linear polynomial leads to a strain measure that cannot capture points of inflection because the polynomial itself has no points of inflection. In contrast, in the nonlocal approach, a linear polynomial kernel function will still capture points of inflection.

In a broader context, the SG filter is one instance of a curve fitting approach. Some of the high level differences between curve-fitting approaches and the NLVC approach to going from displacement values to strains are shown in Figure 2.

2.2. **Digital signal processing.** Other methods of constructing a strain measure are also possible via adapting ideas from digital signal processing (DSP). For example, b-spline interpolation has become very popular because of its high signal to noise ratio and compactness of the support needed to perform interpolation with high order of accuracy [7, 8, 9, 10]. While these methods are used extensively for interpolating image intensity values and computing image gradients, they are less used for computing strains in a similar fashion. Rather than apply the spline-based kernels to the intensity values, one could use as data the displacement values and compute the rates of change necessary for a strain measure. Following this line of reasoning, many of the known relationships between interpolation kernels and the resulting regularity of the reconstructed signal or the order of accuracy can be repurposed to aid in the construction of alternative differentiating filters. The key difference between the DSP approach and the general nonlocal approach is that for DSP, the filters are primarily driven by reducing blurring or aliasing rather than the smoothness considerations of the derivative field. Additionally, DSP theory is predicated on continuity assumptions of the

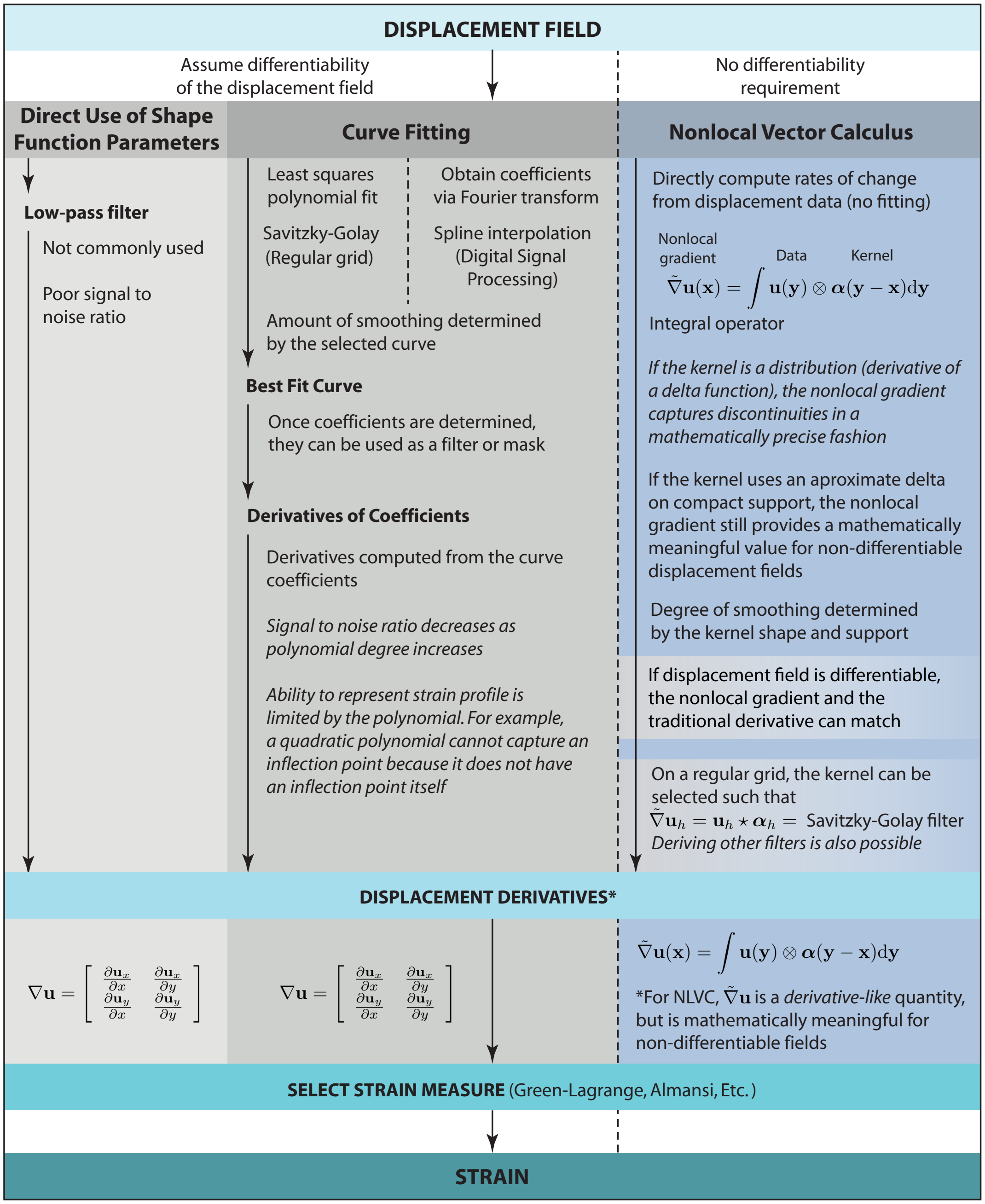

FIGURE 2. Comparison of NLVC with conventional methods.

analogue signal. In a practical sense, this implies that interpolation-based operators may not produce high quality results when used to compute strains because the focus of their development was not on the primary objectives for a strain measure. Further, although DSP ideas and the generalized nonlocal approach to computing strains have a number of similar features, the space of fields appropriate for the nonlocal approach is broader in that it includes discontinuous functions, where as the DSP space does not.

3. Conclusions

We have outlined in this paper, the basic idea behind a new class of strain measures for DIC motivated by NLVC and shown how this framework is different than existing methods. The primary difference between the nonlocal strain measure and SG filters is that curve fitting is not inherent in the nonlocal approach. Also, the nonlocal approach is more general than the straightforward application of DSP ideas to computing derivatives of the displacement field because the nonlocal approach does not assume that the underlying signal is continuous or differentiable. Clearly, a number of implementation details have been omitted from this work as this will be the focus of a forthcoming work on how to construct a nonlocal kernel such that good signal to noise properties are obtained. The purpose of this work was to contextualize the nonlocal approach among other existing methods.

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Correspondence to: Daniel Z. Turner, Multiscale Science, Sandia National Laboratories, P.O. Box 5800; Albuquerque, New Mexico 87185. *E-mail address:* dzturne@sandia.gov, *Phone:* (505) 845-7443

Richard B. Lehoucq, Computational Mathematics, Sandia National Laboratories, P.O. Box 5800; Albuquerque, New Mexico 87185. *E-mail address:* rblehou@sandia.gov, *Phone:* (505) 845-8929

Phillip H. Reu, Thermal/Fluid Experimental Sciences, Sandia National Laboratories, P.O. Box 5800; Albuquerque, New Mexico 87185. *E-mail address:* plreu@sandia.gov, *Phone:* (505) 284-8913