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Fault Tolerance in Adiabatic Quantum Computing

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LDRD

Fault Tolerance

- A computation is *fault tolerant* if its output is unchanged by errors along the way.

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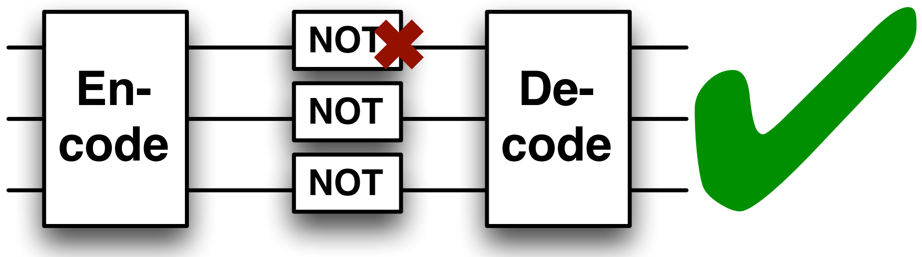
Fault Tolerance

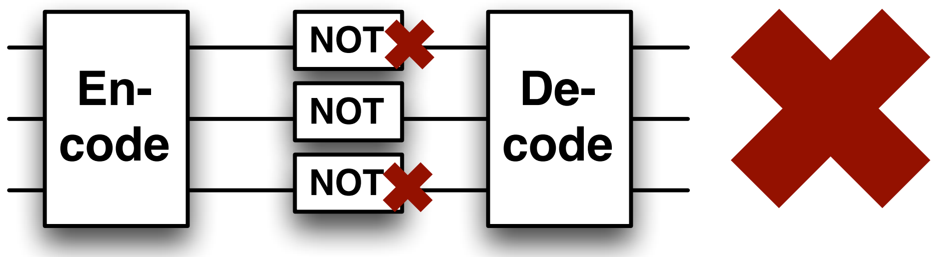
- A computation is *fault tolerant* if [the relevant part of] its output is unchanged by [a certain number/kind of] errors along the way.

- Fault tolerance almost always involves *encoding* the computation into a *logical subsystem.*

$$\{0,1\} \to \{000,111\} \qquad \text{“0”}_{\text{logical}} = \{000,100,010,001\}$$

- FT computations have a *threshold* for error density.





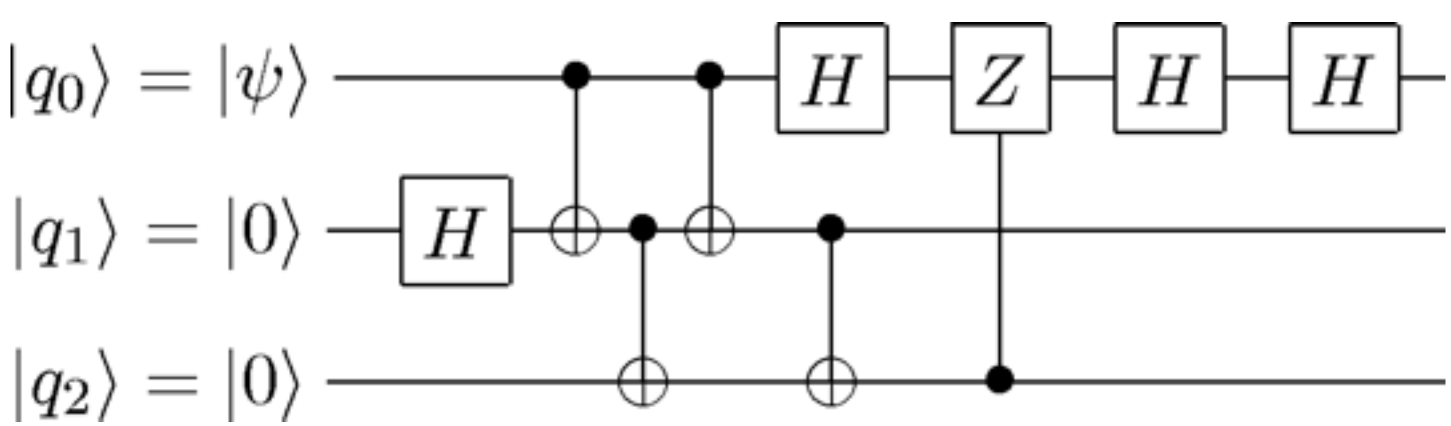
FT in the circuit model

1. The standard error model

- Quantum noise = “what happens in the real world”
  = pretty subtle and complicated!

- Many models. Varying complexity / realism.

- But *most* of FT theory starts with
  weak, **local**, **stochastic**, **i.i.d.**, **depolarizing** errors.



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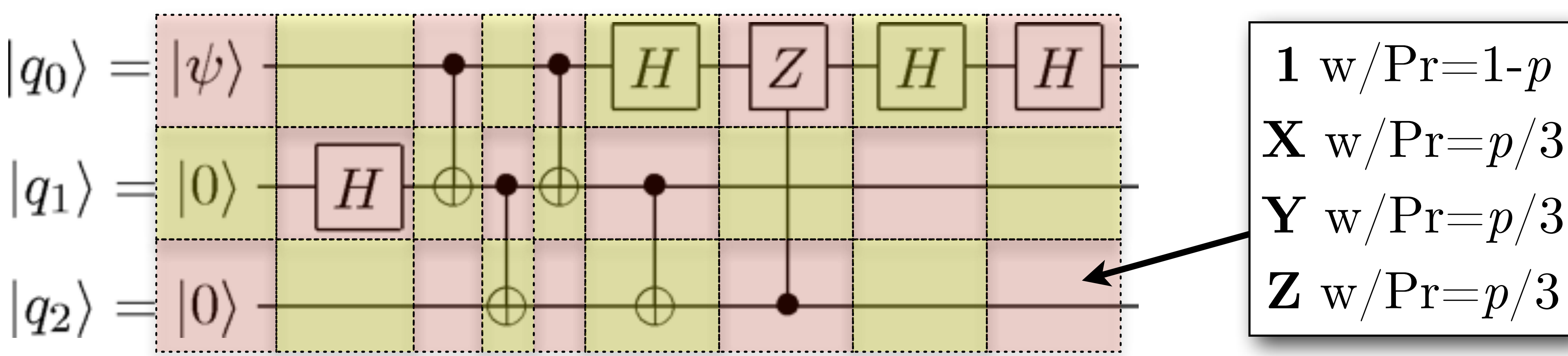
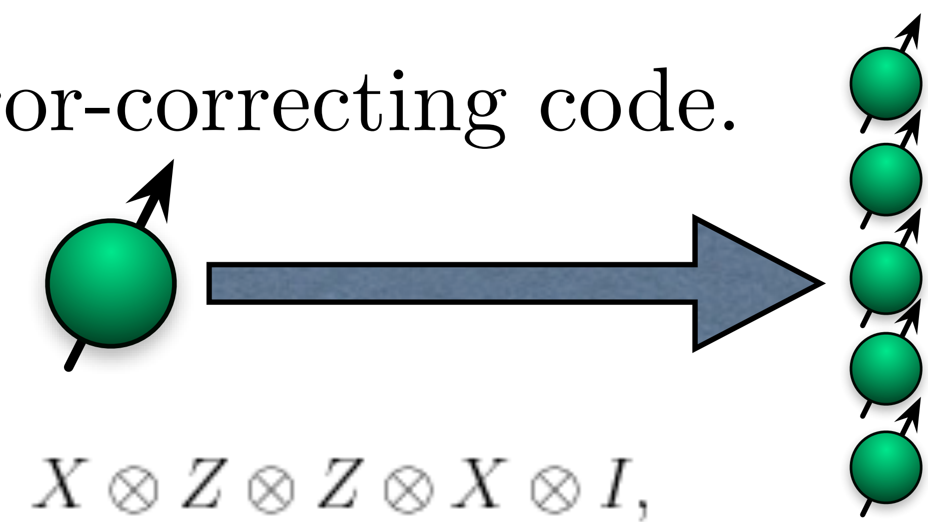


FT in the circuit model

2. Error correcting codes

- Ideal quantum circuits are *reversible*. So errors will propagate to the output and screw it up.

- So we *encode* in a quantum error-correcting code.



- Each 1 qubit ==> n qubits.

- Periodically measure n-1 *parity checks* ("stabilizers") to detect *low-weight* errors.

$$X \otimes Z \otimes Z \otimes X \otimes I,$$
$$I \otimes X \otimes Z \otimes Z \otimes X,$$
$$X \otimes I \otimes X \otimes Z \otimes Z,$$
$$Z \otimes X \otimes I \otimes X \otimes Z.$$

- Once detected, the error can be reversed -- *without* disturbing (or learning about!) the encoded qubit.

FT in the circuit model

3. Fault tolerant logic operations

- Encoding + active error correction ≠ fault tolerance!
- Error correction only corrects *storage* errors.
  (1) Encoded computation requires *logical operations.*
  (2) We do EC using gates. What if *they* have faults?
- Fault tolerant logic possible, but really sophisticated!
  SOME PRINCIPLES:
  1. Do EC/logical operations fast (*constant depth* circuits)
  2. Minimize *error propagation* (avoid FANOUT-type stuff)
  3. Always double-check measurements (time redundancy)
  4. Never decode along the way (makes you vulnerable).

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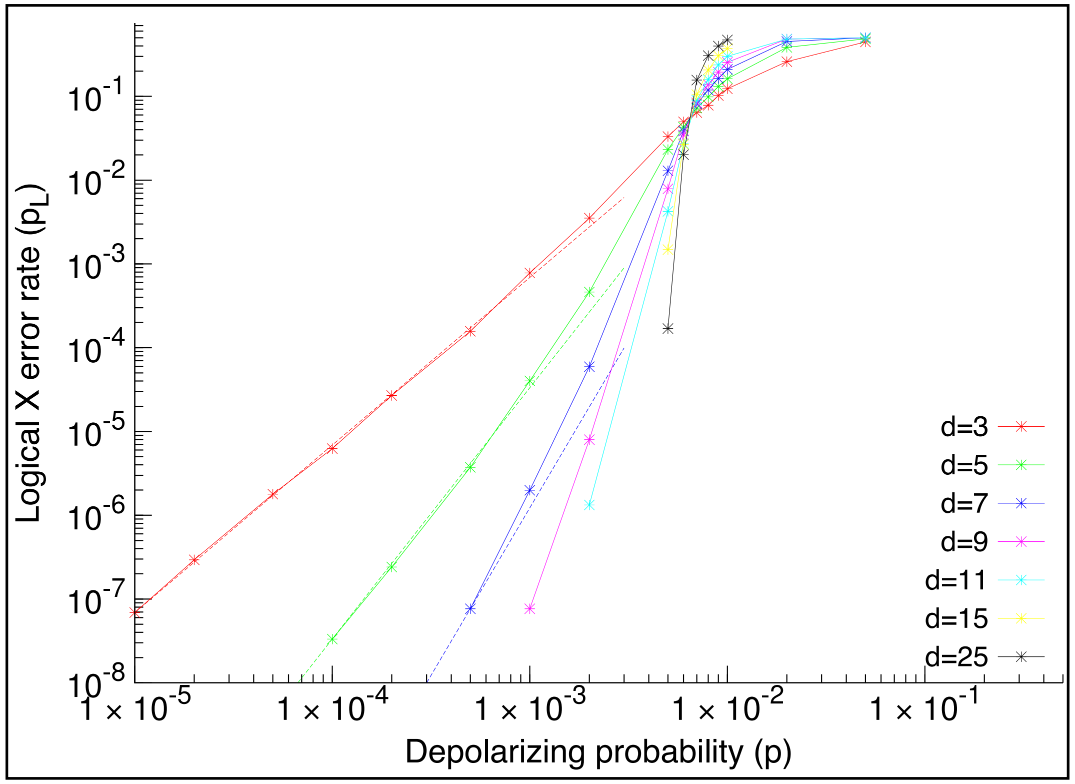
FT in the circuit model

4. Thresholds & overheads

- Fault tolerance *is* possible...
  (at least against the standard error model + a bit more)
  ...but it is *hard* and *expensive*.



- Each scheme has a *threshold* error rate (10^{-2} - 10^{-4}):
  - → Error rate above threshold? **Computation fails!**
  - → Error rate below threshold? **Computation works!**
    - ↳ Encoding requires lots of extra qubits (*overhead*).
      - ↳ Overhead gets very bad near the threshold.

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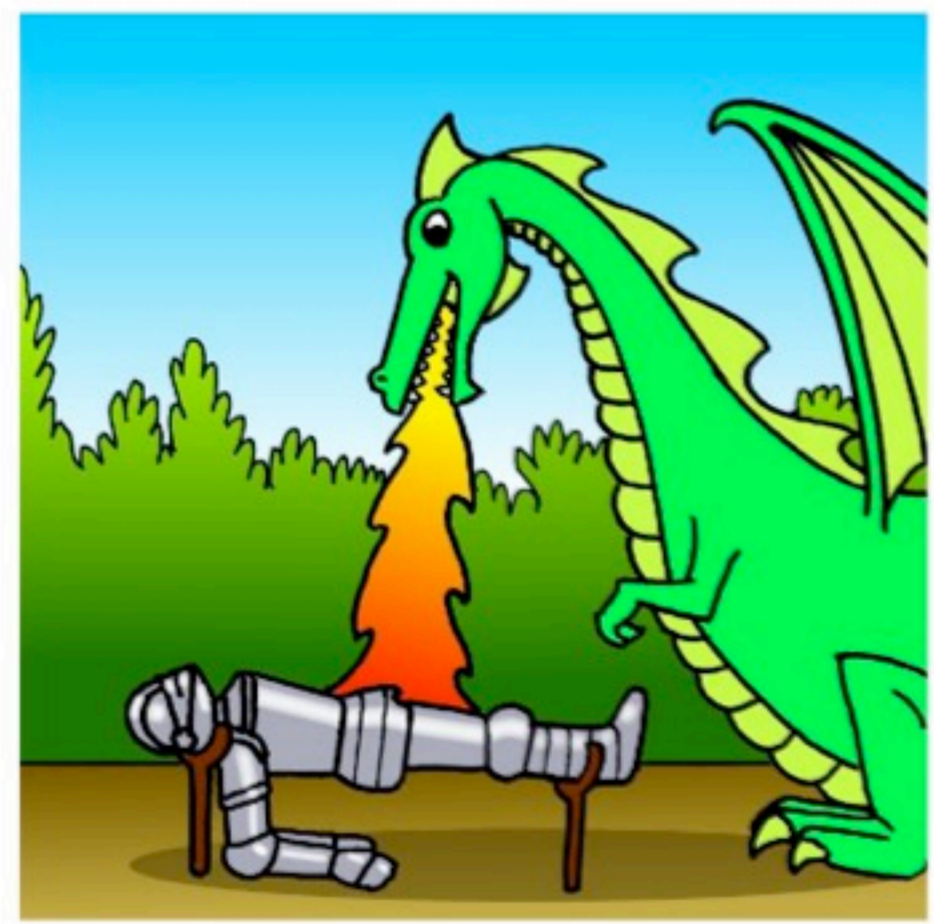
Is AQC intrinsically FT?

- Adiabatic QC is *both*:
  (1) an algorithm
  (2) an architecture (good for that algorithm)
- Viewed as an *architecture*, AQC looks like it ought to be resilient to many forms of error / noise.
- This doesn't mean that it's naturally FT, though!



Dragons knew that when roasting a knight, the the armor really helped to seal in the juices.

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Is AQC intrinsically FT?

1. "History state" AQC is fragile

- If you want to run an arbitrary quantum algorithm (Shor, Grover, etc) as an AQC, then you would use the "History State" construction (Kitaev, Aharonov et al).
- This proves equivalence -- with poly(N) overhead -- of the *noise-free* models (circuits <==> AQC).
- Unfortunately, it requires serializing the circuit...
  ...which completely destroys any fault tolerance it might have (FT requires gates in parallel).
- History state AQC is horribly fragile to errors.

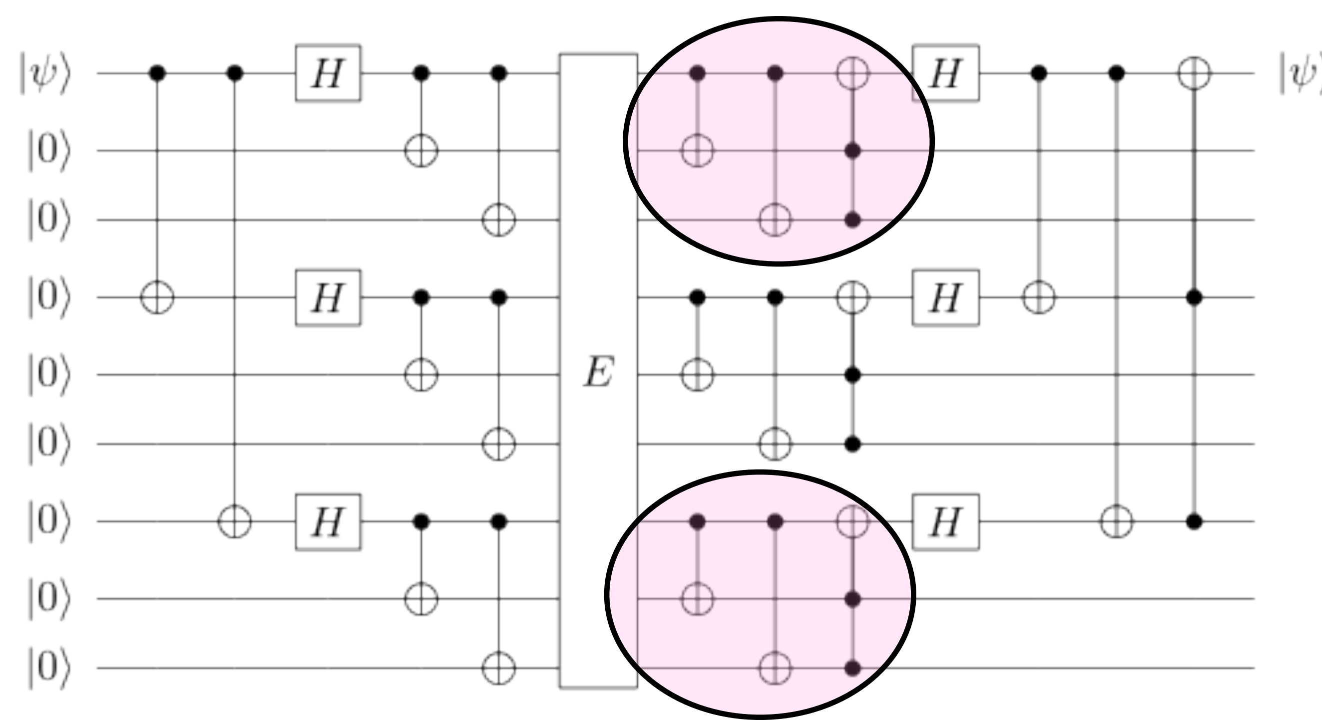
Is AQC intrinsically FT?

2. The enigma of ground state QC

- "Ground State Quantum Computing" is a proposal for mapping [FT] circuits to [FT] adiabatic computations (Ari Mizel, `arXiv:1403.7694`).

- So, is this a fault tolerant adiabatic architecture???

- Compelling counter-argument given by Hastings [1].



[1] https://scirate.com/arxiv/1403.7694

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Is AQC intrinsically FT?

3. Challenges for AQO (annealing)

- Adiabatic quantum optimization seems the most likely to be robust.
- Immune to dephasing and path variations. Gap provides *some* protection against heating.
- Cooling can only help.
- However... as the computation scales up, gap typically gets small, and heating / bitflip errors will become fatal. **Not intrinsically FT**.

Fault Sources in AQC

- No computation can be tolerant to every *conceivable* kind of fault.
- FT constructions are designed against specific error models.
- We need to know what we are defending AQC against!
- This is not a simple question -- and it's probably not the same answer as for the circuit model.

Fault Sources in AQC

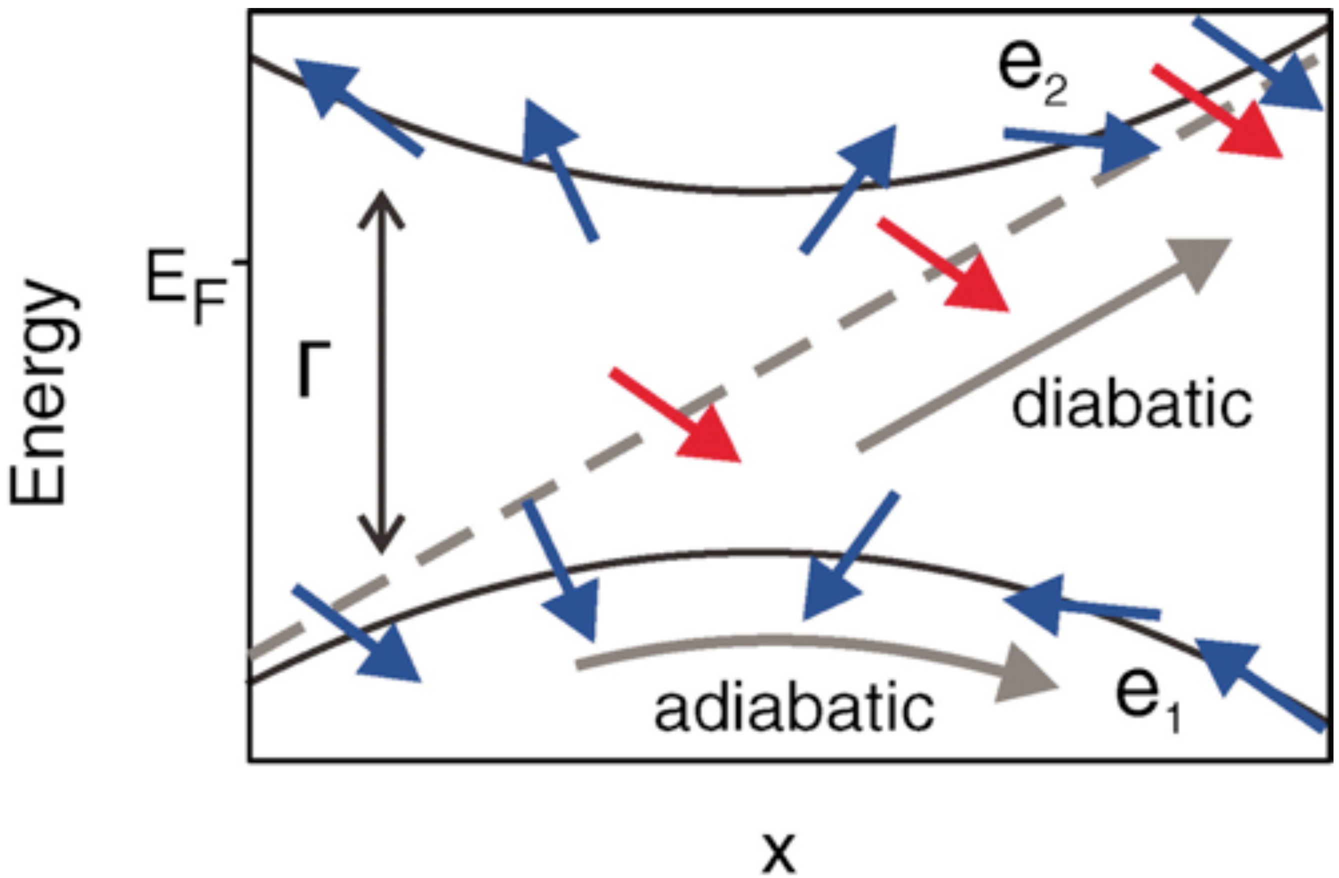
1. Landau-Zener transitions

- The simplest "error" in AQC is a transition out of the ground state because you changed H too fast.

- Not really an error if you just ran the AQC too fast!

- Can be "error" if due to accidental jitter in H(t). This appears as heating.

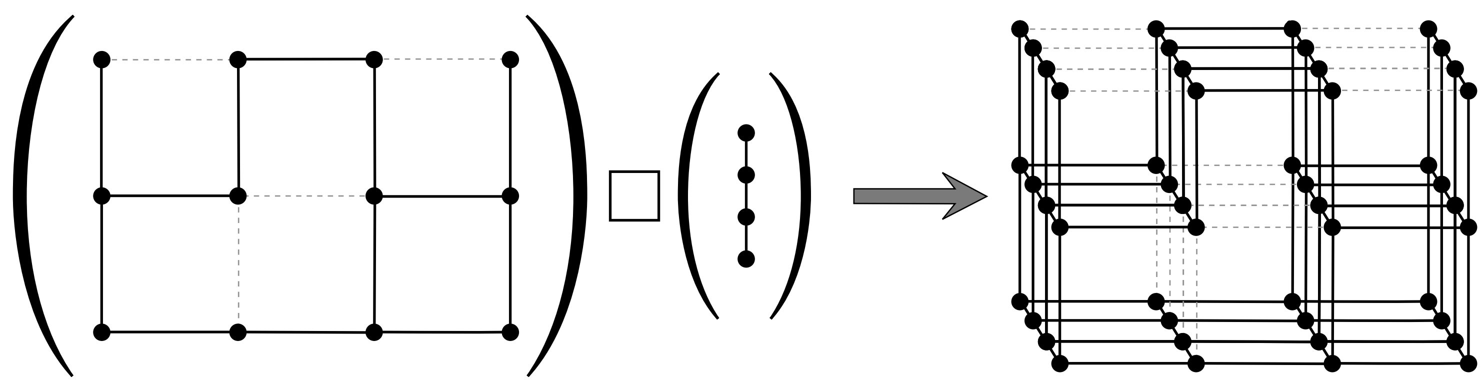
- **Note**: can (& will) jump to higher excited states.



Fault Sources in AQC

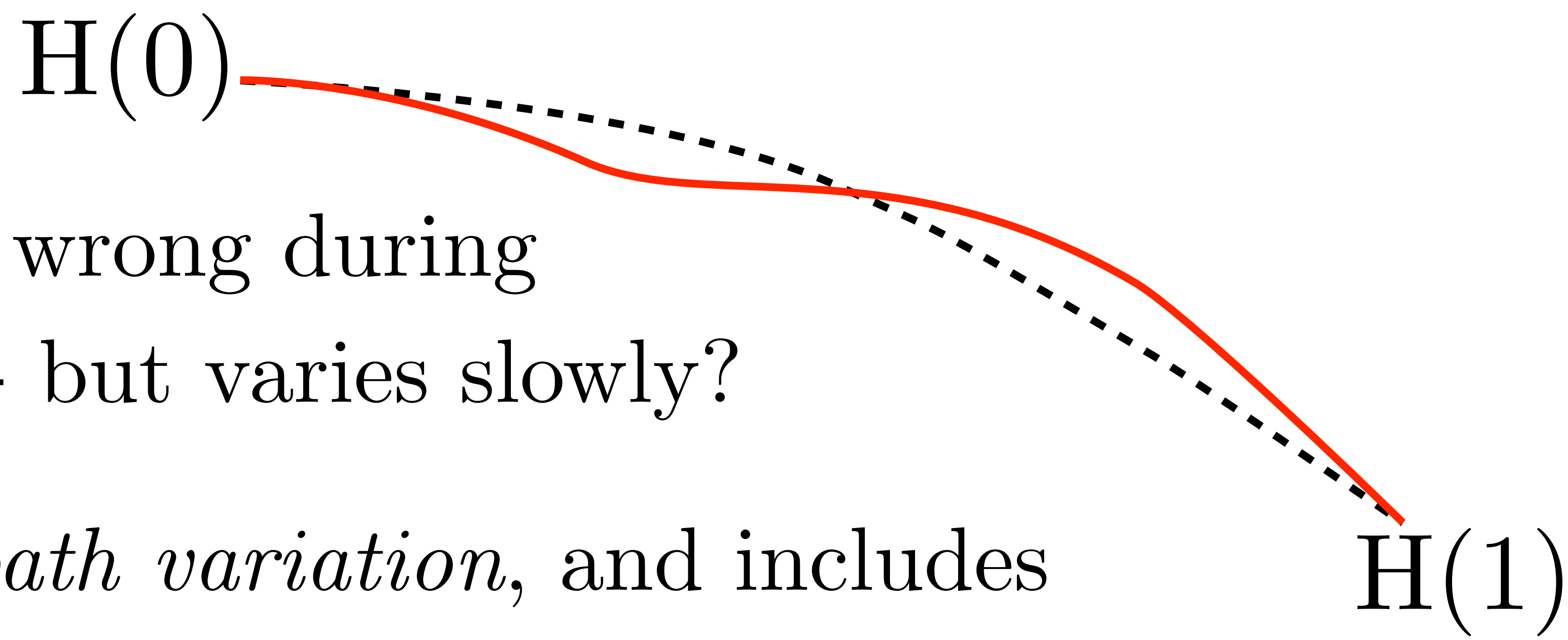
2. Wrong problem Hamiltonian

- Small errors in the coefficients of the problem Hamiltonian => you successfully solve the wrong problem!
- This is a real potential problem with current hardware -- precision for each term is low.
- Can be solved by ferromagnetic repetition coding



Fault Sources in AQC

3. Slow Hamiltonian error (path variation)



- What if the Hamiltonian is wrong during the evolution -- but varies slowly?

- This is called *path variation*, and includes all low-frequency Hamiltonian errors.

- In general, AQC is near-immune to path variation and can be made even more so using standard techniques.

Fault Sources in AQC

4. Realistic coupling to a bath

- The ultimate error model -- covers *everything*:

$$H(t) = H_{AQC}(t) + H_{S\leftrightarrow B} + H_{bath}$$

- Typically reduced to describing *spectral density* of the bath (i.e., bath fluctuates at various freqs).

- This talk: simplify further to a sum of:

(1) A $\boldsymbol{H}_{\mathrm{err}}(\mathrm{t})$ term that is *slow* (low frequency).

(2) Some amount of *white noise* (high frequency).

↳ Produces *stochastic* errors (no time correlation)

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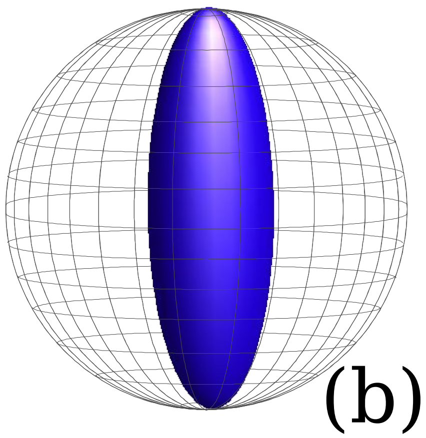
Fault Sources in AQC

5. Stochastic errors

- Hamiltonians generate *amplitudes* for "errors" (i.e., for being in the wrong quantum state).
- But if $H_{\mathrm{err}}(t)$ and $H_{\mathrm{err}}(t')$ are uncorrelated for $t \neq t'$, amplitudes can be replaced with *probabilities*.
- This leads to *stochastic errors* -- e.g., dephasing or depolarization -- just as in the circuit model.
- Dephasing is ok for AQC but energy jumps are not.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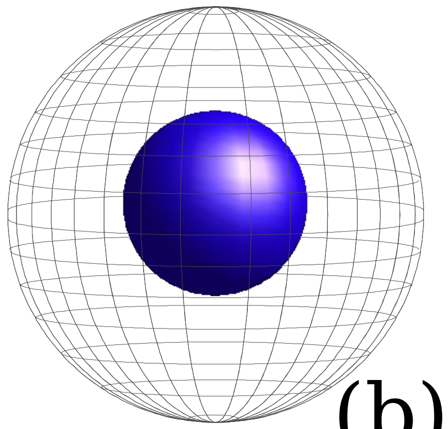
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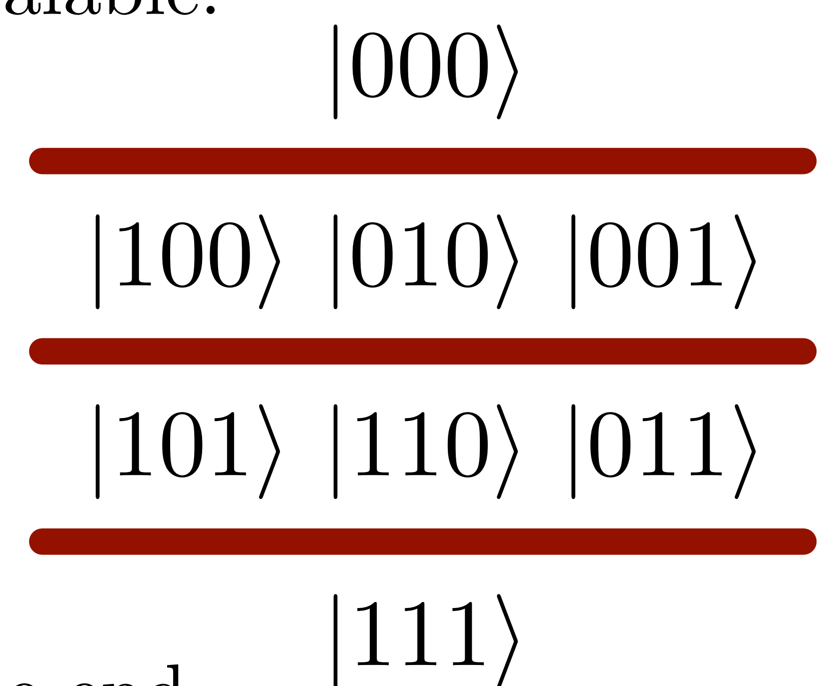
Tools to protect AQCs

- Protecting an AQC against noise (in control or from environment) requires *suppressing* or *correcting* transitions out of the $|0\rangle$ state.

- Simplest: increase H. Unfortunately, not scalable.

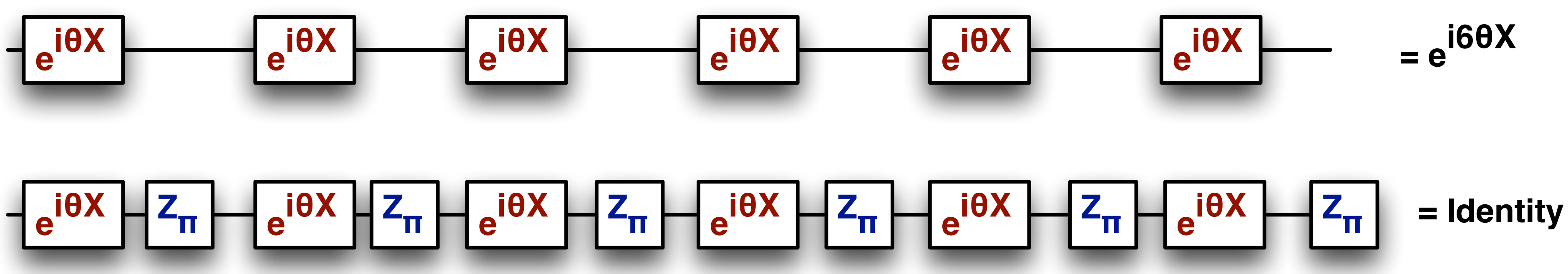
- All other techniques involve:
  (1) *encoding* in an error-correcting code,
  (2) suppressing/correcting jumps out of code space
  (3) decoding (reading out logical bits) at the end.



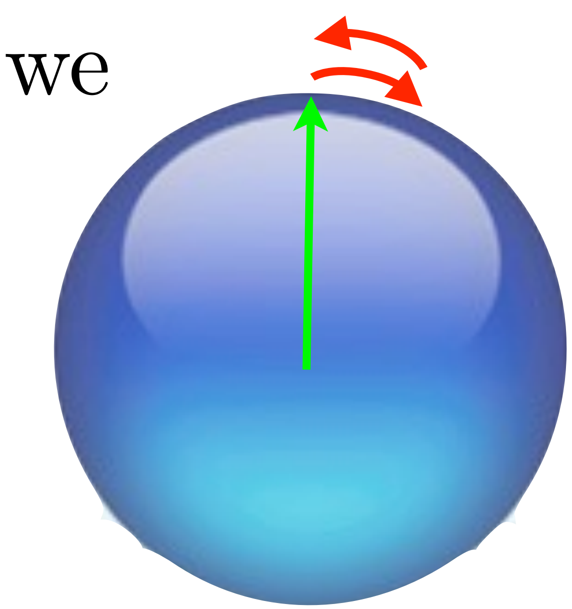
Tools to protect AQCs

1. QECC + dynamical decoupling

- Dynamical decoupling: suppress unwanted Hamiltonian rotations by periodic π flips.



- Given a code whose stabilizers are $\{S_k\}$, we can *suppress* transitions out of the code space by periodically applying the S_k as DD operations. *Requires fast control.*



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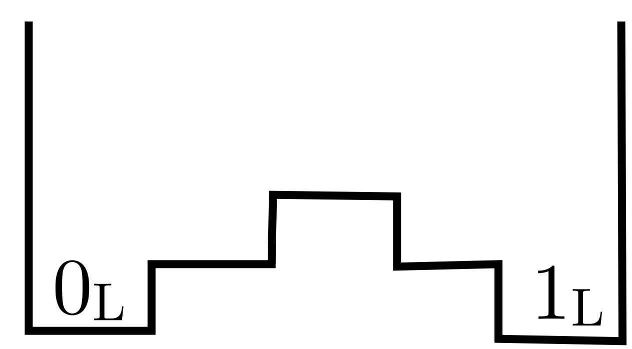
Tools to protect AQCs

2. QECC + penalty Hamiltonians

- We choose a QECC and encode in it:
  - ↳ Prepare each *logical* qubit in the *logical* $|+\rangle$ state.
  - ↳ Evolve by $\mathrm{H}_{logical}$ = sum of *logical* X and Z operators.
- But how do we keep the system in the code subspace (i.e., suppress errors)?
- Add the code stabilizers $\{\boldsymbol{S}_k\}$ to H as a *penalty Hamiltonian*. Code states have 0 energy; error states have E = # of errors.



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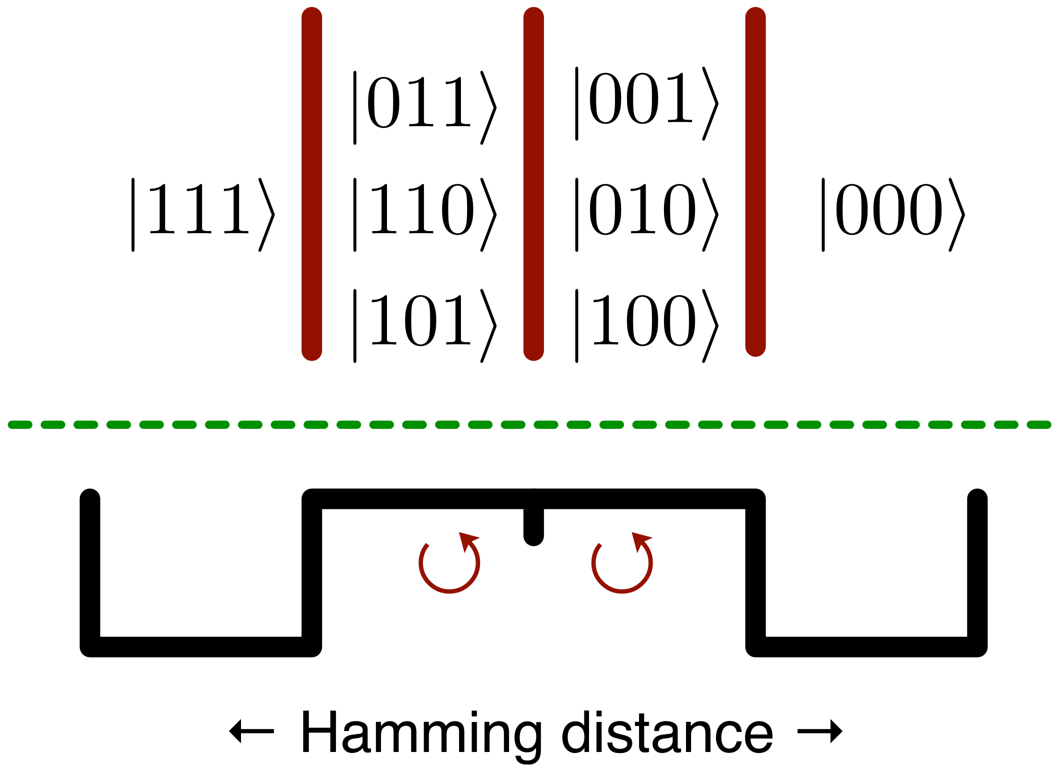
Tools to protect AQCs

3. How/why DD and penalties work

- DD and penalty Hamiltonians ("energy gap protection") *suppress* transitions out of the code space. Instead of *correcting* errors, they prevent them from happening.

- These two tools turn out to be (more or less) the same!

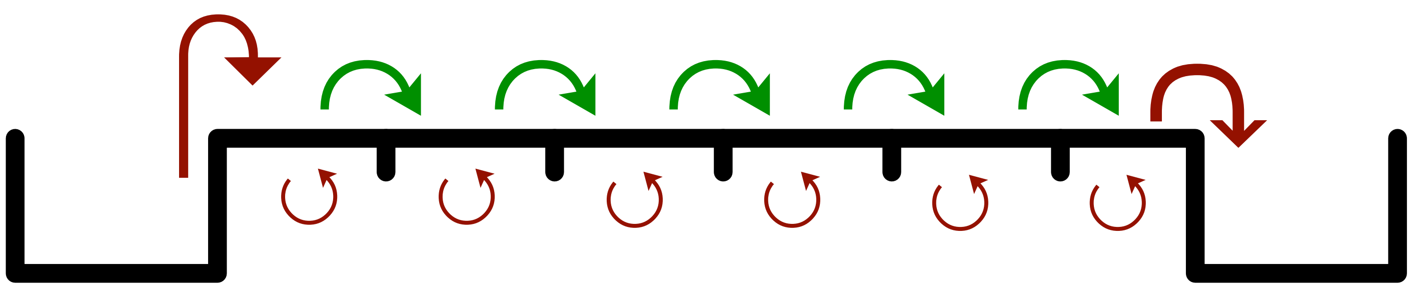
- The phase on "error" states rotates in time as $e^{i\omega t}$ or $(-1)^t$. Destructive interference stops H_{err} from driving transitions.



Tools to protect AQCs

4. Limitations of error suppression

- Suppression isn't perfect:
- (1) Hamiltonians that oscillate in *resonance* with the protection gap can drive transitions.

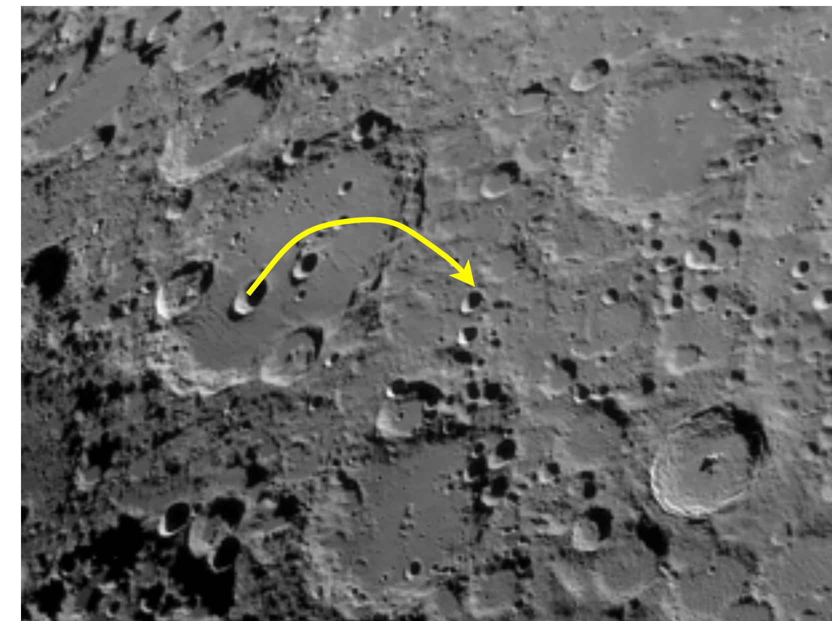


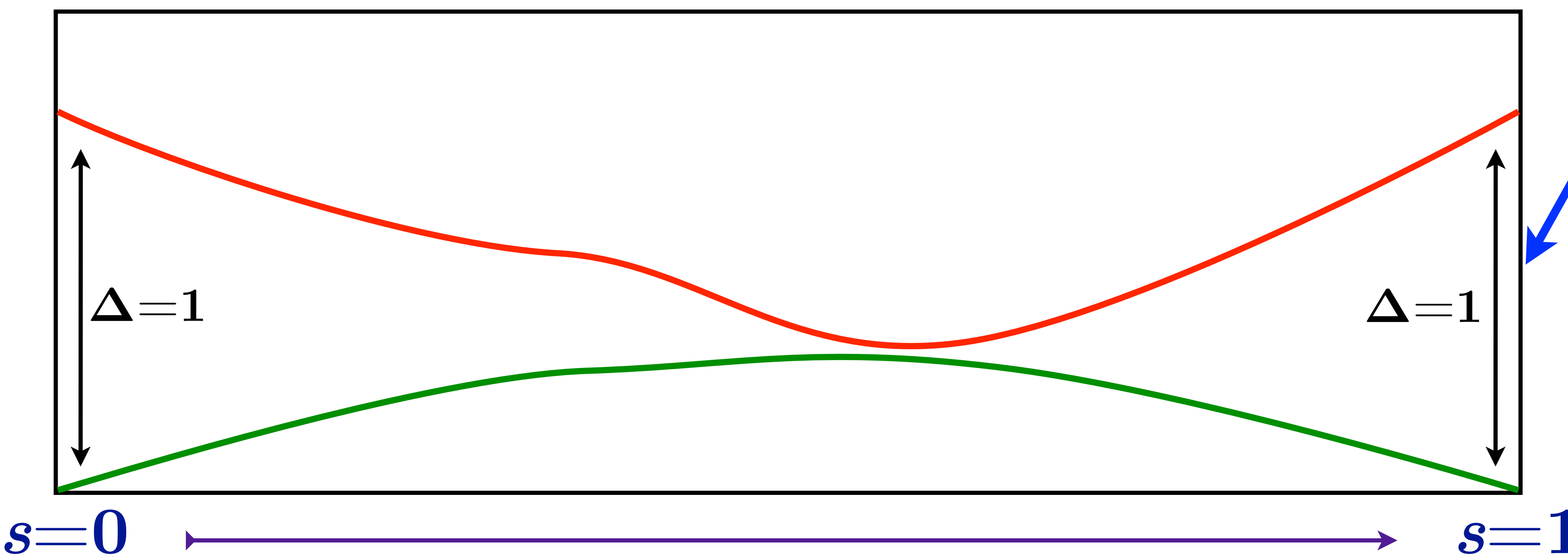
- (2) Once an single error happens, there's no protection against *more* errors ("domino" or "zipper" effect).
- The codes that would block this vulnerability would be *self-correcting quantum memories.*

Tools to protect AQCs

5. Cooling

- Can just cool the AQC?
- Probably not. If these Hamiltonians could be cooled efficiently, why would we do AQC at all? Just use a fridge!



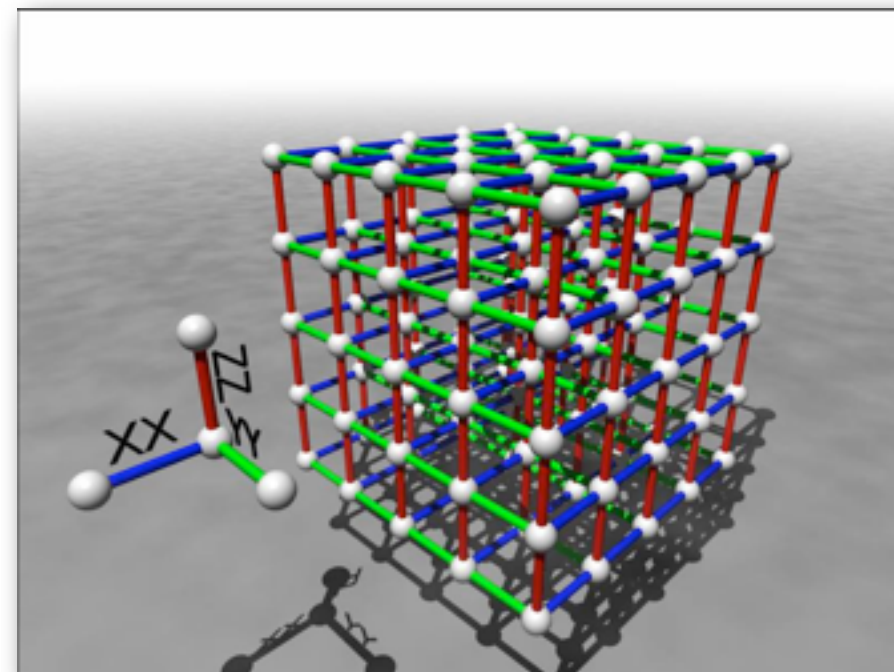


doesn't thermalize fast

Tools to protect AQCs

6. QECC + Cooling

- What if we implement a QECC -- using penalty Hamiltonians -- and then put it in a fridge. Can we rely on cooling to *correct* errors?

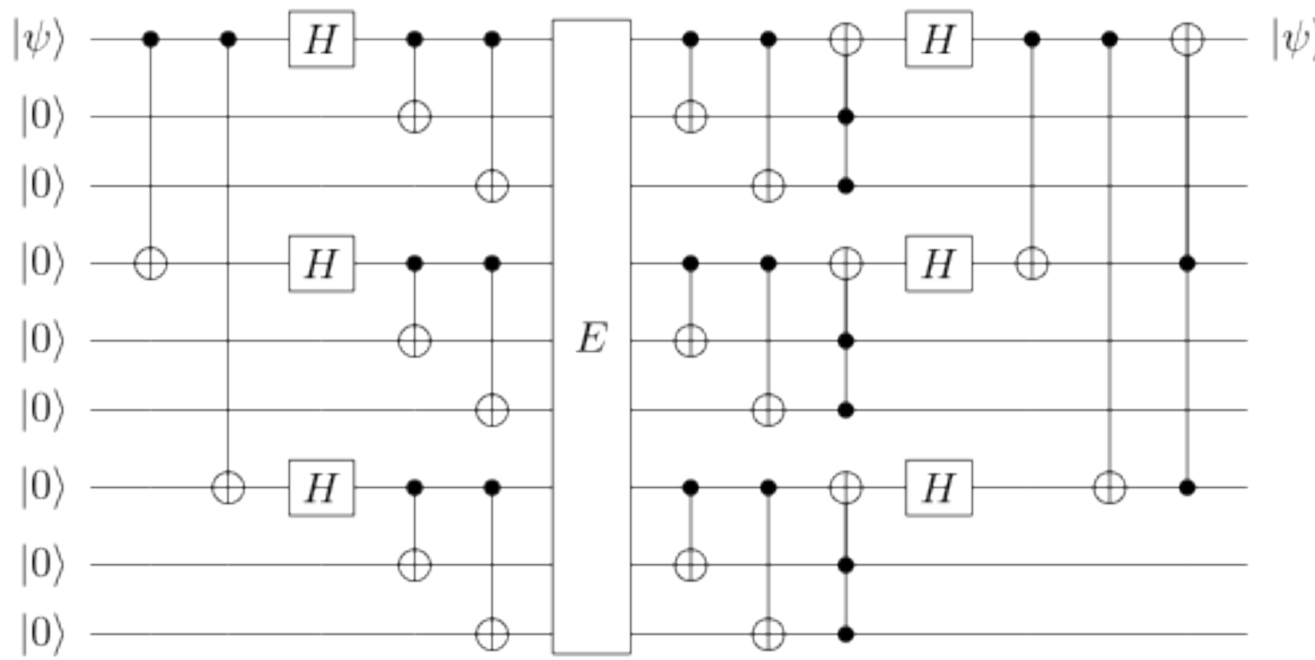


- It depends on the code.
- Only if we invent a *quantum self-correcting memory.*
- Correcting errors in known codes requires <u>clever nonlocal</u> decoding. Natural cooling is <u>dumb</u> & <u>local</u>.

Tools to protect AQCs

6. QECC + Active correction

- What if we encoded our AQC in a QECC...
  ...and then ran an active error correction protocol?



- Yes, this would protect.
- However, it requires all the same resources as the circuit model of QC!
- Actually *computing* adiabatically while actively correcting errors is surprisingly hard (impossible?)

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Efficacy (theory)

1. Dynamical Decoupling

- Should work to suppress noise whose frequency is lower than the DD pulse rate.
- Requires some fast gates and precise control.
- Gets very unwieldy for useful codes: you have to pulse *all* the stabilizers.

$$X \otimes Z \otimes Z \otimes X \otimes I,$$
$$I \otimes X \otimes Z \otimes Z \otimes X,$$
$$X \otimes I \otimes X \otimes Z \otimes Z,$$
$$Z \otimes X \otimes I \otimes X \otimes Z.$$

- In practice, will probably be used together with penalty Hamiltonians (to apply high-weight stabilizers)

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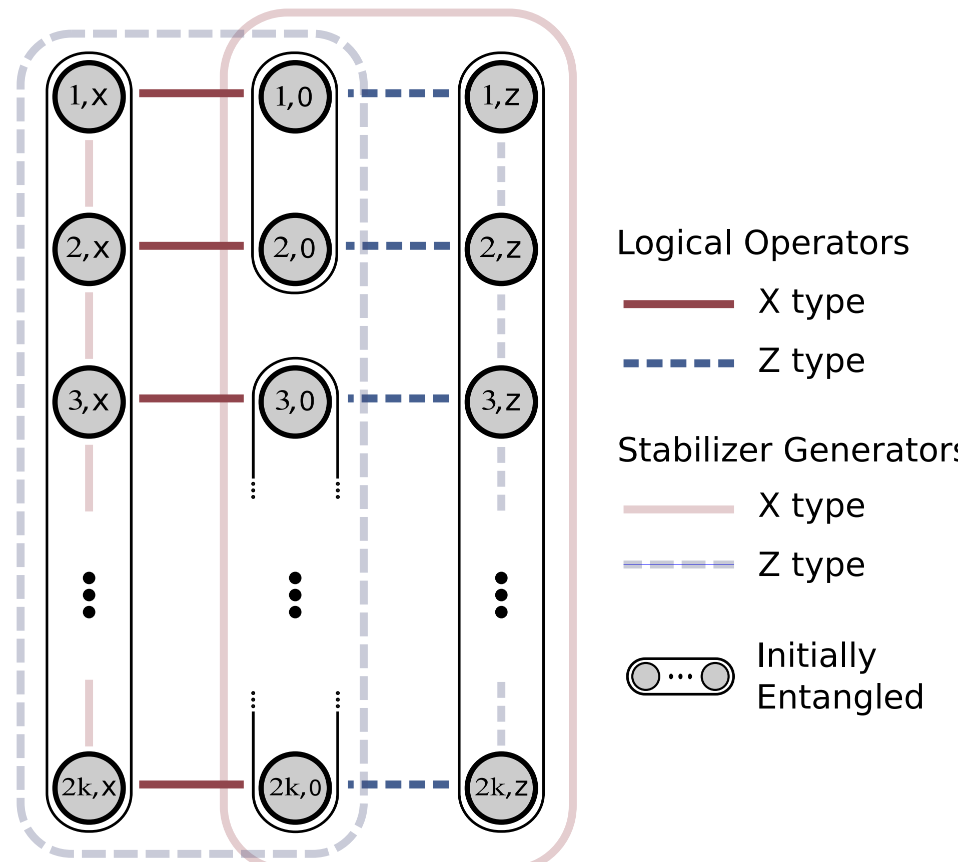
Efficacy (theory)

2. Penalty Hamiltonians

- Like DD, should work to suppress low-frequency errors.
- Limited by strength of the penalty Hamiltonian (not ∞!)
- In practice, hard to find codes with low-weight stabilizers that can be implemented using real-world Hamiltonians. Will probably require hybridization with DD technique.

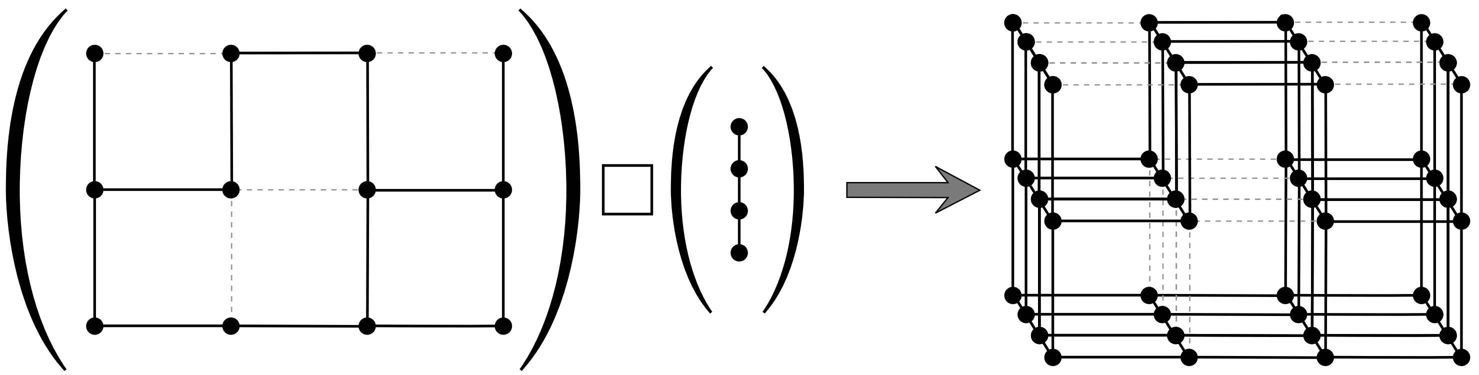


Efficacy (theory)

2b. Encoding the problem Hamiltonian

- Encoding + penalty Hamiltonians can also be used to solve the "wrong problem Hamiltonian" issue.

- Approach: Encode the problem Hamiltonian in a repetition code.



- Basically, this replaces each qubit with a big ferromagnet. The desired Hamiltonian scales up as O(N), whereas errors scale as $O(N^{1/2})$.

- Does not play well with the *dynamics* of AQC.

Efficacy (theory)

3. Cooling

- Unlikely to *solve* our problems in the N$\rightarrow\infty$ limit.
- Clearly will help *somewhat* in practice (you want your device to be cold!)
- Very difficult to separate two effects in practice:
  (1) Cooling adds *classical* annealing (helps, but not quantum!)
  (2) Cooling helps to correct errors, protecting *quantum* advantages.
- It would help if we understood quantum speedups...

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Efficacy (theory)

4. Active Correction

- Would definitely protect code space -- in principle.
- It's the highest priority for *standard* (circuit) QC.
- Requires resources incompatible w/AQC philosophy.
- More "AQC-compatible" if we manage to invent self-correcting quantum memories.
- Major open question (stay tuned):
  Can we actually *compute* adiabatically in a QECC?

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Efficacy (experiment)

Penalty Hamiltonians on D-Wave Two

- To date, there is *one* experimental test (2013-14).
- Pudenz et al used *Quantum Annealing Correction* for random Ising problems on D-Wave Two:
  (1) Encoded problems in 3-qubit repetition code.
  (2) Applied stabilizers (ZZI, IZZ, ZIZ) as penalty H.
  (3) Decoded final state to obtain result of computation.
- Caveat: X terms could not be encoded (X $\rightarrow$ XXX)
  ↳ Penalty suppresses X... its strength β must be tuned.
- Small system, small code -- hard to extrapolate to large *N*!

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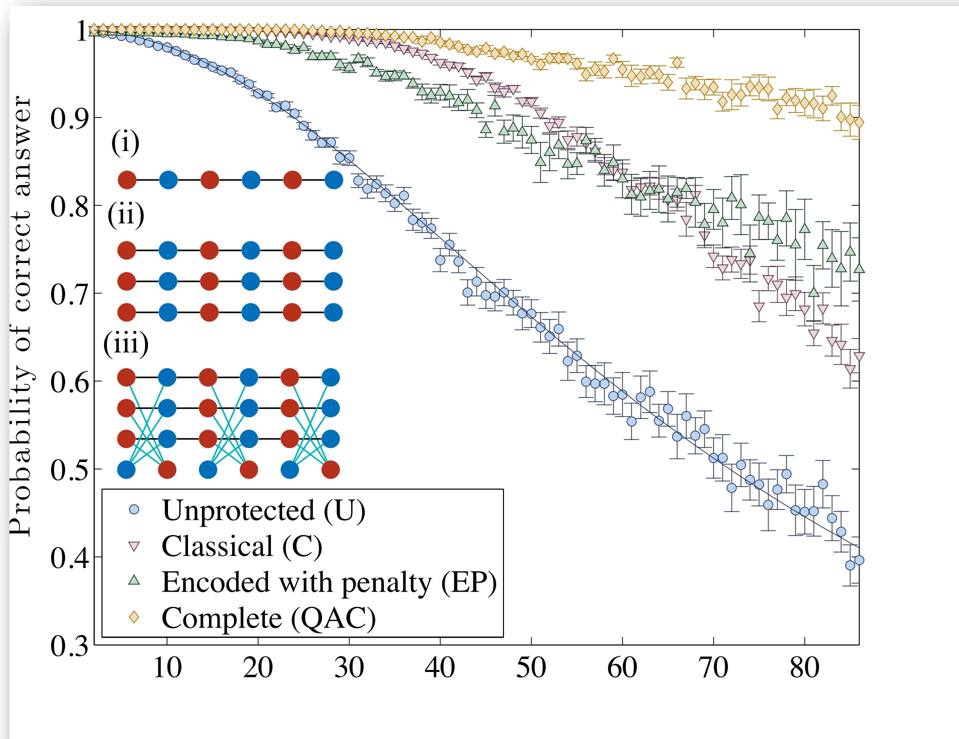
Efficacy (experiment)

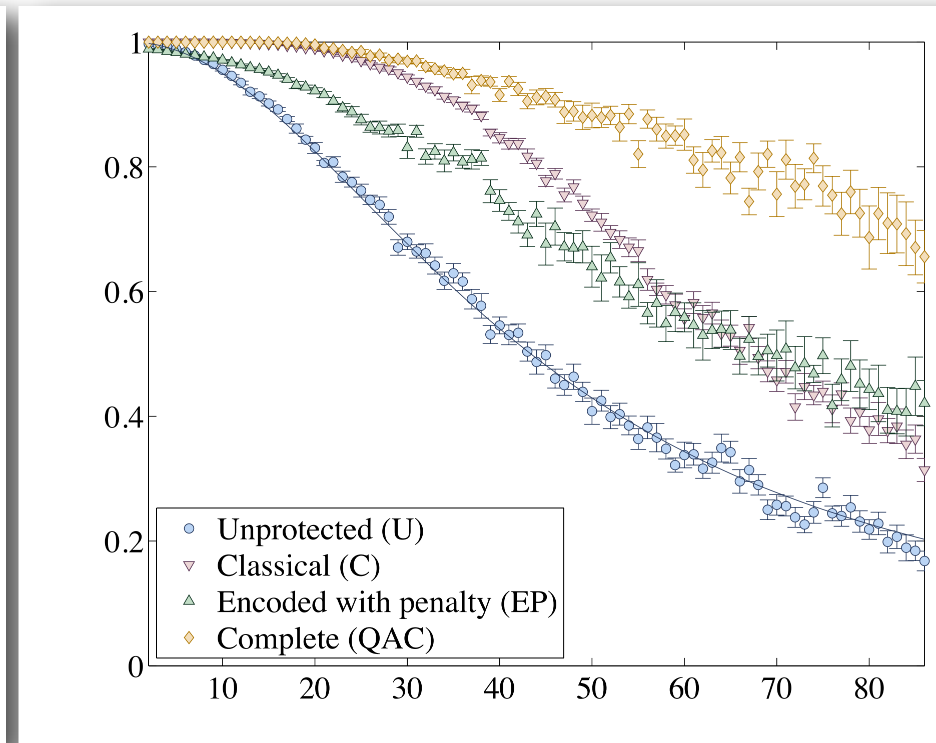
Summary:

1. QAC increases the success probability of the adiabatic computation.

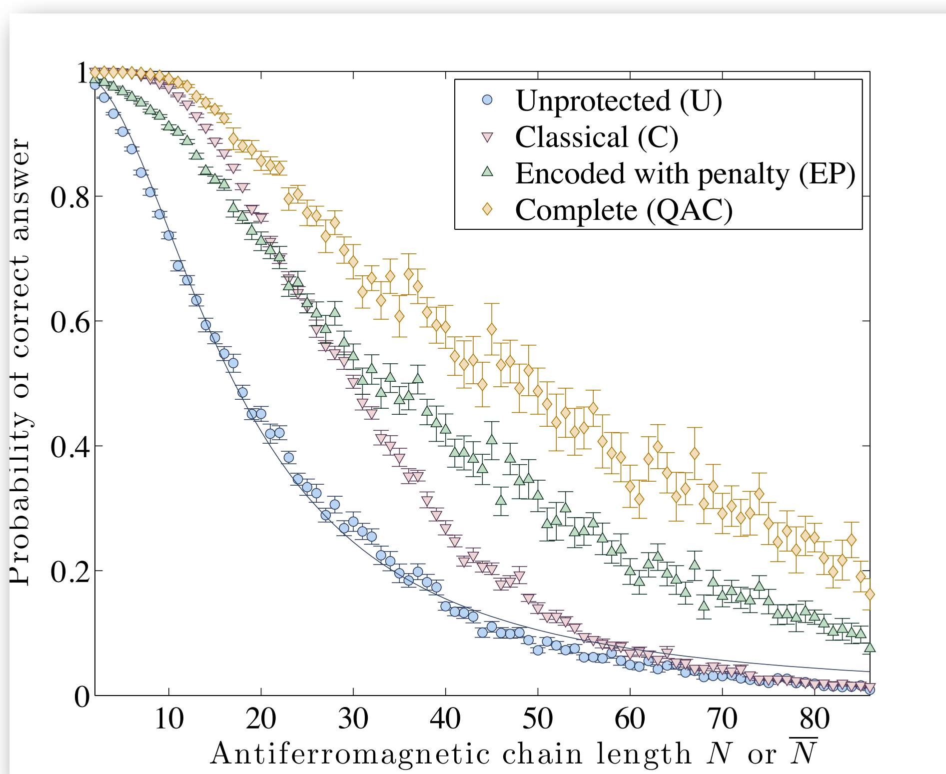
2. Maximum advantage at high problem energy α.



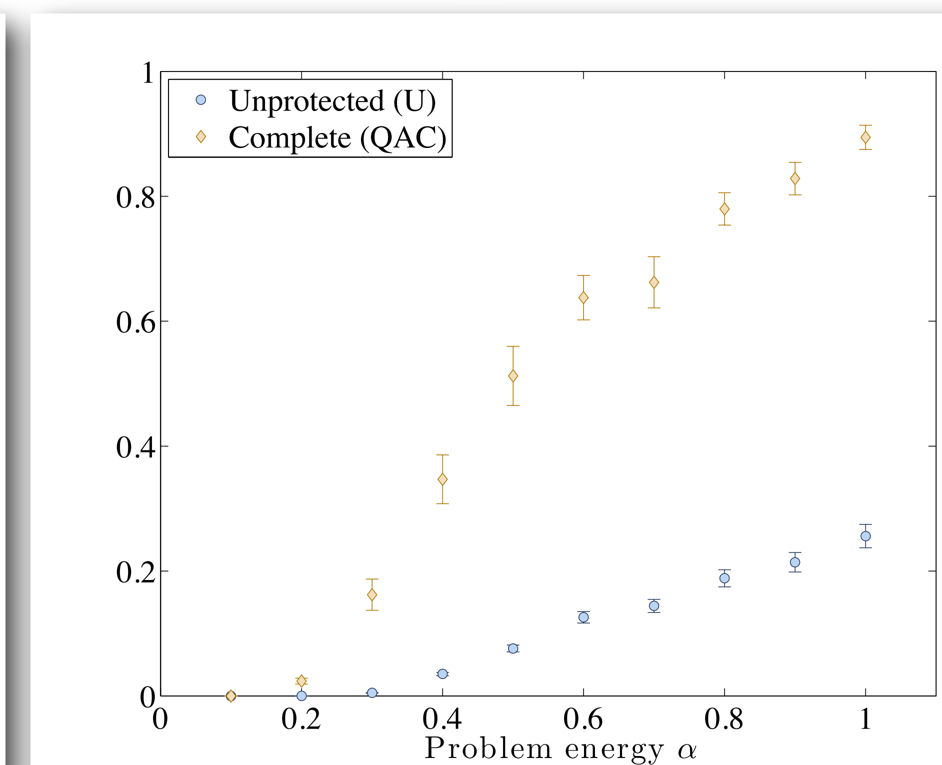
(a) $\alpha = 1$



(b) $\alpha = 0.6$

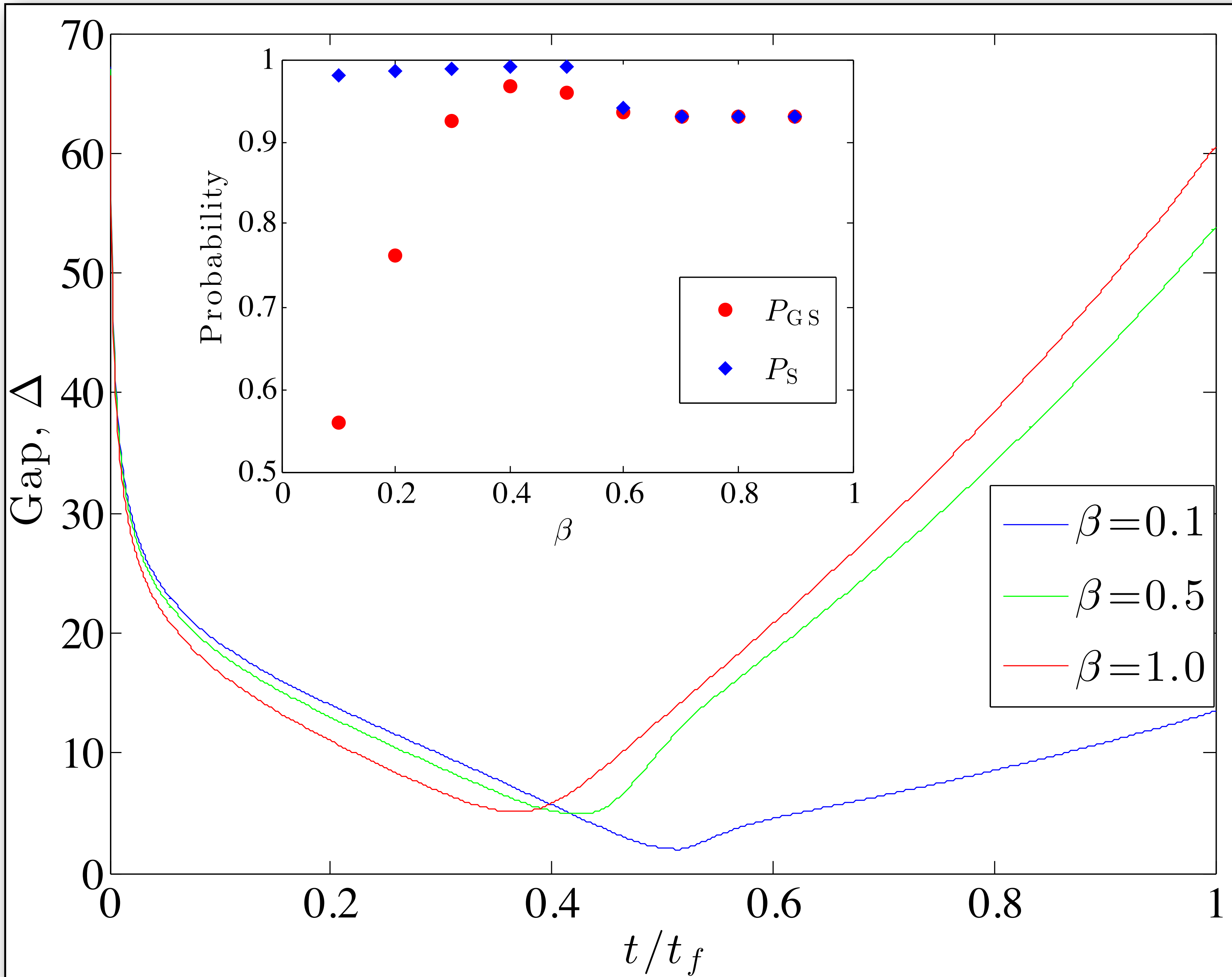


(c) $\alpha = 0.3$



(d) U and QAC *vs* α, $N = \bar{N} = 86$

Efficacy (experiment)



QAC increases the gap of the encoded problem.

Optimizing the gap requires tuning penalty strength β.

Efficacy (experiment)

What's going on?

- **Unambiguous**: QAC strategy improved results.
- **Unambiguous**: *any* N→∞ extrapolation of current experimental results on D-Wave Two is controversial.

- **Ambiguous**: Is the improvement due to
  (1) suppression of errors (allowing more quantum)?
  (2) enlarging the correct energy basin (classical)?
- **Ambiguous**: Will improvements persist as size and code distance increase? (penalty H may suppress X terms in H_{AQC})

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Toward scalable FTAQC?

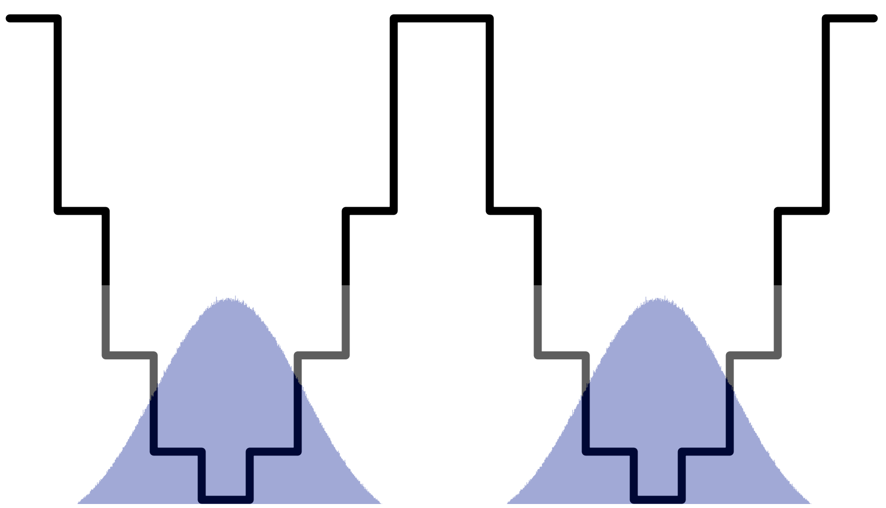
- To be *useful* for solving computational problems, AQC devices need to scale up and get big.
- As size (N) increases, errors become more important => fault tolerance is critical.
- We are now prepared to ask whether fault tolerant AQC is possible as $N \rightarrow \infty$. **Can we prove a threshold theorem, as for the circuit model?**
- Summary: it doesn't look good.

We need active correction

- AQC may have *low* error rates, but is not immune.
- Error suppression techniques only suppress low-frequency errors.
- Real environments usually include all frequencies => stochastic errors will happen at some rate.
- Cooling isn't enough without self-correcting memories.
- **Conclusion**: absent a creative breakthrough, FT will require some form of active (or self-) correction.

Trouble with error spaces

- Fault tolerant error correction doesn't keep the state in the code space!
- Instead, it's like cooling: there are always a few errors (excitations).



- Problem: The encoded *logical* H_{AQC} acts differently on error spaces than on the code space!
  (Some terms change sign -- $H = \mathrm{X+Z} \Longrightarrow H = \mathrm{X\text{-}Z}$)
- Only known fix requires *incredibly* complicated H_{logical}.

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Rapunzel in the tower

- Once we have protected logical qubits...
  ...we need to *compute* with them.
- Computing = dynamics.

- We need to drive *logical operations* that change the logical state in time.
- However... since the whole point of error correction is to *prevent* [unwanted] logical operations... this is hard.
- Have we locked our QI up so well that we can't get it?

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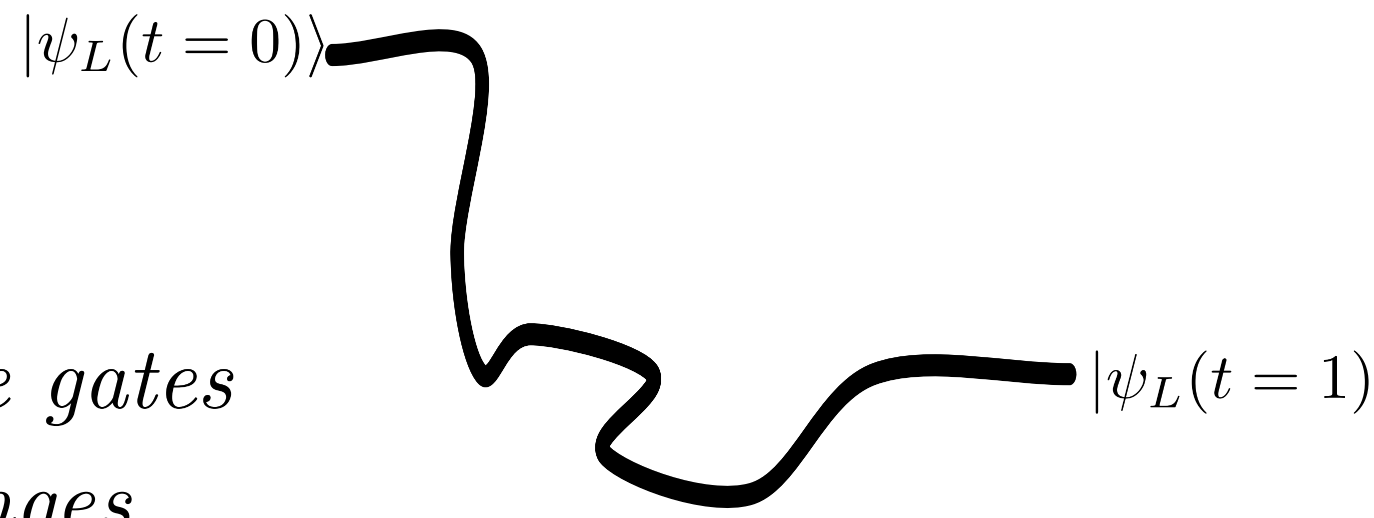
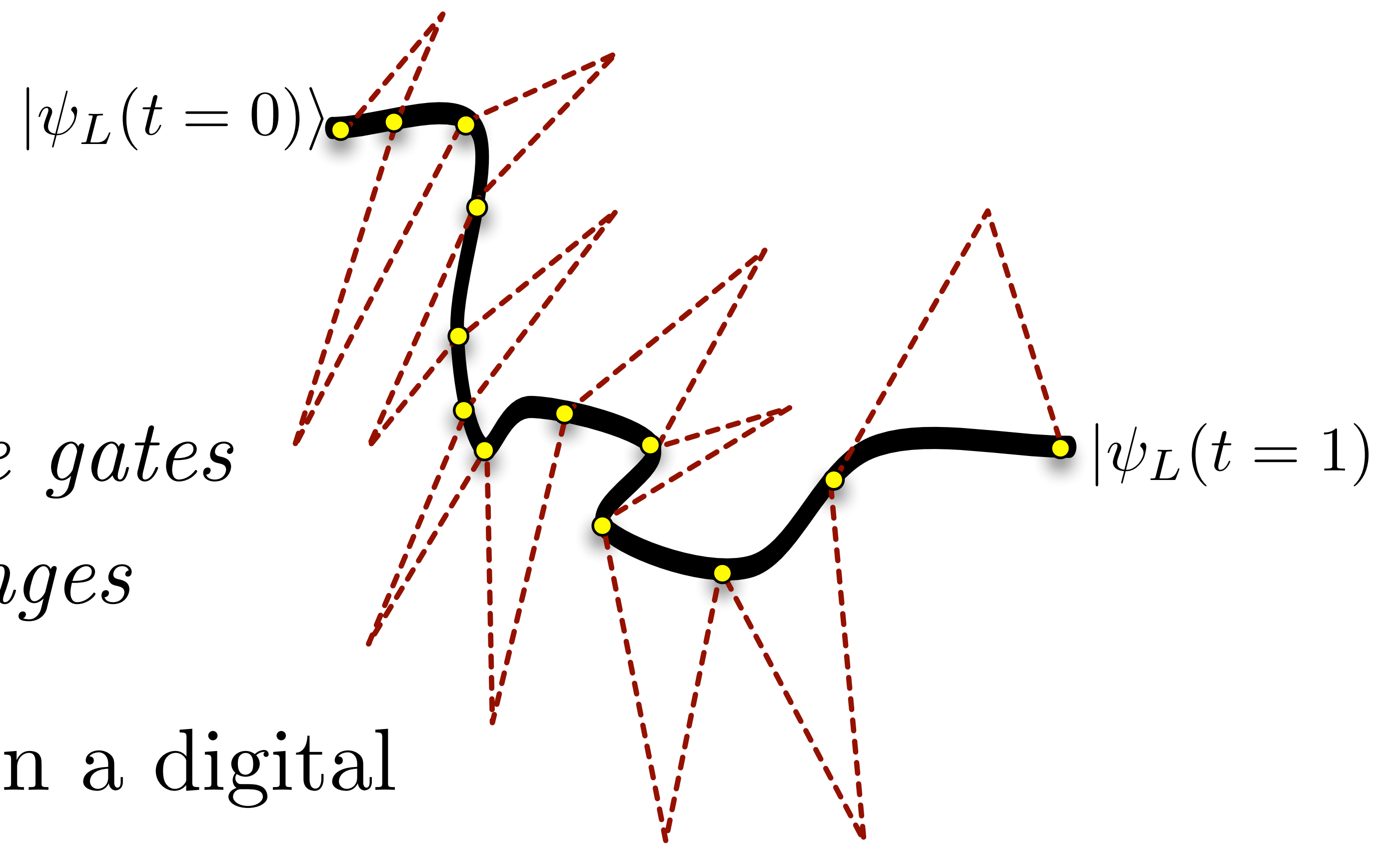
Logical dynamics in AQC

- FTQC always requires logical dynamics:
  - circuit model: *discrete gates*
  - AQC model: *slow changes*



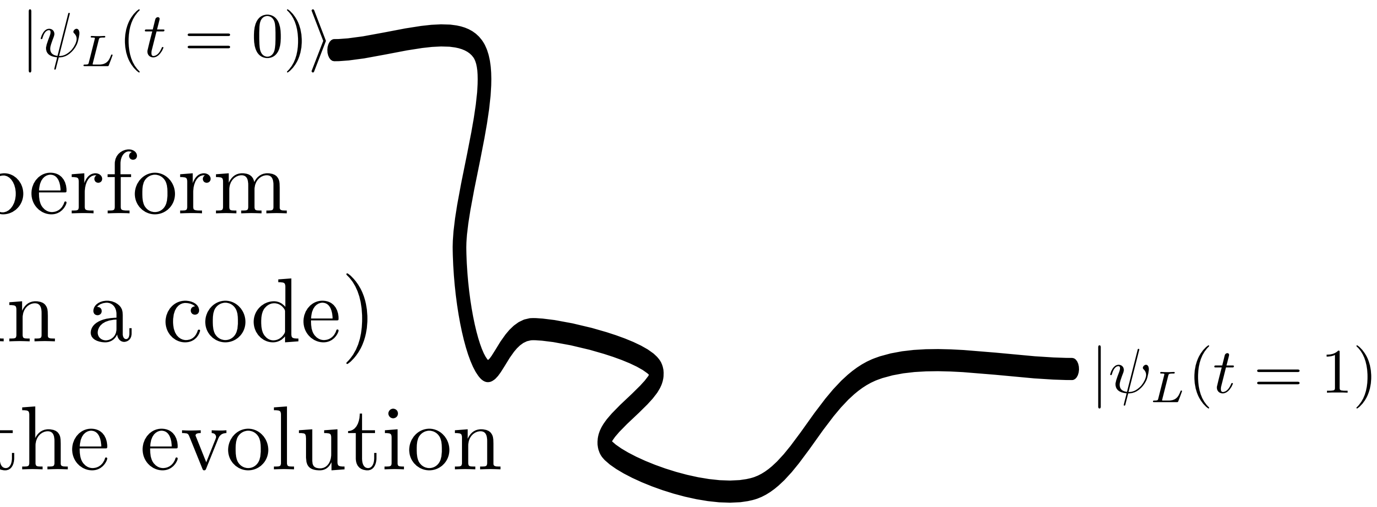
Logical dynamics in AQC



- FTQC always requires logical dynamics:
  - circuit model: *discrete gates*
  - AQC model: *slow changes*

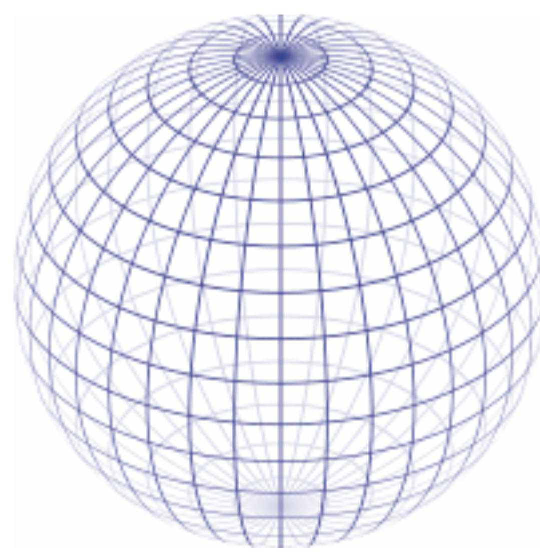
- We can *simulate* AQC on a digital quantum computer, using logical gates.
  This works -- but it's not the point. We want to know if *adiabatic computing* can be fault tolerant, not whether the adiabatic algorithm can be simulated using discrete gates (it can).

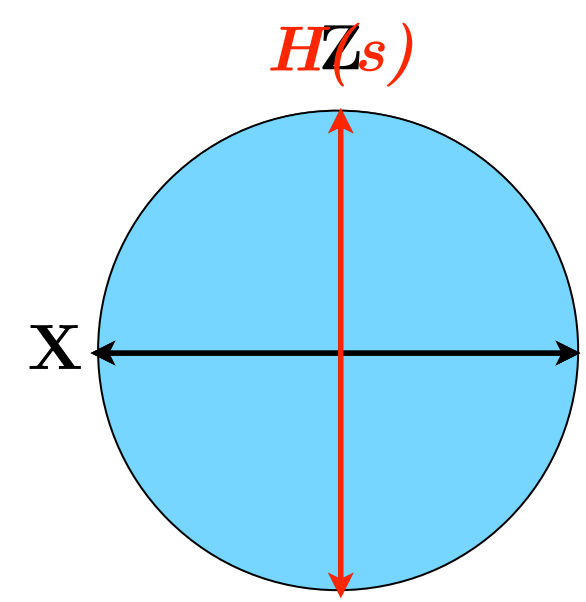
Simulating H_{AQC}



- Key question: Can we perform logical operations (within a code) that *faithfully simulate* the evolution of the ground state of the AQC Hamiltonian (including its smoothness)?

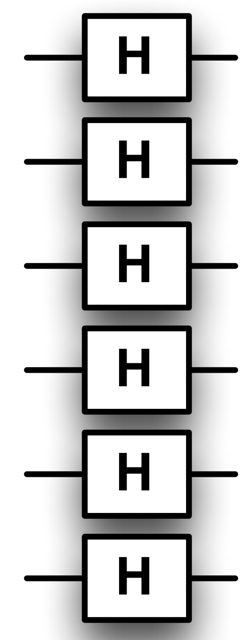
- Requires *continuous* and *non-commuting* logical operations.

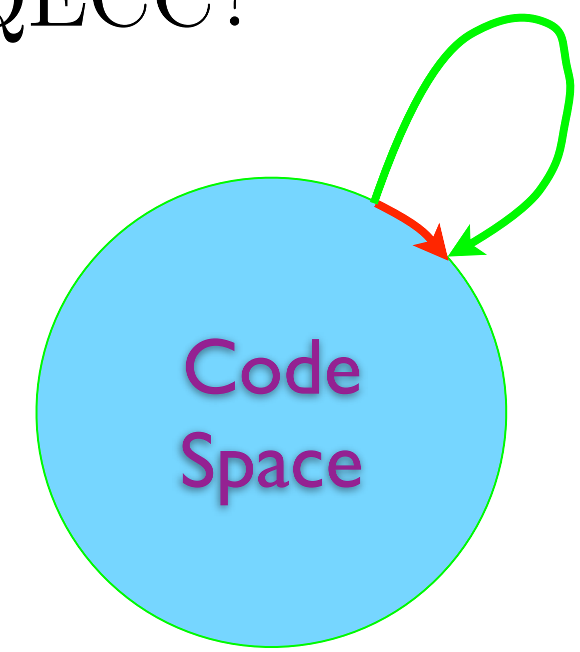




Transversal gates

- What logical operations are "easy" within a QECC?
- Need to apply *physical* qubit operations to:
  (1) get out of the code space, then
  (2) get back in before EC happens.





- Must be done *fast*.
  ↳ *Transversal* gates are great.
  ↳ *Constant-depth* circuits ok.
- But there are some really strong no-go theorems about what logical operations can be performed "fast enough".

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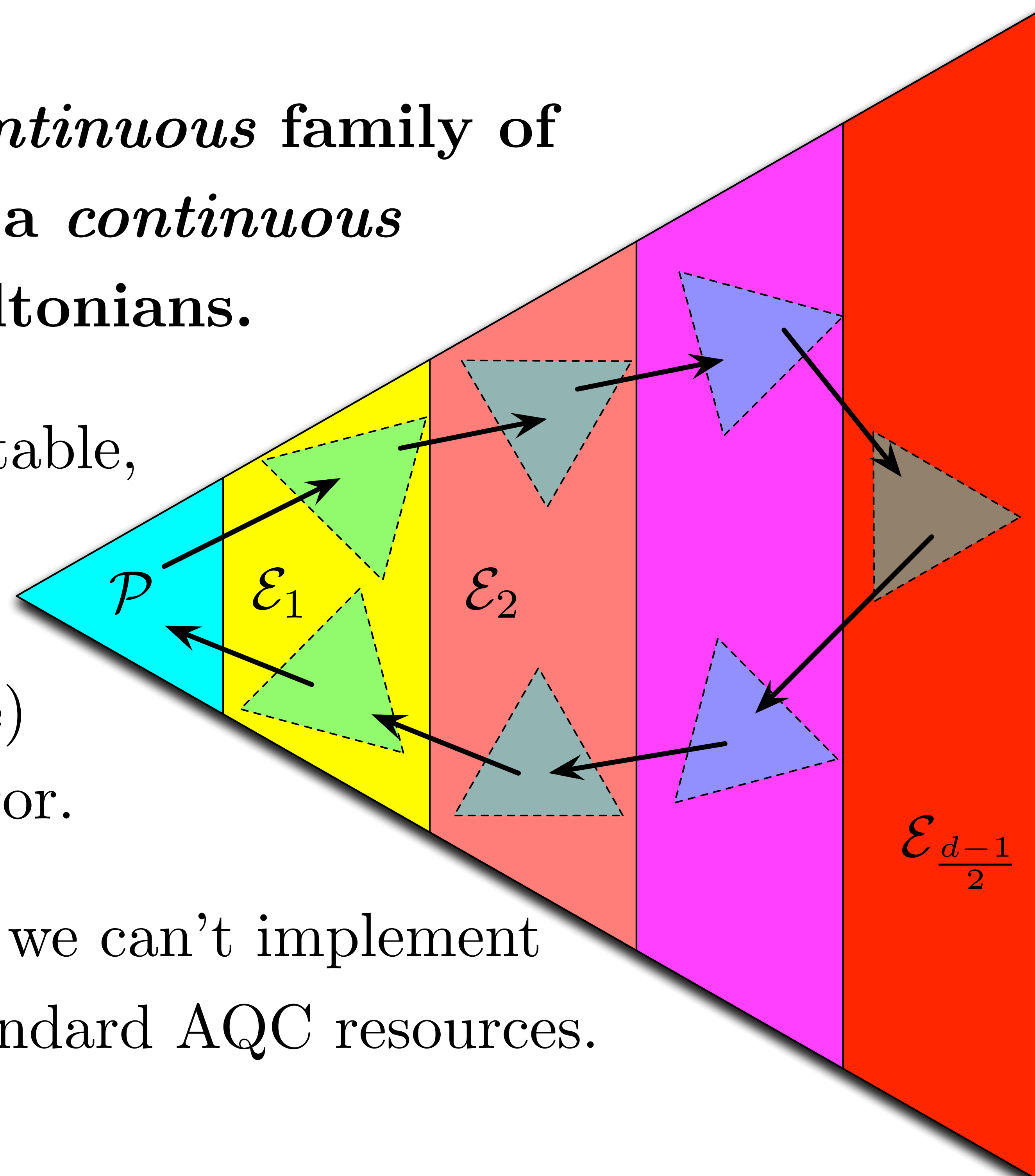
No-go arguments

- **You can't generate a *continuous* family of logical operations using a *continuous* family of physical Hamiltonians.**

- If H is physically implementable, and generates a logical operation U, then $H+\delta H$ (where δH is implementable) just looks like U plus an error.

- This <u>**strongly**</u> implies that we can't implement logical (FT) AQC using standard AQC resources.



No-go arguments

- Can we implement logical AQC (i.e., smooth slow dynamics on the logical qubits) using *any* resources?
- A series of increasingly strong results (Eastin/Knill, Bravyi/Koenig, Pastawski/Yoshida, Belvedere *et al*) put *severe* limitations on what logical operations can be implemented feasibly (constant depth) in various codes.
- **Summary**: as $N \rightarrow \infty$, only *discrete* sets of noncommuting logical unitaries can be applied in constant depth.
- Absent a breakthrough, this seems to rule out FTAQC.

Prospects

- AQC is intrinsically robust to *some* errors, but there are other errors that will demand fault tolerant QEC.
- We have several tools to protect *logical* AQC from noise.
- EC experiments on D-Wave Two have shown some advantages!
- The prospects for N$\rightarrow\infty$ fault tolerant AQC look bad.
- **However**: Even if the AQC *architecture* cannot be made fault tolerant... the adiabatic *algorithm*[*s*] could certainly be simulated on a digital quantum computer.
- This <u>might</u> well be the best way to solve some problems!

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