Applying Model Selection to Quantum State Tomography: Choosing Hilbert Space Dimension

SAND2015-1070C

Travis L Scholten

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Tomography is hard

Doing so in infinite dimensional Hilbert space is harder

Let's make it easier…

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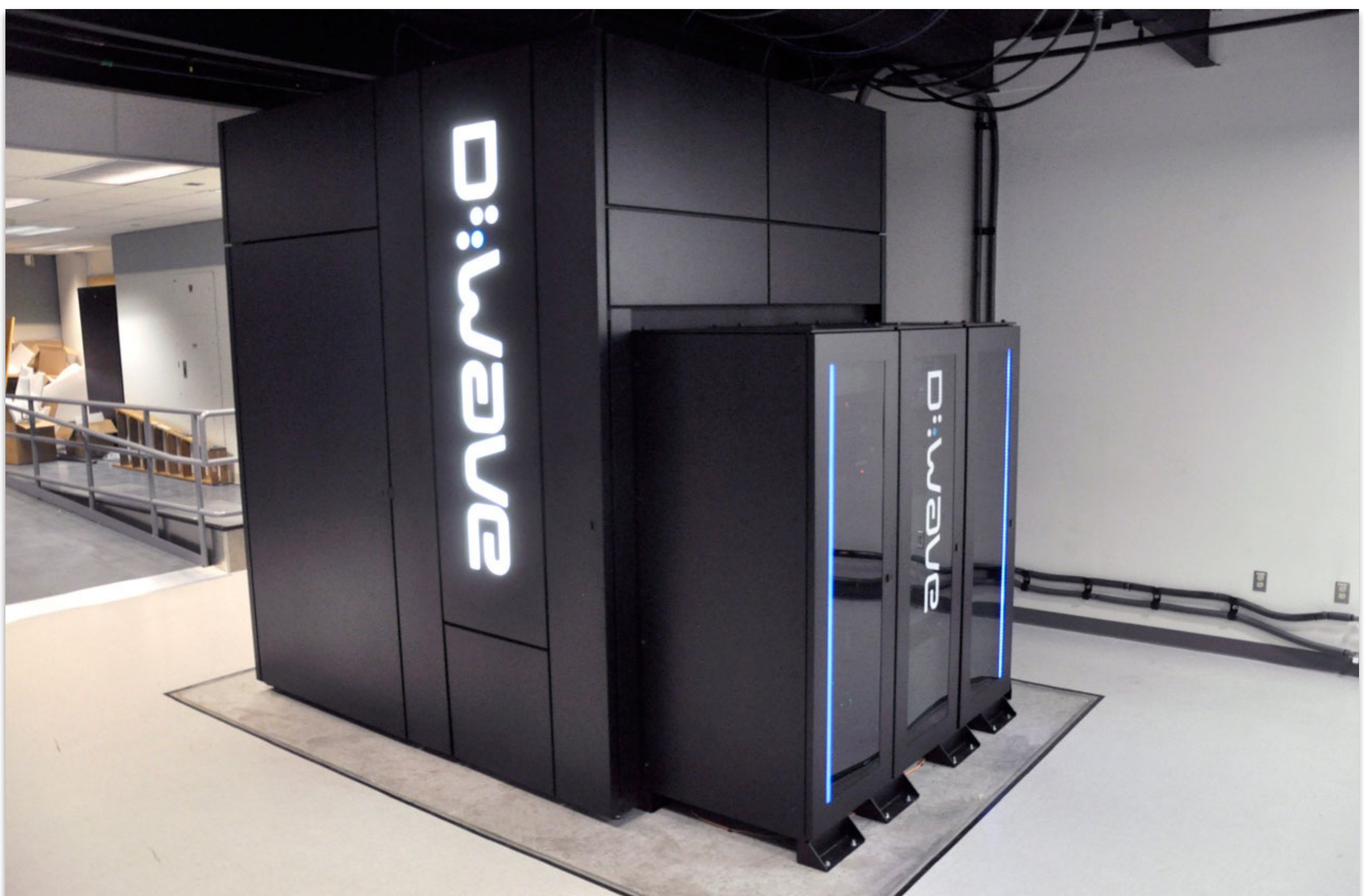
Tomography is hard.

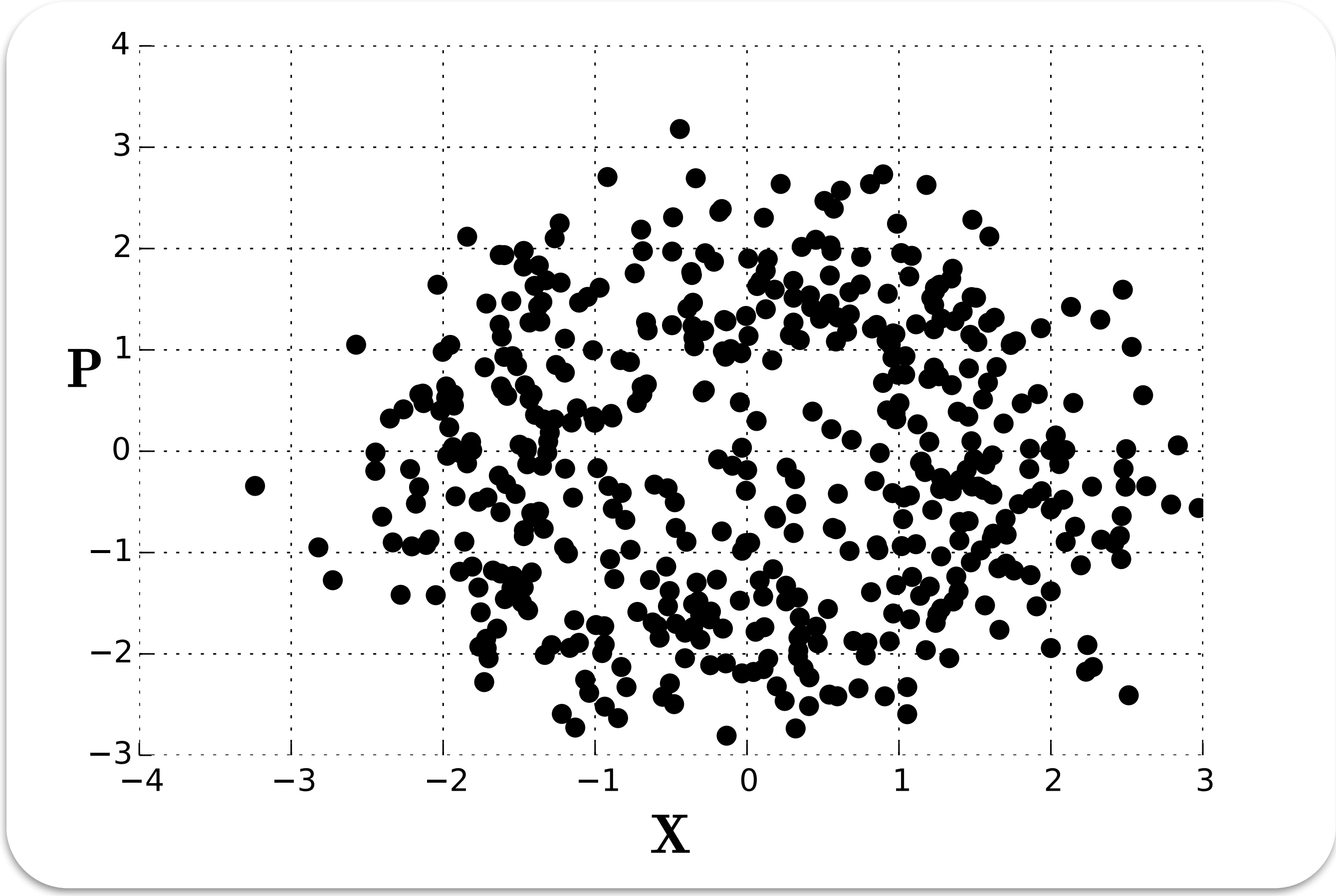
State tomography
Process tomography
Gate Set tomography

We do tomography to
make the unknown knowable



Tomography in infinite dimensional Hilbert space is harder.

484 Simulated Heterodyne
Measurement Outcomes



From measurements on
a **continuous variable system**,
we estimate…

Tomography in infinite dimensional Hilbert space is harder.

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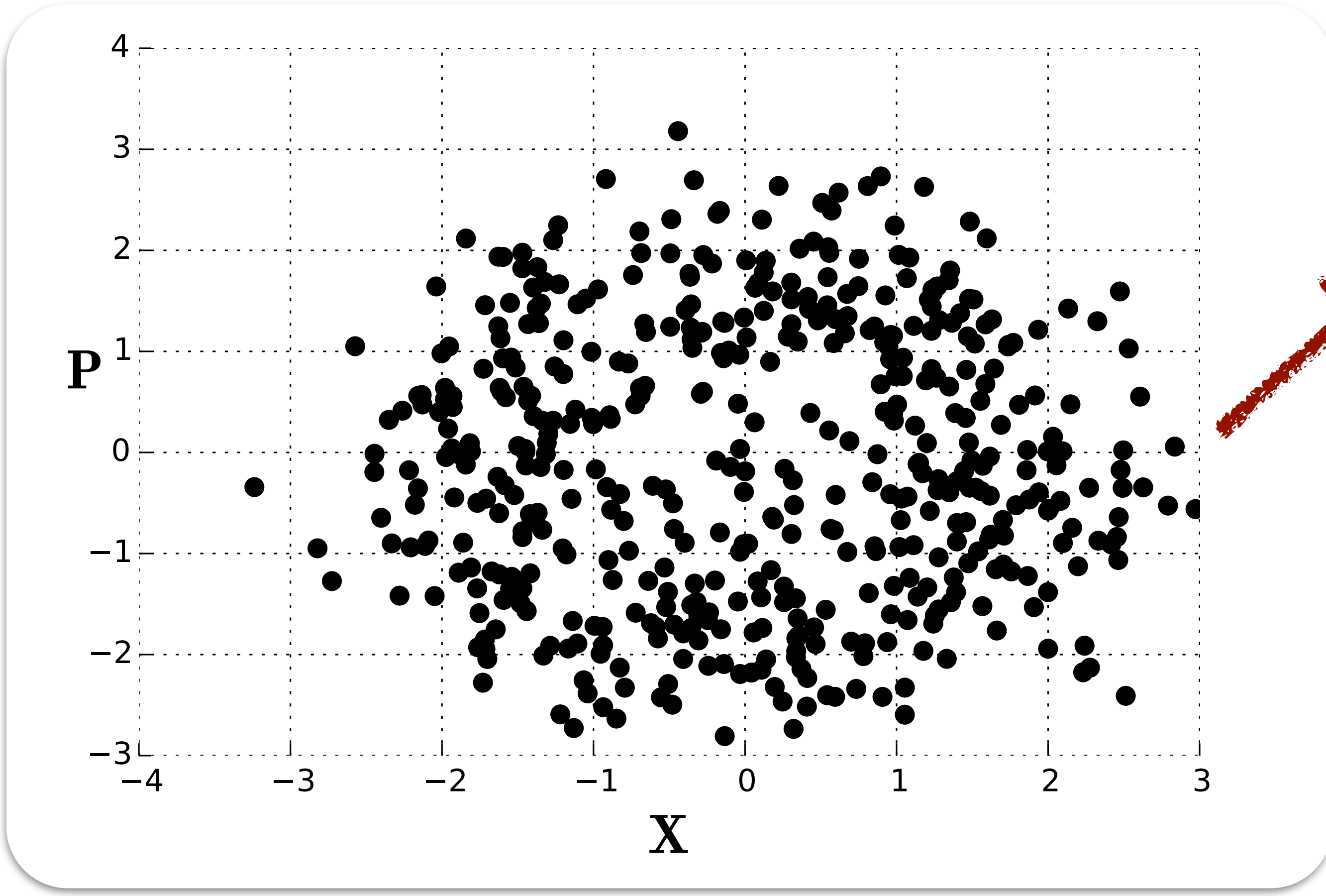

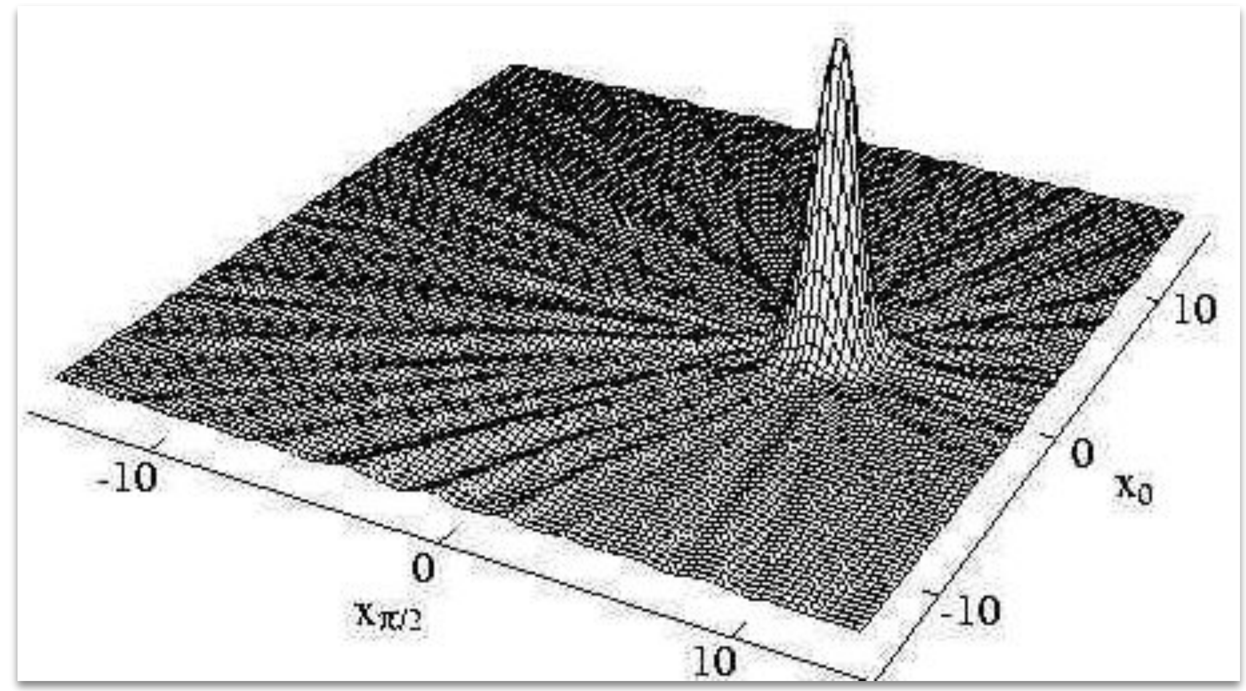

…a Wigner function…

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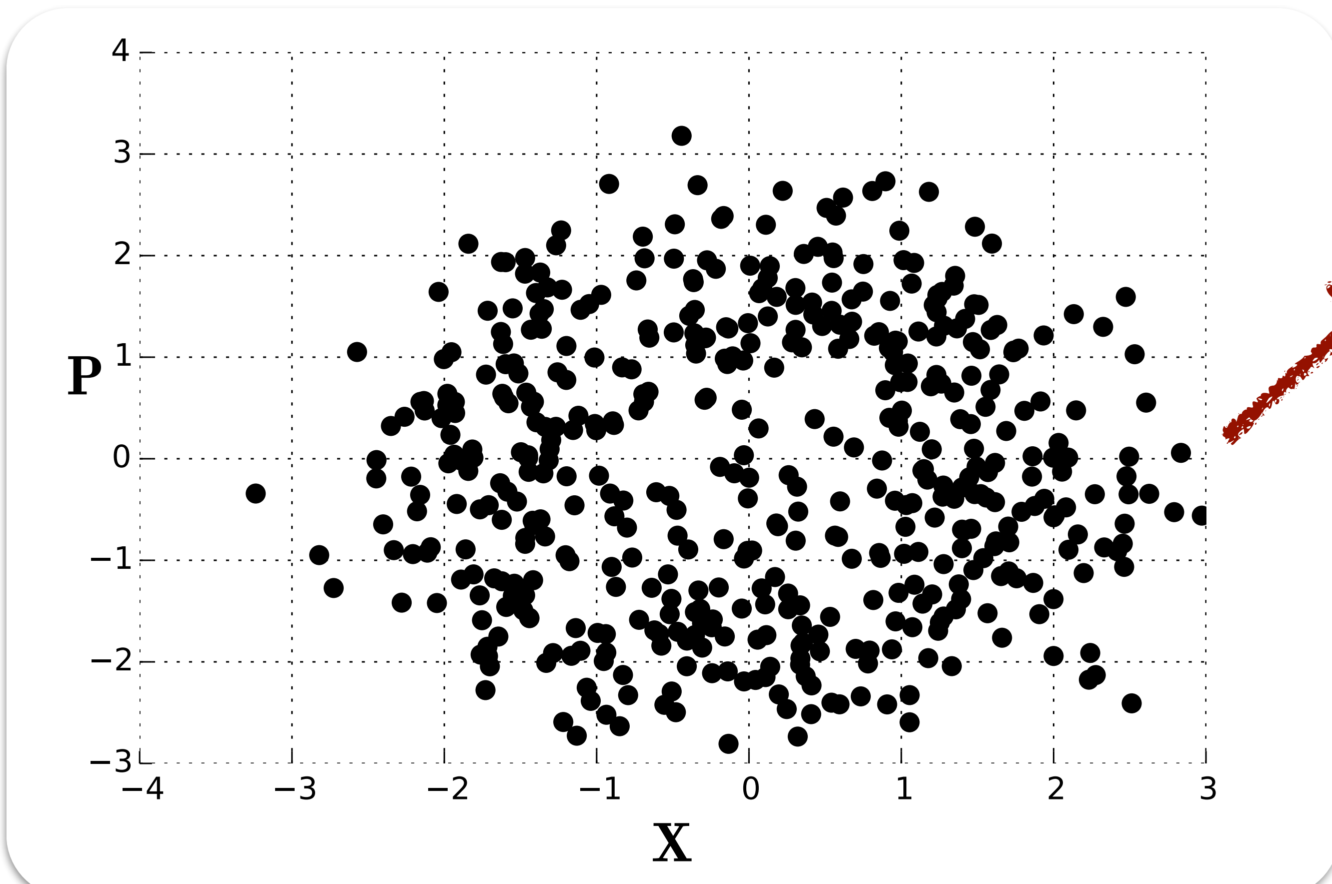
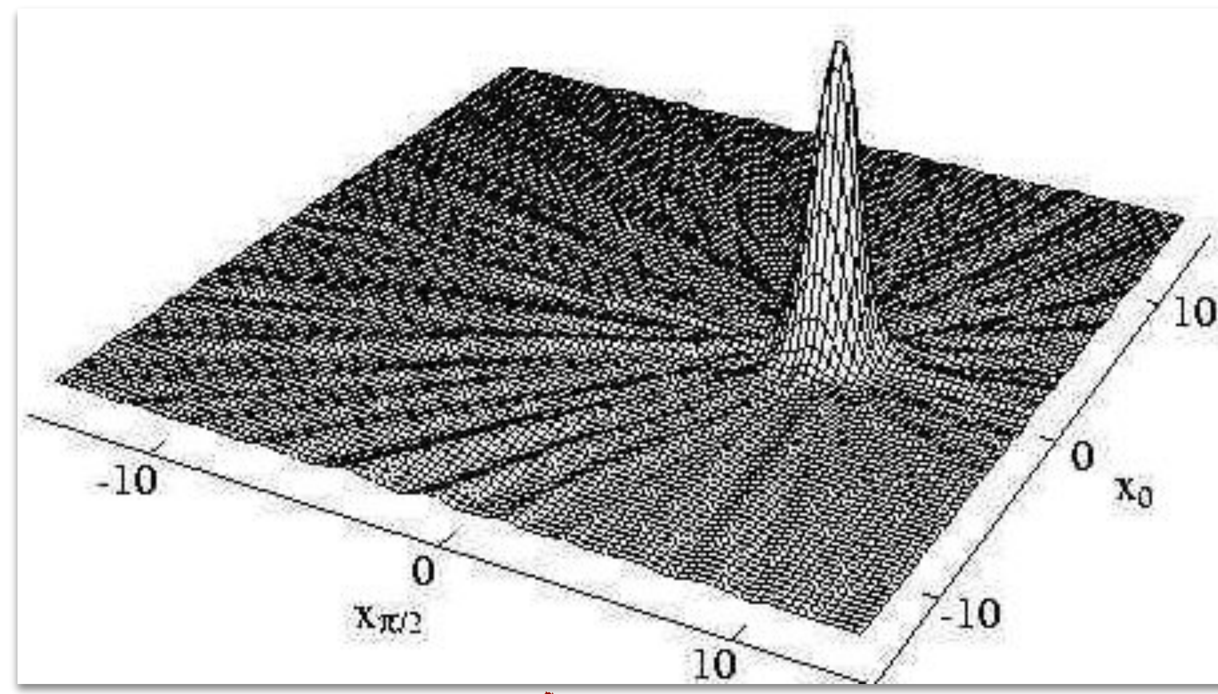




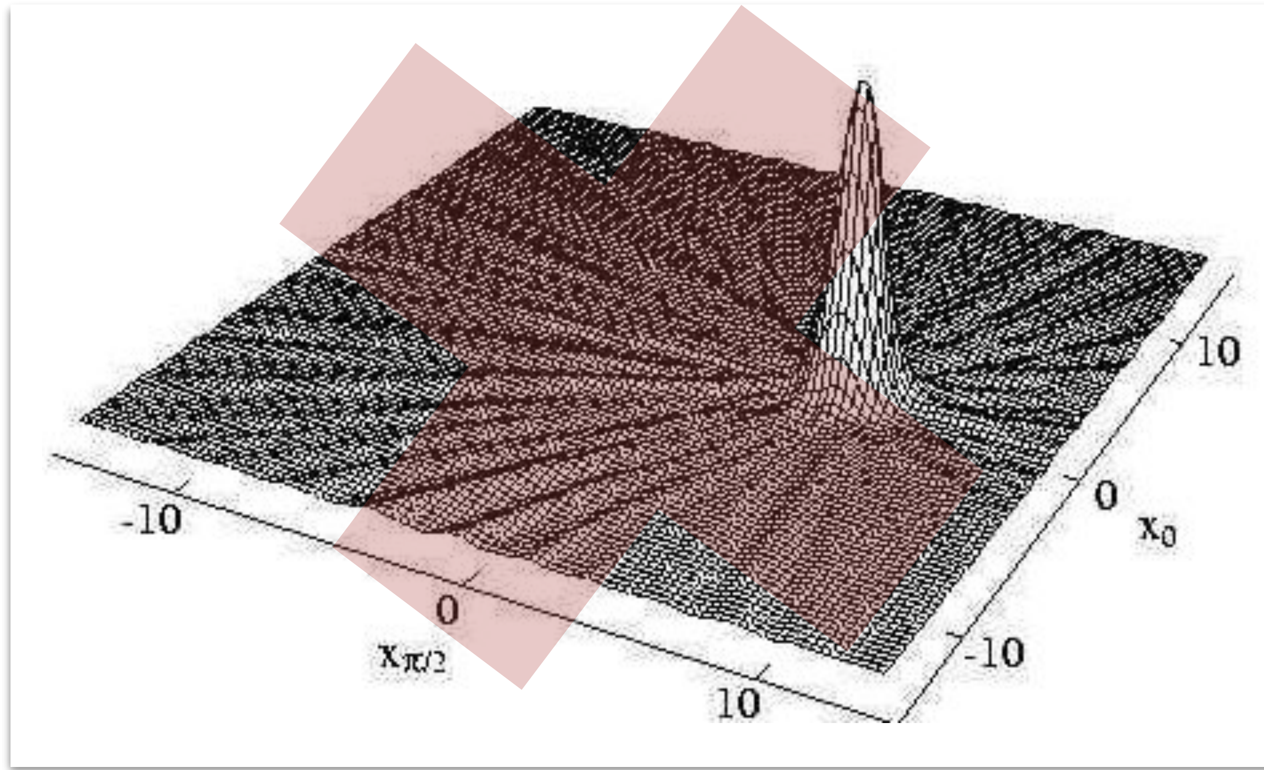
$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \cdots \\ \rho_{10} & \rho_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

…a Wigner function…

…or a density matrix

Tomography in infinite Hilbert space is harder.

Ad-hoc smoothing/binning not always reliable



Infinite parameters estimated from **finite data!**

Don’t use an infinite matrix

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \cdots \\ \rho_{10} & \rho_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Let's make it easier.

Don't use an infinite matrix…

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \cdots \\ \rho_{10} & \rho_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

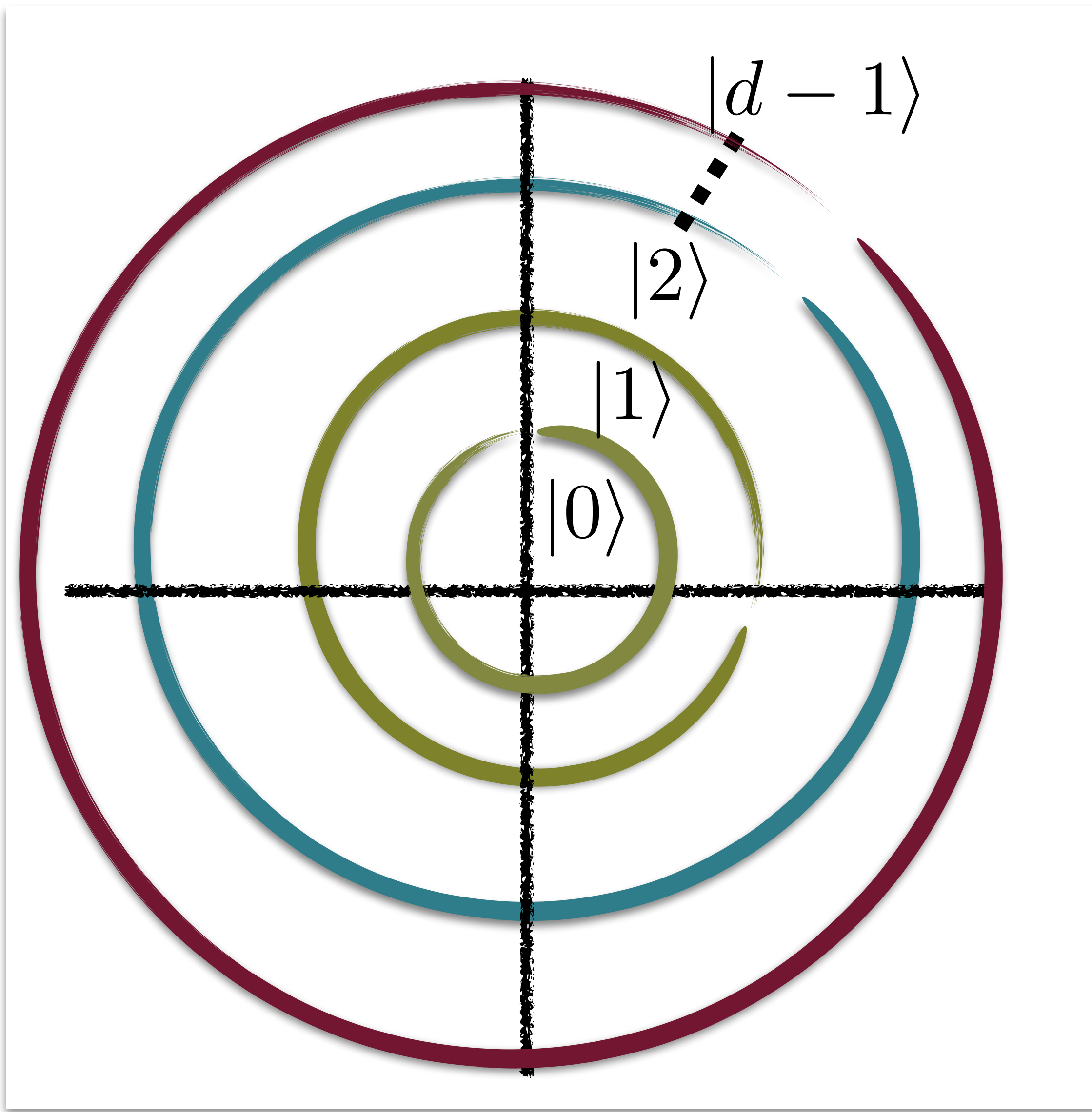
Finite parameters
estimated from **finite data.**

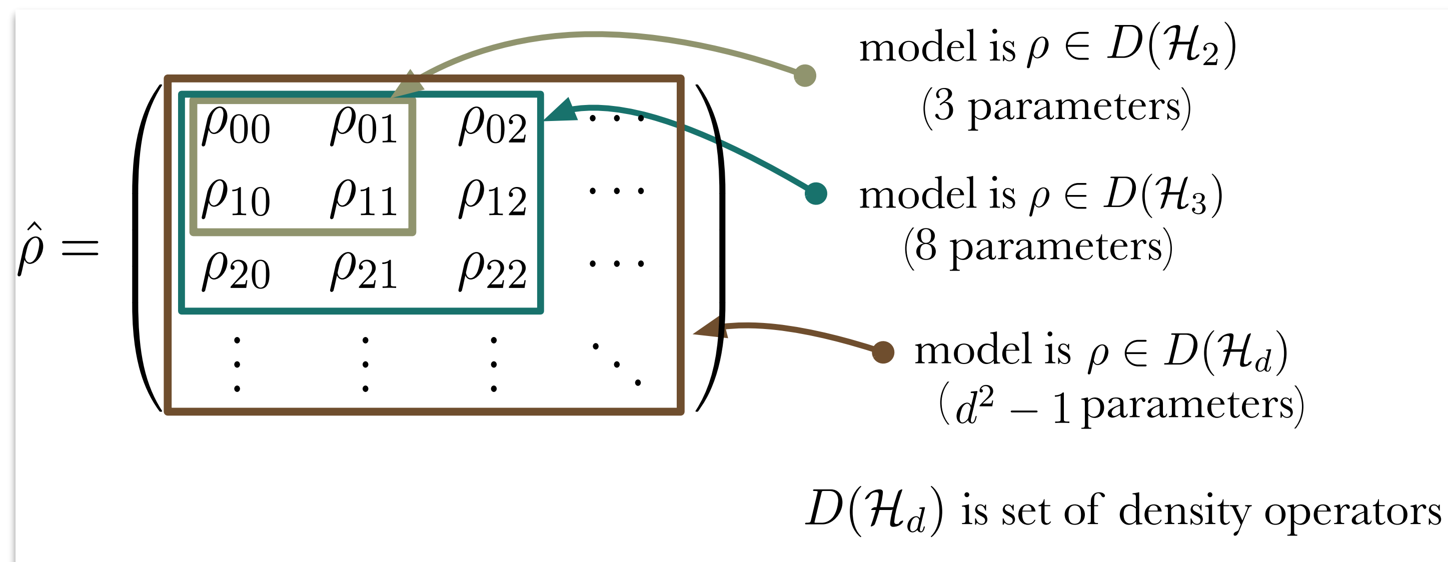
…use a smaller one instead!

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Let's make it easier.

Low energy states can be modeled using a **truncated Fock basis**: $\mathcal{H}_d = \mathrm{span}(|0\rangle, \cdots, |d-1\rangle)$

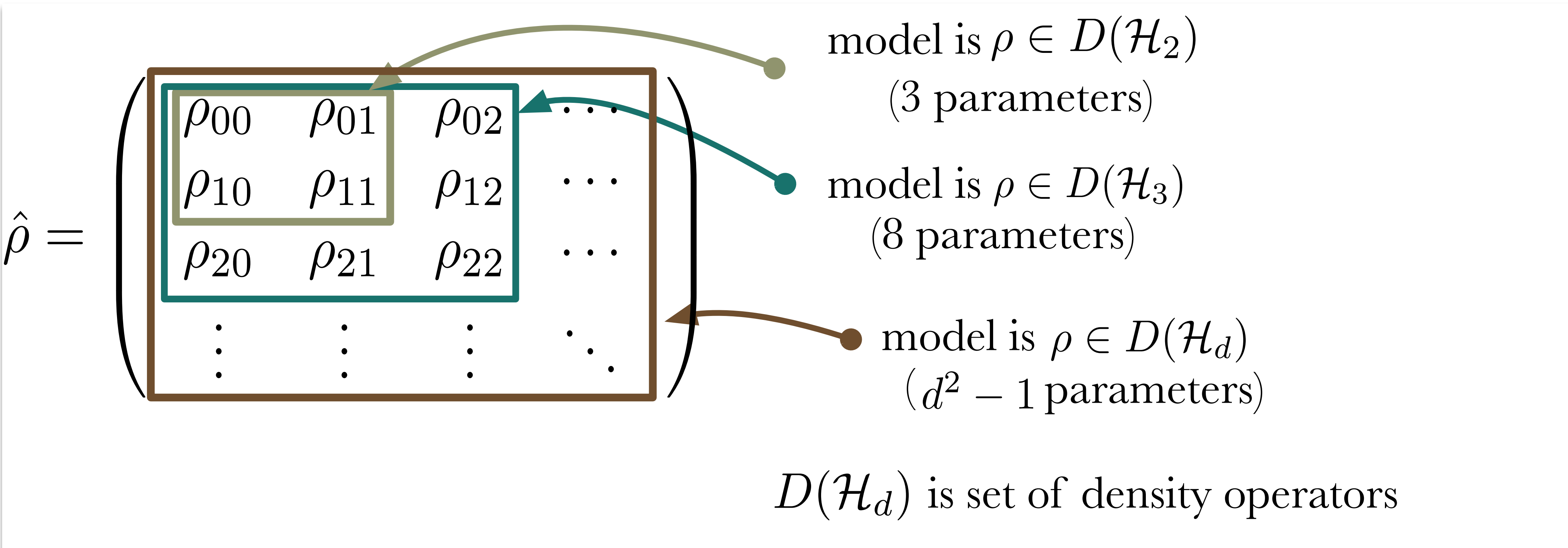




This is where we will do tomography.

Let's make it easier.

We have an **algorithm** for deciding which *d* is best.



1) Take data

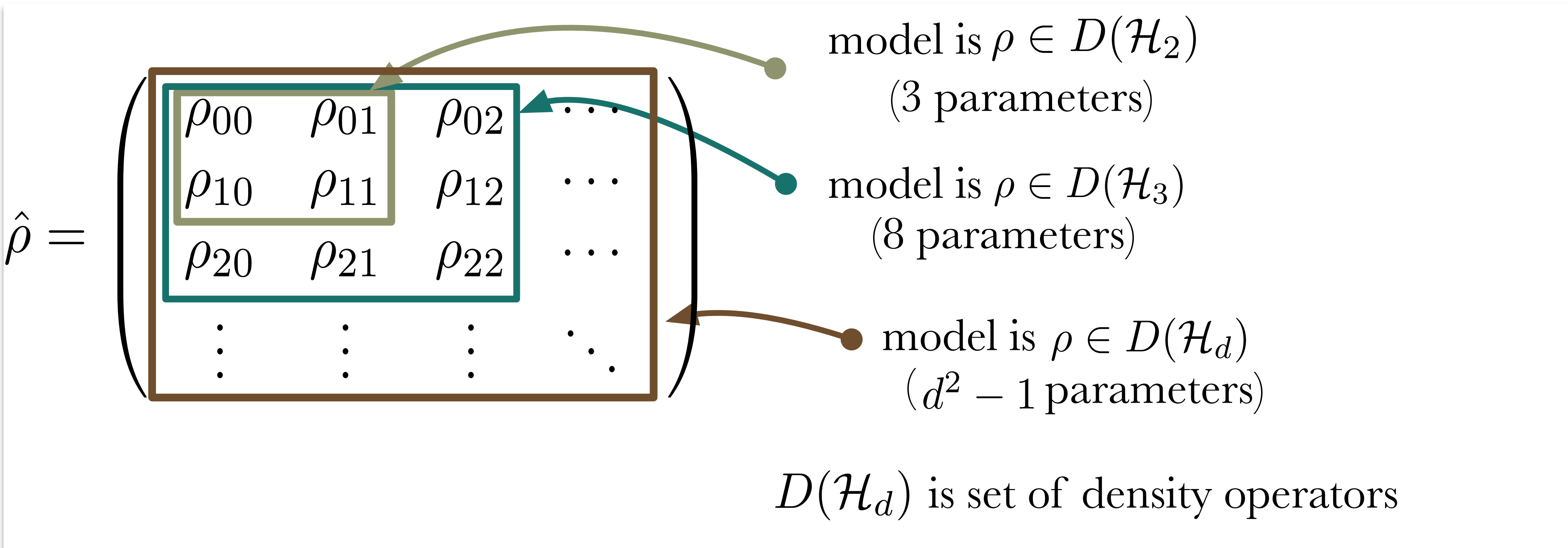
2) For *d* in [2, 3, 4, …], compare model *d* to *d + 1*

3) If model *d + 1* fits significantly better, reject *d*

Otherwise, *d* is where we stop.

Let's make it easier.

We can **quantify** "fitting significantly better".



Goodness of fit = loglikelihood ratio statistic:

$$\lambda(d_1, d_2) = -2\log\left(\frac{\mathcal{L}(d_1)}{\mathcal{L}(d_2)}\right) = -2\log\left(\frac{\max_{\rho\in D(\mathcal{H}_{d_1})} \mathcal{L}(\rho)}{\max_{\rho\in D(\mathcal{H}_{d_2})} \mathcal{L}(\rho)}\right)$$

Let's make it easier.

The **precise** algorithm for "fitting significantly better".

$$\lambda(d_1, d_2) = -2\log\left(\frac{\mathcal{L}(d_1)}{\mathcal{L}(d_2)}\right) = -2\log\left(\frac{\max\limits_{\rho\in D(\mathcal{H}_{d_1})}\mathcal{L}(\rho)}{\max\limits_{\rho\in D(\mathcal{H}_{d_2})}\mathcal{L}(\rho)}\right)$$

1) Take data

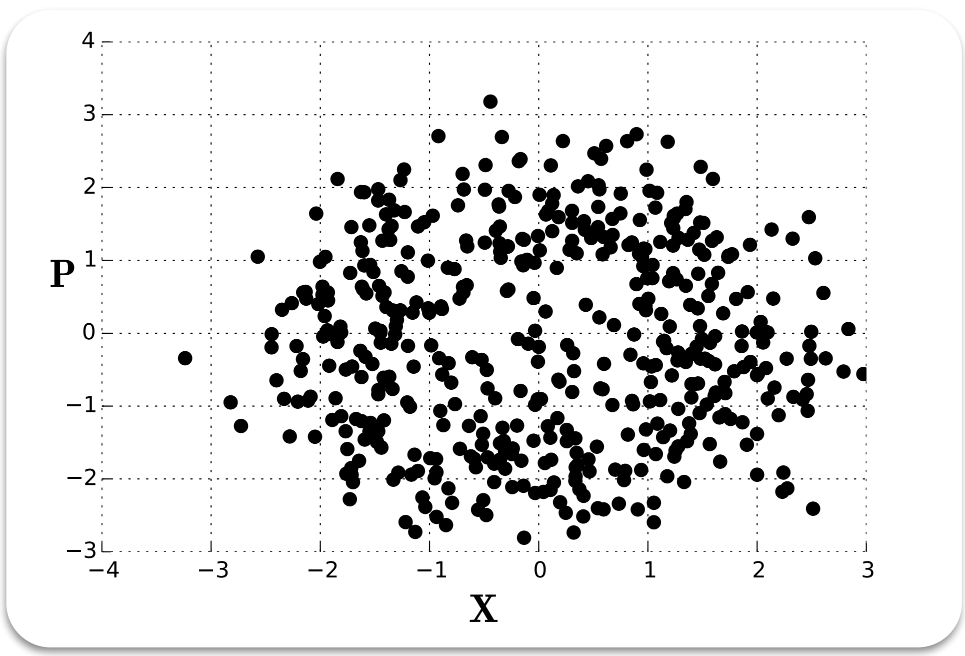
2) Compute $\lambda(d, d+1)$ for *d* in [2, 3, 4, …]

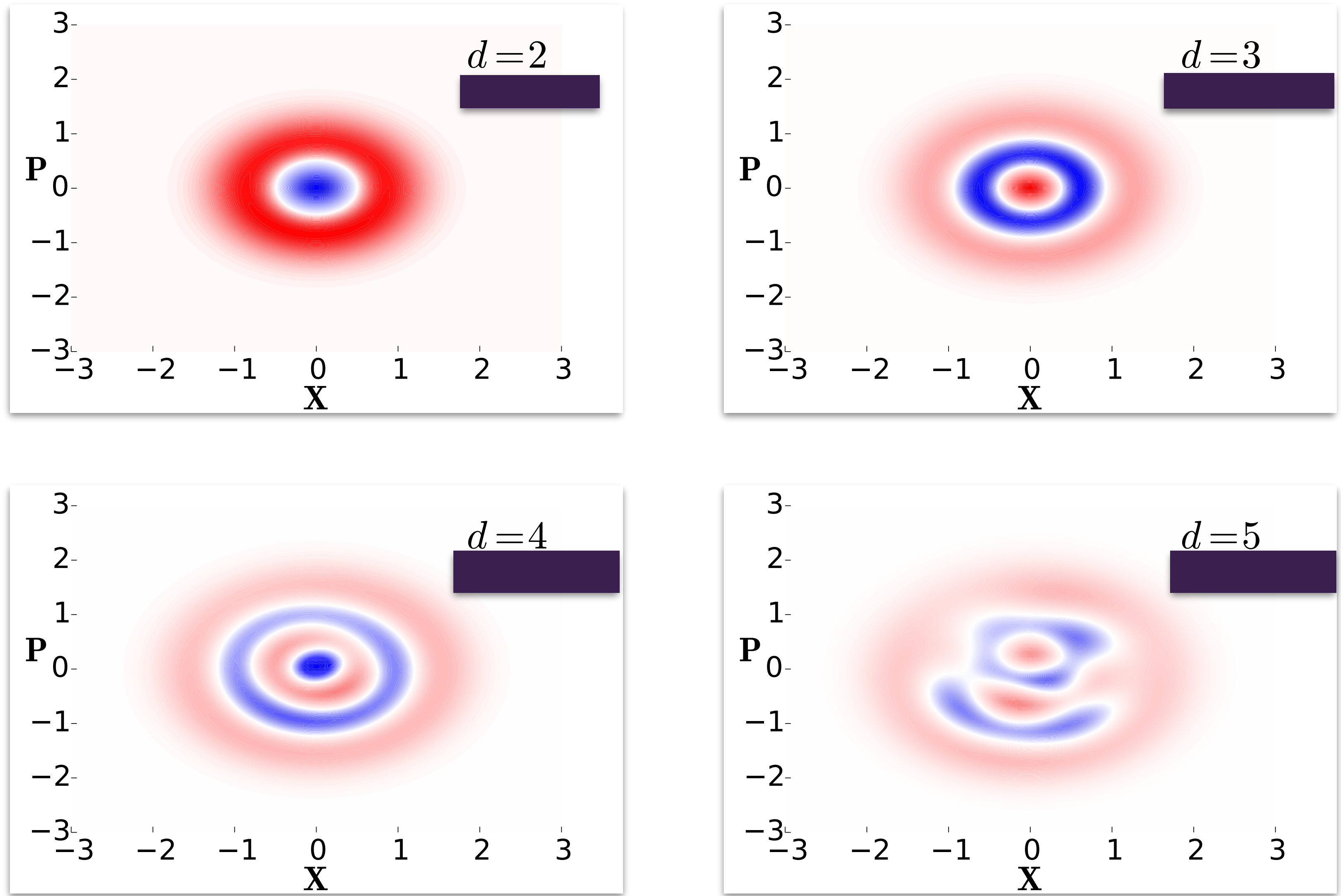
3) If $\lambda(d, d+1)$ greater than threshold, reject *d*
   Otherwise, *d* is where we stop.

Let's make it easier.

An **example** demonstrates use of loglikelihood ratio statistic.

484 Simulated Heterodyne Measurement Outcomes





Can you tell which model fits the best?

Let's make it easier.

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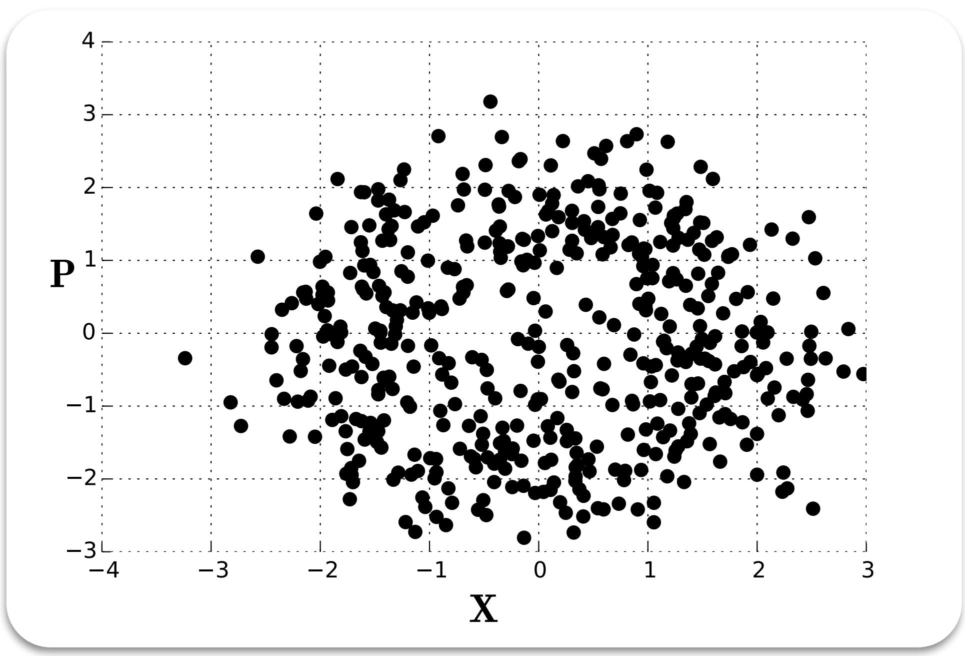

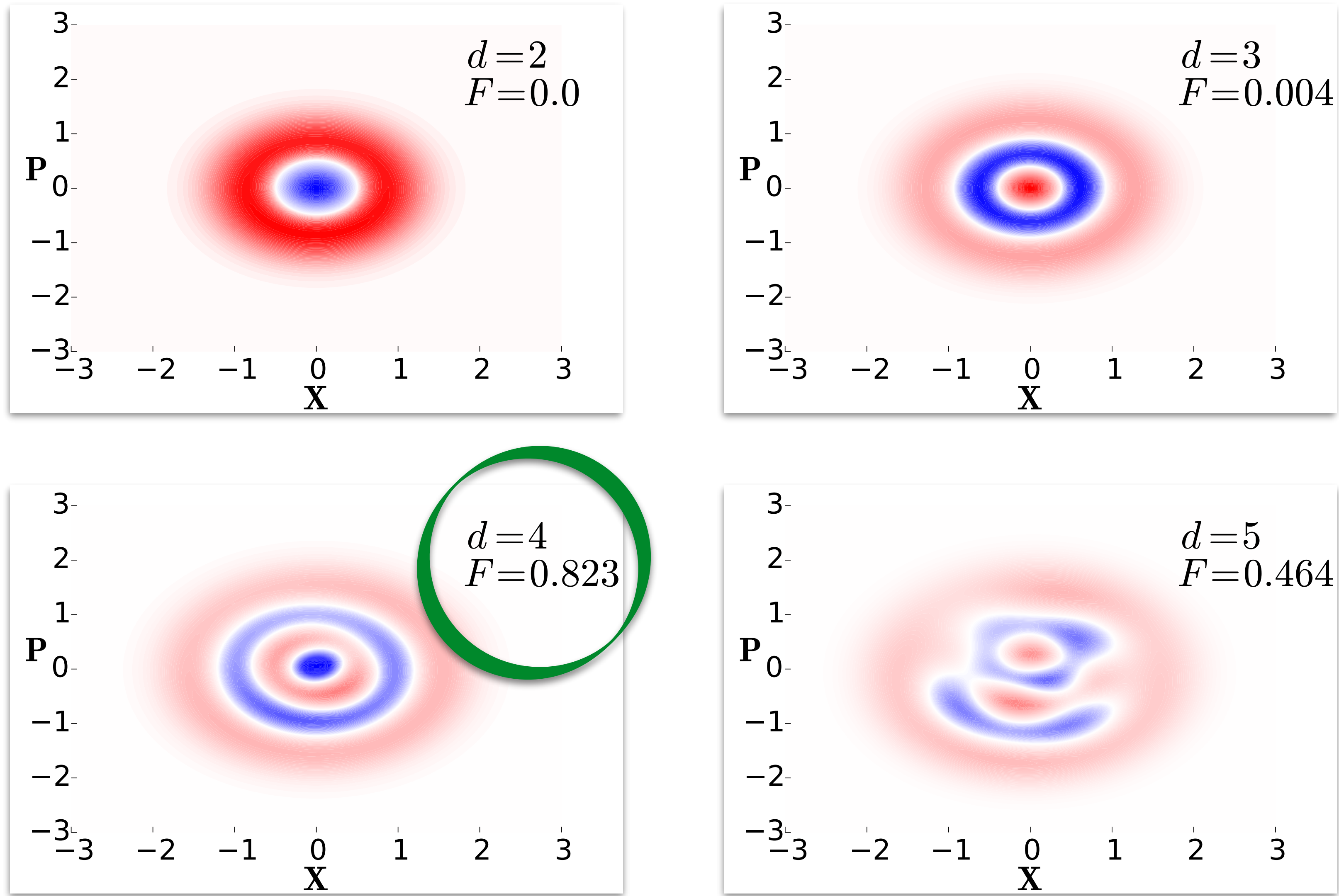

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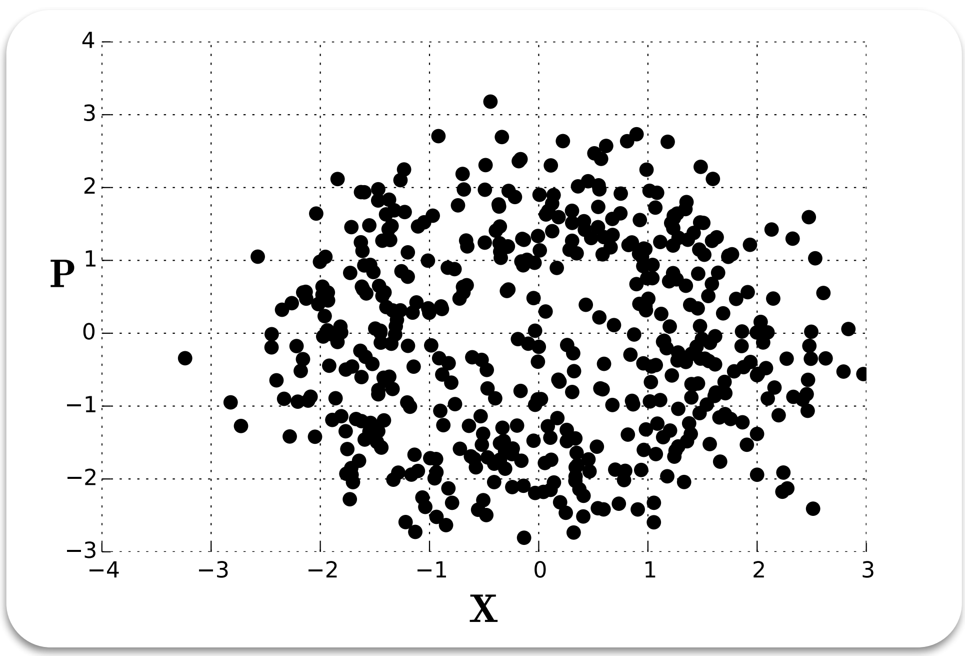
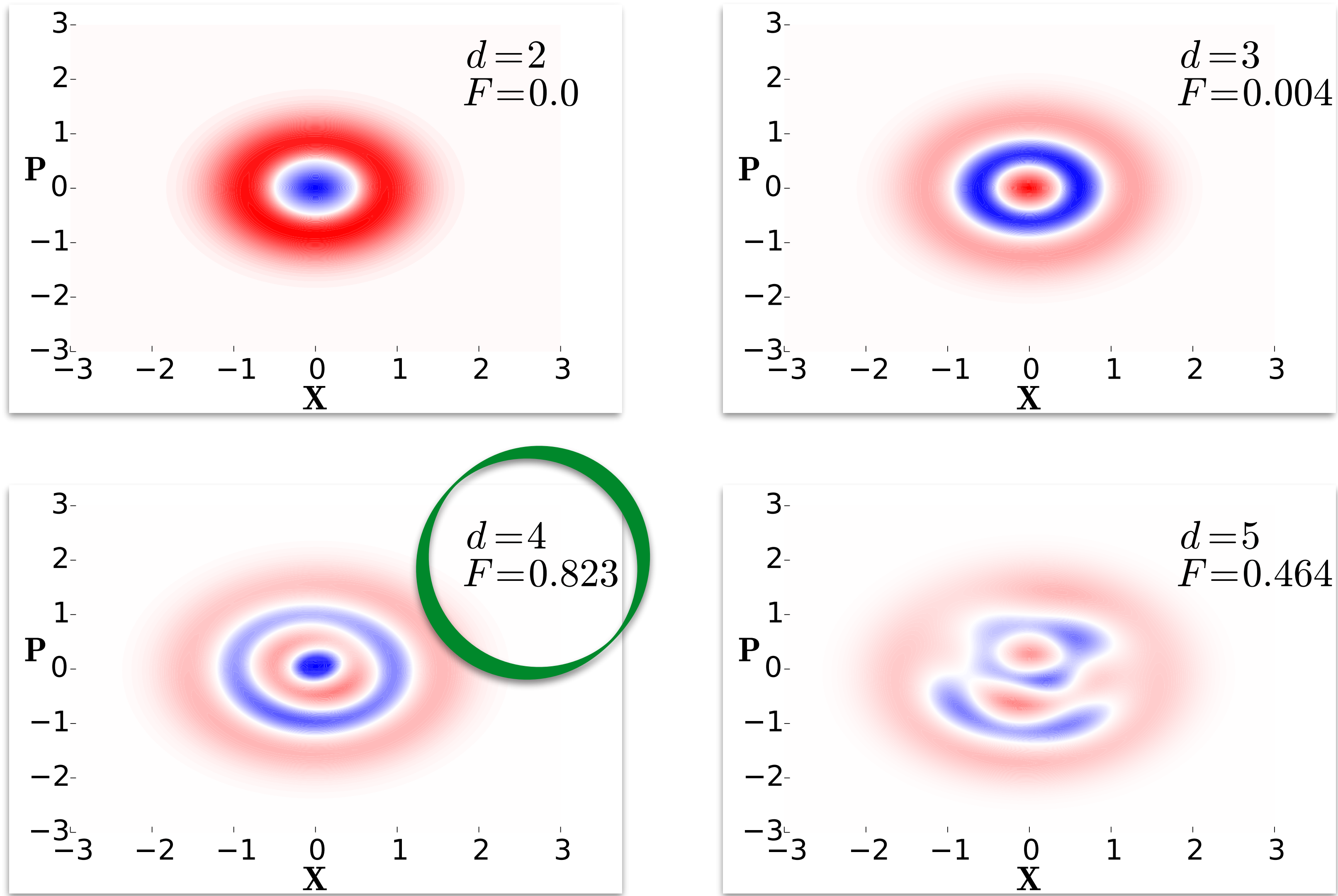


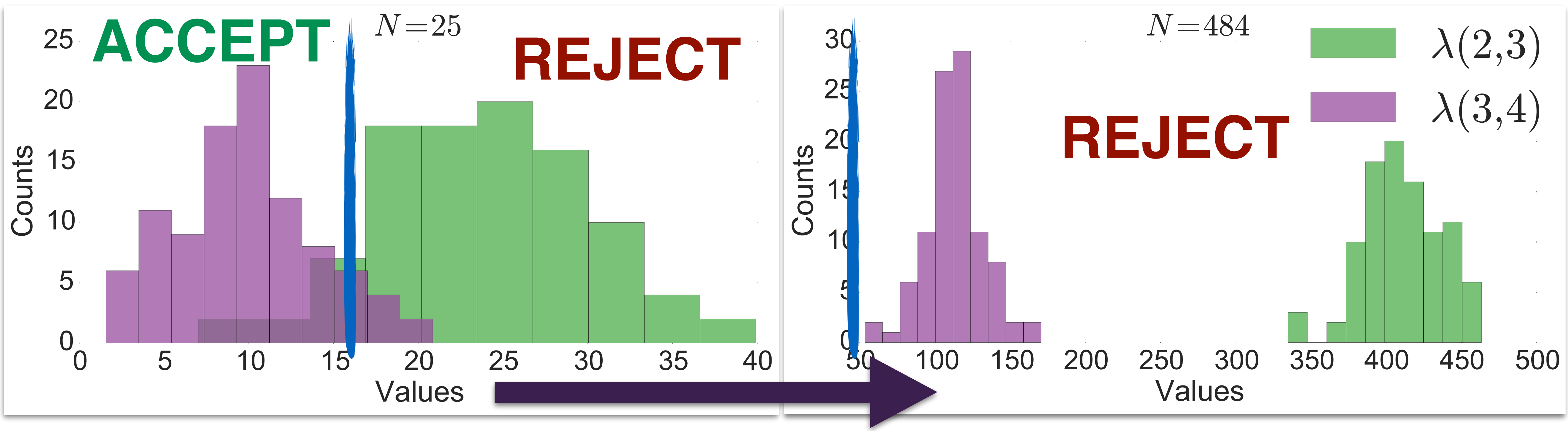


Can you tell which model fits the best?

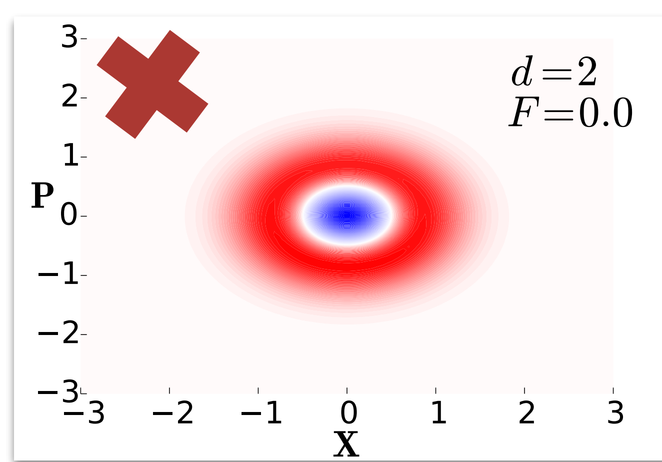
What if you did not know the true state?

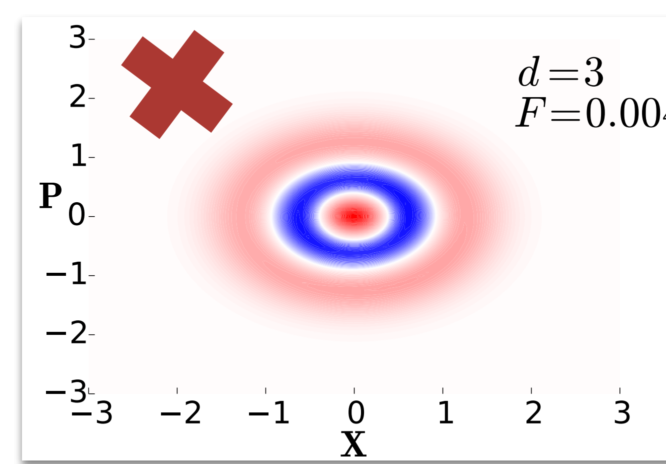
Let's make it easier.

A threshold decides when **smaller models fit worse**.



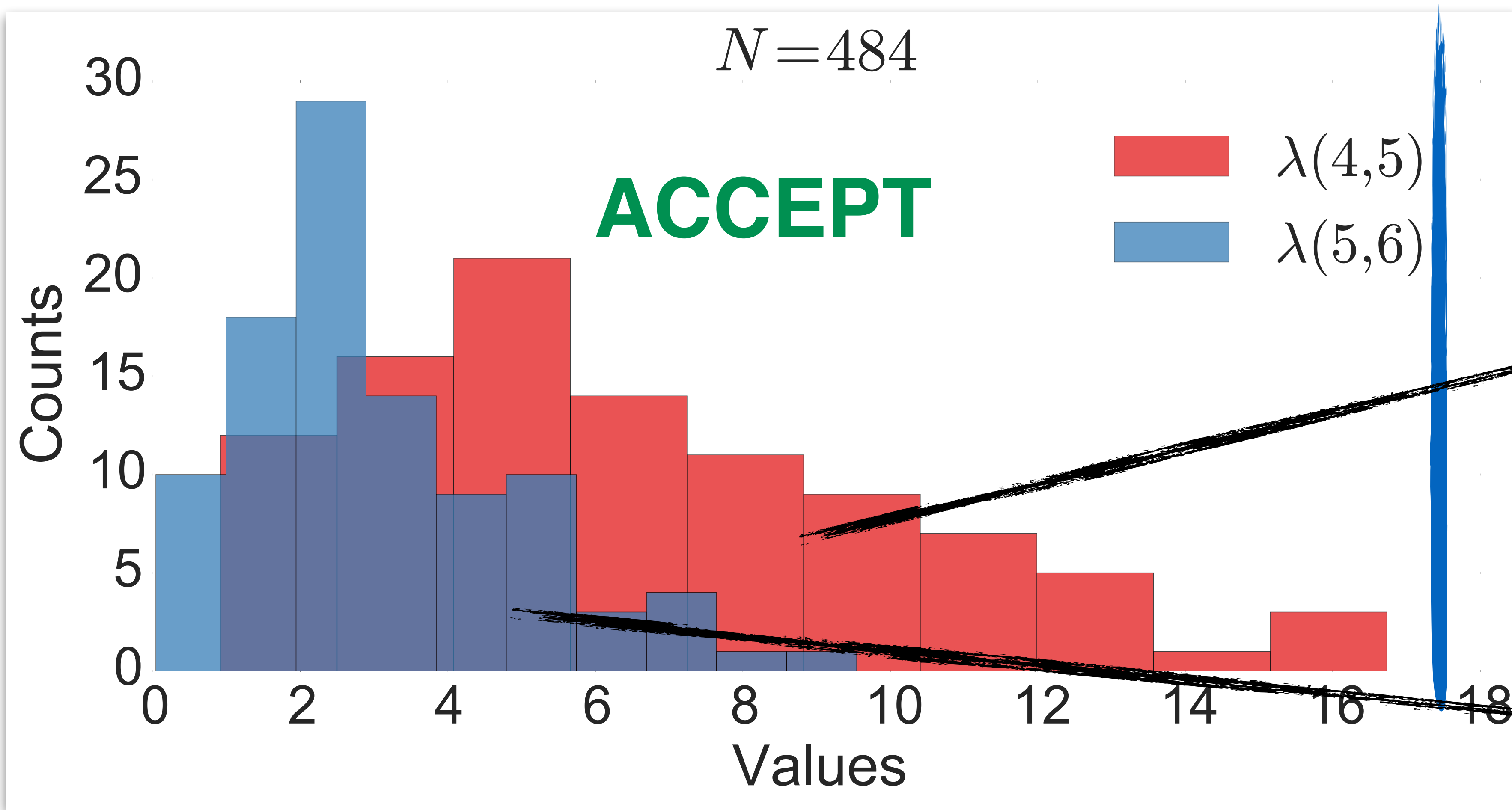
Qubit/qutrit does not well model 4 dimensional system…statistic grows with sample size.

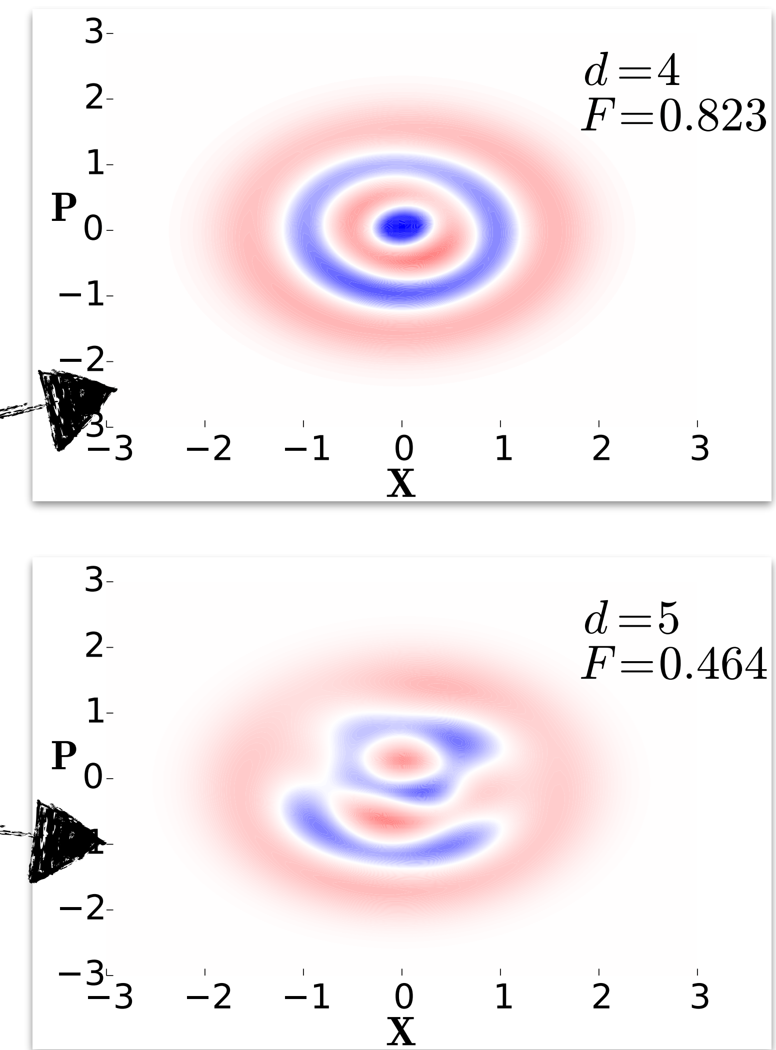




Let's make it easier.

When smaller model **fits well**, all larger models do too.





This is the focus of my current work.

Use numerics to investigate.