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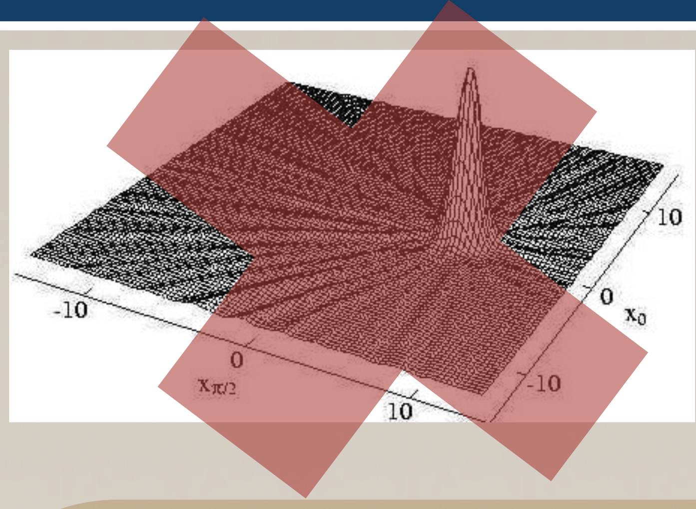

Lost in (Hilbert) Space - Model Selection for Quantum Tomography

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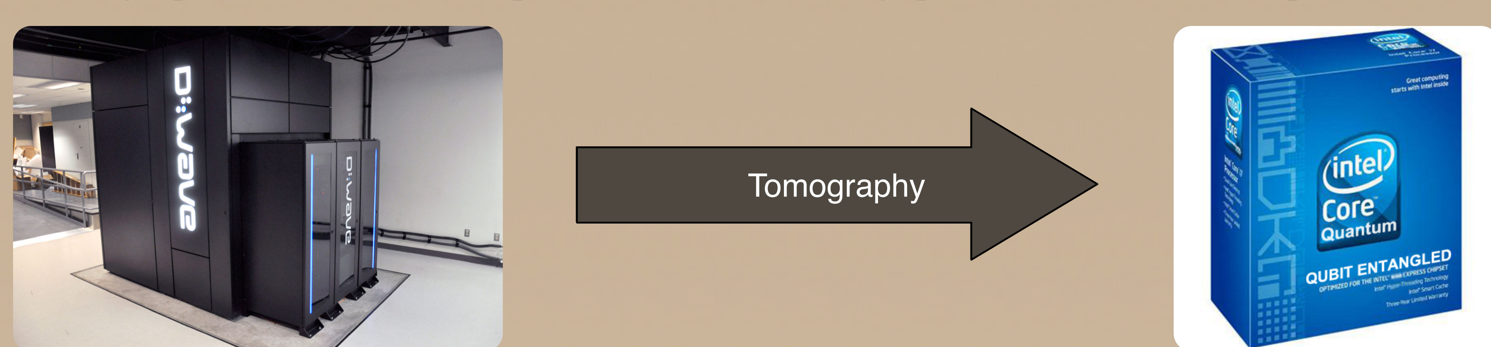
To characterize a system is tomography. However, it can be difficult to do quantum tomography well.

Tomography = set of techniques for characterizing quantum information processors (QIPs)



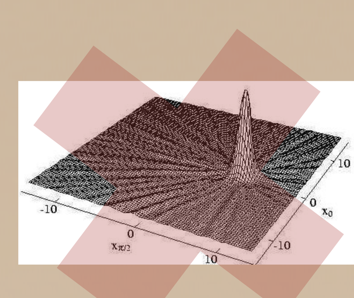
Uncharacterized device - not very useful

Fully characterized/controllable - very useful

State tomography = estimate quantum state inside QIP $\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \cdots \\ \rho_{10} & \rho_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$

Some estimates (CV systems) need matrices with infinite/unknown dimension! What do we do?

$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \cdots \\ \rho_{10} & \rho_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$ Use infinite matrix? Not practical!



Ad hoc methods - smoothing, binning, etc - not always reliable

$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$ Choose an effective (small) Hilbert space dimension

For systems with large/unknown Hilbert space dimension, can we find an effective dimension for which tomography is both accurate and feasible?

We investigate state tomography of continuous variable (CV) systems.

CV system = mode ω of electromagnetic field

Hilbert space: $L^2(\mathbb{R})$ (infinite dimensional!)

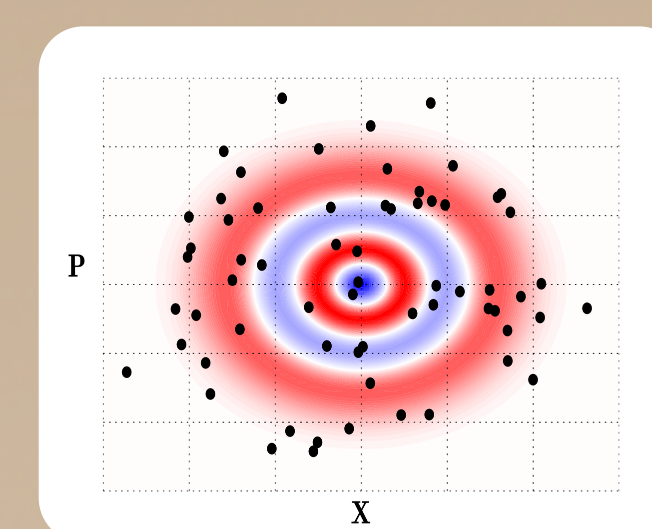
States: $\int \rho(x,x')|x\rangle\langle x'|\,dx\,dx' \longleftrightarrow \sum_{j,k}\rho_{jk}|j\rangle\langle k| \quad H_{SHO}|j\rangle = \hbar\omega\left(j+\frac{1}{2}\right)|j\rangle$

A possible measurement:
heterodyne detection (coherent state projection)

Measure $\mathbf{X},\mathbf{P}$ simultaneously to within $\Delta\mathbf{X}\Delta\mathbf{P} = \frac{\hbar}{2}$

POVM is $\{|\alpha\rangle\langle\alpha|d\alpha\}$ where
$|\alpha\rangle = |x+ip\rangle$ is a coherent state

Infer state (density matrix $\hat{\rho}$
or Wigner function $\hat{W}(x,p)$)
from observed data $\{\alpha_1,\cdots,\alpha_N\}$



Wigner function $W(x,p)$ plotted with heterodyne data (black points)

Low energy states can be *modeled* by density matrices on the Hilbert space $\mathcal{H}_d = \text{span}\{|0\rangle,\cdots|d-1\rangle\}$

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} & \rho_{02} & \cdots \\ \rho_{10} & \rho_{11} & \rho_{12} & \cdots \\ \rho_{20} & \rho_{21} & \rho_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

model is $\rho \in D(\mathcal{H}_2)$ (3 parameters)

model is $\rho \in D(\mathcal{H}_3)$ (8 parameters)

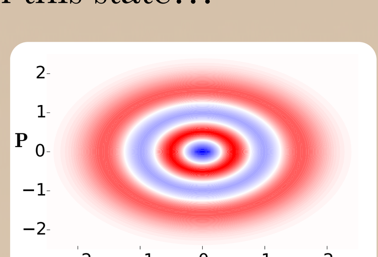
model is $D(\mathcal{H}_d)$ (d^2-1 parameters)

$D(\mathcal{H}_d)$ is set of density operators

For coherent state (heterodyne) tomography of low energy states how do we choose d?

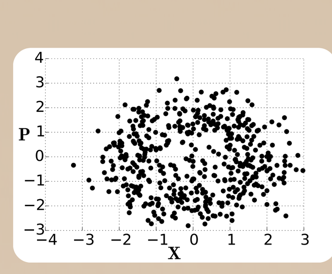
An example of different models in CV tomography.

From this state…



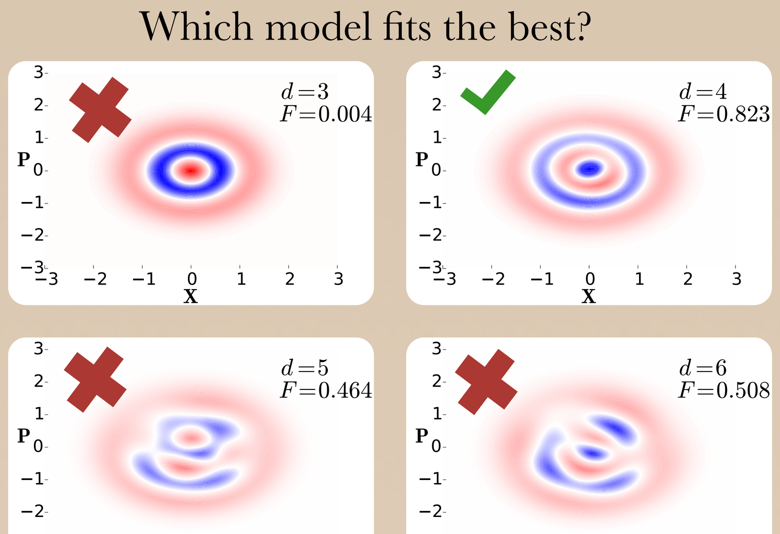
Plot of Wigner function for true state $\rho = .1|0\rangle\langle 0| + .9|3\rangle\langle 3|$

…we simulated this data…



…and made estimates using different models.

Which model fits the best?



Plots of Wigner functions for maximum likelihood estimates. Hilbert space dimension and fidelity with true state are given.

We want to choose a model which *works well*. The true dimension seems to do so. Can we choose it *without* knowing the true state?

Got feedback? Let me know!

@Travis_Sch

+TScholten

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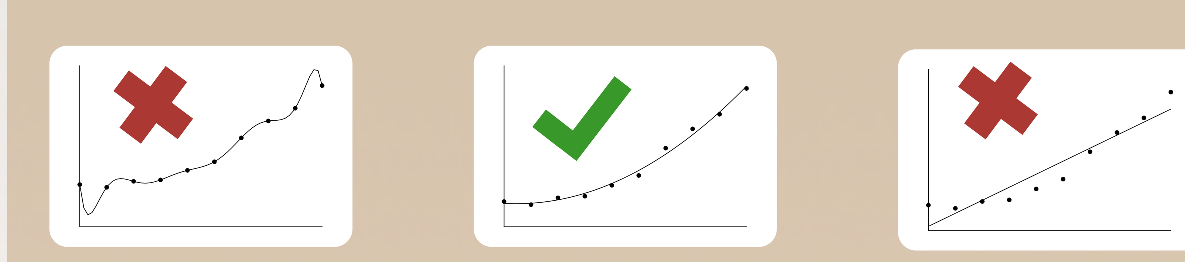
We can use *hypothesis testing* to choose between models in state tomography.

A model is a parametrized manifold of probability distributions

State tomography: Model = set of d dimensional density matrices in $D(\mathcal{H}_d)$

Likelihood function = how well state fits data $\mathcal{L}(\rho) = \text{pr}(\text{data}|\rho)$



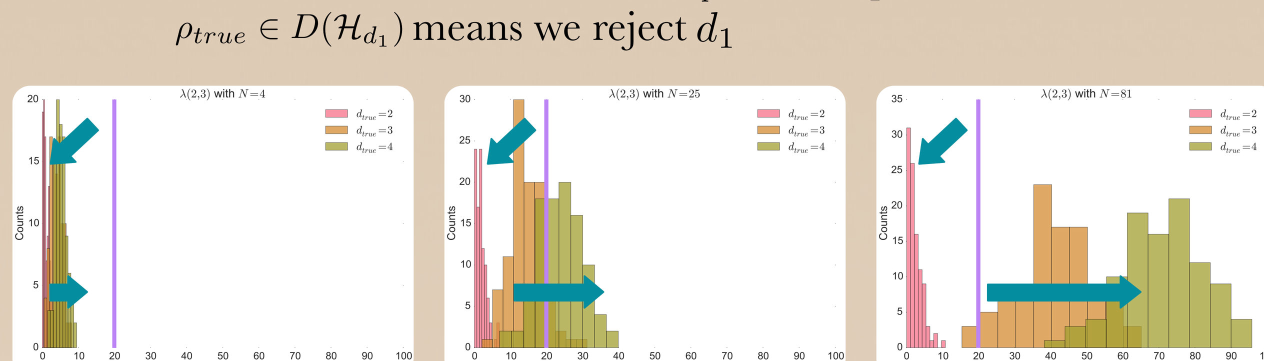
For simple data (above), we can identify overfitting (left) and underfitting (right) by inspection. But for complex problems like tomography, we need rigorous, quantifiable criteria and/or algorithms.

The *loglikelihood ratio statistic* compares two models.

$$\lambda(d_1,d_2) = -2\log\left(\frac{\mathcal{L}(d_1)}{\mathcal{L}(d_2)}\right) = -2\log\left(\frac{\max_{\rho\in D(\mathcal{H}_{d_1})}\mathcal{L}(\rho)}{\max_{\rho\in D(\mathcal{H}_{d_2})}\mathcal{L}(\rho)}\right)$$

λ is a random variable whose distribution depends strongly on whether $\rho_{true} \in D(\mathcal{H}_{d_1})$

Experimental data yielding λ_{exp} not compatible with $\rho_{true} \in D(\mathcal{H}_{d_1})$ means we reject d_1



Histograms of $\lambda(2,3)$ for various d_{true} at three sample sizes N
When smaller model is valid $\lambda(2,3)$ is random but small, not growing with N
When invalid, $\lambda(2,3)$ does grow with N

By setting a *threshold value* λ_{thresh} we define a reliable test:

Reject d_1 when $\lambda_{exp} > \lambda_{thresh}$

Any d_1 not compatible with ρ_{true} will fail test when $N >> 1$

What are good threshold values? How do we *select* a model?

We devise an algorithm for choosing Hilbert space dimension without knowing the true state.

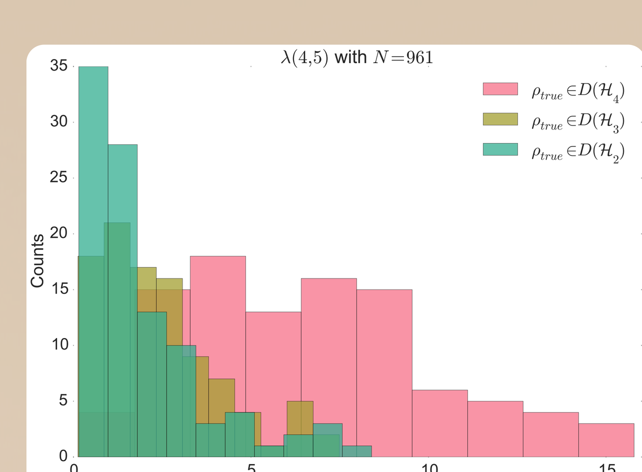
Take N data points → Make a list of $\lambda(d,d+1)$ → Go through list → $\lambda(d,d+1)$ too big? → No: keep $D(\mathcal{H}_d)$ as good model; Yes: reject $D(\mathcal{H}_d)$ as good model; Report smallest d kept

How big is too big?

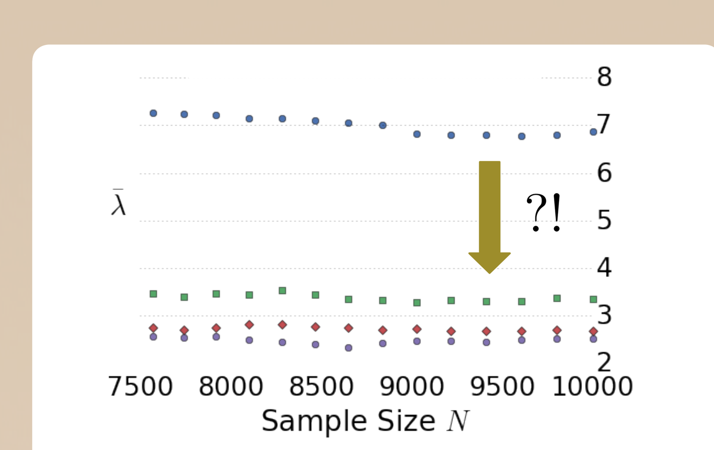
We need to determine good threshold $\lambda_{thresh}(d,d+1)$

If *Wilks theorem* holds, $\lambda(d,d+1)$ is χ^2_{2d+1} distributed — threshold done.

Numerics indicate Wilks theorem *does not work!*



Histograms show distribution of $\lambda(4,5)$ depends on ρ_{true} - a violation of Wilks.



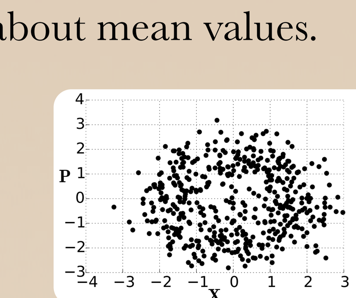
Wilks predicts $\langle\lambda(d,d+1)\rangle = 2d+1$ as $N\to\infty$
We see *very* different behavior- $\bar{\lambda}(d,d+1) \approx 2.5$

We are working towards a generalization of the Wilks theorem that will explain these data and yield good thresholds.

Can we modify Wilks in some way? If so, how?

Start by learning about mean values.

Current idea:
non contributing parameters



The data may not support using all parameters in the Hilbert space. Those which do not fluctuate much from zero may not contribute to the statistic.

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