

Finite-Difference Simulation of Atmospheric Acoustic Sound Through a Complex Meteorological Background Over a Topographically Complex Surface

Neill P. Symons, David Aldridge; Sandia National Laboratories

**D. Keith Wilson; U.S. Army Cold Regions Research and
Engineering Laboratory**

David Marlin, Sandra Collier; U.S. Army Research Laboratory

**Valdimir Ostashev; NOAA Environmental Technology
Laboratory**



Why Finite-Difference?

- **We have developed a high quality algorithm for 3D finite-difference wave propagation in moving-media (wind)**
- **Finite-difference advantages**
 - Can use arbitrary gridded models
 - Point by point variation of material parameters is allowed
 - Directly solve the governing equations—generate all physical arrival types
 - Domain decompositions can be used to efficiently solve FD problems on parallel computers
 - Atmospheric and source dynamics are easily incorporated
- **Disadvantages**
 - Computationally intensive, large (realistic) models require supercomputers
 - Complicated stability and dispersion relations must be understood to run models correctly

Physical and Mathematical Assumptions

- **Linearize equations with respect to small-amplitude acoustic wavefield fluctuations**
 - Fundamental equations of continuum mechanics
 - Balance of mass, momentum, entropy
 - Constitutive relations (ideal fluid)
 - Equation of state (pressure)
- **Ambient medium**
 - Adiabatic (no energy sources or heat transfer between fluid parcels)
 - Details of derivation in 4pPA4 from Nashville meeting and paper in preparation
 - Divergence-free motion (incompressible fluid)
 - Must be enforced by model builder
- **Gravity and pressure gradients ignored in acoustic equations**
 - Uncouples \mathbf{p} and \mathbf{w} from fluctuations in mass density ρ

Acoustic Velocity-Pressure System for Moving-Media

$$\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{w} + (\mathbf{w} \cdot \nabla) \mathbf{v} + b \nabla p = b \mathbf{f}$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \kappa \nabla \cdot \mathbf{w} = \frac{\partial e}{\partial t}$$

Wavefield variables:

$\mathbf{w}(\mathbf{x}, t)$ - particle velocity vector

$p(\mathbf{x}, t)$ - acoustic pressure

Body sources:

$\mathbf{f}(\mathbf{x}, t)$ - force density vector

$e(\mathbf{x}, t)$ - energy density scalar

Acoustic model parameters:

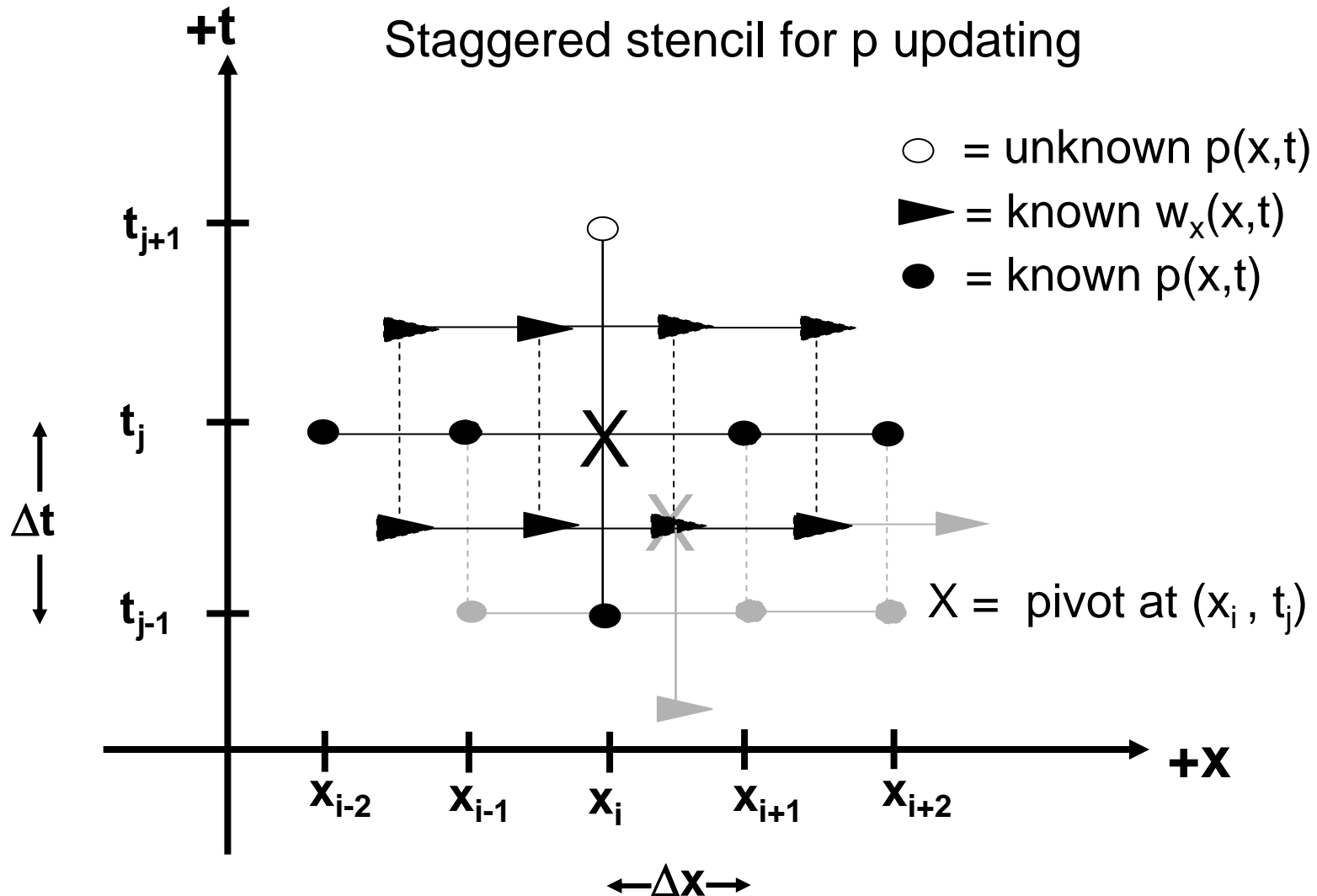
$b(\mathbf{x}, t)$ - mass buoyancy

$\kappa(\mathbf{x}, t)$ - bulk modulus

$\mathbf{v}(\mathbf{x}, t)$ - fluid velocity vector

A system of four, coupled, linear, first-order, partial differential equations.

$O(2,4)$ Staggered-Grid Finite-Differences

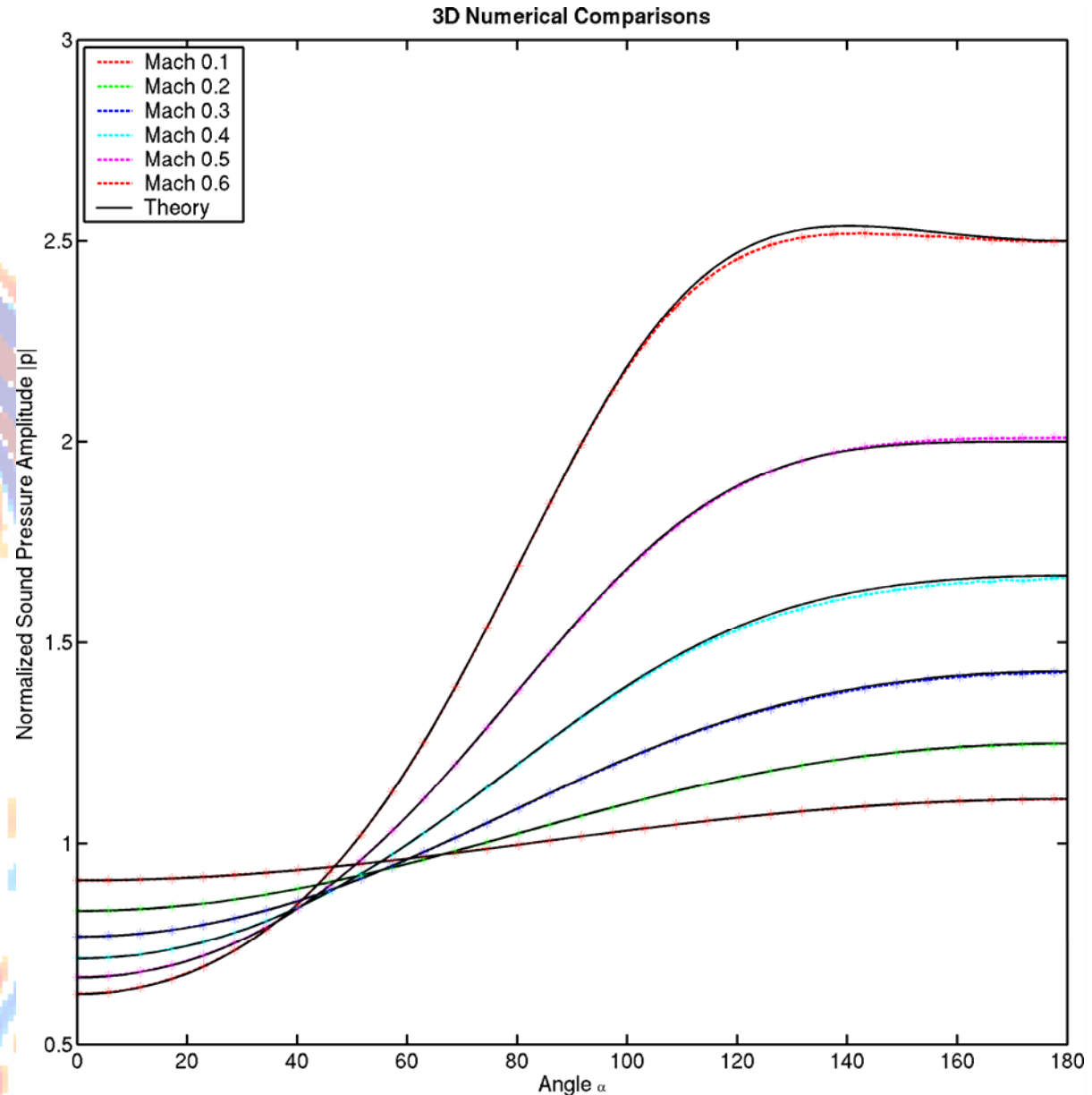


Dispersion and Stability

- **5 nodes/propagating wavelength is the normal rule of thumb with 4th order spatial finite-difference algorithms**
 - Determine the highest frequency in the far field (may be 1 or more differentiations of the source waveform)
 - Determine the slowest velocity in the model and pick a grid spacing such that λ/dx is ~ 5
 - Slowest velocity is the apparent velocity $V(1-Mach)$
 - Can be much smaller than lowest velocity for high wind speeds
 - Can get excessive dispersion for upwind propagation if this is ignored
- **CFL ($\sim [dt \times V_{max}] / dx$) must be less than 1 for stability**
 - Must use the highest apparent velocity $V(1+Mach)$
 - Algorithm will “blow up” if this ignored

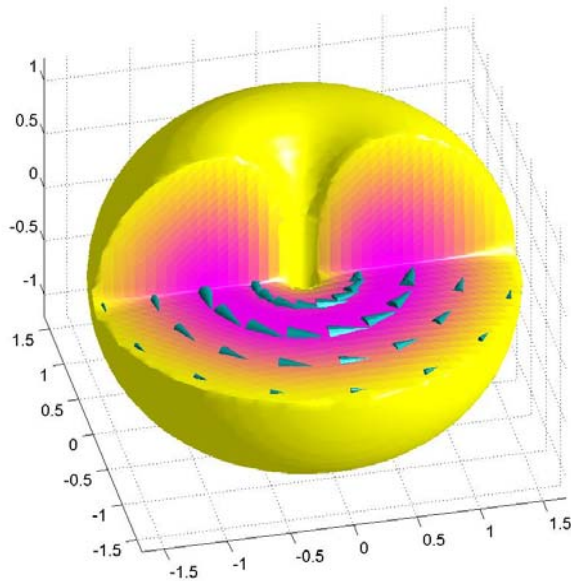
Comparison to Analytic Solution (1)

- **Black lines are the theoretical pressure verses azimuth at a variety of Mach #**
- **Colored lines are the results of this algorithm**
 - Needed to grid up a very fine model to get good results at the higher Mach numbers
- **These are wholespace results**
 - At 50m from the source



Quasi-Wavelet Turbulence

Quasi-wavelets: Technique that mimics the conceptual picture of turbulence consisting of discrete eddies of different sizes.



Like wavelets, they...

- Are based on a self-similar, localized function

Unlike wavelets, they...

- Have random orientations and positions
- Are not required to be zero-mean functions
- Do not form a complete basis

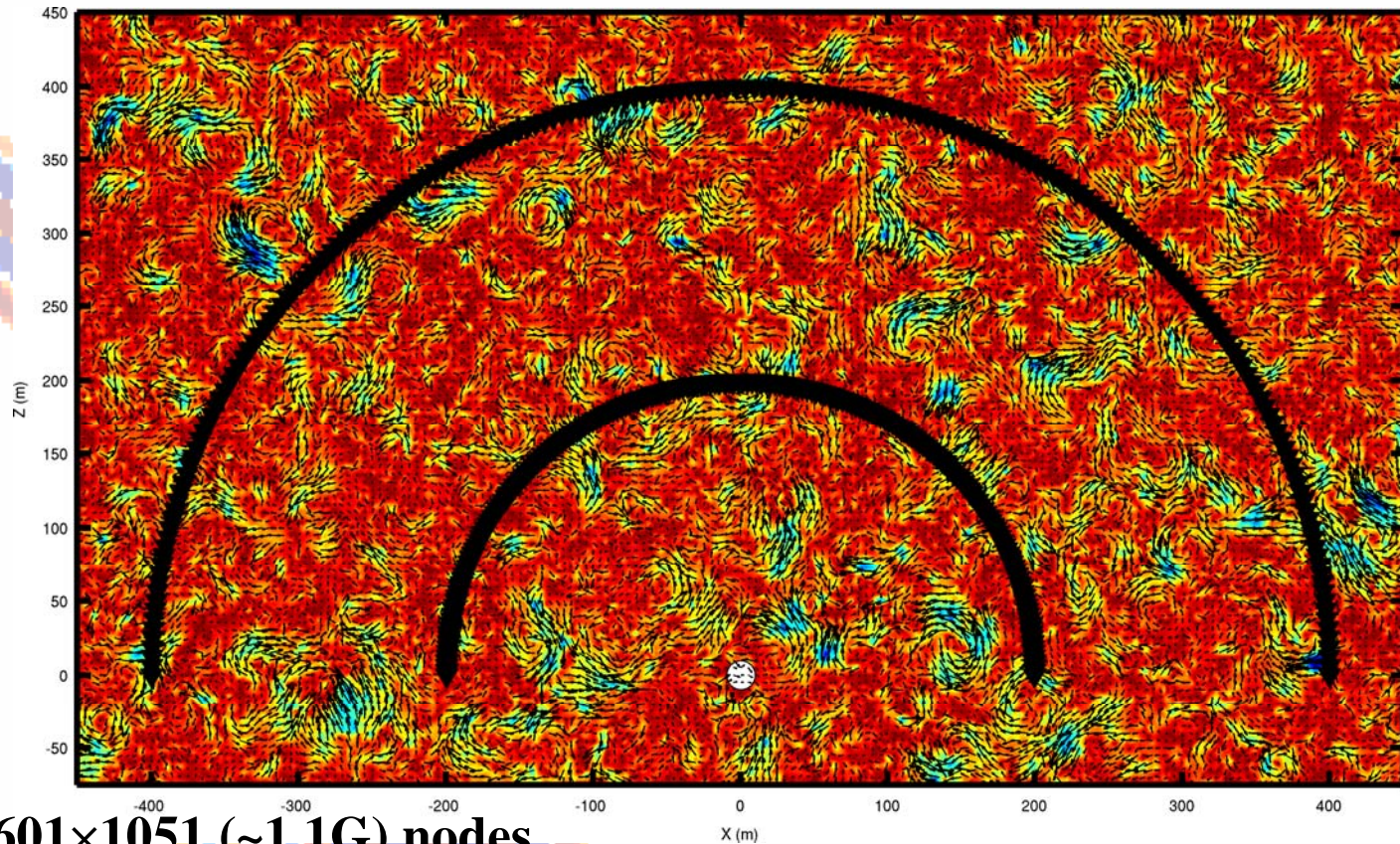
Parallel implementation:

1. Generate a “master list” of QW positions, sizes, and orientations.
2. Each processing element searches the list for the QWs within its domain. Fields associated only with local QWs are calculated.

Test of the QW Implementation

**Receivers at
two fixed
distances
from the
source**

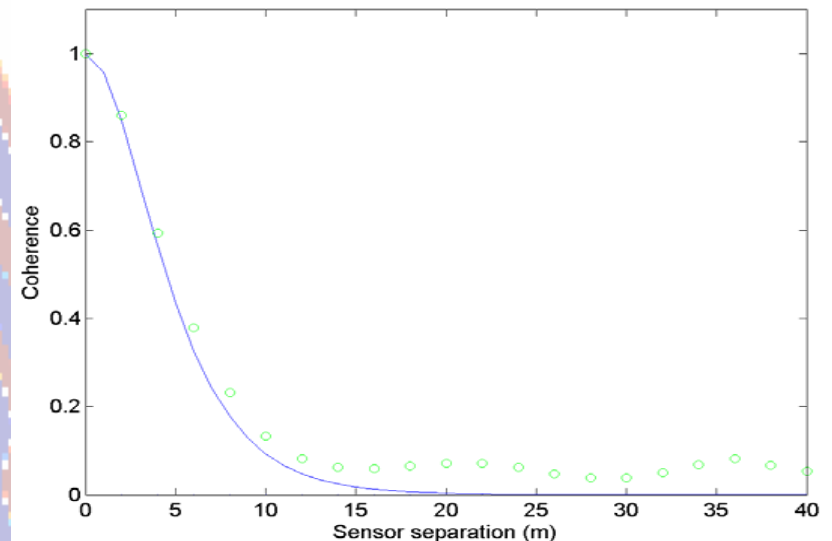
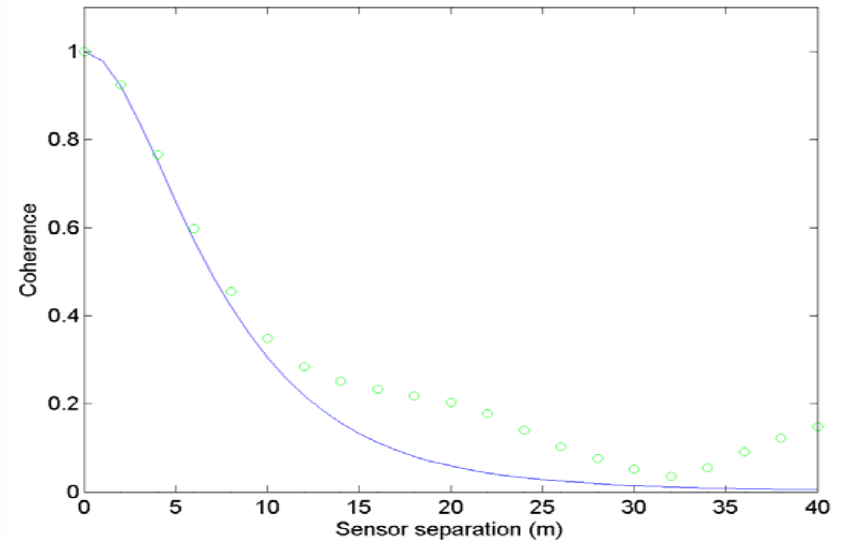
- This is the upper half of a band around a sphere
- Total of 8400 receivers



- **Model is $1801 \times 601 \times 1051$ (~1.1G) nodes**
 - Whole space
 - 590000 QW
 - Kinetic Energy Dissipation Rate 10
 - Size range from 1-10m
 - Maximum Velocity ~10m/s

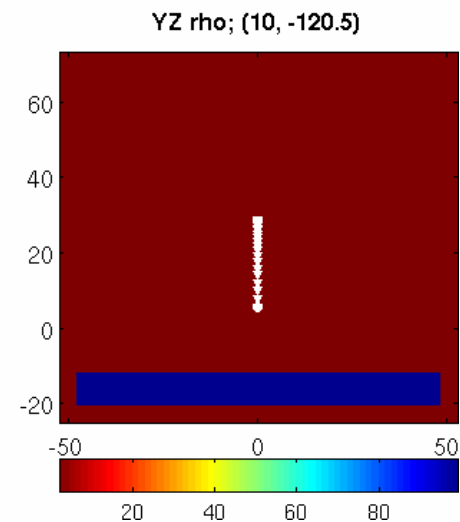
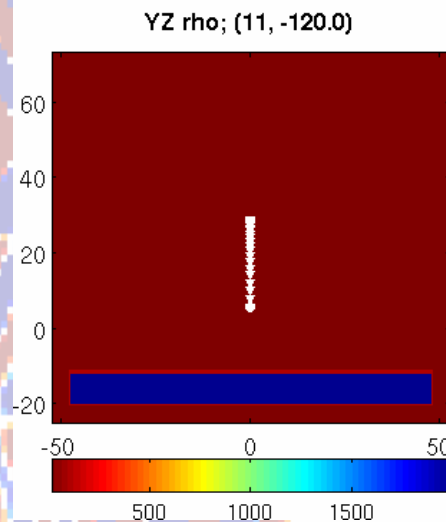
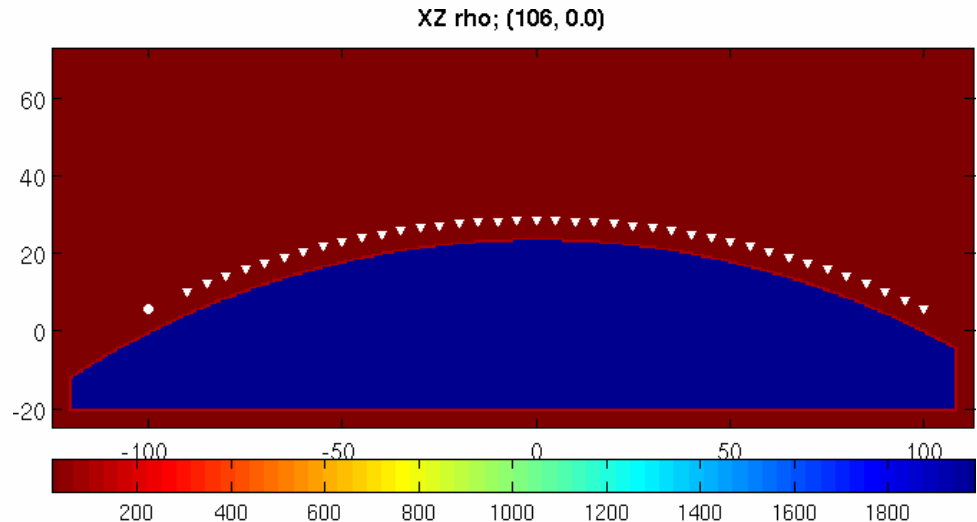
QW Results

- **Fairly good fit to the analytic solution**
 - **Coherence**
 - 200m -11%
 - 400m -15%
 - **Extinction**
 - 200m 26%
 - 400m 7%
- **Over the propagation distances of this test, for a dissipation rate of 10**
 - The extinction is very high
 - The signal phase has essentially zero mean
 - This makes it very difficult to estimate meaningful phase statistics from a small number of samples
 - Results for this scenario probably reflect limitations due to the random nature of the test



Test of Topography

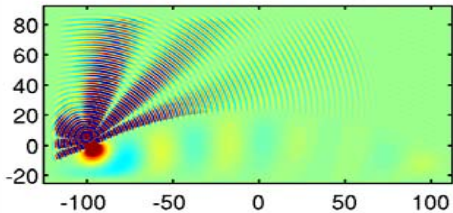
- **2 1/2 D Hill**
 - 3D propagation
 - Cylindrical 2D Hill
- **Receivers at 1m intervals (along the surface)**
- **2 m above the surface**



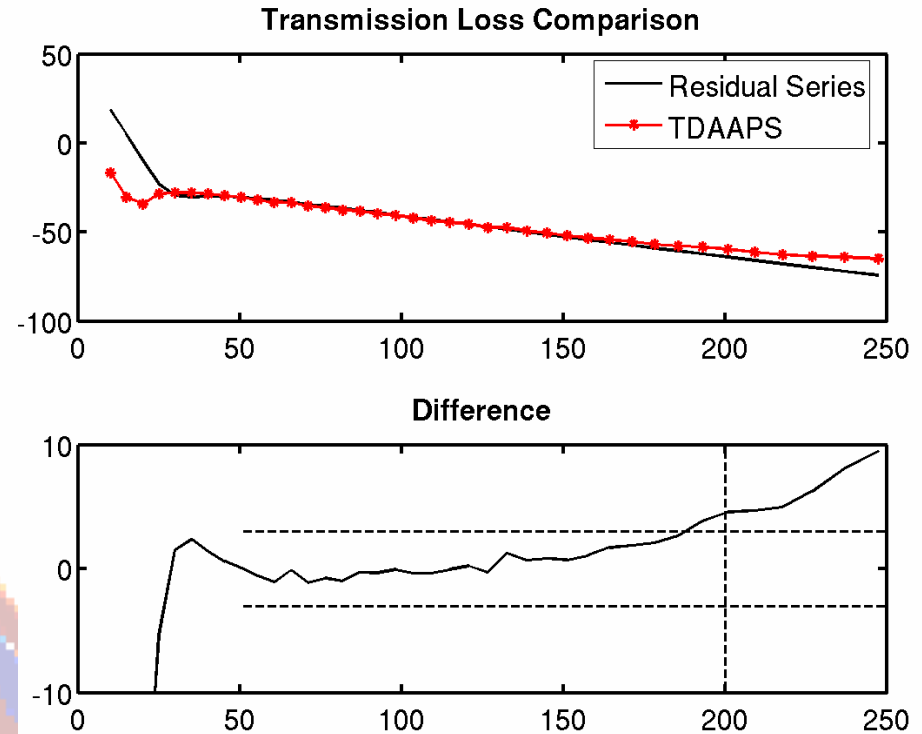
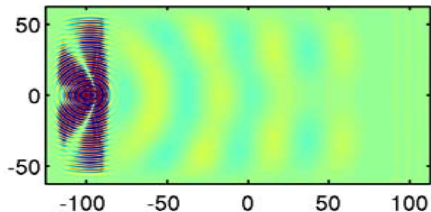
Topography Test Results

- Another difficult test
 - The only energy that reaches the more distant receivers has refracted over the hill
 - We get slightly higher amplitude than predicted
 - Possibly energy that propagates through the hill
 - Possibly energy reflected from boundaries

slice.BH2M100b: XZ.Pressure; 507.5ms



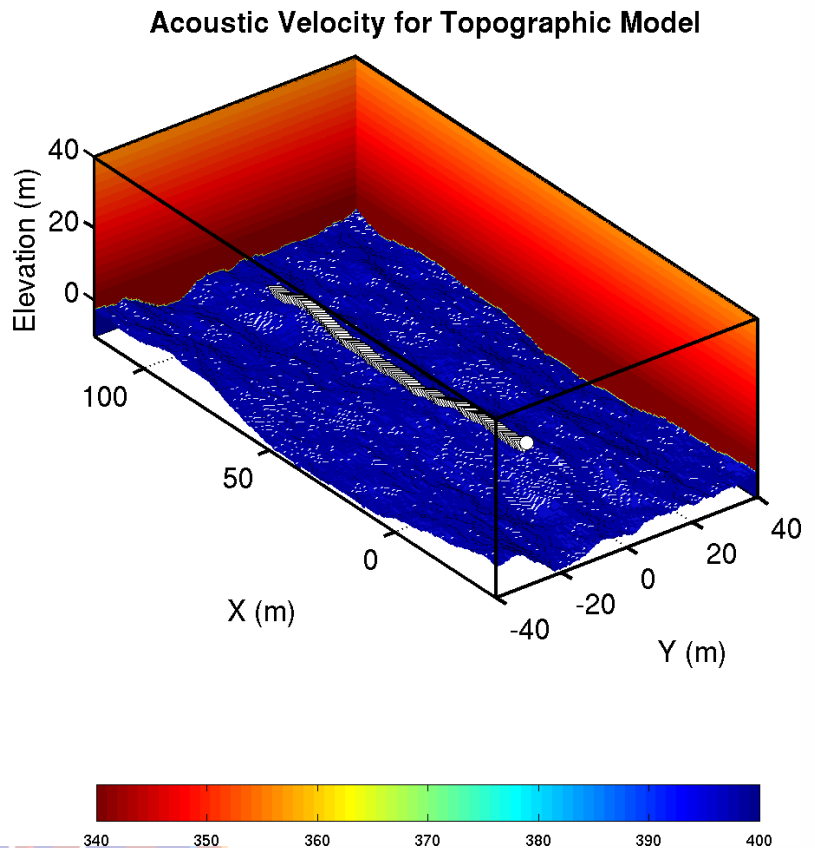
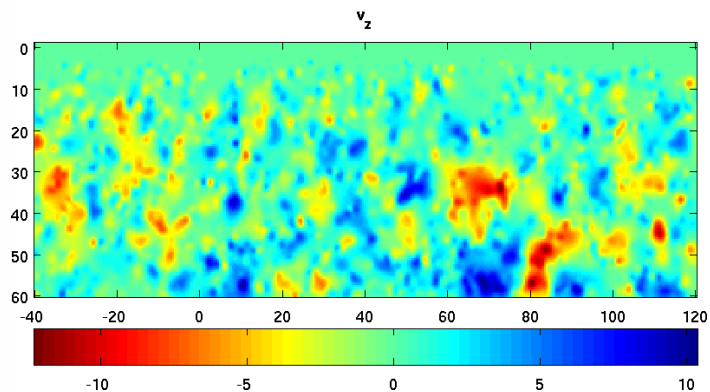
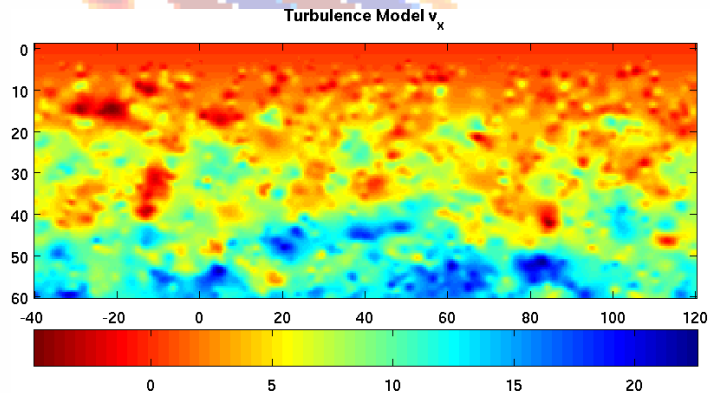
slice.BH2M100b: XY.Pressure; 507.5ms



- Analytic solution
 - Not expected to reproduce the near-source complications
 - M. Berengier, G.A. Daigle, Diffraction of sound above a curved surface having an impedance discontinuity, Journal of the Acoustical Society of America, **84**, September 1988, pp 1055-1060

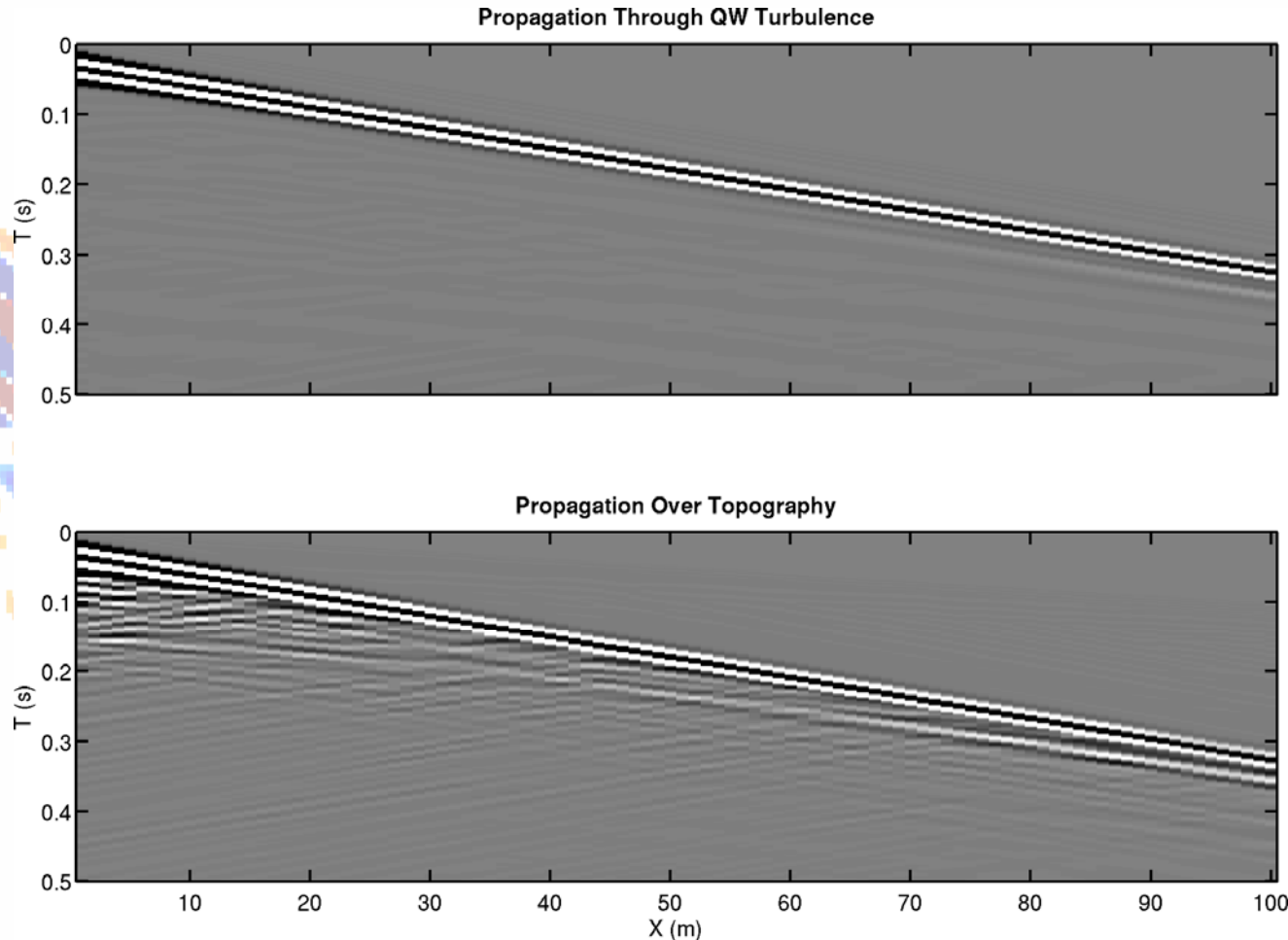
Effect Comparison

- Compare the contribution of turbulence and topography
 - Two effects that are hard to separate in the real world



Comparison

- These results clearly show that for this particular geometry and frequency range the effect on the sound due to the topography is much larger than the effect from the turbulence
- In the topographic model the direct arrival and the scattered energy trailing is of a similar magnitude
- The signal energy scattered by the turbulence, even for the unrealistically intense turbulence used here, is much lower and is barely visible at this plot scale



Conclusions

- **We have successfully implemented FDTD calculations of sound propagation through moving media**
- **Accuracy of the algorithm has been demonstrated for propagation over topography and through turbulence**
- **A highly realistic atmospheric turbulence representation based on quasi-wavelets has been incorporated directly into the algorithm**
- **Demonstrated the utility of modeling to investigate where the complications in our data come from**