Efficient Coefficient Extraction from Doublet Resonances in Microphotonic Resonator Transmission Functions

Adam M. Jones[1,2,*], Anthony L. Lentine[1], Christopher T. DeRose[1], Andrew L. Starbuck[1], Andrew Pomerene[1], and Robert A. Norwood[2]

[1]Applied Photonic Microsystems, Sandia National Laboratories, Albuquerque, NM, 87123, USA
[2]College of Optical Sciences, University of Arizona, Tucson, AZ, 85721, USA
**adajone@sandia.gov*

Abstract: We develop a computationally efficient and robust algorithm to automatically extract the coefficients of doublet resonances and apply this technique to 418 resonances in ring resonator transmission data with a mean RMS deviation of 7.28×10^{-4}.

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1. Introduction

The transmission function of a microphotonic resonator is uniquely dependent on intracavity loss mechanisms and the characteristics of the interaction region used to couple light into and out of the resonator. Fitting to this transmission function enables extremely accurate estimation of the round-trip propagation loss and coupling coefficients of the resonator; this information can be used to improve the design process in fabricating future versions of resonator-based devices, or to estimate propagation loss of the component waveguide system.

We demonstrate a novel algorithm for automatically extracting the six parameters of a doublet resonance in a computationally efficient and robust manner. The algorithm was successfully applied to 418 resonances from 11 devices with a mean RMS deviation of 7.28×10^{-4} including the common, perilous case of misidentification of an unresolved doublet as a singlet resonance. From these parameters we can accurately determine waveguide losses, coupling efficiencies, and quality factors in automated fashion sans an *a priori* estimate of the extracted parameters.

2. Resonator Transmission Functions

In the presence of roughness induced intracavity contradirectional coupling[1], the transmission function is well described as a coherent linear sum of the two singlet resonance transmission functions corresponding to the standing wave modes arising from interference of the forward and backward travelling optical modes of the resonator[1]. Using spatial coupled mode theory[2], a model for the transmission function (T) of a laterally coupled, four-port resonator may be derived in terms of the round-trip loss coefficient (α), phase coefficient (ϕ) and the field transmission coefficient of the coupling region (t)**;** $\phi = 2\pi n_g L$ where n_g is the group index and L is the resonator path length (Eq. 1). Identification of the standing wave modes is accomplished by addition of the subscripts c and s corresponding to the modes of cosinusoidal and sinusoidal azimuthal dependence, respectively.

$$T = \left| \frac{t_c - \alpha_c t_c exp(-i\phi_c)}{1 - \alpha_c t_c^2 exp(-i\phi_c)} + \frac{t_s - \alpha_s t_s exp(-i\phi_s)}{1 - \alpha_s t_s^2 exp(-i\phi_s)} \right|^2 \tag{1}$$

3. Distillation of the Transmission Function

The transmitted intensity is essentially a measurement of the transmission function corrupted by a smoothly varying insertion loss envelope and transmitted intensity noise; fidelity is restored by accounting for these additive signals. Estimation of the loss envelope is achieved here via application of morphological erosion whereby exponential moving average (EMA) filters are applied to the signal in leading and trailing configurations with the maximum value of these filtered signals kept as the local estimate of the loss envelope. This algorithm is then applied in iterative fashion until the resonances are completely eroded and only the underlying loss envelope remains.

Transmitted intensity noise is accounted for in fitting the estimated transmission function ($\hat{T}$) to the experimentally observed data (T); here, experimental results indicate that the noise is correlated and heteroskedastic thus invalidating the use of ordinary least squares (OLS) in optimally estimating the parameter set associated with a given resonance. Detailed analysis of the behavior of transmitted intensity noise in measurements of this type have shown that the local variance is greatest off-resonance with a minimum contribution to the signal seen on-resonance [3]. This allows for use of weighted least squares (WLS) whereby the squared error elements are weighted by an estimate of the local variance with the error function given by $wSSE = \sum_{n=1}^{N} W_n \left(T - \hat{T}\right)^2$.

4. Fitting the Transmission Function

The interplay between the six parameteric variables of Eq. 1 leads to a large number of local minima in the 6D space subtended by $wSSE$. In the case of the doublet resonance, typical approaches such as simulated annealing and genetic algorithms fail to guarantee convergence to the globally optimum solution[4] while the brute force method potentially requires a prohibitive number of initial estimates to produce the desired accuracy.

Explored here is a perturbative approach whereby the 6D estimation problem is decomposed into a set of 3D fits to alternating halves of T. Initially, the left and right halves are separately fit to singlet resonances with the better of the two fits retained. Further iterations consist of estimating the parameters of one singlet by those obtained in the previous iteration with the parameters of the other singlet left as parameteric variables in the current iteration. This alternating fitting scheme perturbatively erodes the total fitting error and eventually converges to some optimal fit assumed to lie within the same basin of attraction[5] as the optimal solution. A final solution is obtained via application of a gradient-descent solver to the full 6D problem starting from the previously obtained result.

5. Experimental Method and Results

Two port SOI ring resonators with 50μm diameters, 400nm widths, a 3μm bottom oxide, and 230nm thick device layer were fabricated at Sandia National Laboratories'[6] MESA facility using a silicon photonics process developed therein. In total, 418 resonances from eleven devices were analyzed; example application of our algorithm is demonstrated in Fig. 1with the fits to several resonance types presented in Fig. 2.

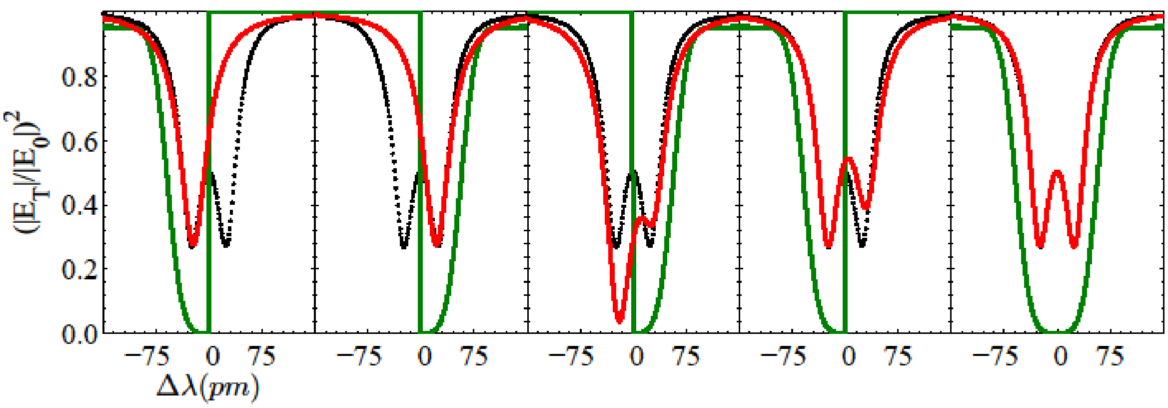

Fig. 1. Application of fitting algorithm to doublet resonance data (black dots) with weighting function (green line) and current fit (red line). Shown (from left to right) are initial fits to the left and right halves of the resonance, iterative fits to the left, then right resonances, and the final fit to the transmission function.

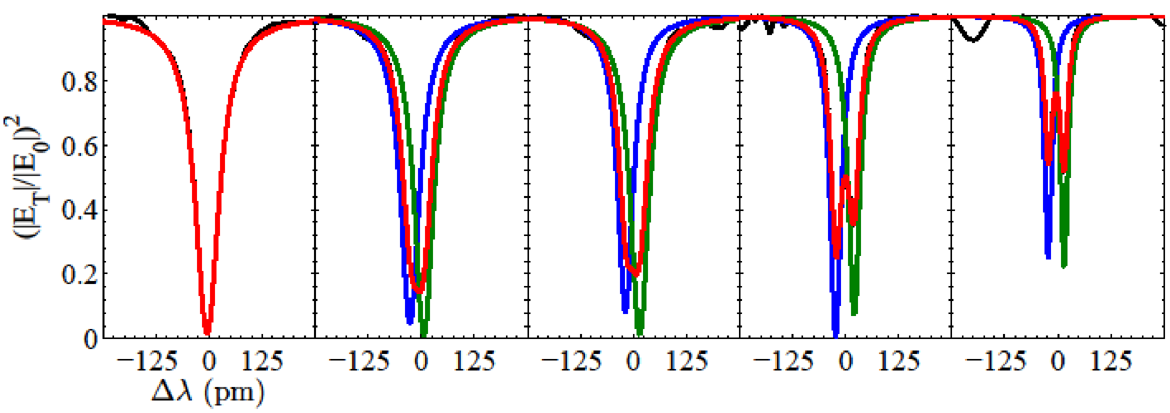

Fig. 2. From left to right: singlet, apparent singlet, unresolved doublet, asymmetric doublet, and symmetric doublet resonances. Experimental data (black dots), left resonance (blue line), right resonance (green line), and transmission function fit (red line) shown.

7. References

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[6] Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.