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Exploring the Existence of Null Spaces in Mediated-Reality Supplemented Methods to X-ray Attenuation Estimation

Edward S. Jimenez[1] **and Kyle R. Thompson**[2]

Sandia National Laboratories
[1]Software Systems R&D
[2]Structural Dynamics and X-ray/Non-Destructive Evaluation
PO Box 5800; Mail Stop 0933
Albuquerque, NM 87185-0933
(505) 284-9690; Fax: (505) 844-2018; E-mail: esjimen@sandia.gov

1 ABSTRACT

Previous efforts to estimate energy dependent x-ray attenuation information for various materials given a traditional radiograph formed from a Bremsstrahlung source profile have shown that the estimation degraded by the non-linearities in the system as well as the potential presence of null spaces in the approximation of the system. This work will explore simulations of the image formation operator to identify numerical stability and computational challenges associated with estimating the attenuation information of various materials using a variety of energy configurations including narrow bandwidth detectors and sources.

2 INTRODUCTION

This work investigates the computational challenges of using mediated-reality-based approaches to estimating attenuation profiles with respect to energy for the purpose of identifying materials in radiographs. Previous work by Jimenez et. al. [2, 3, 5] have explored various approaches with varying degrees of success.

The first approach [5] was an exploratory method that acquired multiple radiographs using a traditional system with a Bremsstrahlung source and then attempted to calculate effective attenuation at an effective energy. This method could reasonably estimate low Z materials such as water and polyethylene, but severely underestimated higher Z materials such as lead, tin, and copper. This work, along with inspiration from [1,4,6,7], lead to the work presented in [3], in which basis functions were explored; these basis functions would create a finite vector space to search using simulation-based optimization via direct search method. Various basis functions were presented, and although promising, none of the methods presented could reasonably estimate attenuation profiles. The final piece of work [2] leveraged basis functions and a multi-channel detector and performed an optimization with respect to each bin and was able to reasonably reconstruct the attenuation; however, due to the suspected numerical inaccuracies due to stability, approximation of the imaging system, along with choice of basis functions, there was a high prevalence of artefacts present in the estimations.

This work refers to null spaces as regions or materials in the object space that cannot be resolved for a given x-ray energy. If the system at a given energy cannot measure a given path in object space due to either a lack of penetration or insufficient attenuation then the estimation task is not possible. This work will explore varying energies, materials, and path lengths to attempt to understand the numerical behaviour and to determine a path forward in identifying a suitable system configuration.

3 APPROACH

The x-ray imaging system is represented as:

$$\overrightarrow{g} = \mathscr{H} f(\mathbf{X}, \mu(\mathbf{X})),$$

where $\mathscr{H}$ is the continuous-to-discrete imaging operator, $f(\mathbf{X}, \mu(\mathbf{X}))$ is an object of finite support occupying a region $\mathbf{X}$ in $\mathbb{R}^3$ with the position dependent linear attenuation profile across the relevant energy range for the given imaging system, the vector $\overrightarrow{g}$ is the discretized measurements from the imaging system.

Rather than modelling an image from a traditional Bremsstrahlung source to a single channel image, this work will integrate images from very narrow energy bins across the energy profile and study signal and attenuation properties and behaviour by observing signal absorption and recovery of the attenuation profile with respect to energy. Since this work focuses on the numerical aspects of the imaging system, the 'detector' for each energy bin is assumed to be ideal and the imaging system is made to be noise-free. The digital phantom being imaged consists of 11 sphere of homogeneous material in a circular arrangement as shown in figure 1. For a given energy bin ε, the image will be modelled as:

$$I_j(\varepsilon) = I_0(\varepsilon)\Delta I e^{-(\mu_a(\varepsilon)x_a + \mu(\varepsilon)x)},$$

where $I_0(\varepsilon)$ is the initial photon intensity at energy ε, ΔI the the width of the energy bin, $\mu_a(\varepsilon)$ is the attenuation of air (ambient temperature and sea-level pressure) at energy ε, $\mu(\varepsilon)$ is the attenuation of the digital phantom, x is the length of the path from the source to the j^{th} pixel that intersects the phantom, and x_a is the complementary length to x.

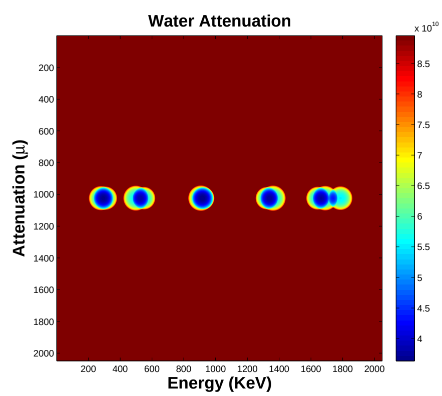

Figure 1: Simulated Radiograph

4 IMPLEMENTATION

This work was coded using MATLAB R2014a (ver. 8.3.0.532). The system simulated a 450keV tungsten target source with the profile shown in figure 2. The materials of interest include: Water, Polyethylene, Copper, Tin, and Lead. These materials were chosen as they were studied in the previous works. Each energy bin is 1keV wide, the detector consists of 2048×2048 pixels with a pitch of 200μm. The source is placed 226cm and is centered and perpendicular with respect to the detector. The center-of-mass of the phantom is located 188cm from the source and centered with respect to the detector.

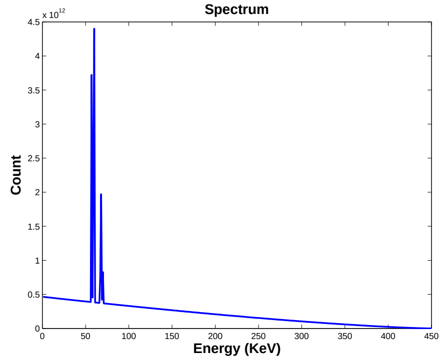

Figure 2: 450keV tungsten target Bremsstrahlung profile.

5 PRELIMINARY RESULTS

Figure 3 shows the mean signal absorption with respect to energy; as expected, lower Z materials have significantly less absorption as energy increases. Figure 4 shows the mean relative error with respect to energy in performing a straightforward energy bin-wise estimation of the attenuation; note that the relative error varies by 2 to 3 orders-of-magnitude, with error being much higher Z materials. Additionally, it seems that error spikes in the neighborhood of the k-edges for the high Z materials, thus implying a difficulty in resolving k-edges. Figure 5 contains the mean signal absorption with respect to thickness, as expected, absorption percentage increases with thickness; however, for high Z materials show essentially complete absorption at a thickness greater than $2cm$. Finally, figure 6 contains the mean relative error with respect to thickness; interestingly, the mean error is generally decreasing for all materials, somewhat counter-intuitive when taken together with figure 5.

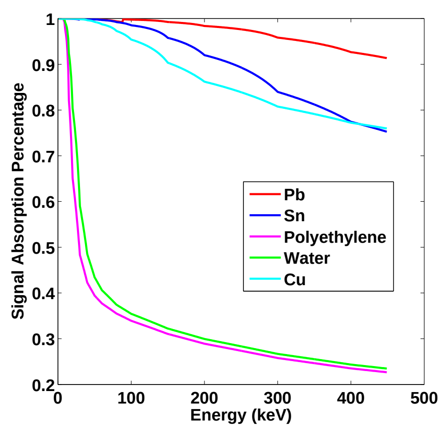

Figure 3: Mean signal absorption with respect to energy.

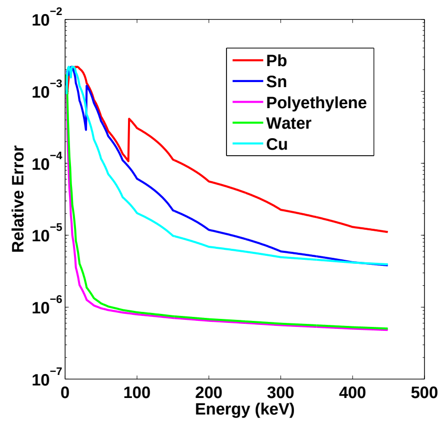

Figure 4: Mean relative error with respect to energy.

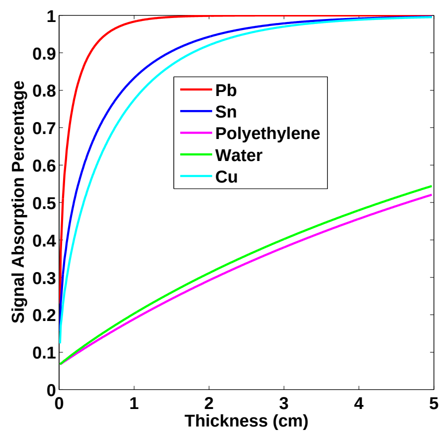

Figure 5: Mean signal absorption with respect to thickness.

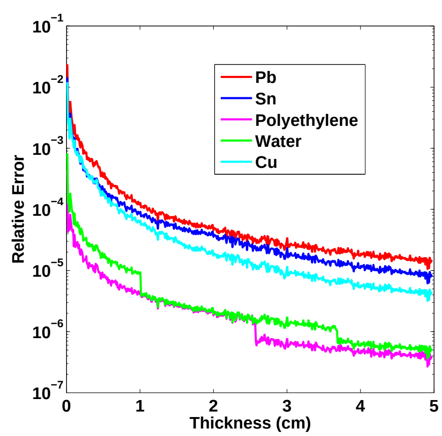

Figure 6: Mean relative error with respect to thickness.

6 CONCLUSION

This work has explored signal absorption properties and relative error behaviour in attenuation estimation. For the work done by Jimenez et. al. [5], the elevated relative error for higher Z materials may explain the consistent underestimation of effective attenuation. Also, the error spikes about the k-edges may create a problem for direct calculations of attenuation estimations; this is encouraging for simulation-based optimization approaches, such as [2], as many of them avoid a direct calculation of the attenuation profile altogether. The dramatic signal absorption for high Z materials in energy and thickness even at relatively high energies creates further questions about numerical challenges as this will almost certainly lead to signal-to-noise issues and thus stability and estimability challenges. The high demand for extracting more information from radiographs for various inspection and quality assurance applications is driving much innovation and this work shows that there is plenty of investigations that need to be made. Additionally, with the rise of energy discriminating imaging detectors, even more information will be demanded.

7 ACKNOWLEDGEMENTS

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