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Internal (Annular) and Compressible External (Flat Plate) Turbulent Flow Heat Transfer Correlations

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Internal (Annular) and Compressible External (Flat Plate) Turbulent Flow Heat Transfer Correlations

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ABSTRACT

Here we provide a discussion regarding the applicability of a family of traditional heat transfer correlation based models for several (unit level) heat transfer problems associated with flight heat transfer estimates and internal flow heat transfer associated with an experimental simulation design (Dobranich 2014). Variability between semi-empirical free-flight models suggests relative differences for heat transfer coefficients on the order of 10%, while the internal annular flow behavior is larger with differences on the order of 20%. We emphasize that these expressions are strictly valid only for the geometries they have been derived for e.g. the fully developed annular flow or simple external flow problems.

Though, the application of flat plate skin friction estimate to cylindrical bodies is a traditional procedure to estimate skin friction and heat transfer, an over-prediction bias is often observed using these approximations for missile type bodies. As a correction for this over-estimate trend, we discuss a simple scaling reduction factor for flat plate turbulent skin friction and heat transfer solutions (correlations) applied to blunt bodies of revolution at zero angle of attack. The method estimates the ratio between axisymmetric and 2-d stagnation point heat transfer skin friction and Stanton number solution expressions for sub-turbulent Reynolds numbers $<1x10^4$. This factor is assumed to also directly influence the flat plate results applied to the cylindrical portion of the flow and the flat plate correlations are modified by

this factor. Results using this correction are in overall agreement with CFD and classical measurements based correlation correction approaches for the cylindrical portion of the body.

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NOMENCLATURE

Symbols

a	Dimensionless model constant
B	Stagnation point inviscid flow constant; U=Bx
c	Locally defined constant
D	Cylinder/sphere diameter
C_f	Skin friction $C_f = \dfrac{2\tau_w}{\rho U^2}$
C_{f_0}	Undisturbed free-stream skin friction
const	Constant
f	Dependent similarity variable
I	Turbulence intensity (absolute value)
K	Clauser turbulent viscosity constant
L	Streamwise length scale
L_{lam}	Streamwise location extent of laminar stagnation point
m	Similarity/Faulkner-Skan model coefficient
M	Free-stream Mach number
Nu	Nusselt Number
Pr	Prandtl number
Re	Reynolds number
Re_x	Streamwise flat plate Reynolds number
Re_{x_t}	Transition Reynolds number
Re_δ	Boundary layer thickness R
Re_θ	Momentum thickness Reynolds number
St	Stanton number
t	time
u	Streamwise turbulent mean flow
U	Free stream turbulent mean flow velocity
u’	Root Mean Square (RMS) streamwise velocity fluctuation amplitude

W	Local turbulent velocity scale
x	Streamwise spatial coordinate
x^*	x/δ
y	Cross-stream spatial coordinate
y^*	y/δ

Greek

α	Turbulence power law constant
ξ	y/δ
δ	Boundary layer thickness
δ_0	Dimensionless constant for boundary layer thickness approximation $\delta=\delta_0 x$
δ^*	Displacement thickness
δ^+	Boundary layer thickness inner law length scale $\delta^+ = \frac{\delta v^*}{\nu_w}$
η	Local scaled similarity
κ	Von Karman constant $\kappa=0.41$
$\tilde{\kappa}$	Pulsatile flow modified Von Karman constant
ν	Kinematic viscosity
ω	Frequency
ω_0	Dimensionless frequency $\omega_0 = \frac{\omega\delta}{U}$
Φ	Power Spectral Density, i.e. spectra
ρ	Density
τ	Shear stress
τ	Auto-correlation time separation
θ	Momentum thickness

Subscripts/Superscripts

inc	Incompressible

FT	Fully Turbulent
max	Maximum
os	Laminar-turbulent pressure “over-shoot”
pp	Pressure PSD
rms	Root Mean Square (RMS)
s	Steady
turb	Turbulent
t, tran	Transition
T	Turbulent
vehicle	Reentry vehicle
w	Wall
∞	Steady free-stream constant

I. INTRODUCTION

Here we provide a discussion of the applicability of a family of traditional heat transfer correlation based models for several basic heat transfer problems associated with flight heat transfer estimates and internal flow heat transfer associated with an experimental simulation design (Dobranich 2014). Variability between semi-empirical free-flight models suggests relative differences for heat transfer coefficients on the order of 10%, while the internal annular flow behavior is larger with differences on the order of 20%. We emphasize that these expressions are valid for the geometries they have been derived for e.g. the fully developed annular flow or simple external flow problems. Typically, the the actual geometry for an applied problem can be more complex which may correspondingly modify the heat transfer behavior. While it may be possible to capture higher order effects using analytical models, it is perhaps more realistic to utilize CFD based simulation results to understand the heat transfer effects that result for the actual physical geometry.

Though, the application of flat plate skin friction estimate to cylindrical bodies is a common first-order modeling procedure to estimate skin friction and heat transfer, an over-prediction bias is often observed using these approximations for missile type bodies. To provide a correction for this effect we develop a simple scaling factor for flat plate turbulent skin friction and heat transfer solutions (correlations) applied to blunt bodies of revolution at zero angle of attack. The method estimates the ratio between axisymmetric and 2-d stagnation point heat transfer skin friction and Stanton number solution expressions for sub-turbulent Reynolds numbers $<1 \times 10^4$. This factor is assumed to also directly influence the flat plate results applied to the cylindrical portion of the flow and the flat plate correlations are modified by this factor. Results seem to be in basic agreement with CFD (and classical measurements) based correlation correction approaches (J. Smith) for the cylindrical portion of the body.

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II. ANALYSIS/RESULTS

Correlation based models are developed for external compressible flows and internal annular flows. A missile body heat transfer correction is then discussed.

A. Compressible External Flow

The model is described by the family of correlations (Holman 1986) (for Stanton number):

$$St_x = \begin{cases} 0.332(\mathrm{Re}_x^*)^{-1/2}\,\mathrm{Pr}^{-2/3} & ; \quad \mathrm{Re}_x < 5x10^5 \\ 0.0296(\mathrm{Re}_x^*)^{-1/5}\,\mathrm{Pr}^{-2/3} & ; \quad 5x10^5 < \mathrm{Re}_x < 1x10^7 \\ 0.185(\log_{10}(\mathrm{Re}_x^*))^{-2.584} & ; \quad 1x10^7 < \mathrm{Re}_x < 1x10^9 \end{cases} \tag{1}$$

Where the Reynold's number is based upon a "reference temperature" T* computed as:

$$\mathrm{Re}_x^* = \frac{\rho^* U_e x}{\mu^*} = \frac{\mu_e}{\mu^*}\frac{\rho^*}{\rho_e}\frac{\rho_e U_e x}{\mu_e} \approx \left(\frac{T_e}{T_*}\right)^{1.67} \mathrm{Re}_x \tag{2}$$

With the reference temperature (White 2006) computed as:

$$\frac{T_*}{T_w} = 1 + 0.5(1 - \frac{T_e}{T_w}) + 0.22(\frac{T_{aw}}{T_w} - \frac{T_e}{T_w}) \tag{3}$$

The adiabatic wall temperature is given by:

$$\frac{T_{aw}}{T_w} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)\left(\frac{T_e}{T_w}\right) \rightarrow \frac{T_e}{T_w} = \frac{\dfrac{T_{aw}}{T_w}}{\left(1 + \dfrac{\gamma - 1}{2} M_e^2\right)} \tag{4}$$

This suite of equations provided a basic estimate for the Stanton number for the compressible flow over a flat plate.

For compressible flat plate flow, an alternative family of closures is available based upon the theory-based models of Van Driest (White 2006). These models estimate the Stanton number via the Reynold's analogy through the local skin friction. This model, similar to the reference temperature method serves as an effective extension to incompressible skin friction theory, where the modeled extension follows from a density dependent turbulence closure (Prandtl's mixing length) with the Crocco-Busemann law (an approximate energy equation integral as the thermal model). The basic expression for the skin friction can be written:

$$C_f = \frac{1}{F_c} C_{f\_inc}(\mathrm{Re}_{x\_\mathrm{mod}}) \tag{5}$$

Where the Reynold's number is based upon a "reference temperature" T* computed as:

$$\mathrm{Re}_{x\_\mathrm{mod}} = \mathrm{Re}_x \frac{\mu_e}{\mu^*} F_c^{-1} \approx \mathrm{Re}_x \left(\frac{T_e}{T_*}\right)^{1.67} F_c^{-1} \tag{6}$$

and F_c is given by:

$$F_c = \frac{\dfrac{T_{aw}}{T_e} - 1}{(\arcsin(A) + \arcsin(B))^2} \tag{7}$$

$A \equiv \dfrac{2a^2 - b}{(b^2 + 4a^2)^{1/2}}$; $B \equiv \dfrac{b}{(b^2 + 4a^2)^{1/2}}$ where:

$$a = \left(\frac{\gamma - 1}{2} M_e^2 \frac{T_e}{T_w}\right)^{1/2} \quad ; \quad b = \frac{T_{aw}}{T_w} - 1 \tag{8}$$

and $\dfrac{T_{aw}}{T_e} = \dfrac{T_{aw}}{T_w}\dfrac{T_w}{T_e}$ with $\dfrac{T_{aw}}{T_w} = \left(1 + \dfrac{\gamma - 1}{2} M_e^2\right)\left(\dfrac{T_e}{T_w}\right)$. This formulation for the skin friction is undeniably more complex that the reference temperature approach, but tends to have the benefit that the formulation is based upon (approximate) first principle arguments. To complete the approach one requires: (1) an incompressible skin friction model and a model (analogy) to relate skin friction to the heat transfer rate.

The simplest (and according to White 2006) most accurate incompressible model is provided by $C_{f\_inc} = \dfrac{0.455}{\ln^2(0.006\,\mathrm{Re}_x)}$. This model, which is based on turbulent inner-law approximations, provides an excellent estimate for the incompressible skin friction for a flat plate for a wide range of Reynolds numbers.

To compute the Stanton number (dimensionless heat transfer coefficient) we typically invoke one of the analogies (Reynold's being the most common) between momentum transfer and heat transfer. A typical Reynold's number analogy (White 2006) would permit us to write:

$$St = 0.5 C_f \,\mathrm{Pr}^{2/3} \tag{9}$$

We note that the derivation of the analogy suggests that: $St = \dfrac{1}{2} C_f$ and the modification of the constant and the Prandtl number effects are semi-empirical modifications. Other closure procedures are possible. A model based upon the analogy between the momentum law-of-the-wall and the temperature law-of-the-wall (White 1988):

$$\frac{U}{v^*} = \frac{1}{\kappa}\ln\left(\frac{\delta v^*}{\nu_w}\right) + B$$

$$\frac{\rho_e c_p (T_w - T_e) v^*}{q_w^"} = \frac{1}{\kappa}\ln\left(\frac{\Delta v^*}{\nu_w}\right) + A \tag{10}$$

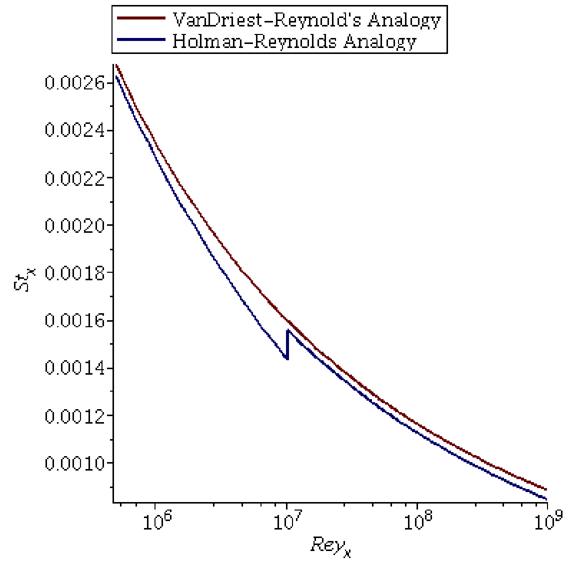
where δ and Δ are the hydrodynamic and thermal boundary layer thicknesses, respectively. The empirical constants B and A follow directly from measurements/empirical modeling with: B=5 and A=12.7Pr$^{2/3}$-7.7. Subtracting the two expressions, introducing $\frac{v^*}{U_e} = \frac{\sqrt{2}}{2} C_f^{1/2}$ and solving for $St = \frac{q_w^"}{\rho_e c_p U_e (T_w - T_e)}$ gives:

$$St_x = \frac{\frac{1}{2} C_f}{1 + [A - B - \frac{1}{\kappa}\ln(\frac{\Delta}{\delta})][\frac{1}{2} C_f]^{1/2}} \tag{11}$$

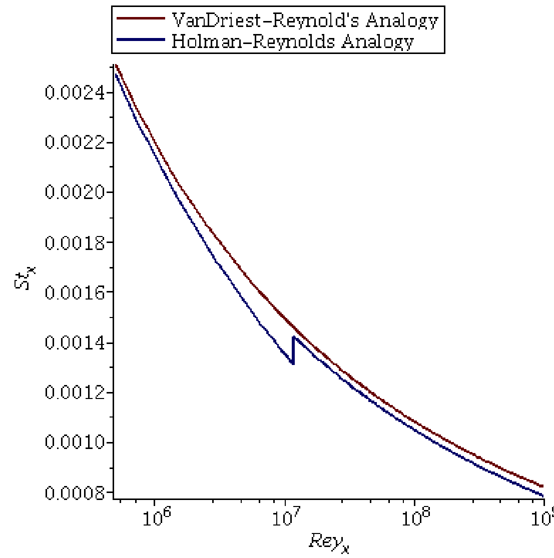
Since $\frac{1}{\kappa}\ln(\frac{\Delta}{\delta}) << 1$ we obtain $St_x = \frac{\frac{1}{2} C_f}{1 + 12.7[\Pr^{2/3} - 1][\frac{1}{2} C_f]^{1/2}}$. While this expression has the potential to be of considerable accuracy, in keeping with equations (1)-(4) we will use the simpler Reynolds analogy as described by equation (9).

Let’s then compare the behavior and trend of equations (1)-(4) versus (5)-(9) for several Mach numbers in figure 1. From figure 1., it is apparent that the Stanton number estimates basically follow the same trend with a slight (maximum of about 10%) relative difference between the piecewise continuous Holman model and the continuous White/Van-Driest model. While Holman recommends the piecewise-model for the lower Reynold’s number regime, it is perhaps unnecessary since the higher Reynold’s number branch which is based upon the Shultz-Grunow correlation is known to be accurate for low Reynolds numbers as well.

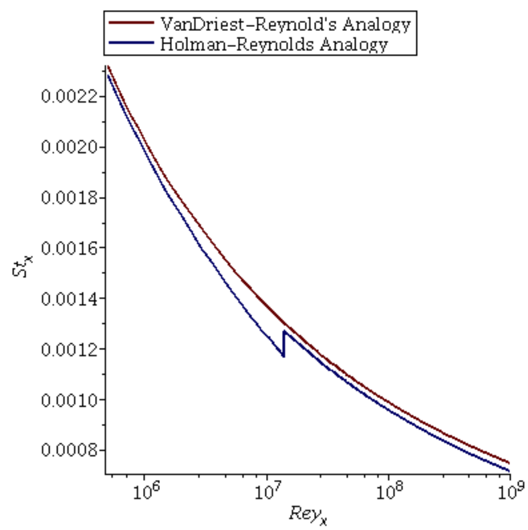
Regardless, we believe that the inherent error associated with model of this class, is on the order of 10% and it follows that there is likely little reason to be concerned regarding the viability of the Holman correlation to achieve the goal of providing “engineering accuracy” analytical estimates for the B61 flight regime.



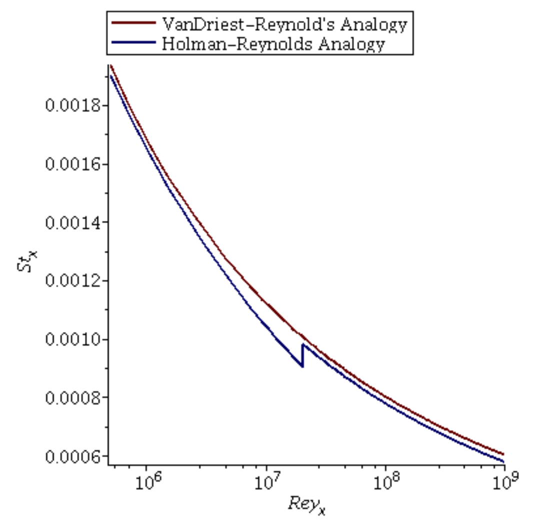
(a) M=0.3 (max. err. 10%)



(b) M=0.7 (max. err. 9%)



(c) M=1. (max. err. 9%)



(d) M=1.5 (max. err. 7%)

Figure 1. Comparison between Holman (1986) recommended high speed model (piecewise continuous/reference temperature) and White/Van-Driest (2006) compressible formulation for several Mach numbers. Maximum error is associated with piecewise correlation transition

B. Low Speed (Incompressible) Fully-Developed Flow Annular Model

Engineering correlations are utilized in a second situation to provide local heat transfer estimates for a thermal loading analog. Here flow and heat transfer are characterized by an annular (between bomb case and an enclosing shroud. The annular model identified follows from Incropera and De Witt and is based upon a turbulent a simple circular pipe/duct flow approximation where the annular effective diameter is computed via the classical hydraulic diameter approximation as the difference between the outer and inner diameter. Explicitly these approximations (Gnielinski 1976) (See Incropera and De Witt 1990) take the form:

$$Nu_{Dh} = \frac{\left(\frac{f}{8}\right)(\mathrm{Re}_{Dh} - 1000)\,\mathrm{Pr}}{1 + 12.7[\mathrm{Pr}^{2/3} - 1][\frac{f}{8}]^{1/2}} \tag{12}$$

Where "f" is the Darcy friction factor with $f=4C_f$. As suggested an expression that related the hydraulic diameter is given as: $D_h = D_o - D_i$. The corresponding Nusselt number Nu_D and the Reynold's numbers Re_D are based upon the hydraulic diameter as: $Nu_{Dh} = \frac{D_h h}{k}$ $\mathrm{Re}_{Dh} = \frac{\rho D_h U}{\mu}$. Notice that the ratio of the inner diameter to the outer diameter, i.e. $a = \frac{D_i}{D_o}$ is inherent to the hydraulic diameter as:

$$\begin{aligned} D_h &= (1 - \frac{D_i}{D_o})D_0 = (1-a)D_o \\ \mathrm{Re}_{Dh} &= (1-a)\,\mathrm{Re}_D \end{aligned} \tag{13}$$

The required Darcy friction factor follows from:

$$f = \frac{1}{(0.79 \ln \mathrm{Re}_{Dh} - 1.64)^2} \tag{14}$$

Due to the importance of this flow problem considerable work has been done to extend the Nusselt number correlation to more explicitly model concentric annular flow. Gnielinski (2009) provided the more recent correlation specifically modified to include the effect of the annular geometry as captured through the inner to outer shell diameters as $\frac{D_i}{D_o}$.

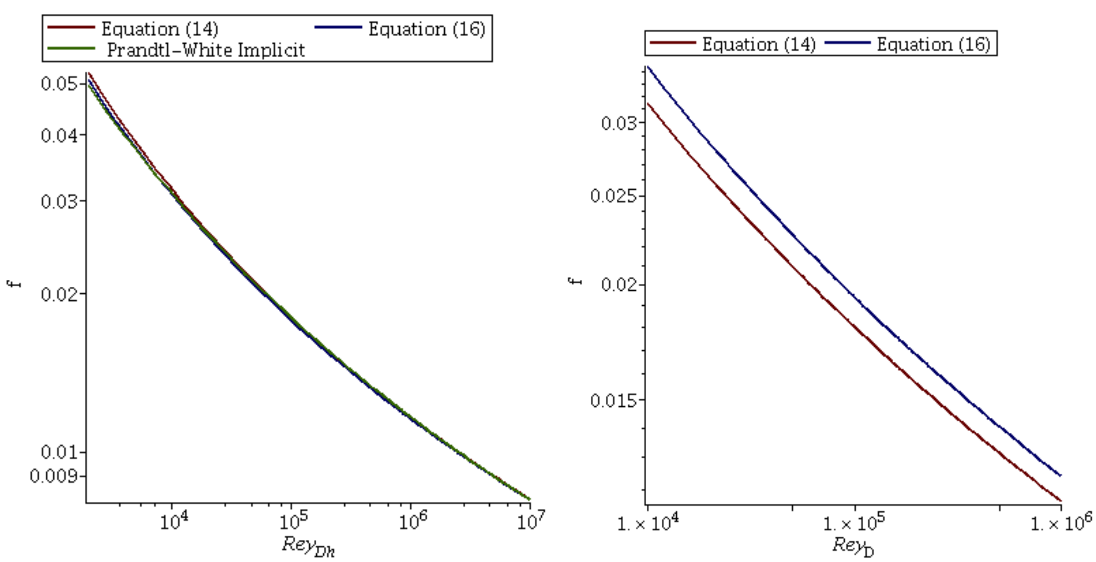
$$Nu_{Dh} = \frac{\left(\frac{f}{8}\right) \mathrm{Re}_{Dh} \Pr(0.75a^{-0.17})}{k_1 + 12.7[\Pr^{2/3} - 1][\frac{f}{8}]^{1/2}} \quad ; \quad k_1 = 1.07 + \frac{900}{\mathrm{Re}_{Dh}} - \frac{0.63}{(1+10\Pr)} \tag{15}$$

With

$$f = \frac{1}{(1.8\log_{10}\mathrm{Re}^* - 1.5)^2} \qquad \mathrm{Re}^* = \mathrm{Re}_{Dh} \frac{(1+a^2)\ln a + (1-a^2)}{(1-a^2)\ln a} \tag{16}$$

Where a is $a = \frac{D_i}{D_o}$ and Re_D is the Reynold's number based on the outer diameter D_o.

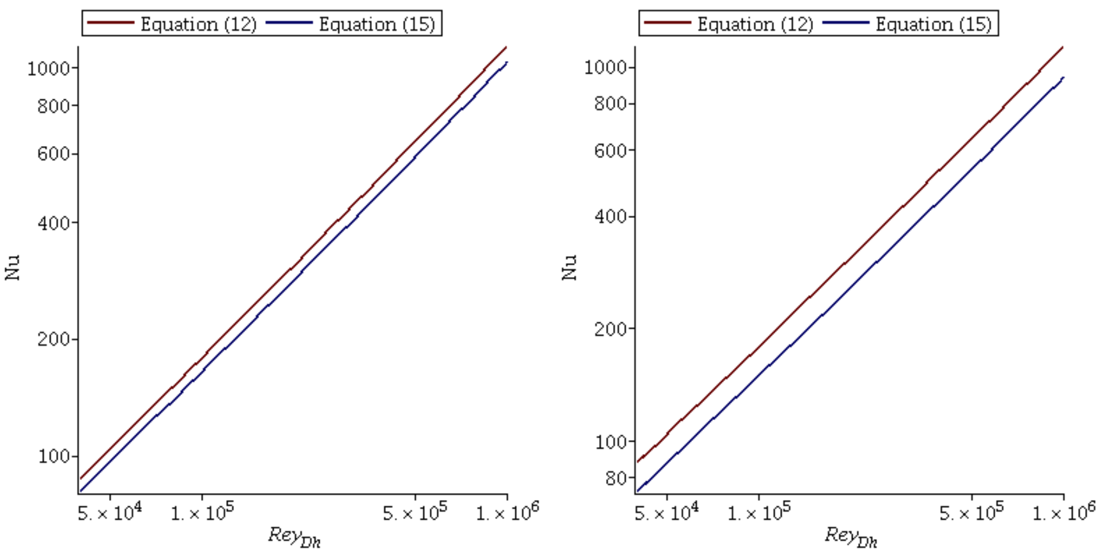
Let's begin our modeling effort by examining the Darcy friction factor results for circular tubes as compared to equation (14), equation (16), using a=0, and the classical smooth tube model (White 2006): $f^{-1/2} = 2.0\log_{10}(\mathrm{Re}_{Dh} f^{1/2}) - 0.8$. The latter expression is of course implicit in "f" but is solvable numerically (or using Lambert "W" functions). In figure 2 (a). we examine the Darcy friction factor results as computed using these expressions for Re_D. In figure 2 (b) we examine Darcy friction factor results for circular tubes as compared to equation (14), equation (16), using a=1/2



(a) a=0 (b) a=9/10

Figure 2. (a): Comparison between Darcy friction factor models: equation (14), equation (16) and hydraulic diameter model for a=Di/D0=0 and (b): a=Di/D0=9/10

Figure 2. provides a useful comparison between Darcy friction factor models: equation (14), equation (16) and $f^{-1/2} = 2.0\log_{10}(\mathrm{Re}_D f^{1/2}) - 0.8$ for $a = \dfrac{D_i}{D_o} = 0$ and (b): equation (14), equation (16) $a = \dfrac{D_i}{D_o} = \dfrac{9}{10}$ While certainly the friction factor is a component of the heat transfer law vi the Chilton-Colburn analogy, there is value in simply comparing the Nusselt number correlation as described by equation (12) and equation (15). Figure 3. compares these results for a=1/2 and a=9/10. For a=1/2, the maximum relative error is 8% and while for a=9/10 the maximum relative error is 20%.



(a) a=1/2 (b) a=9/10

Figure 3. Nusselt number values for two "a" ratios (a) a=1/2, maximum relative error=8% and (b) a=9/10, maximum relative error=20%

Gnielinski (2011) modified the expressions like equation (12) and (14) to (1) better account for the annular flow effect in a direct manner and (2) capture more recent correlation information. It is suggested that equation (15) and (16) provide a "best" model for turbulent heat transfer in an annulus. Following from the preceding discussion then, the heat transfer rate may be lower than the current simulation by as much as 20%. This degree of reduction may (or may not) be of importance in the associated thermal test design computation.

Due to the consistent bias of the Gnielinski "improved" correlation there is clear value in considering other "classical" annular models to better gain a sense of the other correlation based models. Two classical approaches are worth consider. Dirker and Meyer (2003) and Kays (2005) suggest a simple expression of the form:

$$Nu_{Dh} = 0.022\,\mathrm{Re}_{Dh}^{0.8}\,\mathrm{Pr}^{0.6} \tag{17}$$

A related class of model follows from Petukhov and Roizen (1964) who simply suggest:

$$Nu_{Dh\_improved} = 0.86a^{-0.16} Nu_{Dh} \tag{18}$$

Where $Nu_{Dh\_improved}$ is and "improved" Nusselt number expression and Nu_{Dh} is a Nusselt number for an equivalent tube based upon the hydraulic diameter. An obvious choice to compute Nu_{Dh} is simply equation (12) supplemented by equation (14). This expression is explicitly based upon heat transfer from an inner cylinder with an insulated outer condition. This modeling provides some sense of where the $a^{-0.17}$ term in equation (15) is based. Notice that the differences between methods are on the order of about 20%.

We plot these results for the four models in figure 4.

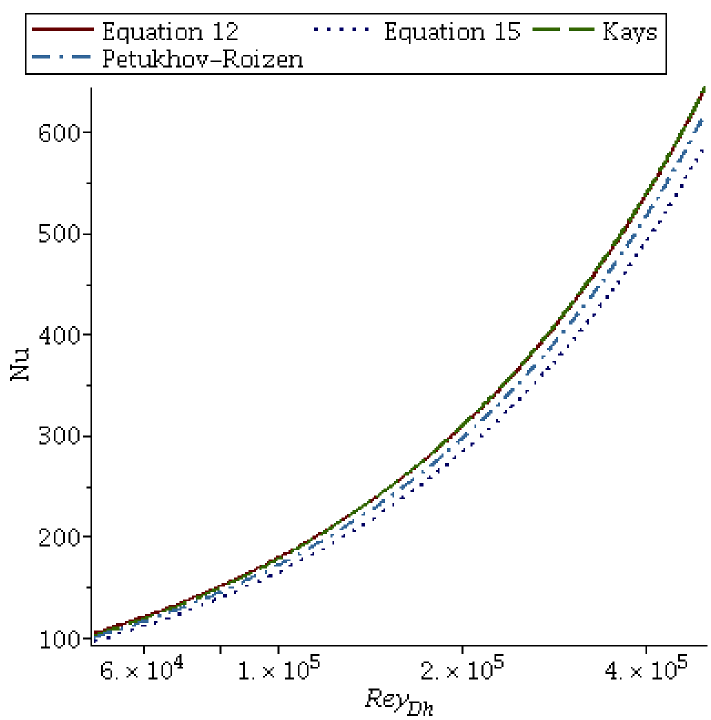


Figure 4. Comparison between Gnielinski (1976) for a=9/10, equation (12); Gnielinski (2009) equation (15); Kays et. al. (2005) and Petukhov and Roizen (1964).

In Figure 4 Equation (12) and the Kays model are indistinguishable while equation (15) and Petukhov and Roizen yield reduced Nusselt number values. The preceding discussions have sought to analyze the simplest possible (unit level) heat transfer problems associated with flight heat transfer estimates and internal flow heat transfer associated with an experimental simulation design. Variability between semi-empirical free-flight models suggests relative differences for heat transfer coefficients on the order of 10%, while the internal annular flow behavior is larger with differences on the order of 20%. We emphasize that these expressions are valid for the geometries they have been derived for e.g. the fully developed annular flow.

We conclude by examining some "high temperature" correction effects. Property variation in the supporting models becomes a significant issue when a modeling heating or cooling problem with a large variation in the wall temperature versus the bulk temperature. Traditionally, one evaluates all properties at the film temperature, i.e.:

$$T_f = \frac{1}{2}(T_w + T_b) \rightarrow \frac{T_f}{T_b} = \frac{1}{2}(\theta + 1) \tag{19}$$

where $\theta = \frac{T_w}{T_b}$. For the heating case, i.e. $\theta > 1$ the Nusselt number is reduced as compared to the $\theta = 1$ model. Indeed Petukhov (1970) suggests that $\frac{Nu|_{\theta=2}}{Nu|_{\theta=1}} = 0.72$ and $\frac{Nu|_{\theta=3}}{Nu|_{\theta=1}} = 0.575$ implying that the effect of temperature variation can be significant. To model this effect, it is common to include explicit expressions such as:

$$\frac{Nu|_{\theta}}{Nu|_{\theta=1}} = \theta^n \tag{20}$$

Here "n" is an empirical closure of the form n≈-0.47 though -0.5 is often utilized. This expression would be consistent with the Gnielinski (2009) modification.

C. Scaling Factor Correction for Flat Plate Friction/Heat Transfer Correlations Applied to Blunt/Cylinder Bodies

A common method used to estimate turbulent skin friction and (via the Reynolds hypothesis) heat transfer is the application of flat plate based skin friction factors applied to aerodynamic bodies (usually bodies of revolution, axi-symmetric etc.). For bodies where boundary layer thickness is small relative to body diameter and there are minimal pressure gradient effects the flat plate model can be used directly.

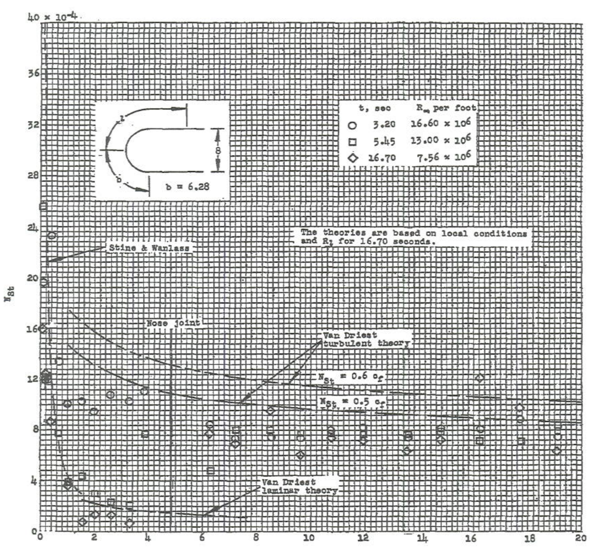
Specific corrections based upon the Mangler transformation are available for conical body flow behavior. For turbulent flow, both skin friction and heat transfer tend to be slightly (15%) increased within conical regions.

While these methods have proven to be useful for a wide range of problems, there are situations where the flat plate method over-predicts the heat transfer or skin friction. As an example of the over prediction, consider the flight test study by (*Garland and Chautin NACA RML57D04a 1957; ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20050019361.pdf)* (These were rocket propelled model flight tests. M=2.0-3.88, see figure 1). Garland and Chautin state:

> *"Laminar, transitional, and turbulent heat-transfer coefficients were measured and the following results noted: The laminar data agreed with theory for a hemisphere-cylinder body.* ***The turbulent data, on the cylinder, were consistently lower than predicted by the turbulent heat-transfer theory for a flat plate."***

Figure 5 provides a typical M=2.5 comparison between the estimated (Van Driest theory) and measurement for heat transfer.

(a) Flight vehicle on launcher



(b) Heat transfer estimate M=2.5

Figure 5. Flight model tests for hemispherical/cylinder vehicle suggesting overprediction (20%) of heat transfer by classical flat plate (Van Driest) method.

Review of the data suggests that the measurements are approximately 75-80% of the predicted turbulent flat plate methods (Van Driest). Garland and Chautin did not identify a correction approach for this discrepancy.

A possible reason for this over prediction follows from the "local" nature of the correlation solutions, i.e. the correlations have no specific mechanism to include upstream flow field information. An expression for skin friction such as: $C_f \approx 0.027\,\mathrm{Re}_x^{-1/7}$ (White 2006) derived by assuming turbulent flow from a known origin, i.e. x=0 has no information regarding potential upstream effects. The flow is completely determined by Re_x the local Reynolds number.

Certainly simple minded corrections using virtual origin corrections are known. A simple example involves imposition of a modified Reynolds number as: $\mathrm{Re}_{x\_\mathrm{mod}} = \mathrm{Re}_x - \mathrm{Re}_{tr} + \mathrm{Re}_{x0}$ where Re_{tr} is the transition Reynolds number and Rex_0 is a turbulent virtual origin Reynolds number constant determined by demanding continuous boundary layer thickness (or momentum thickness) at the transition location. For an incompressible boundary layer, equating momentum thicknesses would take the form:

$$0.664\,\mathrm{Re}_{tr}^{1/2} = 0.0155\,\mathrm{Re}_{x0}^{6/7} \quad (21)$$

Which a "typical" transition Reynolds number of Re_{tr}=5E5 would imply that Re_{x0}=168000. We can then easily estimate the skin friction as: $C_f \approx 0.027\,\mathrm{Re}_{x\_\mathrm{mod}}^{-1/7}$. Notice that this correlation has (at minimum) Let's plot this result in figure 6. As shown in the figure, the turbulent skin friction is (slightly) larger than the uncorrected value where the maximum increase is on the order of 15%. For larger values the modified model recovers the standard approach.

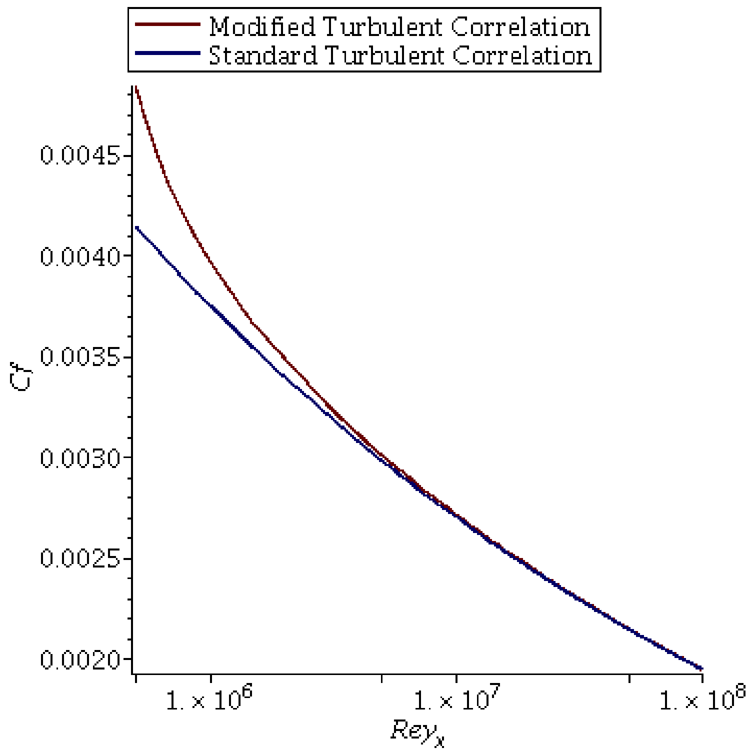


Figure 6. Modified flat plate skin friction correlation Cf=0.027Re-1/7x_modusing virtual origin approach suggesting that a skin friction origin does not correct for over prediction.

Obviously, a virtual origin approach fails to explain the over prediction of skin friction some aerodynamic bodies. Let's consider how one may apply a CFD model to an aero-body flow. A simple approach is to ignore transition effects and model the entire flow field using fully-turbulent behavior. While a laminar zone with an attendant transition model may be more "correct" the inherent low Reynolds number behavior of typical turbulence models may permit reasonable behavior which quickly recovers the physical turbulent flow portion anyway.

Additionally, the initial condition for a body of revolution may have a region marked by a strong positive pressure gradient (accelerating flow) whether it be a stagnation point (blunt) or a sharp conical flow region. Since we are ultimately most interested in heat transfer, let's consider the laminar and turbulent Stanton number for a 2-d stagnation point flow. The laminar Stanton number is given by:

$$St_{lam} = c\,\mathrm{Re}_x^{-1/2}\,\mathrm{Pr}^{-2/3} \qquad 0.55 < c < 1 \tag{22}$$

We use c=0.77. Correspondingly, a turbulent value is estimated as:

$$St_{turb} = 0.095\,\mathrm{Re}_x^{-1/4}\,\mathrm{Pr}^{-2/3}$$

$$Nu_{turb} = 0.095\,\mathrm{Re}_x^{3/4}\,\mathrm{Pr}^{1/3} \tag{23}$$

Remarkably for $1E3<\mathrm{Re}_x<1E4$ these two heat transfer estimate are broadly comparable providing analytical support to the use of turbulent flow CFD modeling approaches for low Reynolds number regions. Equation (3) (which is new) seems to provide moderate to good heat transfer estimates for turbulent stagnation point problems.

Equation (3) has been derived using an extension of the classical Faulkner-Skan approach approximately valid for turbulent flows (DeChant 2015). Thus in keeping with the laminar Faulkner-Skan development which includes a parameterized range of stagnation to flat plate flows we can generalize equation (3) as:

$$St_{turb} = \frac{6m+1}{7}\frac{0.095}{\mathrm{Re}_x^{3/28m+1/7}}\mathrm{Pr}^{-2/3} \tag{24}$$

Where m=1 for stagnation point flow and m=0 for flat plate. Notice that for m=1 equation (3) is recovered while for m= 0 $St_{turb} = 0.0136\,\mathrm{Re}_x^{-1/7}\,\mathrm{Pr}^{-2/3}$. This flat plate heat transfer result compares reasonably well to either the Holman piecewise approximation or the "best" approximation (White 2006) with $St_{turb} = \frac{1}{2}\frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)}\mathrm{Pr}^{-2/3}$. Though there is some scatter in these results (and White criticizes the lower Reynolds number portion of the Holman correlation) any one of them is perfectly acceptable as an estimate for flat plate heat transfer.

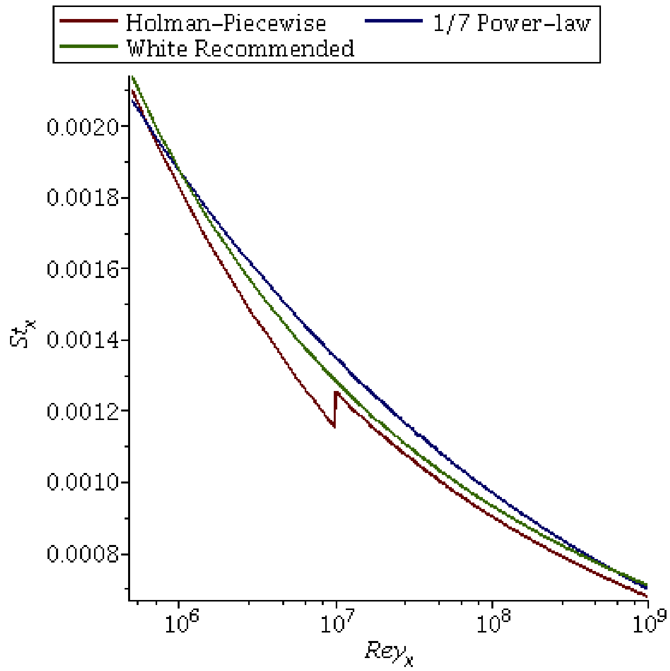


Figure 7. Incompressible flat plate heat transfer estimates using piecewise Holman, 1/7 power law and White's method for skin friction and Reynolds analogy. Any of these correlations is adequate for engineering modeling,

Of more interest to us is application of equation (4) for an "m" value intermediate the endpoint 0<m<1. Indeed, using the known Mangler transformation change of variables m_{2d}=1/3m_{axi} we can represent an axi-symmetric stagnation point flow, m_{axi}=1 as:

$$St_{turb} = \frac{3}{7} 0.095 \, \mathrm{Re}_x^{-5/28} \, \mathrm{Pr}^{-2/3} \tag{25}$$

Then if we compute the ratio for the 3-d axi-symmetric stagnation point versus the 2-d axi-symmetric value we obtain:

$$\frac{St_{turb\_axi}}{St_{turb\_2d}} = \frac{3}{7} \mathrm{Re}_x^{1/14} \tag{26}$$

Equation (6) is only valid near a stagnation point and we restrict the Reynolds number Re_x to be on the order of 10^3 to 10^4 where $\left.\frac{St_{turb\_axi}}{St_{turb\_2d}}\right|_{\mathrm{Re}_x=1E3} \approx 0.7$ and $\left.\frac{St_{turb\_axi}}{St_{turb\_2d}}\right|_{\mathrm{Re}_x=1E4} \approx 0.83$. A simple arithmetic average of the correction values gives an average of 0.76.

Slender aerodynamic bodies of revolution have only a small portion of their flow field that is characterized by strong pressure gradient behavior near the low Reynolds number nose; the rest is certainly better modeled using the zero pressure gradient flat plate expressions. **We hypothesize, however, that the large pressure gradient, axi-symmetric, low Reynolds number flow portion modifies the fundamental behavior of the zero pressure gradient portion as well.** So that one could write:

$$St_{body} = \left(\left. \frac{St_{turb\_axi}}{St_{turb\_2d}} \right|_{\mathrm{Re}_x = ave} \right) St_{plate} \approx 0.76 St_{plate} \tag{27}$$

This 75% reduction is caused not only by the axi-symmetric/3d relieving effect but also by the modification of the flow field within the axi-symmetric stagnation pressure region, suggesting a possible mechanism for the observed offset.

Let's examine how this reduction might compare to the correlation expression suggested by J. Smith (Smith 2014). Smith modifies the basic flat plate correlation model: $St = 0.0296\,\mathrm{Re}_x^{-1/5}\,\mathrm{Pr}^{-2/3}$ as:

$$St_{new\_corr} = \frac{0.0237}{1 + 0.1 M_\infty^2} \left(\frac{\mathrm{Re}_x}{f_m} \right)^{-1/5} \mathrm{Pr}^{-2/3} \qquad f_m \approx 1.3 \tag{28}$$

If we consider a transonic case with M_∞=1 we can collect constants to write: $St_{new\_corr} = 0.0227\,\mathrm{Re}_x^{-1/5}\,\mathrm{Pr}^{-2/3}$. Comparing this to the original correlation we find:

$$\frac{St_{new\_corr}}{St_{corr}} = \frac{0.0227}{0.0296} = 0.77 \tag{29}$$

We note that the good agreement between correlation correction and our correction procedure is serendipity. We list the correction associated with the Smith correction:

Mach number; M	$\frac{St_{new\_corr}}{St_{corr}}$
0.6	0.81
1.0	0.77
1.6	0.67

Table 1 Smith (2014) correlation correction factor (new correlation)/(old correlation) for several Mach numbers. Our scaling analysis, i.e. equation (7) is (currently) Mach independent.

The preceding discussion suggests that there might be a connection between blunt body flow behavior near the nose of missile bodies and the skin friction estimates on the cylindrical zero pressure gradient portion. A simple scaling law is suggested. Additional examination of CFD/experimental data would seem appropriate. Moreover a more formal examination of boundary layer flows which start in stagnation/large pressure gradient regions may be useful.

III. CONCLUSIONS

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Here we provide a brief discussion of the applicability of a family of traditional heat transfer correlation based models for several (unit level) heat transfer problems associated with flight heat transfer estimates and internal flow heat transfer associated with an experimental simulation design. Variability between semi-empirical free-flight models suggests relative differences for heat transfer coefficients on the order of 10%, while the internal annular flow behavior is larger with differences on the order of 20%. We emphasize that these expressions are valid for the geometries they have been derived for e.g. the fully developed annular flow or simple external flow problems.

Further we have discuss a simple scaling reduction factor for flat plate turbulent skin friction and heat transfer solutions (correlations) applied to blunt bodies of revolution at zero angle of attack. The method estimates the ratio between axisymmetric and 2-d stagnation point heat transfer skin friction and Stanton number solution expressions for sub-turbulent Reynolds numbers $<1x10^4$. This factor is assumed to also directly influence the flat plate results applied to the cylindrical portion of the flow and the flat plate correlations are modified by this factor. Results are in basic agreement with CFD (and classical measurements) based correlation correction approaches for the cylindrical portion of the body.

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